Investments in structural safety: the compatibility between the economic and societal optimum solutions

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Abstract: To optimize the investment level in projects that improve structural safety, two approaches are commonly used. While Lifetime Cost Optimization (LCO) balances upfront investments against a reduction in uncertain future failure costs, the Life Quality Index (LQI) balances monetary expenditure against changes in life expectancy. The LQI methodology is often considered as a boundary condition for LCO by requiring that the safety investment does not result in a reduction of the LQI (LQI net benefit criterion). However, for safety investments defined by a continuous design parameter, the net benefit criterion is often a weak requirement and a maximization of the LQI should be pursued. In this paper the LQI maximum societal benefit criterion is introduced and its relationship with the LCO criterion is investigated. Results indicate that for a societal decision maker the LCO and LQI optimum design criteria are identical when the costs related to human losses are evaluated in accordance with the Societal Willingness to Pay concept. Any other evaluation method necessarily results in a suboptimal LQI and an unnecessary loss of societal welfare.

1 Introduction

Contemporary society emphasizes the importance of safe and sustainable construction. A common methodology is to minimize the total expected costs during the lifetime of the structure. This methodology is referred to as Lifetime Cost Optimization (LCO). An alternative methodology is to assess the societal acceptability of decisions by considering their effect on the life-expectancy e and the gross domestic product g through the compound indicator of the Life Quality Index (LQI).

Application of the LQI to structural safety has found considerable support in the scientific community. However, generally only the LQI net benefit criterion is considered, defining a lower bound for the safety investment. Often this results in a very wide range of acceptable designs, prompting many to believe that the societal acceptability criterion of the LQI should not be explicitly considered when assessing the optimum level of structural safety.

However, the fundamental concepts underlying the LQI require that the level of safety investment is determined for which the LQI is maximal, i.e. maximizing societal welfare. This 'LQI maximum societal benefit criterion' is derived further and a comparison is made with the LCO evaluation and with the LQI net benefit criterion. The conceptual application example at the end of the paper illustrates the different concepts and their interaction.

2 The present value of future costs and benefits

Engineering problems in the field of structural safety are associated with the stochastic incurrence of costs and benefits during the lifetime of the structure. The incurrence of damages for example relates to the uncertain exposure to extreme events, and to the probability of failure given the extreme event. Furthermore, if the structure is systematically renewed or repaired after failure, the renewed structure is again exposed to possible damaging events. This sequence of renewed exposures is referred to as a renewal process. A systematic derivation of the fundamental equations underlying renewal processes can be found in [7].

In summary, all the stochastically incurred costs and benefits have to be discounted to their present value by a continuous discount rate γ. Describing the occurrence of failure by a homogeneous Poisson process with renewal rate λ given by the annual probability of failure P_f, and considering the costs H_i incurred at the i^{th} renewal (i.e. failure) to be independent and identically distributed with mean value μ_H, the expected present value of the damage cost D is given by (1). When the time horizon t_{max} approaches infinity, eq. (1) reduces to the well-known result (2) mentioned for example in [8].

$$E[D] = \mu_H \int_0^{t_{\max}} P_f \exp(-\gamma t) dt = \mu_H \frac{P_f}{\gamma}\left(1-\exp(-\gamma t_{\max})\right) \tag{1}$$

$$\lim_{t_{\max}\to\infty} E[D] = \mu_H \frac{P_f}{\gamma} \tag{2}$$

As indicated by (1) and (2) and considering the derivations further in this paper, a traditional cost optimization or LQI evaluation based on expected costs does not require knowledge of the variability of H_i. Decision making based on expected costs is however rational and moral only when considering situations where no possibility of ruin exists, see for example the concurrent arguments made by Taleb in [9]. If catastrophic – or more general: unacceptable – losses are possible, the necessary actions for eliminating these possibilities should be identified first, before continuing for example to a cost-optimization based on expected losses.

In the remainder of this paper the time horizon t_{max} for the assessment will be considered as infinite, based on the observation that for large finite time horizons the present value contribution of costs incurred at t_{max} is very small, diminishing the difference between a large but finite time horizon and an infinite time horizon. Considering for example a discount rate of 3%, a cost H_i incurred 100 years in the future has a present value of only $0.05H_i$. Furthermore, as indicated in [1] the assumption of an infinite time horizon is especially reasonable when considering a portfolio of buildings from a regulatory perspective (societal decision maker).

3 Lifetime Cost Optimization (LCO)

Lifetime Cost Optimization (LCO) determines the level of safety investment in the design and construction stage which minimizes the total cost over the lifetime of the structure (maximizes total utility), taking into account the uncertain future occurrence of damage due to exposure to adverse events. The basic formulation for the lifetime utility is given by (3), with Z the total utility, B the benefit derived from the structure's existence, C the initial construction cost, D

the damage cost due to adverse events during the lifetime of the structure, and $\boldsymbol{\theta}$ the vector of design parameters considered for optimization [8]. In the remainder of this paper only a single design parameter θ will be considered for simplicity. As the goal of the optimization is to maximize Z, the optimum design criterion is given by (4).

$$Z(\boldsymbol{\theta}) = B(\boldsymbol{\theta}) - C(\boldsymbol{\theta}) - D(\boldsymbol{\theta}) \tag{3}$$

$$\frac{dZ(\theta)}{d\theta} = 0 \tag{4}$$

The benefit B accrues over the lifetime of the structure. When considering a pure safety investment, the benefit derived from the structure's existence relates to the avoidance of failure costs. Consider for example a situation where an annual failure probability $P_{f,0}$ exists in absence of any safety investments. The decision to make a safety investment will then result in a change of regime from a situation governed by the reference failure probability to a situation governed by the new annual failure probability $P_f(\theta)$. Considering the avoidance of the reference state failure costs as a benefit derived from the structure's existence, the expected present value evaluation of B is given by (5) as an application of (2), with μ_M the expected direct and indirect material losses associated with a failure event and μ_F the expected costs associated with human losses. The right-hand equality in (5) is given by rewriting μ_M and μ_F as a product of the exposed population N, the annual GDP per capita g and failure cost indicators ξ_M and ξ_F. The cost-indicators ξ_M and ξ_F have the advantage of indicating the severity of the damages in respect to the monetary capacity of the exposed population.

$$B(\theta) = (\mu_M + \mu_F)\frac{P_{f,0}}{\gamma} = Ng(\xi_M + \xi_F)\frac{P_{f,0}}{\gamma} \tag{5}$$

The initial construction cost C is realized at the start of the structure's existence. The ratio of C to the exposed population N and the GDP g defines ξ_C (6). Naturally, ξ_C is a function of θ. The damage cost D relates to the uncertain failure costs incurred in the new regime considering the residual failure probability $P_f(\theta)$. Evaluating the expected repair cost μ_R for the structure as a ratio r of the initial construction cost, the expected present value evaluation of D is given by (7). Here r is assumed independent of θ for simplicity.

$$C(\theta) = Ng\xi_C(\theta) \tag{6}$$

$$D(\theta) = Ng\left(r\xi_C(\theta) + \xi_M + \xi_F\right)\frac{P_f(\theta)}{\gamma} \tag{7}$$

The above equations allow elaboration of the LCO optimum design criterion (4) to (8), where the θ-dependency of ξ_C and P_f has been omitted in order not to overburden the notations.

$$\frac{d\xi_C}{d\theta}\left(1 + \frac{rP_f}{\gamma}\right) + \frac{r\xi_C + \xi_M + \xi_F}{\gamma}\frac{dP_f}{d\theta} = 0 \tag{8}$$

4 The LQI and its application to decision making

4.1 Introduction

The Life Quality Index (LQI) as introduced by Nathwani et al. [4] provides a powerful compound social indicator which can be applied to evaluate the societal acceptability of decisions related to Life Safety. As stated by Lind et al. [3], the underlying goal of safety investments and risk management should be to cost-effectively improve the overall societal welfare. Consequently, key parameters incorporated in the LQI are the annual GDP per capita g and the life expectancy e. These parameters are considered together with an exponent q defining the trade-off between work and leisure to form the LQI index (9). Derivations of the LQI index are given amongst others by [4], [6] and [8]. Note that slightly different definitions of the exponent q exist, with a recent definition given in [6]. These differences are, however, of little importance for the discussions in the current paper.

$$LQI = g^q e \tag{9}$$

4.2 The LQI net benefit criterion

Investment in a safety measure will generally result in a change of the LQI, as given by (10). Generally, the investment results in a reduction dg of the GDP, while on the other hand resulting in a small increase de of the life expectancy e. If both dg and de apply on a yearly basis (10) can be evaluated directly (see [4] for examples). The societal acceptability of the measure is then defined by requiring that the change in LQI is positive, resulting in (11) as the common formulation of the "LQI net benefit criterion". When $dLQI$ is zero, the safety measure has no net benefit for society, but neither results in a loss of societal welfare.

$$dLQI = qg^{q-1}edg + g^q de \tag{10}$$

$$\frac{dLQI}{LQI} = q\frac{dg}{g} + \frac{de}{e} \geq 0 \tag{11}$$

In the field of structural safety the costs and benefits accrue stochastically over time, and therefore the present value of (11) needs to be considered. Denoting with PV(.) the present value evaluation and considering both q and g to be constant, results in (12).

$$PV\left(\frac{dLQI}{LQI}\right) = \int_0^{t_{\max}} \left(q\frac{dg}{g} + \frac{de}{e}\right)\exp(-\gamma t)dt = \frac{q}{g}\int_0^{t_{\max}} dg\exp(-\gamma t)dt + \int_0^{t_{\max}} \frac{de}{e}\exp(-\gamma t)dt \tag{12}$$

The present value evaluation of dg is equivalent to the per capita present value of the monetary costs and benefits accrued up to t_{max}. This results in (13) (considering the evaluations above in Section 3 and an infinite time horizon t_{max}, while dividing by N for a per capita evaluation and omitting the costs to human life which are evaluated separately through de).

$$\int_0^{\infty} dg\exp(-\gamma t)dt = \frac{1}{N}\left(Ng\left(\frac{\xi_M}{\gamma}P_{f,0} - \xi_C - \frac{r\xi_C + \xi_M}{\gamma}P_f\right)\right) \tag{13}$$

As discussed in [1], [4], [8], the relative change in life expectancy can be related to a change in mortality dM, through the demographic constant C_{FM}, giving (14) with N_f the expected number of casualties in case of a failure event. This results in (15) for the present value of de/e.

$$\frac{de}{e} \approx C_{FM} dM \approx C_{FM} \frac{N_f}{N} \left(P_{f,0} - P_f \right) \tag{14}$$

$$\int_0^{\infty} \frac{de}{e} \exp(-\gamma t) dt = C_{FM} \frac{N_f}{N} \frac{P_{f,0} - P_f}{\gamma} \tag{15}$$

Requiring that the present value of $dLQI$ is positive, the LQI net benefit criterion for safety investments, where costs and benefits stochastically accrue in the future, is given by (16).

$$q\left(\frac{\xi_M}{\gamma} P_{f,0} - \xi_C - \frac{r\xi_C + \xi_M}{\gamma} P_f \right) + C_{FM} \frac{N_f}{N} \frac{P_{f,0} - P_f}{\gamma} \geq 0 \tag{16}$$

4.3 The LQI maximum societal benefit criterion

As applied in [4], the LQI net benefit criterion is a rational tool for evaluating the acceptability of binary safety decisions (e.g. "is the proposed 'seat cushion flammability regulation' beneficial for society?" see [4]). The LQI net benefit criterion has, however, also found general acceptance in the literature on structural safety. When considering a single continuous safety parameter, θ, the LQI net benefit criterion is used to define a lower bound, θ_{min} (see e.g. [1] and [8]), leaving the field open for an LCO evaluation for $\theta \geq \theta_{min}$.

The underlying goal of the LQI application is, however, to maximize societal welfare, in agreement with the fundamental principle stated by Lind et al. [3] that safety investments should improve overall societal welfare. Therefore, when determining the appropriate level of investment in a safety parameter θ, the present value of $dLQI$ / LQI as given in (12) should be maximized. This consideration results in the general 'LQI maximum societal benefit criterion' of (17), resulting in (18) when taking into account (13) and (15).

$$\frac{d}{d\theta}\left(PV\left(\frac{dLQI}{LQI} \right) \right) = \frac{d}{d\theta}\left(\frac{q}{g} \int_0^{t_{\max}} dg \exp(-\gamma t) dt + \int_0^{t_{\max}} \frac{de}{e} \exp(-\gamma t) dt \right) = 0 \tag{17}$$

$$\frac{d}{d\theta}\left(q\left(\frac{\xi_M}{\gamma} P_{f,0} - \xi_C - \frac{r\xi_C + \xi_M}{\gamma} P_f \right) + C_{FM} \frac{N_f}{N} \frac{P_{f,0} - P_f}{\gamma} \right) = 0 \tag{18}$$

Elaborating (18) gives (19), which has been arranged to follow the same structure as the LCO criterion of (8).

$$\frac{d\xi_C}{d\theta}\left(1 + \frac{rP_f}{\gamma} \right) + \frac{r\xi_C + \xi_M + \frac{C_{FM} N_f}{qN}}{\gamma} \frac{dP_f}{d\theta} = 0 \tag{19}$$

5 Comparison between LCO and LQI optimum solutions

Comparing the LCO and LQI optimum design criteria of (8) and (19), similarities are observed. The parameters applied in both equations may, however, have different values, depending on the viewpoint of the decision maker. For example, the parameter ξ_M associated with direct and indirect material losses will generally be evaluated differently by different private stakeholders, and will have yet another value from a societal viewpoint. Considering the background of the LQI as an indicator for societal welfare, the LQI is necessarily evaluated from a societal perspective. More precisely, the LQI should be evaluated from the perspective of the group of persons bearing the costs and reaping the benefits of the safety measure (note that this does not imply that every person bearing the costs also has to be a beneficiary).

Often the investment in structural safety is fully determined by a societal decision maker. On the one hand the performance specifications for large safety projects (e.g. surge barriers) will generally be set by a governmental decision, while on the other hand the safety levels obtained in common structures are (in general) defined through the applicable codes, standards and regulatory requirements. In current practice a private evaluation of the optimum safety level is conceivable only for exceptional situations. Consequently, the societal decision maker is the most relevant point of view both for the LCO and the LQI assessment. For the societal decision maker, the evaluation of the parameters ξ_C, ξ_M, r, and γ are the same in both (8) and (19). Comparing (8) and (19) for a societal decision maker, the LCO and LQI optimum design criteria are identical if Equation (20) holds for evaluating the human consequences.

$$\xi_F = \frac{C_{FM} N_f}{qN} \tag{20}$$

Multiplying (20) with Ng gives (21), an evaluation of the expected costs μ_F associated with losses of human lives and limbs in case of a failure event. Considering failure events to occur with renewal rate λ, the present value evaluation of (21) is given by (22), in accordance with Section 2 above. The approximations on the right hand of (22) are introduced through (14). Multiplying all members of the equation with γ and acknowledging that $\mu_F \cdot \lambda$ is monetarily equal to N times an expected annual loss dg of GDP per capita, results in (23). The right hand equality in (23) is identical to the Societal Willingness to Pay (SWTP) as derived by Pandey and Nathwani [5], proving that the LCO and LQI optimum design solutions of (8) and (19) are identical for a societal decision maker when the costs μ_F (i.e. ξ_F) associated with losses to human life and limb are evaluated in accordance with the SWTP principle.

$$Ng\xi_F = \mu_F = \frac{Ng}{q}\frac{C_{FM} N_f}{N} \tag{21}$$

$$\frac{Ng\xi_F\lambda}{\gamma} = \frac{\mu_F\lambda}{\gamma} = \frac{Ng}{q\gamma}\frac{C_{FM} N_f}{N}\lambda \approx \frac{Ng}{q\gamma}C_{FM}dM \approx \frac{Ng}{q\gamma}\frac{de}{e} \tag{22}$$

$$Ng\xi_F\lambda = \mu_F\lambda = Ndg = \frac{Ng}{q}\frac{de}{e} \tag{23}$$

The above indicates a perfect compatibility of the LCO and LQI optimum design solutions when evaluating loss to human life and limb in accordance with the SWTP. Other LQI-based methods for evaluating losses to human life and limb exist (see the literature study in [1]), but these different evaluations will necessarily result in a suboptimal LQI.

Note that the conclusions above are valid only when investment costs (ξ_C), repair costs (r), material damages (ξ_M) and discount rates (γ) are the same in both the LCO and LQI evaluation. As mentioned earlier, this will generally not be the case for a private decision maker. Currently, the compatibility of a private LCO with the LQI is considered to result from applying the LQI net benefit criterion as a boundary condition. This is however not recommendable, as discussed further. To discern a possible way forward, consider that amongst others the variability of costs and the uncertainty with respect to the discount rate imply that the future LQI realization is stochastic. Consequently, decision support methods as in [10] can be applied to determine an acceptable range for the decision parameter θ based, for example, on a maximum acceptable deviation between the optimum LQI and the LQI resulting from the private decision. Within this acceptable range, private LCO evaluations can be considered acceptable. This is not further elaborated here.

6 Discussion on the application of the LQI net benefit criterion

Recent literature applying the LQI in the field of structural safety considers the LQI net benefit criterion of Equation (16) as defining a lower bound for the safety investment, see e.g. [1] and [8]. Furthermore, the LQI net benefit criterion is often applied neglecting the benefit term (i.e. the reduction in material damages). Occasionally even the present value evaluation is omitted. A number of objections can be raised to this application:

- When considering investments in a continuous safety parameter the LQI net benefit criterion may result in a very wide range of acceptable designs, i.e. accepting design options far from the societal optimum, resulting in unnecessary loss of life.
- When costs are stochastically incurred over the lifetime of the structure, the present value of the future material losses and (reduction in) risk to human life and limb have to be considered. Neglecting the present value evaluation will result in a distortion of the LQI cost-benefit evaluation.
- The LQI net benefit criterion of (16) is dependent on the reference failure probability $P_{f,0}$. The acceptability of a final safety design according to the net benefit criterion may therefore depend on any intermediate safety levels obtained. See Section 7.3 for an application of this path-dependency.
- Neglecting the reduction of material damages when evaluating the LQI net benefit criterion results in a weakening of the criterion (i.e. accepting lower levels of safety investment). Consider for example a situation where an expensive safety investment monetarily ‘pays for itself’ by reducing the present value of future material losses. In this situation every associated risk reduction with respect to human life and limb effectively comes at zero cost, and therefore this safety investment should be implemented in accordance with the LQI principle, even if the ‘crude’ investment per life year saved is very high.

7 Example application

7.1 Problem statement

The example application presented here is an adaptation of the flood protection problem in [8]. A town has an annual probability of flooding $P_{f,0}$ of 0.1 due to the bad shape of the existing dams. New dams will be constructed and the optimum dam height should be determined. Values of relevant parameters are given in Table 1. For the newly constructed dam, the annual probability of flooding P_f will be governed by the dam height through (24). The construction cost is a function of the dam height through (25).

Table 1: Parameters governing the flood protection problem

Parameter	**Symbol**	**Value**	**Dim.**
Dam height	h	TBD	m
Overtopping parameter	b	3	m
Dam cost per m^3	c	150	USD/m^3
Dam length	L	10000	m
Relative repair cost dam after overtopping	r	0.2	-
GDP per capita	g	25000	USD
Work-leisure trade-off factor	q	0.143	-
Discount rate	γ	0.05	1/year
Population distribution constant	C_{FM}	19.2	-
Expected number of casualties in case of a flood	N_f	50	persons
Total population town	N	200000	persons
Expected direct and indirect material losses due to flooding	μ_M	$5 \cdot 10^7$	USD

$$P_f = \exp\left(-\left(\frac{h}{b}\right)^3\right) \tag{24}$$

$$C(h) = ch^2L \tag{25}$$

7.2 LCO and LQI evaluation

The total utility Z_{LCO} is evaluated in function of the dam height in accordance with (3) and the derivations in Section 3. Results are visualized in Figure 1 for different ξ_F. The optimum dam heights h_{LCO} according to the LCO optimum design criterion of (8) are indicated for each ξ_F. Furthermore, Figure 1 visualizes the present value evaluation of $dLQI$ / LQI according to (12) and the derivations in Section 4.2. The minimum required dam height h_{min} according to the LQI net benefit criterion of (16) is indicated in the graph, as well as the optimum dam height h_{LQI} according to the LQI maximum societal benefit criterion. As visualized in Figure 1, the optimum heights h_{LCO} and h_{LQI} match when ξ_F = 0.0336, i.e. when ξ_F is determined through (20). As in [8], the LQI net benefit criterion is found to impose only a very limited restriction on h (discussion further in Section 7.3). For completeness, note that the LQI net benefit criterion of (16) is positive only in the range {4.01m; 17.04m}. In casu only the lower bound h_{min} is of importance as the upper bound is considered outside the range of reasonable values for h.

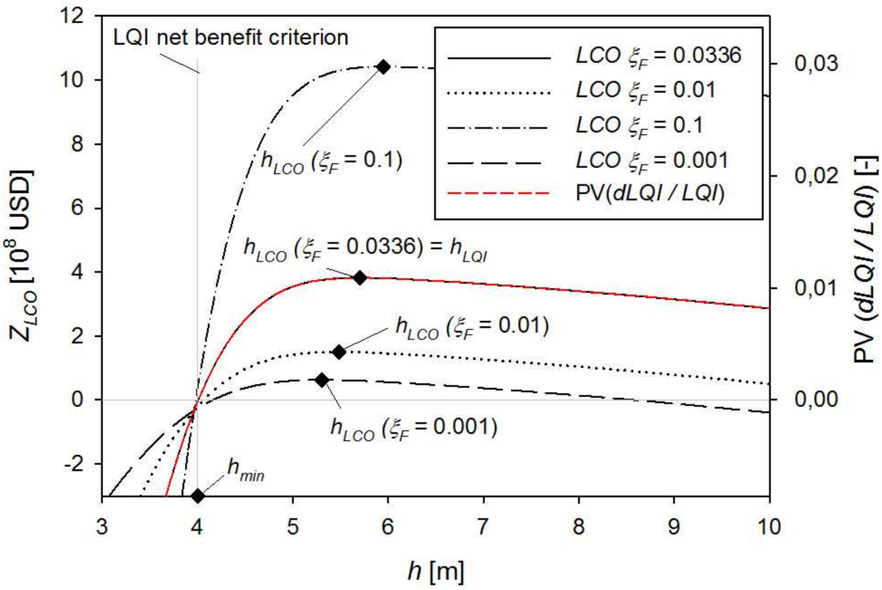

Figure 1: Total utility Z_{LCO} and LCO optimum design (h_{LCO}) in function of ξ_F. Present value evaluation of dLQI / LQI and LQI optimum design (h_{LQI})

7.3 Discussion on the LQI net benefit criterion

In this Section ξ_F is considered in accordance with (20), resulting in $h_{LCO} = h_{LQI}$ and the total utility Z_{LCO} being (up to a constant factor) equal to $PV(dLQI\ /\ LQI)$. Consequently, the LQI net benefit criterion is identical to the requirement $Z_{LCO} \geq 0$. Results are visualized in Figure 2 considering different $P_{f,0}$. For large $P_{f,0}$ the net benefit criterion results in a low value for h_{min}, far from the optimum value h_{LQI}. It can be argued that this weak requirement is not compatible with the philosophy underlying the LQI. For small $P_{f,0}$ however the upper bound h_{max} for the net benefit criterion comes into play as well (i.e. $P_{f,0}$ = 0.02). And for even smaller $P_{f,0}$ (i.e. $P_{f,0}$ = 0.01) the net benefit criterion indicates that no acceptable dam height exists as all investments result in $Z_{LCO} < 0$. Note however that in the simplified example, no deterioration or damage of the existing dams has been considered (i.e. the decision to build a new dam was a priori assumed as a given). Therefore, the result for $P_{f,0}$ = 0.01 should be considered as a theoretical and illustrative result only. Application of the LQI maximum societal benefit criterion is, on the other hand, independent of $P_{f,0}$. In conclusion, it is suggested that the LQI maximum societal benefit criterion is more relevant for guiding investment in a safety parameter, while the net benefit criterion is most beneficial to support binary decision making. Referring to the result for $P_{f,0}$ = 0.01 in Figure 2, the binary evaluation indicates that currently no investments should be made (for the simplified model described above). When the decision to build a new dam is a given, the LQI maximum societal benefit criterion guides the decision.

8 Conclusions

Application of the LQI net benefit criterion in the field of structural safety may result in a weak requirement serving as a boundary condition for LCO with respect to a continuous design parameter. In agreement with the philosophy of maximizing societal welfare, which underlies the LQI, an LQI maximum societal benefit criterion is presented. For a societal decision maker the resultant LQI optimum safety investment is identical to the optimum safety investment obtained through LCO if the costs associated with human losses are evaluated according to the Societal Willingness to Pay concept. Other methodologies necessarily result in a suboptimal present value for the LQI change resulting from the safety investment. Further developments are required for integrating the societal LQI evaluation with private LCO. The current concept of applying the LQI net benefit criterion as a lower bound for the private LCO may be too weak a requirement.

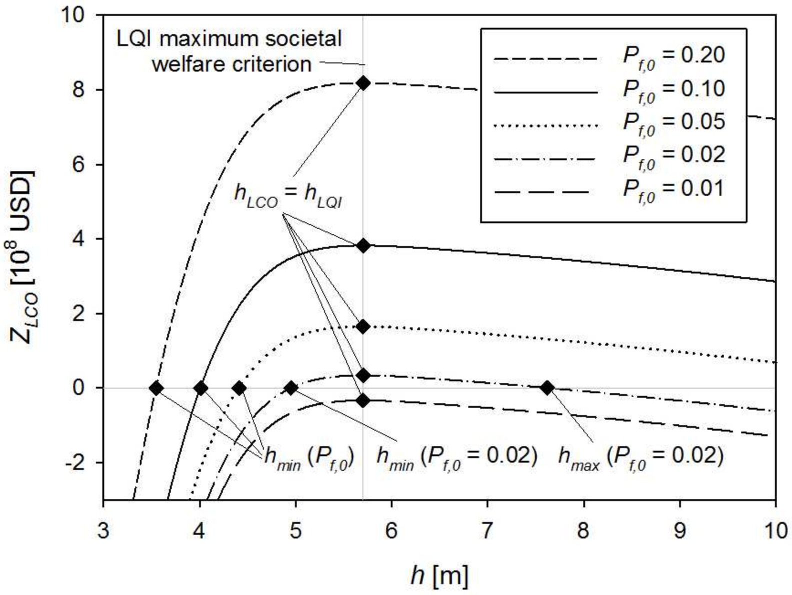

Figure 2: Total utility Z_{LCO} for ξ_F = 0.0336 for different $P_{f,0}$. Minimum dam height h_{min} (LQI net benefit criterion) and $h_{min,}$, optimum dam height $h_{LCO} = h_{LQI}$ (LQI maximum societal benefit criterion).

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