A non-linear 2-way coupling between DualSPHysics and a wave propagation model

Tim Verbrugghe, A. Kortenhaus, P. Troch
dept. of Civil Engineering
Ghent University
Ghent, Belgium
timl.verbrugghe@ugent.be

J.M. Domínguez
EPHYSLAB Environmental Physics Laboratory
Universidade de Vigo
Ourense, Spain

***Abstract*—Wave energy converters (WECs) need to be deployed in large numbers in an array layout in order to have a significant power production. Each WEC has an impact on the incoming wave field, diffracting, reflecting and radiating waves. Simulating the wave transformations within and around a WEC farm is complex; it is difficult to simulate both near field and far field effects with a single numerical model, with relatively fast computing times. Within this research a numerical tool is developed to model both near-field and far-field wave transformations caused by WECs. The tool consists of coupling a wave-structure interaction solver, based on the Smoothed Particle Hydrodynamics (SPH) formulation and a wave propagation model, based on the potential flow theory. The coupling is performed within an OpenMPI environment, with a python shell controlling the data transfers. A 2D proof-of-concept is introduced to demonstrate the ability of the model to propagate non-linear waves and model floating bodies with high accuracy.**

I. INTRODUCTION

The deployment of multiple floating wave energy converters (WEC) at an offshore location influences the incident wave field by reflection, diffraction and radiation. The superposition of these phenomena results in a complex perturbed wave field [1]. In order to accurately simulate wave propagation through a WEC farm, both near field and far field effects need to be accounted for. This can be achieved by coupling of different solvers for the near field and far field. For linear simulation over variable bathymetry, there have been studies coupling a wave propagation solver and a BEM potential flow solver [2], [3]. However, real sea and storm conditions are characterized by irregular, 3D waves with the occurrence of non-linear effects. For this reason a novel coupling methodology is suggested, where a fast wave propagation model (OceanWave3D [4]) accounts for the far field effects and an accurate wave-structure interaction model, based on the smoothed particle hydrodynamics formulation (DualSPHysics [5]) is applied for the near field effects. The combined model allows for simulation of WEC devices in higher order irregular waves and more extreme wave conditions.

Smoothed particle hydrodynamics (SPH) is a flexible Lagrangian and mesh-less technique for computational fluid dynamics. The Lagrangian reference frame of SPH makes it useful in solving problems with large deformations and distorted free surfaces. In comparison with other numerical methods, the SPH formulation is simple and robust [6]. SPH has been successfully applied to a number of free-surface problems that involve wave breaking and splashing [7], [8]. The impact between a rigid body and water has been studied in [9]. A fixed cylinder in a wave train and forced motion of cylinders generating waves is mentioned in [10], while floating bodies in waves have been successfully studied in 2D [11]. 3D problems of wave generation by a heaving cone and a floating body in waves undergoing predominantly heave motion are investigated in [12]. The latter has also indicated that there is a large benefit of calculating with a variable particle mass distribution.

Regarding coupling methodologies, The SPH solver has been applied in a study where a one-way coupling was realized between a wave propagation model and the SPH model [13]. However, in this research, the coupling information is shared in a two-way principle, resulting in a more accurate solution. The SPH model receives detailed information on the wave kinematics from the wave propagation model, while the transformed kinematics resulting from the wave-structure interaction and the perturbed free surface are transferred back to the wave propagation model.

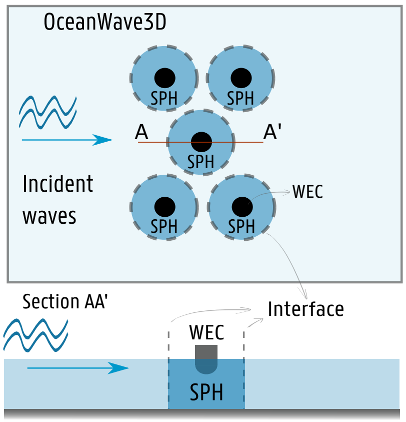

Fig. 1. Top view and longitudinal section of the coupling principle. Circular SPH zones with a WEC device in the center are coupled to a wave propagation solver. Information is transferred at the interfaces in front and at the back of the device

Ideally, the coupling is performed as illustrated in Fig. 1. A large domain is set up for propagating fully non-linear, short-crested 3D waves. In the center zone, a WEC farm is installed. Each device is modeled in a circular SPH zone, with a custom designed interface for exchanging information between the two models. Currently, the 2-way coupling model is limited to two dimensions. In this study, the methodology of the coupling model is discussed, as well as the results of a proof-of-concept. In the second chapter of this paper, the methodology of the coupling is given. A detailed description of the applied models is provided, followed by an explanation of the coupling algorithm. Next, the results of a proof-of concept model are discussed in chapter 3. The focus is put on simulations in linear and non-linear waves, as well as the response of a heaving WEC. Lastly, concluding remarks are made in chapter 4.

II. METHODOLOGY

A. Wave Propagation Model

The first part of the numerical tool is the wave propagation model. This model propagates the incident waves and calculates the interaction with the diffracted and radiated waves coming from Nemoh. The open-source fully non-linear potential flow solver OceanWave3D is used for this task [4], [14]. It is aimed at closing the performance gap between traditional Boussinesq-type models and volume-based solvers such as the fully nonlinear potential flow model and enables fast (near) real-time hydrodynamics calculations.

The fully nonlinear potential flow problem for waves on a fluid of variable depth is applied to find the free surface elevations on a 3D grid. The evolution of the free surface is governed by the kinematic and dynamic boundary conditions:

$$\partial_t \eta = -\nabla\eta \cdot \nabla\tilde{\Phi} + \tilde{w}(1 + \nabla\eta \cdot \nabla\eta) \qquad (1)$$

$$\partial_t \tilde{\Phi} = -g\eta - \frac{1}{2}\left(\nabla\tilde{\Phi} \cdot \nabla\tilde{\Phi} - \tilde{w}^2(1 + \nabla\eta \cdot \nabla\eta)\right) \qquad (2)$$

These are expressed in function of the free surface quantities $\tilde{\Phi} = \Phi(x, y, \eta, t)$ and $\tilde{w} = \partial_z \left.\Phi\right|_{z=\eta}$.

The problem is discretized using a method of lines approach and for the time-integration of the free-surface conditions, a classical explicit four-stage, fourth-order Runge-Kutta scheme is employed. Spatial derivatives are replaced by the discrete counterparts using the high-order finite difference method and nonlinear terms are treated by direct product approximations at the collocation points. At the structural boundaries of the domain, i.e. at the bottom and wall sides, Neumann conditions are imposed. In the current version of the numerical tool, non-linear effects are not considered. Consequently, the linear superposition method can be applied and the total transformed wavefield can be created by summation of the independent surface elevations and potentials.

B. Wave-Structure Interaction Model

The software used for the detailed modelling of the wave-structure interactions is DualSPHysics [5]. It applies the SPH formulation to model the hydrodynamics, where particles represent the flow, interact with structures and can exhibit large deformation with moving boundaries. Originally, the code was written in Fortran and only available to compute on CPUs (SPHysics), leading to a very high computational cost. However, in recent years Graphics Processing Units (GPUs) have appeared as a cheap alternative to accelerate numerical models. GPUs are designed to manage huge amounts of data and their computing power has developed to be much faster than conventional CPUs in certain cases. Since SPH methods have an algorithmic structure, very open to parallellism, the computing power of GPUs can be also applied to SPH methods. DualSPHyiscs is created specifically with GPUs in mind, giving the user the choice to calculate on either a CUDA-enabled GPU or a CPU. Within this research, both the CPU and GPU version of DualSPHysics is used to run the 2-way coupled wave propagation model. In table I, the specific hardware is detailed.

TABLE I
AVAILABLE COMPUTING POWER

	CPU	**GPU**
Brand	Intel	Nvidia
Type	i7 6700	GTX 1070
Cores	4	1920
Memory	32 GB	8 GB
Clock Speed	3,4 GHz	1,5 GHz

C. Coupling methodology

1) Coupling principle: As mentioned before, SPH simulations are very computationally intensive. The data output required from a WEC SPH model is often limited to a zone closely spaced around the floating WEC. However, there is a spatial need for wave generation and wave absorption, around $3-4$ wavelengths long. This leads to a significant increase in water particles, and thus higher computation times. Moreover, wave generation techniques available in DualSPHysics are limited to first and second order wave generation by using piston-type or flap-type wave paddles. This generation type requires a certain propagation length before the full kinematics and surface elevation are developed. Within this research, the objective is to simulate higher-order (up to 5^{th}) irregular short-crested waves in a domain which is as small as possible.

In an attempt to answer both the problem of speed and the problem of wave generation, a coupling methodology as illustrated in Fig. 2 is developed. As a first proof-of-concept, a 2D wave flume is created where waves are propagated within a 2-way coupled model. In the large computational domain, fully non-linear waves are generated by a wave propagation software package called OceanWave3D. This tool supplies the model with both the surface elevation and horizontal and vertical wave kinematics over a varying bathymetry. The fully non-linear potential flow equations are solved over a rectangular grid, which is split up in vertical layers. Within the center of the OceanWave3D model, a small SPH model is nested with moving boundaries at both sides of its domain. The moving

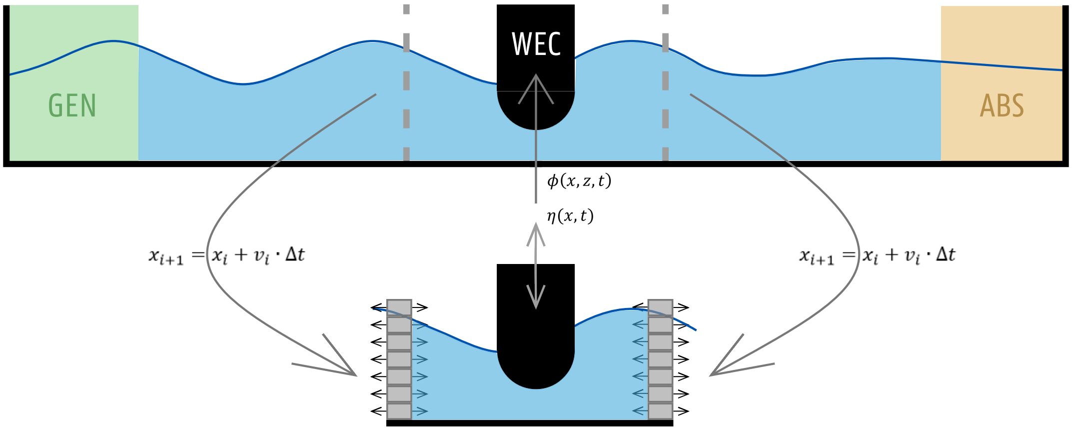

Fig. 2. Principle of 2D coupling between Oceanwave3D and DualSPHysics.

boundaries are a stack of rectangular blocks, with a height equal to the SPH particle size dp. By integrating the velocity profile, supplied by OceanWave3D, the horizontal position of the boundary blocks can easily be found by applying Equation 3:

$$x_{i+1} = x_i + v_i \cdot \Delta t \tag{3}$$

For every time step in OceanWave3D Δt_{OW3D}, the SPH model is run with a significantly smaller (variable) time step Δt_{SPH}. When the SPH simulation time equals Δt_{OW3D}, information is transferred back to OceanWave3D. Specifically, the surface elevation is returned together with an estimate of the complete wave potential. This is done by integrating the horizontal water velocities over the OceanWave3D grid locations. This estimate is obtained by assuming irrotational flow and expressing Equation 4:

$$u = \frac{\partial \phi}{\partial x} \tag{4}$$

To ensure a smooth transition between the DualSPHysics free surface and the OceanWave3D free surface, relaxation zones are applied (see Fig. 3). The applied relaxation function f_{rel} is given in Equation 5, with L the length of the relaxation zone.

$$f_{rel} = \left(\frac{x - x_0}{L} \right)^{3.5} \tag{5}$$

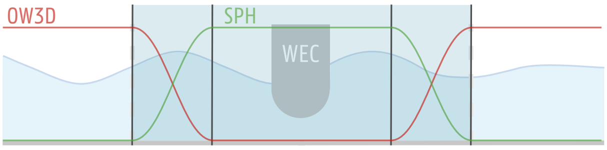

Fig. 3. Sketch of relaxation zones providing a smooth transition between the OceanWave3D domain and the DualSPHysics domain (SPH zone is indicated with light blue background).

2) Coupling Implementation: The coupled model is run over 3 processes within an openMPI implementation (see Fig. 4). The first process is a python code which is used for the set-up of the model and processing of data transferred from one model to another. The second process is the OceanWave3D numerical wave flume in which openMPI is used to send the velocity profile at the the OW3D-SPH interface to the python process. After the SPH simulation, this process receives surface elevation and velocity information from the python process. The third process is the DualSPHysics process. Here, the position of the moving boundary blocks is received by the python process. After the simulation, the velocity is interpolated on the OceanWave3D grid and sent to the python process.

3) Coupling algorithm: In practice, the coupling algorithm is coded as follows. First, a main python script is run to set up the computation. The following variables are initialized:

- Flume parameters: length, depth, coupling zone, mesh size, particle size
- Wave parameters: height, period, wavetype
- WEC paramaters: shape, size

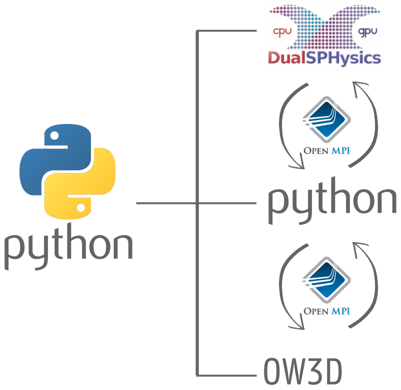

Fig. 4. Program structure of 2D coupling between Oceanwave3D and DualSPHysics.

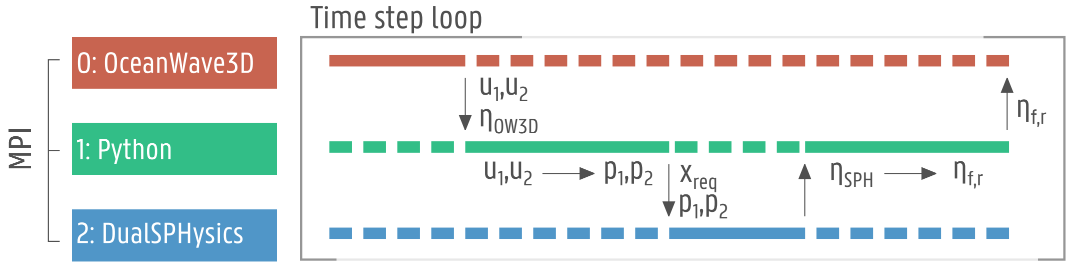

Fig. 5. Coding algorithm of coupling between Oceanwave3D and DualSPHysics.

- Simulation parameters: duration, timestep, coupling type

Next, both the input files for OceanWave3D and DualSPHysics are dynamically created based on the selected input parameters. A dedicated simulation folder is created and the SPH run is initialized by calling GenCase. Next, the main coupled program is started by issuing an MPI run where three tasks are divided over three processes, as illustrated in Fig. 5:

- process 0: OceanWave3D simulation
- process 1: python (data processing, communication hub)
- process 2: DualSPHysics simulation

First, the OceanWave3D propagation model is run for one time step. At the end of the timestep, the horizontal water velocities at the coupling zone boundaries u_1, u_2, and the surface elevation η_{OW3D} are sent to the python process with the MPI Send command. Since the SPH simulation has much more particles in the vertical direction than OceanWave3d has vertical layers, the velocities need to be integrated and interpolated to boundary block positions p_1, p_2 for the SPH simulation. These block positions are sent to the DualSPHysics process, together with the x-coordinates x_{req} of the OceanWave3D grid points which lay within the coupling zone. Next, DualSPHysics is run with a duration, equal to the OceanWave3D time step. Within the SPH simulation, the surface elevation at the x_{req} locations is calculated with the built-in interpolation routine. When the SPH model has run for a duration equal to the OceanWave3D timestep, the free surface elevation η_{SPH} is sent back to the python process. Here, the relaxation function given in equation 5 is applied to the head and tail of the free surface array, to ensure a smooth transition between the OceanWave3D and DualSPHysics solution (see Fig. 3). Additionally, a Savitzky-Golay filter is applied to η_{SPH} to mitigate the irregularities with sizes smaller than the smoothing length $h = 1.2 \cdot dp \cdot \sqrt{2}$, caused by the interpolation routine. This filtered signal $\eta_{f,r}$ is finally sent back to OceanWave3D, where its original solution η_{OW3D} is overwritten with the new SPH free surface.

III. PROOF-OF-CONCEPT

A. Test description

In this section, the described 2-way coupled model is applied to model a series of propagating waves. The waves are selected based on their linear or non-linear characteristics, as described by the diagram of Le Méhauté [15]. A selection of 4 waves is made, which encompasses all possible non-linear and linear theories. The specific characteristics such as wave height, wave period, water depth and SPH particle size are listed in Table II). A minimum particle size of at least $1/8^{th}$ of the wave height is selected. The validity of each wave theory is illustrated in Fig. 6. In a second simulation, the 2-way coupled model is applied to compare the response of a heaving WEC device to experimental data, obtained in the wave flume at the department's facility.

TABLE II
WAVE CONDITIONS

	Theory	**H [m]**	**T [s]**	**d [m]**	**dp [m]**
I	Linear	0.02	1.5	1.0	0.002
II	Stokes 2^{nd}	0.08	2.0	1.0	0.01
III	Stokes 3^{rd}	0.15	2.0	0.7	0.01
IV	Stream function	0.06	2.0	0.3	0.005

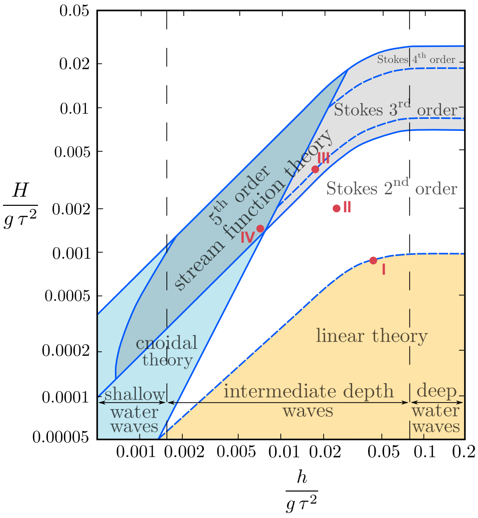

Fig. 6. Selection of waves tested with the 2-way coupled model

The numerical domain is illustrated in Fig. 7. It consists of a $2D$ wave flume with a length of $50.0m$ and a varying water depth according to the wave conditions. The waves are generated within OceanWave3D in a relaxation zone with a length of $20m$. At the end of the OceanWave3D domain, a wave absorption zone is installed with the same length. The SPH zone is located between $x = 20.0m$ and $x = 25.0m$.

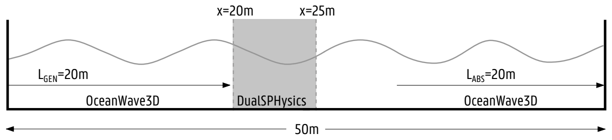

Fig. 7. Numerical domain of the test set-up

B. Results: wave propagation

In this section, the results of the application of the 2-way coupled model to propagate waves over a fixed bathymetry are given. For each selected wave from Table II, a time-series is shown where three wave signals are compared:

- Theoretical wave signal, generated with WaveLab [16]
- Simulated wave signal with OceanWave3D
- Simulated wave signal with 2-way coupled model

By analizing the wave signal in the time-domain, typical non-linear wave characteristics such as wave steepness, asymmetry and higher wave troughs can be checked for accurate reproduction.

1) Wave I - Linear: The first wave is a linear wave with a wave height $H = 0.02m$, wave period $T = 1.5s$ in a water depth of $d = 1.0m$. Since the wave falls within the linear zone, the surface elevation should match the theoretical profile, defined in Equation 6:

$$\eta(t) = \frac{H}{2}\cos\left(\frac{2\pi}{T}\cdot t\right) \tag{6}$$

2) Wave II - Stokes 2^{nd} Order: The second wave is a non-linear wave with a wave height $H = 0.08m$, wave period $T = 2.0s$ in a water depth of $d = 1.0m$. This results in a wave profile described by the Stokes 2^{nd} order wave theory, as defined by Equations 7 and 8.

$$\begin{aligned} \eta(t) &= a\left[\cos\theta + ka\frac{3-\sigma^2}{4\sigma^3}\cos 2\theta\right] + \mathcal{O}\left((ka)^3\right) && (7)\\ H &= 2a\left(1+\frac{3}{8}k^2a^2\right) && (8)\\ \sigma &= \tanh kh && (9)\\ \theta &= \frac{2\pi}{T}\cdot t && (10) \end{aligned}$$

3) Wave III - Stokes 3^{rd} Order: The third wave is a non-linear wave with a wave height $H = 0.15m$, wave period $T = 2.0s$ in a water depth of $d = 0.7m$. This results in a wave profile described by the Stokes 3^{rd} order wave theory, defined in Equation 11.

$$\eta(t) = a\left[\cos\theta + \frac{ka}{2}\cos 2\theta + \frac{3(ka)^2}{8}\cos 3\theta\right] + \mathcal{O}\left((ka)^4\right) \tag{11}$$

4) Wave IV - Stream Function: The last wave is a stream function wave with a wave height $H = 0.06m$, wave period $T = 2.0s$ in a water depth of $d = 0.3m$. The Stream function wave theory was developed by Dean and Dalrymple [8]. The method involves computing a series solution to the fully non-linear water wave problem, involving the Laplace equation with two non-linear free surface boundary conditions (constant pressure and a wave height constraint).

The results of the wave propagation simulations are summarized in Fig. 8 and Table III. In Fig. 8, the surface elevation of all 4 simulated waves is compared to a theoretical solution and a stand-alone OceanWave3D run. In general, the simulations show a very good correspondence. Both the wave crest and wave trough are very close to the theoretical results. The asymmetry of the wave profile in the non-linear wave types is also reproduced. The accuracy of the results is quantified by analysing the surface elevation at the wave crests and the wave troughs. The 2-way coupled SPH model is compared to the theoretical model by calculating the ratio of the wave amplitudes (equation 12) and the relative difference expressed in the smoothing length $h = 1.2 \cdot dp \cdot \sqrt{2}$ (equation 13):

$$\begin{aligned} acc_{\%} &= \frac{a_{SPH}}{a_{Theory}}\cdot 100 && (12)\\ acc_h &= \frac{|a_{SPH} - a_{Theory}|}{h} && (13) \end{aligned}$$

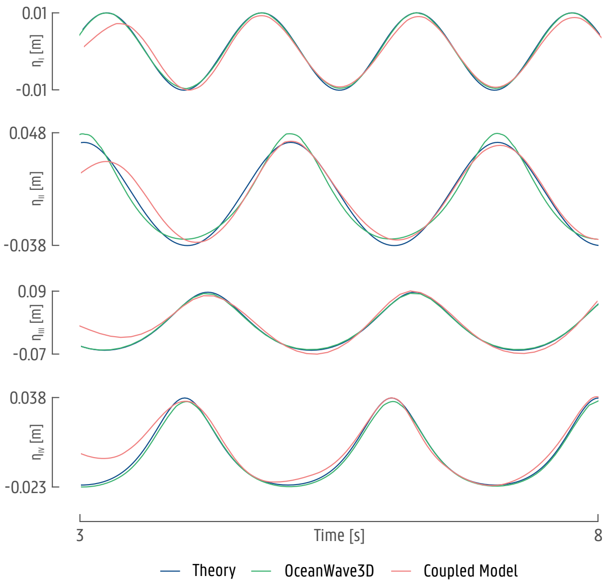

Fig. 8. Comparison of the surface elevation for all 4 simulated waves, between the theoretical profile, a stand-alone OceanWave3D run and a 2-way coupled model run.

The results indicate that the propagated waves are very close to the theoretical values. relative errors range from 1% to 14%, while the errors expressed in smoothing length h range from $0.03h$ to $0.53h$. The maximum error of 14% is registered at the wave trough of the 3^{rd} order Stokes wave. This error is equivalent to $0.53h$, which is still within the acceptable error of $1h$ for an SPH simulation. In general, the wave crest is better reproduced than the wave trough. Additionally, the wave trough is more underestimated than overestimated.

TABLE III
ACCURACY OF COUPLED MODEL RESULTS WITH RESPECT TO THEORETICAL SURFACE ELEVATION

	Crest		**Trough**	
Wave	$acc_{\%}$	acc_h	$acc_{\%}$	acc_h
I	93%	0.21	94%	0.18
II	99%	0.03	89%	0.24
III	103%	0.18	114%	0.53
IV	103%	0.12	91%	0.24

C. Results: WEC response

Up to now, only wave propagation within the 2-way coupled model was tested. The main objective of the model is to use it to simulate floating WECs in non-linear wave conditions. The typical wave-structure interactions such as reflection, radiation and diffraction should be calculated in the SPH model and transferred to the OceanWave3D domain. For this reason, a test is performed where the response of a heaving WEC to a regular wave train is simulated. The test has been carried out in our physical wave flume and was used to validate a numerical OpenFOAM wave flume in [17]. The incident wave has a wave height of $H = 0.04m$, a wave period of $T = 1.6s$ in a water depth of $d = 0.7m$. The wave profile can be described by the Stokes 2^{nd} order wave theory. The WEC is a heaving buoy with a cylindrical top part and a spherical bottom (see Fig. 9), previously used in the WECwakes array experiments [1]. It has a diameter $D = 0.315m$, a draft $T = 0.3232m$ and a mass $m = 21.24kg$. The device is restricted to a heaving motion by sliding over a vertical rod. Friction is minimized by using Teflon bearings.

The WEC response is given in Fig. 10. The heaving motion of the WEC in the SPH simulation corresponds to the experimental data, but does have a 25% higher amplitude. This can be assigned to two reasons:

1) The SPH model does not take into account the vertical shaft over which the WEC is moving. This will induce extra friction in the system and dampen the heaving amplitude.
2) The incident wave at the location of the WEC also has a higher amplitude than the experimental data

The wavefield around the heaving WEC is also compared to the experimental data in Fig. 11. The surface elevation is registered at 4 locations: 3 in front of the WEC, and 1 behind (see Table IV). At the first two locations, the difference between the model and the data remains below $0.44h$. At the 3^{rd} location however, at $0.35m$ in front of the WEC, there is a significant difference of $1.5h$. This error is only visible close to the device, and can possibly due to reflection of the wave against the WEC hull. Additionally, the simulation is 2D, while the experiment has a real 3D character. In reality, the incident wave will be diffracted around the cylindrical hull, while this is not the case in a 2D model.

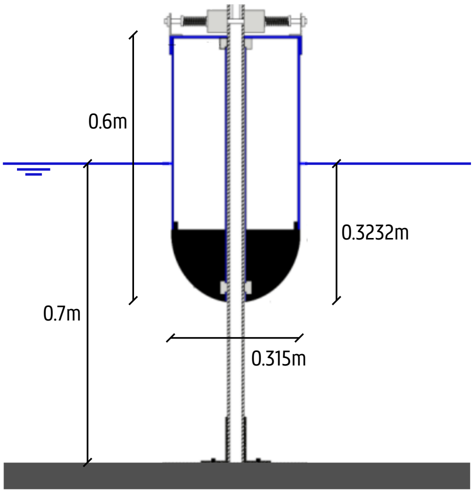

Fig. 9. Experimental WEC device

TABLE IV
LOCATION OF WAVE GAUGES WITH RESPECT TO THE WEC CENTER

Location	Distance from WEC
1	$-1.1m$
2	$-0.7m$
3	$-0.35m$
4	$1.3m$

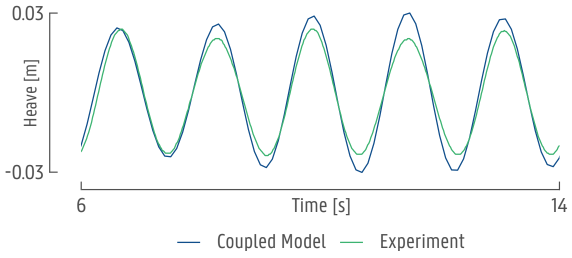

Fig. 10. Comparison of heaving motion of WEC in the coupled SPH model with experimental data

IV. CONCLUSION

In this paper, a novel 2-way coupling methodology between the 3D wave propagation model OceanWave3D and the SPH solver DualSPHysics was introduced. The coupled model consists of a nested SPH zone within a larger wave propagation

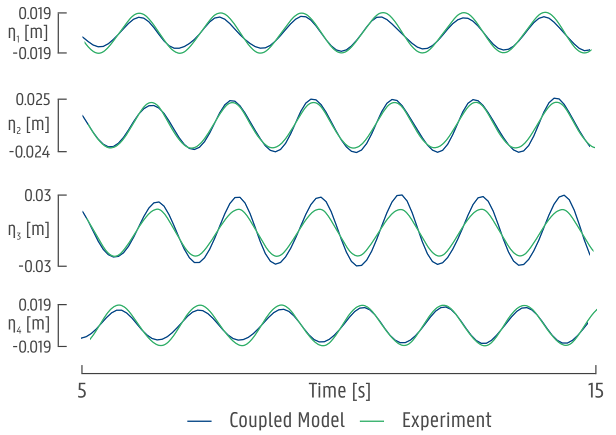

Fig. 11. Comparison of the wavefield at 4 locations near a heaving WEC in the coupled SPH model with experimental data

domain. At both boundaries of the SPH model, there is an interface with OceanWave3D. Here, boundary blocks are moved horizontally, matching the orbital velocities underneath the wavefield calculated with OceanWave3D. This results in wave generation at the left side of the SPH zone and wave absorption on the right side. Within the SPH zone, the surface elevation and wave kinematics are registered and sent back to the wave propagation model.

The 2-way coupled model is programmed within an OpenMPI environment, where 3 subprocesses are transferring and processing data. Next to an OceanWave3D and DualSPHysics process, there is a python process, directing the information transfer and processing the data before sending it to the dedicated subprocess. The code is compiled to run on both CPUs and GPUs.

A proof-of-concept 2D coupled model is introduced to demonstrate the capabilities of the 2-way coupling. First, wave propagation of 4 different linear and non-linear wave types is performed, resulting in a very high accuracy with errors always lower than 1 smoothing length h. Secondly, a heaving WEC body is positioned in the centre of the coupling zone. The results are compared to experimental data obtained in a wave flume. The results show a good agreement with errors...

In future work, the 2-way coupled model will be adapted and expanded with the following features:

- The moving boundary blocks will receive velocity information from OceanWave3D based on their updated position troughout the simulation, instead of from the fixed interface position. This will lead to more accurate results, specifically for strongly non-linear waves;
- Quantifying the reflection in the coupling zone for long simulations;
- The model will be extended to a 3D domain. First, focus will be put on long-crested waves within square coupling zones. On a long term, coupling of short-crested irregular waves within circular coupling zones should be implemented;
- More thorough validation of the 2-way coupled model with experimental data, e.g. from the WECwakes project [1];

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