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Sound Radiation Modes of a Tire on a Reflecting Surface

J Stuart Bolton
Purdue University, bolton@purdue.edu

Kiho Yum

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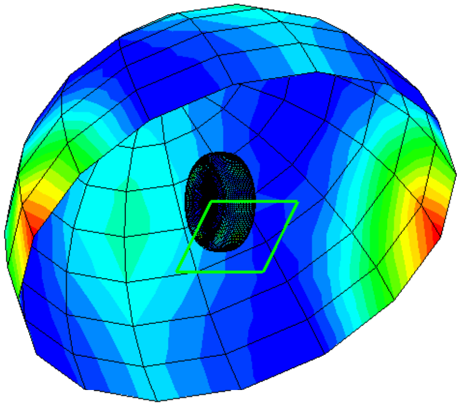
Sound Radiation Modes of a Tire on a Reflecting Surface

Kiho Yum and **J. Stuart Bolton**

Ray W. Herrick Laboratories
Mechanical Engineering
Purdue university

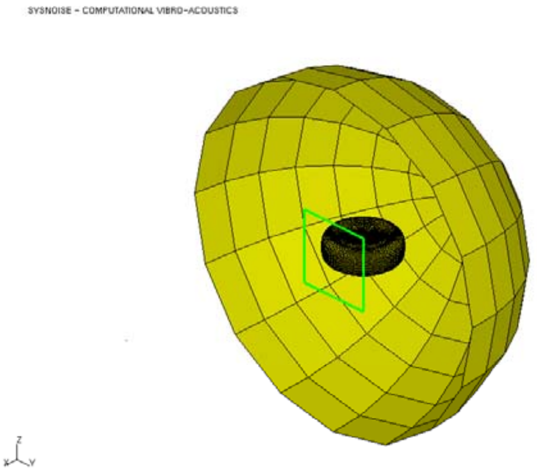
Sound Radiation from a Tire

- ■ **Significance of Tire Noise**
  - one of main sources in automotive noise, especially pass-by noise
- ■ **Generation Mechanism of Tire Noise**
  - **Radial vibration by tread impact**
  - Tangential vibration by tread adhesion (slip/stick)
  - Air pumped out and sucked in
  - Amplification by **horn effect**
  - Tire cavity resonance



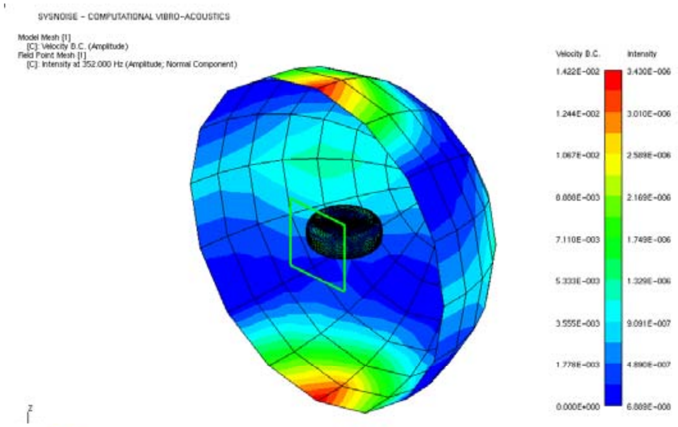
- ■ **Objective: sound radiation from a tire**
  - To investigate 3-D radiation characteristics resulting from a tire and ground geometry using **Acoustic Radiation Modal Analysis**
  - To identify the relationship between structural wave propagation and its radiation characteristics

Analysis Procedure



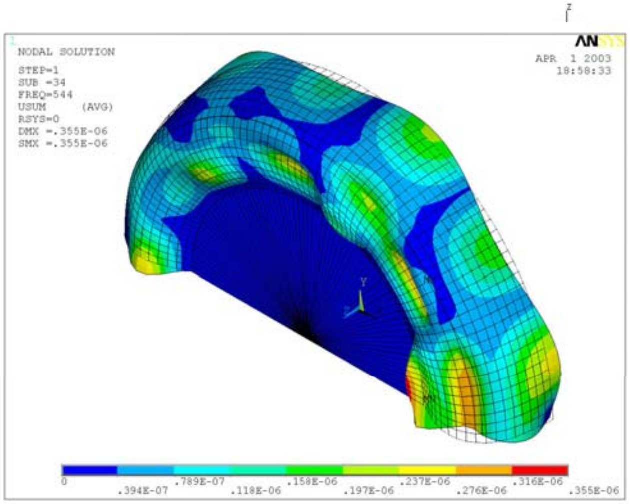
[Direct BEM]

Acoustic Transfer Vector
Acoustic Radiation Mode
calculation



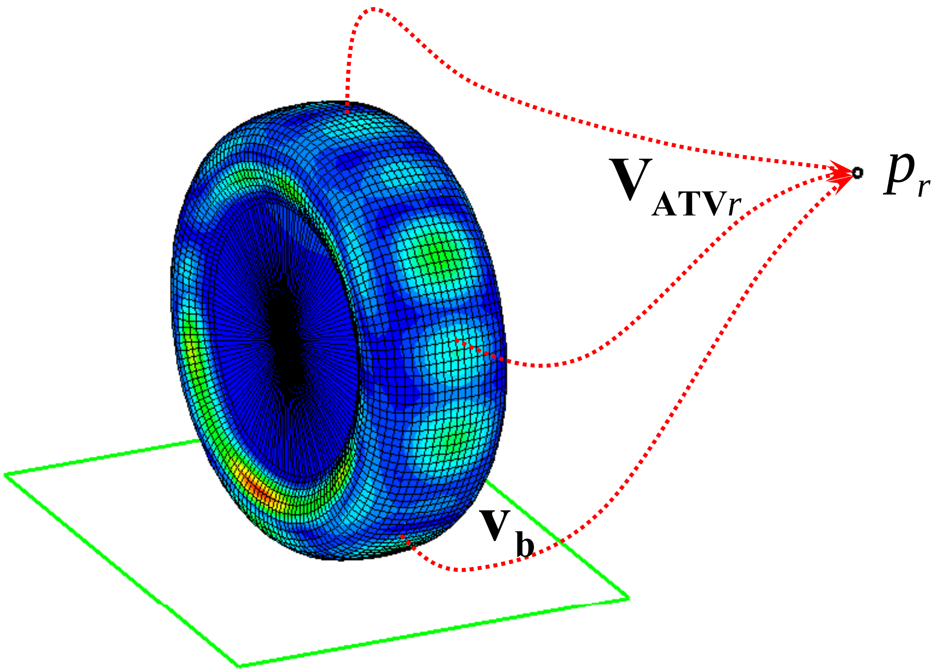
SPL & Sound Intensity
on a hemisphere
surrounding a tire

Sound Power
Radiation Efficiency
Radiation Mode Contribution



[Structural Harmonic FEM]
Surface normal velocity

Acoustic Transfer Vector (ATV)



$$\boxed{p_r = \mathbf{V}_{\mathrm{ATV}r}^{\mathrm{T}} \mathbf{v}_{\mathrm{b}}}$$

- relationship between surface normal velocities and radiated sound pressure in frequency domain
- dependent on geometry of vibrating surface, field point location and physical properties of acoustic medium

Acoustic Transfer Vector (ATV)

■ Helmholtz integral equation

$$p(\vec{x})\alpha(\vec{x}) = \int_S p(\vec{y})\frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega\int_S v(\vec{y})G(\vec{x}|\vec{y})dS_y$$

■ Discretization

- On the surface: $\mathbf{A}\underline{\mathbf{p}_b} = \mathbf{B}\underline{\mathbf{v}_b}$
- In far-field: $\underline{\underline{p_r}} = \mathbf{d}^{\mathrm{T}}\underline{\mathbf{p}_b} + \mathbf{m}^{\mathrm{T}}\underline{\mathbf{v}_b}$

pressure
at a field point

pressure & normal velocity
on the boundary

$$\boxed{p_r = \mathbf{V}^{\mathrm{T}}_{\mathrm{ATV}r}\mathbf{v}_{\mathbf{b}}}$$

$$\boxed{\mathbf{p} = \mathbf{V}^{\mathrm{T}}_{\mathrm{ATV}}\mathbf{v}_{\mathbf{b}}}$$

pressure matrix at all field points
on the recovery surface

- ▸ $\mathbf{V}^{\mathrm{T}}_{\mathrm{ATV}r} = \mathbf{d}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{B} + \mathbf{m}^{\mathrm{T}}$ Acoustic Transfer Vector (ATV)
- ▸ $\mathbf{V}^{\mathrm{T}}_{\mathrm{ATV}}$: Acoustic Transfer Matrix

Sound Radiation Mode

- **Radiated sound power in far-field**

$$W = \sum_{r=1}^{R} \frac{\left|p_r\right|^2}{2\rho c} S_r = \sum_{r=1}^{R} \frac{p_r^{*} p_r}{2\rho c} S_r \xrightarrow[\substack{\text{apply} \\ \text{ATV relationship}}]{} W = \sum_{r=1}^{R} \frac{\mathbf{v}_b^{\mathbf{H}} \mathbf{V}_{\mathrm{ATV}r}^{*} \mathbf{V}_{\mathrm{ATV}r}^{\mathbf{T}} \mathbf{v}_b}{2\rho c} S_r = \mathbf{v}_b^{\mathbf{H}} \mathbf{R} \mathbf{v}_b$$

- $\mathbf{R} = \sum_{r=1}^{R} \frac{\mathbf{V}_{\mathrm{ATV}r}^{*} \mathbf{V}_{\mathrm{ATV}r}^{\mathbf{T}}}{2\rho c} S_r$: **Radiation Resistance Matrix**

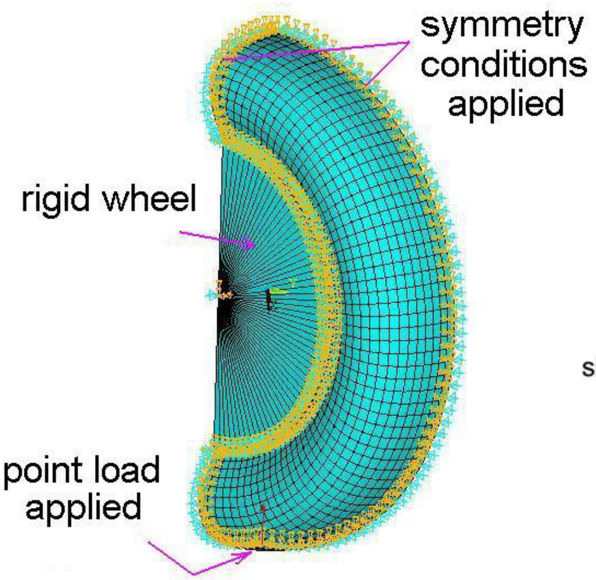
- **Sound Radiation Mode**: resulting from eigenvector decomposition of radiation resistance matrix

$$\boxed{\mathbf{R} = \mathbf{Q}^{\mathbf{H}} \boldsymbol{\Lambda} \mathbf{Q}}$$

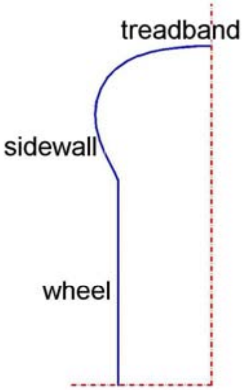
  - normalized eigenvector $\mathbf{Q}$: **Sound Radiation Mode**
  - eigenvalue $\boldsymbol{\Lambda}$: proportional to radiation efficiency

Structural FE Analysis

Tire Model



quarter tire model



tire cross-sectional shape

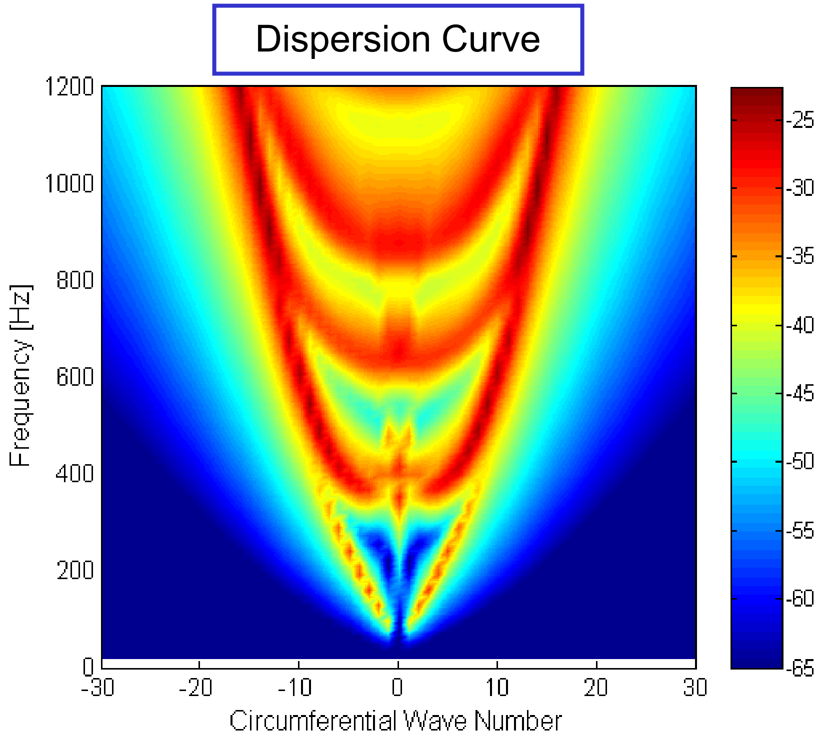
- based on 205/70R14.
- quarter tire model used to reduce calculation cost.
- orthotropic material properties applied on the tread band and sidewall. (provided by Continental Tire Co.)
- inflation pressure: 20 psi

Structural Harmonic Analysis

- Full Matrix Method performed using ANSYS.
- Harmonic point source was applied at the point of contact with the ground.

Structural FE Results

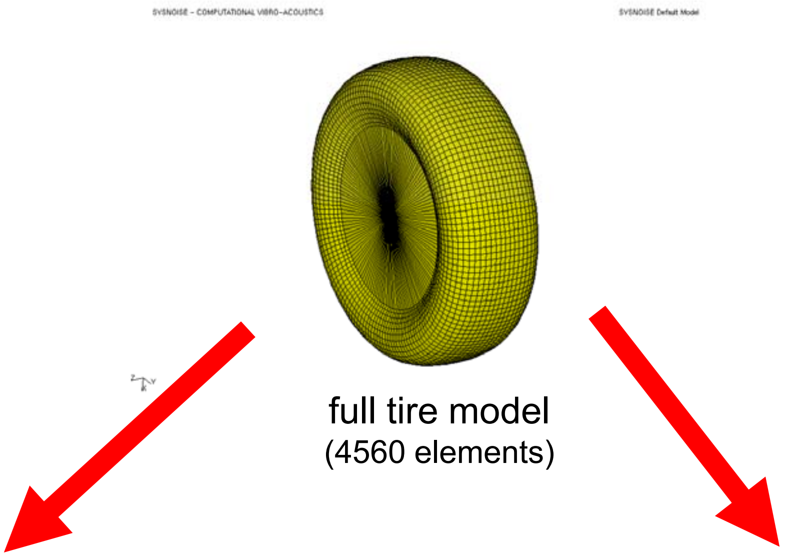
- Wave number decomposition



Dispersion Curve

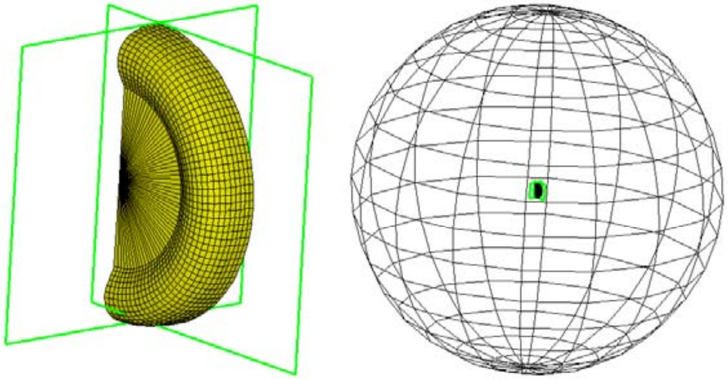
- Circumferential wave number decomposition of structural velocities resulting from the harmonic FE analysis in the space-frequency domain was performed.
- Dispersion Relationship
  - **longitudinal wave**
    - high phase speed
    - first mode appears at the ring frequency
  - **flexural wave**
    - low phase and group speed
    - related to cross-sectional propagating wave

Radiation BE Model

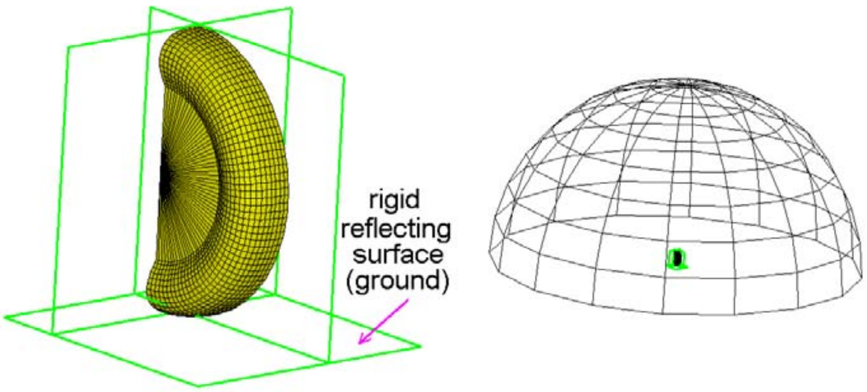


full tire model
(4560 elements)

- using Direct BEM in SYSNOISE.
- quarter tire model used in FE analysis (ANSYS) was imported.
- R7.5 sphere space (hemisphere) field points used for Pass-By Noise test.
- For reflecting surface radiation case, reflecting surface was modeled as rigid.



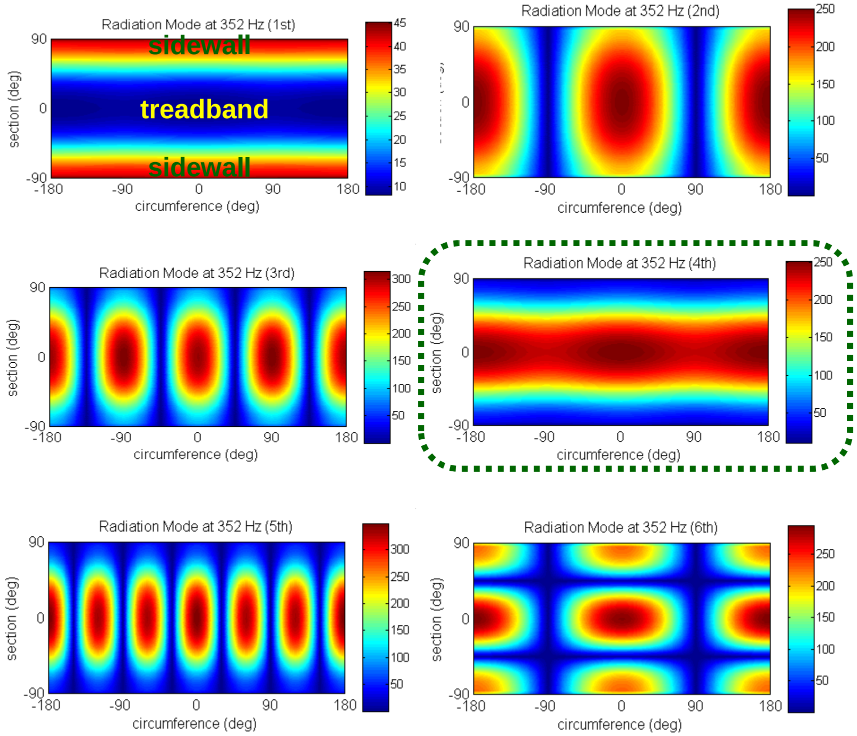
[free space radiation]



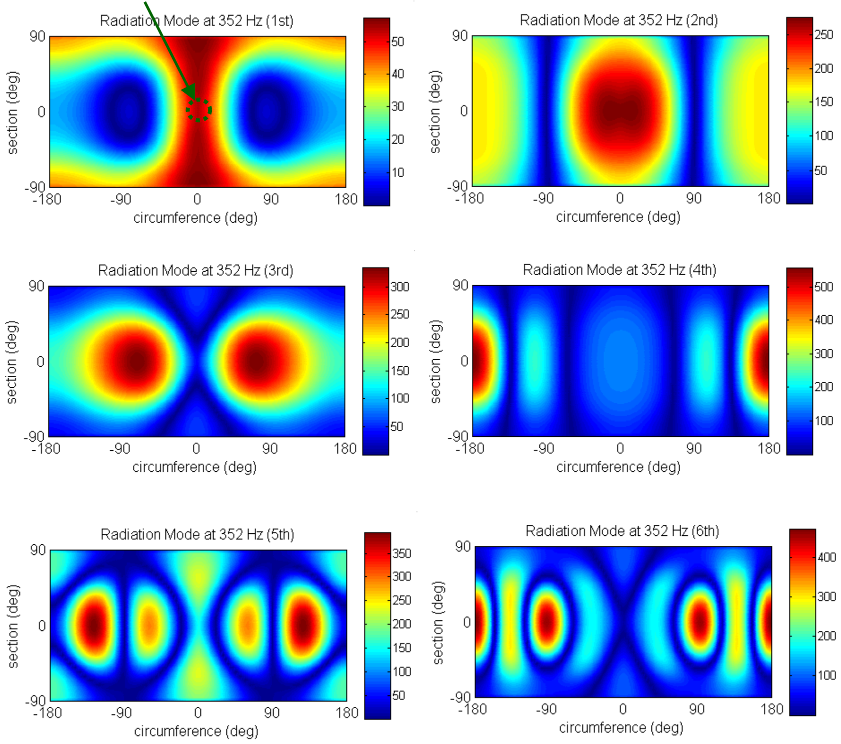
[reflecting surface radiation]

Sound Radiation Mode (352 Hz)

[free space radiation]



[reflecting surface radiation]



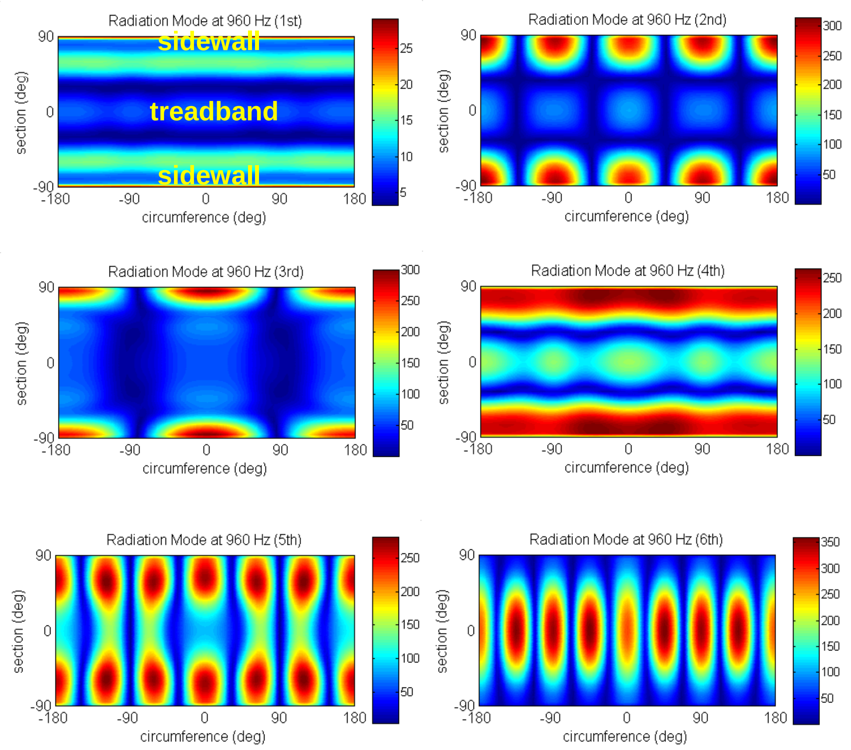
point contacting with reflecting surface

- 1st mode: sidewall dominant
- 4th mode: ring mode on treadband

- 1st & 2nd mode:
  similar with free space radiation case
  but peak added on the contact patch area

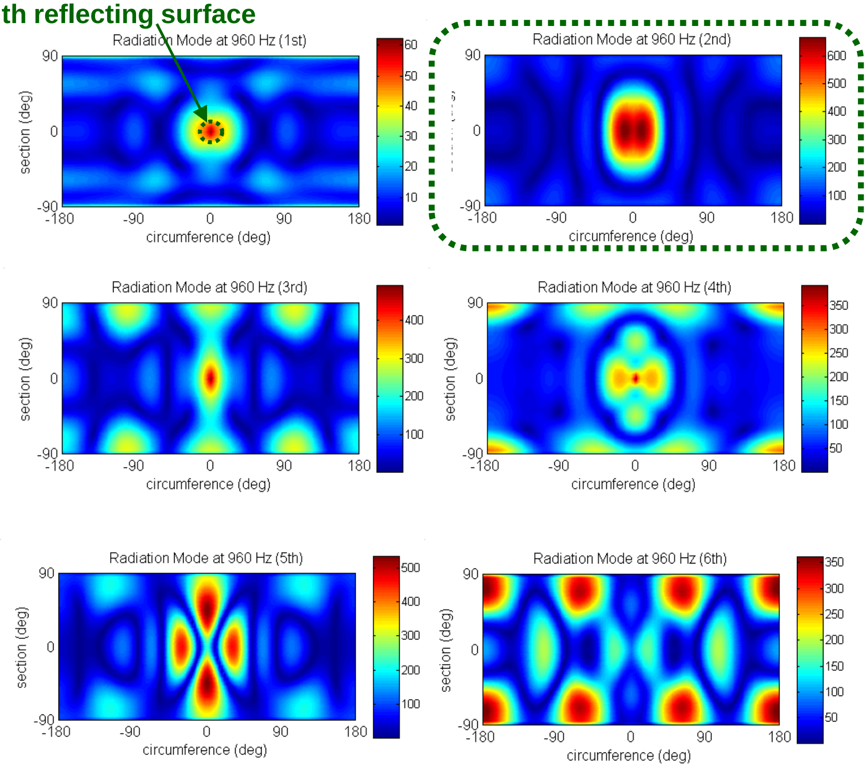
Sound Radiation Mode (960 Hz)

[free space radiation]



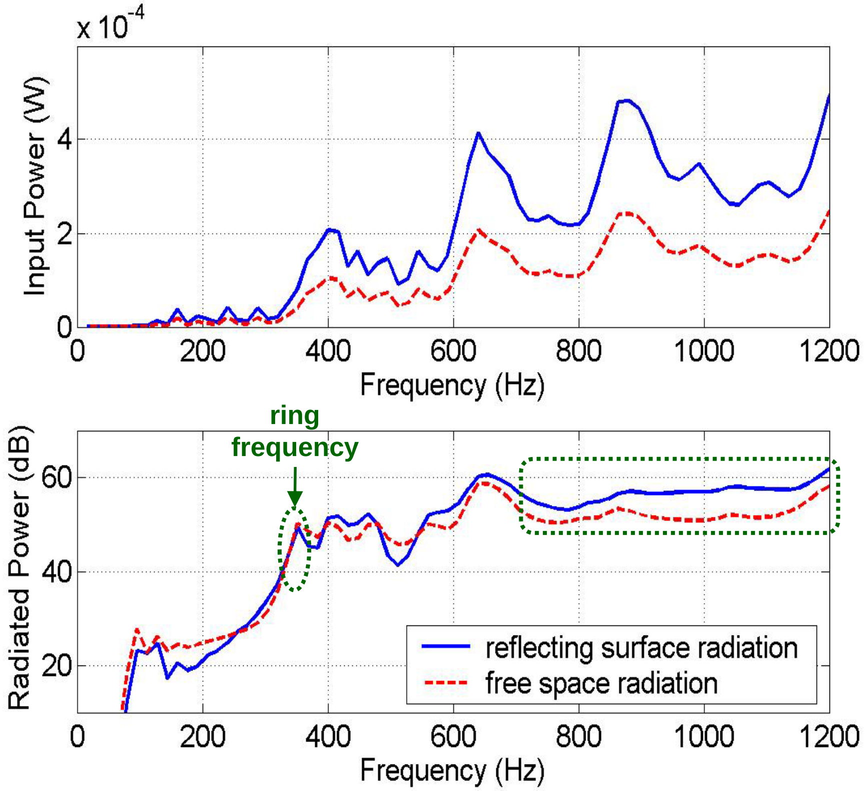
[reflecting surface radiation]

point contacting
with reflecting surface



- 1st - 5th mode:
  peaks located in the contact patch area

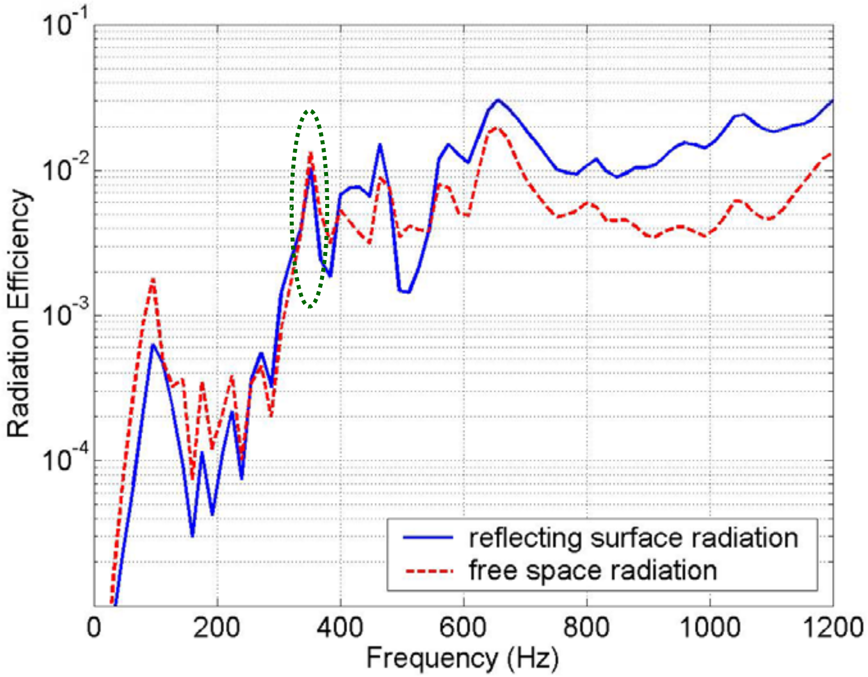
Radiated Sound Power



- Input power
  - Input power of reflecting surface radiation case is twice than that of free radiation case.
  - Peaks match cut-on frequencies of flexural waves.

- Radiated sound power
  - Radiated power peaks don’t match those of input power.
  - The peak at 352 Hz relates to ‘**ring frequency**’.
  - Radiated power for reflecting surface radiation case is amplified above 700 Hz due to ‘**horn effect**’.

Radiation Efficiency



- Definition

  : ratio of radiated power to input power

$$\sigma = \frac{W}{\rho c S_y < \left| v(\vec{y}) \right|^2 >}$$

where < > denotes space average

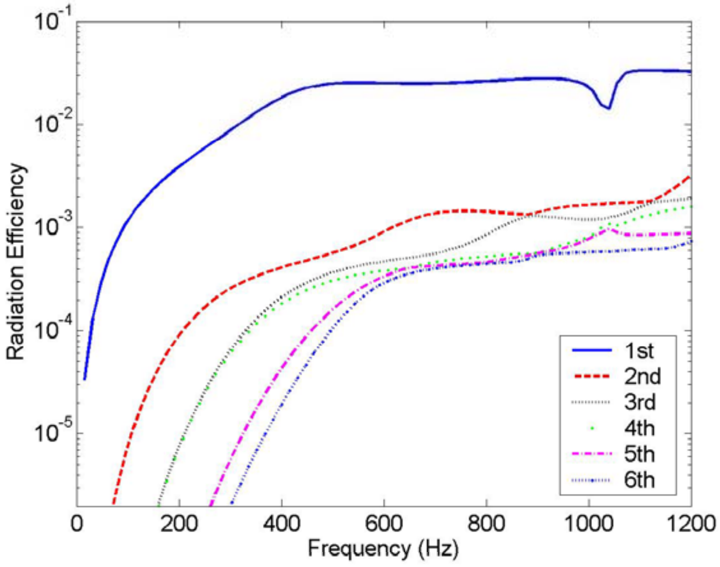
- Radiation characteristics
  - High radiation efficiency characteristics appears at ‘**ring frequency**’, 352 Hz, for both radiation cases.
  - Radiated power for reflecting surface radiation case is amplified above 700 Hz due to ‘**horn effect**’.

Radiation Efficiency of Radiation Mode

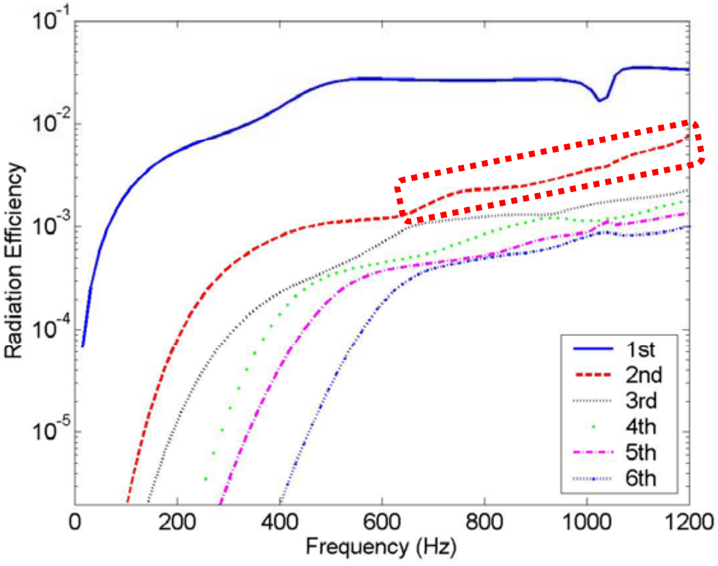
- Radiation efficiency of each radiation mode for a unit surface normal velocity

: $\sigma_n = \dfrac{\lambda_n}{\rho c S_y}$ proportional to eigenvalue of radiation resistance matrix

[free space radiation]



[reflecting surface radiation]



- Radiation efficiency of the 2nd mode of the reflecting surface case is higher above 700 Hz.

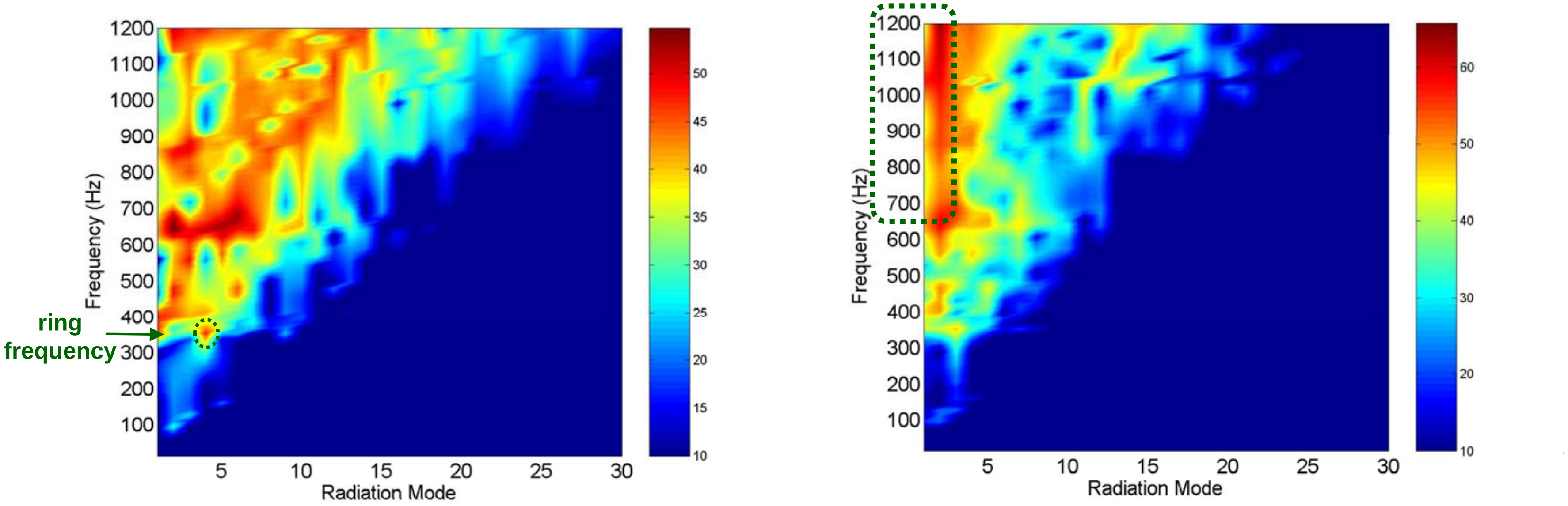
→ **strong radiation region from the contact patch area**

Sound Power Contribution of Radiation Mode

- Sound power contribution of each radiation mode when combined with structural velocities

$$W = \mathbf{v}_b^{\mathbf{H}} \mathbf{Q}^{\mathbf{H}} \mathbf{\Lambda} \mathbf{Q} \mathbf{v}_b = \mathbf{y}^{\mathbf{H}} \mathbf{\Lambda} \mathbf{y} = \sum_{n=1}^{N} W_n = \sum_{n=1}^{N} \lambda_n |y_n|^2$$

[free space radiation] [reflecting surface radiation]



- Free space radiation: mode number with high contribution increases as frequency increases.
- Reflecting surface radiation: **2nd mode is dominant above 700 Hz**.

Summary and Conclusion

- Radiation characteristics of a 3-D tire model in contact with a reflecting surface and enclosed by a hemispherical recovery surface were studied by using acoustic radiation modes.
- The sound radiation resulting from the structural wave propagation was investigated.
- Sound radiation mode is good guide in tire structural noise control.
- Most tire vibration does not contribute to sound radiation.
- The fast longitudinal wave propagating through the treadband contributes on sound radiation at the tire’s ring frequency.
- The 2nd radiation mode above 700 Hz is principally responsible for the horn effect in the presence of reflecting surface.