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The Implementation of Spherical Acoustical Holography

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1. Introduction

1.1 Contents of presentation

- Introduction
- Literature review
- Basic theory
- Coefficient filtering
- Experimental implementation
- Summary

1.2. Motivation

- Compact sources are conveniently enclosed by spherical measurement surface.
- Spherical geometry allows for a finite and compact expansion of the sound field in terms of spherical harmonics.
- Compact, realistic vibrators are sometimes more closely modeled by using a spherical wave expansion than by using planar or cylindrical expansions.
- No error due to spatial truncation of the sound field in spherical acoustic holography unlike the other type of holography.

2. Literature Review – Introduction (1)

- Nearfield planar, cylindrical, and spherical, boundary element holography procedures depending on shape of measurement surface.
- Start from the solution of the homogeneous wave equation
- Use appropriate Green's function, or propagator, for forward projections, away from the sound source
- Use the inverse of the Green's function or propagator, for backward projections, closer to the sound source
- The complete 3-D acoustical properties can be obtained from the measurements on the hologram surface

2. Spherical Holography (2)

- Weinreich and Arnold [8] were the first to develop procedures for experiments.
- There is no error related to spatial truncation.
- The measured pressure distribution is expressed as a finite sum of spherical harmonic coefficients.
- The spherical harmonic coefficients are “filtered” to remove poorly determined high order components.
- The propagators consist of spherical Hankel functions, which depend on radius and spherical harmonics.

3. Basic Theory of Spherical Holography (1)

- **Sound pressure**

$$p(M,\omega)=\sum_{n=0}^{+\infty} h_n(kr) \sum_{m=0}^{n} [A_{nm}Y_{nm}^{+}(\theta,\phi)+B_{nm}Y_{nm}^{-}(\theta,\phi)]$$

Propagation term **Two-dimensional modal functions**

- ***h_n(kr)* – spherical Hankel function (~ exp(*j*kr*)/*r*)**
- **Spherical harmonics**

$$Y_{nm}{}^{+}(\theta,\phi)=\cos(m\phi)\sin^{m}(\theta)\frac{d^{m}P_n(\cos\theta)}{d(\cos\theta)^{m}}$$

$$Y_{nm}{}^{-}(\theta,\phi)=\sin(m\phi)\sin^{m}(\theta)\frac{d^{m}P_n(\cos\theta)}{d(\cos\theta)^{m}}$$

3. Basic theory of spherical holography (2)

$$p(M,\omega)=\sum_{n=0}^{+\infty} h_n(kr)\sum_{m=0}^{n}[A_{nm}Y_{nm}^{+}(\theta,\phi)+B_{nm}Y_{nm}^{-}(\theta,\phi)]$$

- **Identification of Spherical Harmonic Coefficients**

$$A_{nm}=\frac{(2n+1)}{4\pi}\varepsilon_m\frac{(n-m)!}{(n+m)!}\frac{1}{h_n(kr)}\int_0^{2\pi}d\phi\int_0^{\pi}p(M,\omega)Y_{nm}{}^{+}(\theta,\phi)\sin(\theta)d\theta$$

$$B_{nm}=\frac{(2n+1)}{4\pi}\varepsilon_m\frac{(n-m)!}{(n+m)!}\frac{1}{h_n(kr)}\int_0^{2\pi}d\phi\int_0^{\pi}p(M,\omega)Y_{nm}{}^{-}(\theta,\phi)\sin(\theta)d\theta$$

- **Since spherical harmonics are orthogonal**

Projection procedure

Pressure measured on spherical surface

↓

Estimate A_{nm}'s and B_{nm}'s based on measurements at r_i

↓

Filter insignificant coefficients

↓

Use filtered A_{nm}'s and B_{nm}'s to reconstruct sound field at other radii external to source

4. Filtering of coefficients

4.1 Motivation

- The noisy high order field components can be amplified more than the relatively well determined low order components of the measured pressure, particularly during backward projections.
- The pressure field may be completely dominated by noise after backward projection.
- The filtering of noisy measured pressures before or during backward projection is the most important procedure to ensure the stability and accuracy of spherical acoustical holography.

4.2 Types of filtering procedure

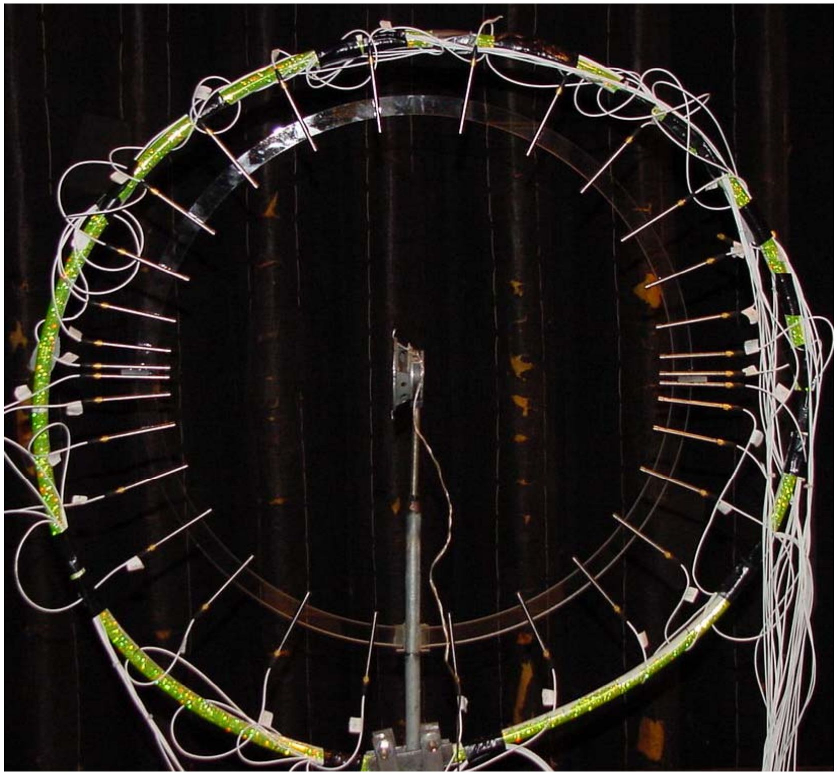
- **Ten filtering methods were applied to one of: spherical harmonic coefficients, the radiated sound power, the magnitude of the measured pressure, the transfer matrix, or a combination of these.**
  - Power filtering
  - Pressure filtering
  - Coefficient filtering
  - Spherical harmonic coefficients truncation
  - Power filtering truncation
  - Coefficient filtering truncation
  - Pressure filtering truncation
  - SVD filtering without area weighting
  - SVD filtering with area weighting
  - SVD of transfer matrix

- **Power filtering**

  - Judged to be the best filtering method among the procedures examined.
  - Compute sound power radiated by each component of the series expression of the sound field.
  - The components of the sound field that make the smallest contribution to the radiated power are individually eliminated.
  - When the sound power associated with a particular spherical harmonic coefficient is smaller than the maximum sound power component by a certain level (e.g., 40 dB), that coefficient was set to zero in the reconstruction process.

5. Experimental implementation

Microphone arrays



Microphone spacing determined by Gaussian Integration Procedure

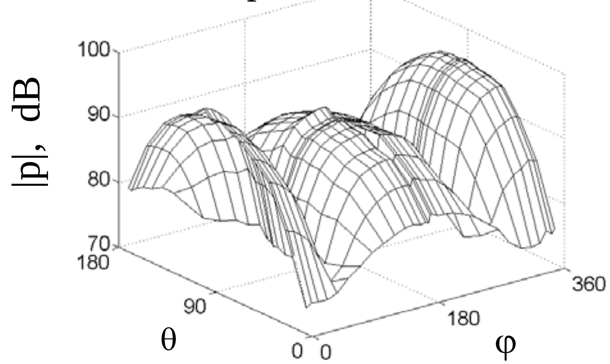
Single loudspeaker source

Two loudspeakers source

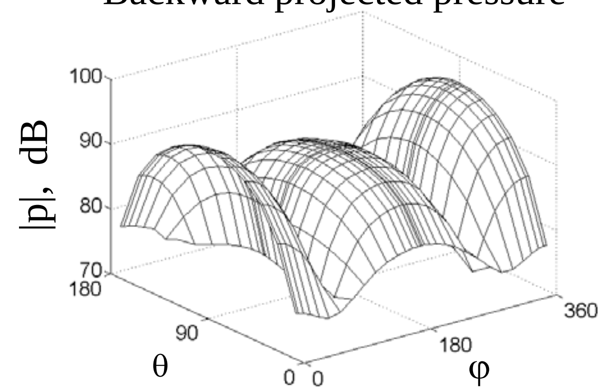
5.2 Experimental results

- **Single loudspeaker (f=500 Hz)**

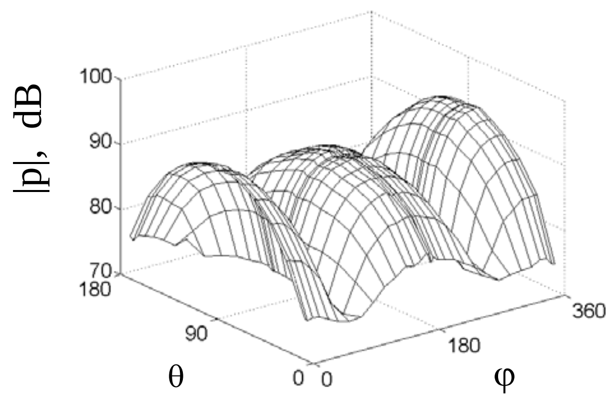
Measured pressure, r_1=0.240 m



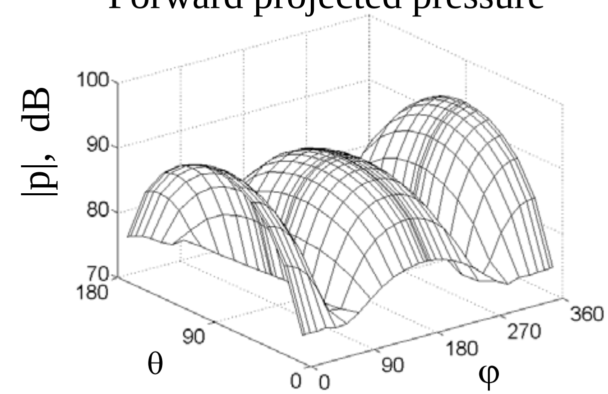
Backward projected pressure



Measured pressure, r_2=0.291 m

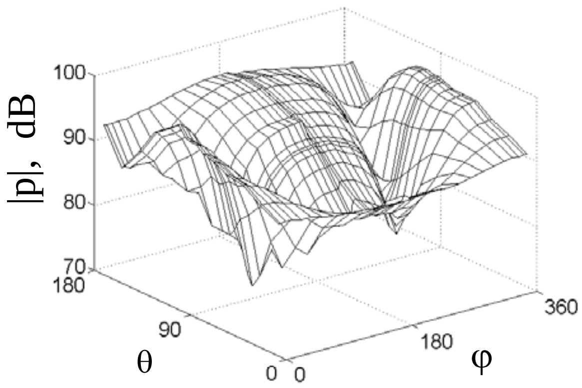


Forward projected pressure

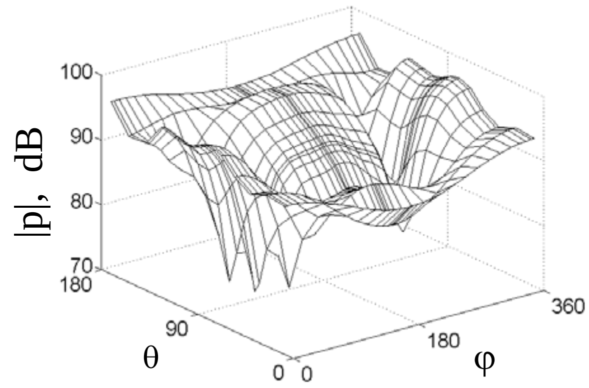


- **Two loudspeakers (f=500 Hz)**

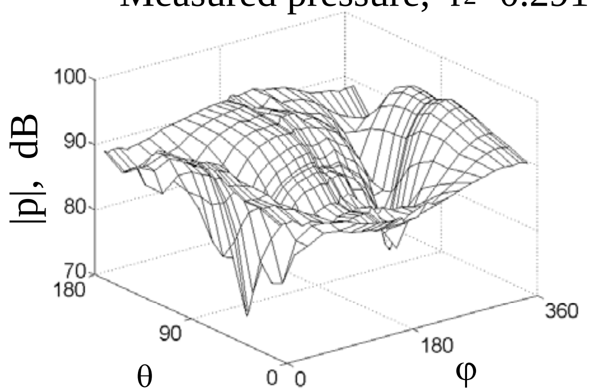
Measured pressure, r_1=0.240 m



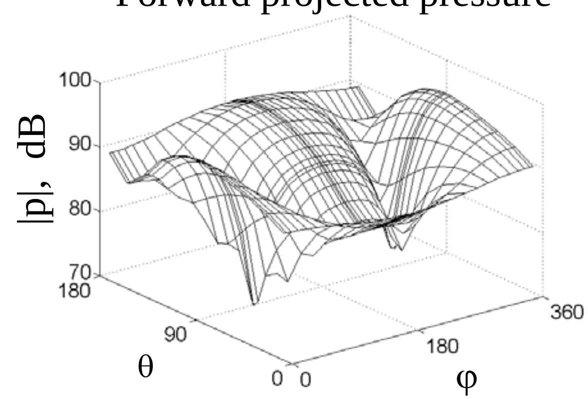
Backward projected pressure



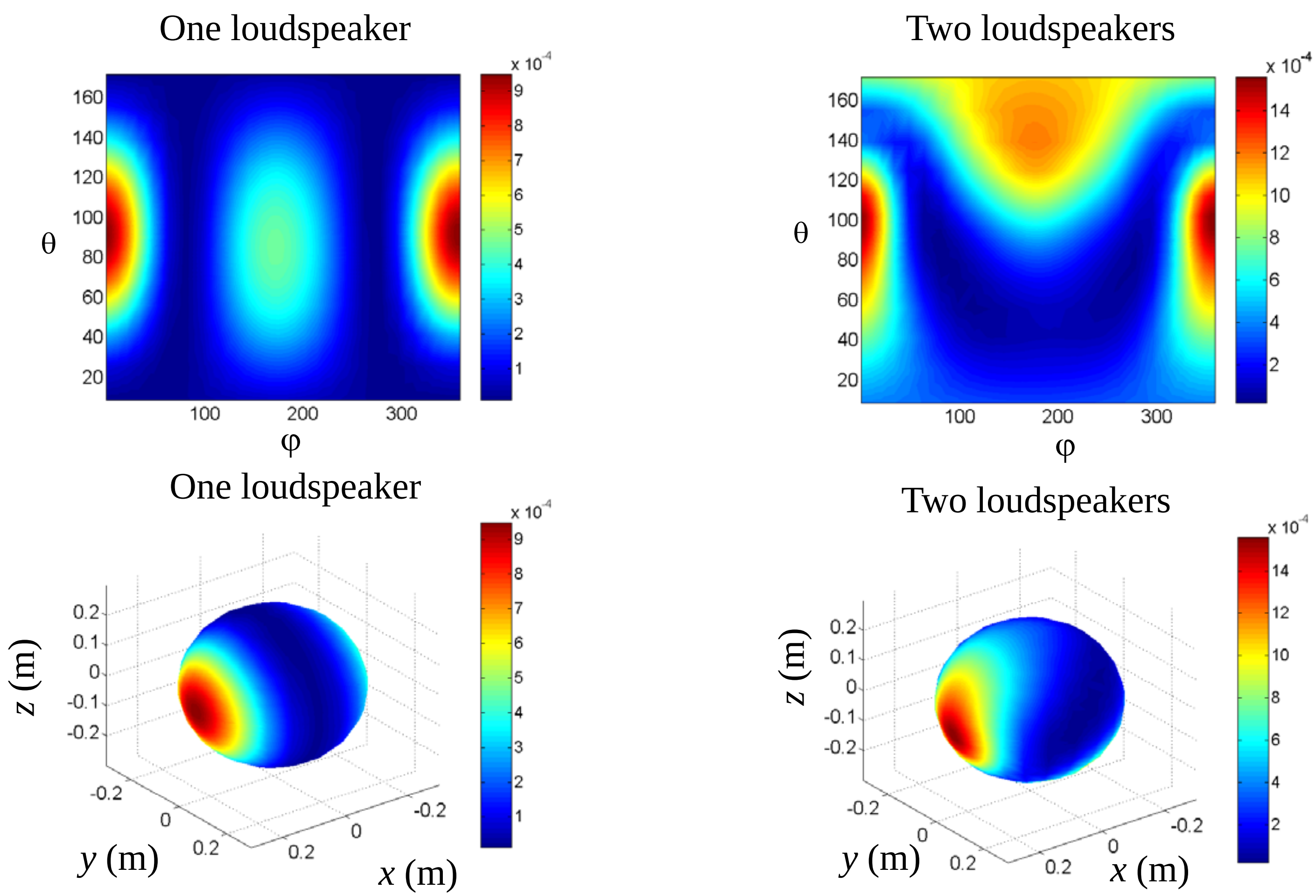
Measured pressure, r_2=0.291 m



Forward projected pressure



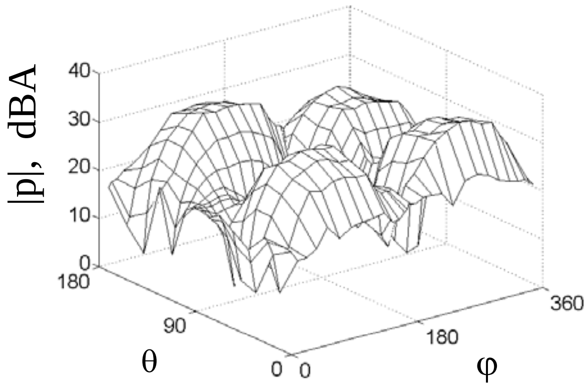
- **Intensity calculated (f=500 Hz, r_1=0.240 m)**



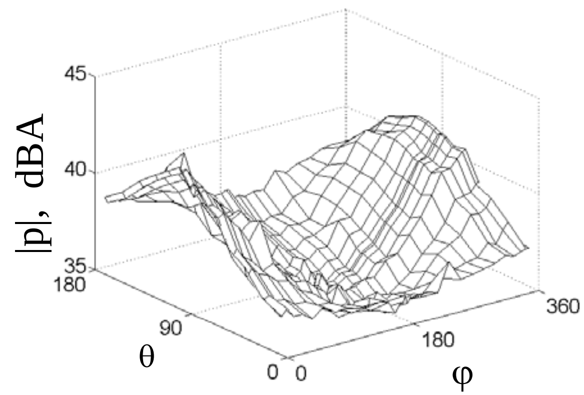
Small air compressor source

- **Small air compressor (Measured pressure, A-weighted)**

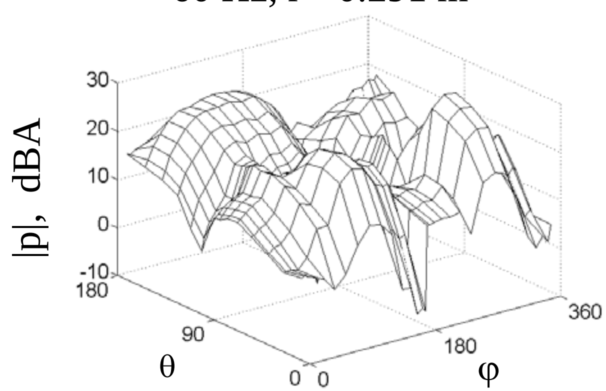
60 Hz, r_1=0.240 m



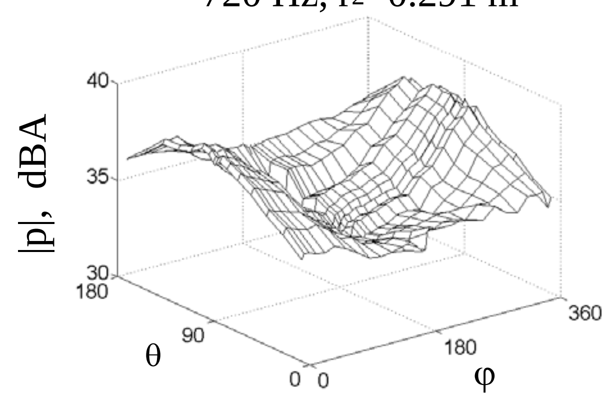
720 Hz, r_1=0.240 m



60 Hz, r_2=0.291 m

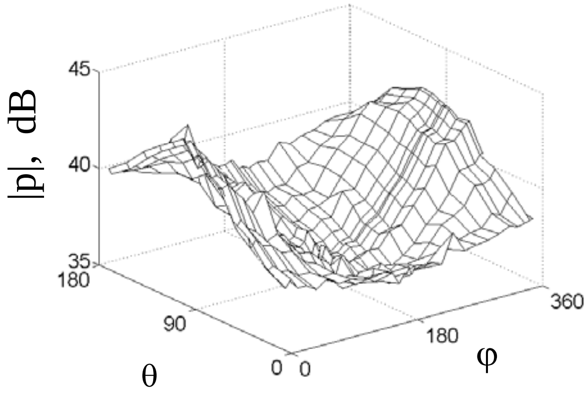


720 Hz, r_2=0.291 m

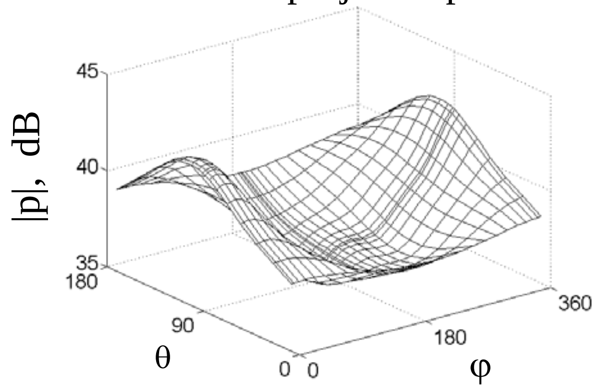


- **Small air compressor (f=720 Hz)**

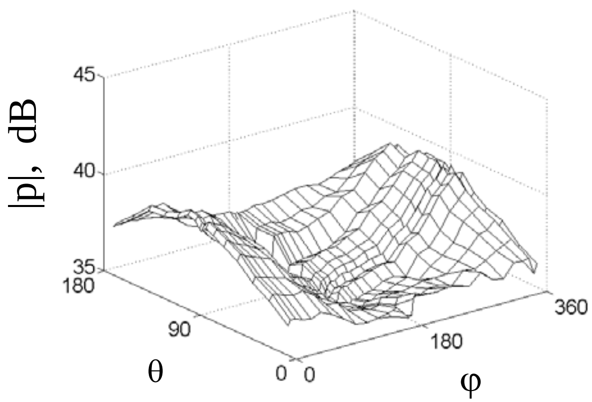
Measured pressure, r_1=0.240 m



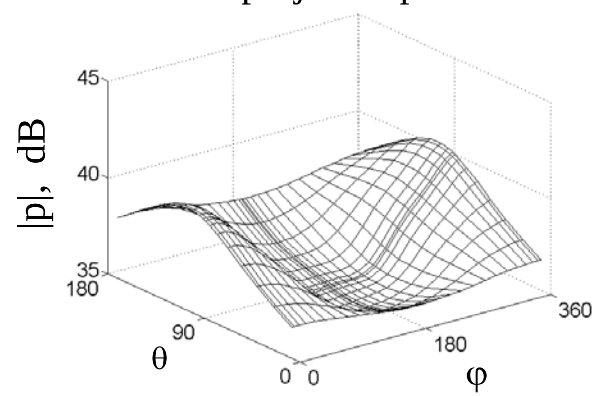
Backward projected pressure



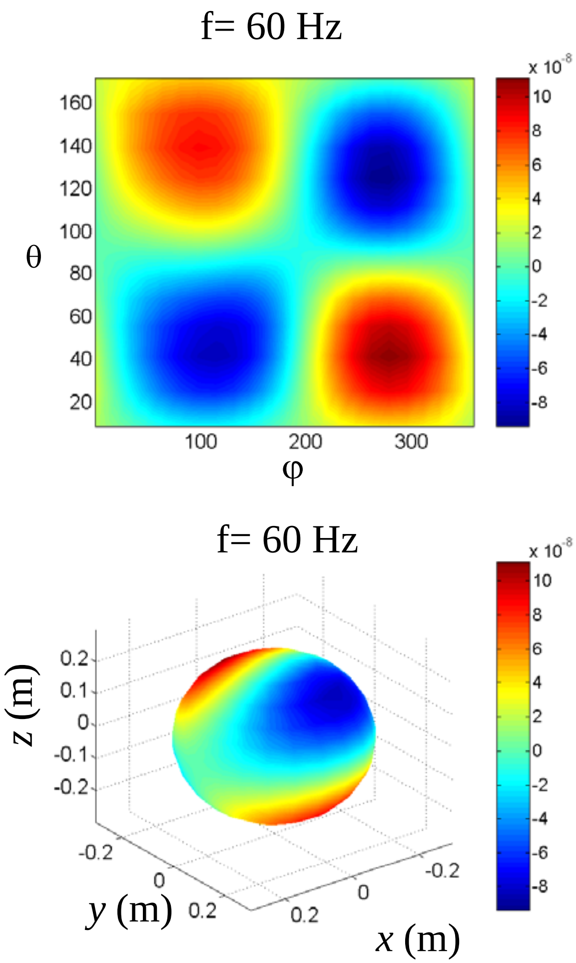
Measured pressure, r_2=0.291 m



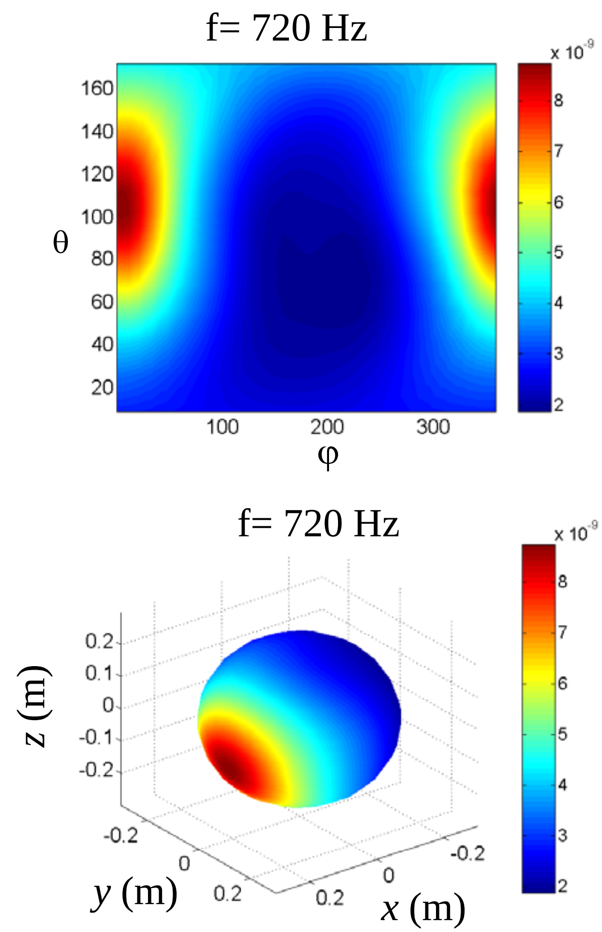
Forward projected pressure



- **Intensity calculated (small air compressor, r_1=0.240 m)**



Radiation from Shell



Radiation from Exit

Summary

- Sound field on spherical surface decomposed into spherical harmonics, each of which has known propagation characteristics
- Sound field can be projected inwards or outwards
- Backward projection is corrupted by noisy high order coefficients
- Coefficient filtering procedures must be implemented to ensure successful back projection