A Search Group Algorithm for Wind and Wave Farm Layout Optimization

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***Abstract*— A Search Group Algorithm (SGA) is presented and applied on both Wind and Wave Farm Layout Optimization. SGA allows calculating the optimal geometric layout of the devices within farms, in order to achieve an optimal power output. At the same time, device interactions are taken into account and the minimal distances between the devices are respected (e.g. necessary for maintenance).**

The SGA performance is compared to that of other algorithms found in the literature for both wind and wave farms, providing improved solutions for all designs used here as benchmarking cases. However, for complex wind farms with a large number of turbines in a restricted area, the efficiency rates of the optimal farm layouts decrease strongly.

Regarding wave farms, we propose the combination of a novel WEC interaction method for deriving the diffraction transfer matrix applied to multi-body interactions in water waves, with the SGA. This combination allows to determine the optimal WEC positions in large farms at a reasonable computational cost. We aim at a further implementation of a cost function to investigate the influence of the farm layout on capital and maintenance costs. With these insights, the optimal power-cost layout can be determined and a comparison can be made between a wind and a wave farm. Other applications of the SGA can focus on farms of floating wind turbines, or co-located wind-wave farms.

***Keywords*— Renewable energy, wind farm/park, wave farm/park/array, layout optimisation, metaheuristic algorithm, floating device interactions within farms**

I. INTRODUCTION

Renewable energy is gaining more and more importance worldwide. In contrast to the traditional energy resources, renewables such as wind, solar and wave energy will never run out. Apart from their well-known environmental benefits, the decrease in the production cost of several of them during the last years has stimulated growth in the renewable energy sector.

Amongst the alternative energy sources, wind is one of the most promising ones. It's a reliable and affordable energy source and has become a pillar of the electricity production systems based on renewables, in many countries. Next to wind, various other renewable energy sources are advancing as well. Wave energy is one of these resources with a huge potential, which can play a crucial role in the diversification of the energy supply worldwide.

To exploit energy from wind or ocean waves, large numbers of wind turbines (abbreviated as WTs) or wave energy converters (abbreviated as WECs) are placed in the same area, often called a "*wind farm*" or a "*wave farm*" (these are called "*wind or wave parks*" as well, while the latter can be composed by "*WEC arrays*"). This also allows reducing costs regarding practical issues such as grid connection and maintenance. The capacity of these farms depends on many factors, for example the specific geometrical, geographical and wave/wind loading characteristics of the installation site and the type and number of devices employed.

However, the performance of such wind or wave farms can be significantly influenced by the geometrical positioning of the devices, that is, the geometrical layout of the farm. An inadequate farm layout design can lead to a smaller efficiency of the entire farm in harvesting energy, and to higher costs (e.g. maintenance costs).

In a wind farm a single WT influences other turbines located downstream, through the so-called "*wake effects*" in the lee of each turbine [1]. It is important to note that due to the presence of wake effects, the generated power of a wind farm is generally lower than the sum of power produced by the individual turbines as if they would operate at the site in isolation (negative effect of the turbine interactions within a wind farm).

However, in the case of a WEC farm, the absorbed power from the incoming waves (and thus the generated electricity) can be affected positively. Both numerical (e.g. [2]-[5]) and experimental studies ([6]-[7]) have shown that the particular geometrical farm layout can lead to destructive, but also to constructive interactions between the WECs. The latter results in a wave farm total power output which exceeds the sum of the power absorbed by the individual WECs in isolation. Therefore, the total power output of a wave farm is affected by the interactions between the WECs, which comprise in general, the waves reflected or radiated by other WECs. As mentioned above, these WEC interactions within a farm may, depending on the geometrical layout, result in a significant decrease or increase of the total power production. Hence it is

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of great importance to choose the layout with care in order to minimize destructive and maximize constructive effects [8].

In this paper, the above described interactions between the (wind or wave) farm devices and/or wake effects will be referred to as "*farm effects*".

Wind farm layout optimization (abbreviated as WFLO) or WEC array layout optimization (abbreviated as WALO) problems intend to find the best position for each device in the farm. This layout optimization is performed in order to reduce the destructive farm effects or even, in the case of WEC farms, to increase the constructive farm effects. Such layout optimization problems are generally nonlinear and non-convex, and therefore, it is important to apply the right kind of optimization algorithms, such as metaheuristic ones.

In the available literature, several algorithms applied on WFLO can be found. A common solution consists of applying the genetic algorithm like [9] and [10]. More recently, other algorithms have been implemented as well. For example, an evolutionary strategy algorithm to maximize the generated power has been developed by [11], which has been applied on several circular wind farms. The study presented in [12] however, focusses on the development of the imperialist competitive algorithm and its application on the same wind farms.

The examples in [11] and [12] both have the potential to become benchmark cases for WFLO problems, since they can be easily reproduced based on the presented data and they include the basic principles of WFLO problems.

The available literature on application of optimization algorithms on WALO problems is less extensive. Most of the studies assume a pre-determined geometrical layout, however [13] applied two methods to determine optimal WEC farm layout configurations. Specifically, a Parabolic Intersection and the Matlab Genetic Algorithm toolbox are applied, where the latter, using reactively tuned devices, results in the highest interaction factors. The WEC array interaction factor —$\bar{q}$-factor— as described in literature ([6], [8], [14]-[18]) is a measure that quantifies the effect of intra-array interactions on the power absorption of a WEC array. The interaction factor is the ratio of the total power from the entire WEC array to that of the same number of WECs in isolation. In [19] a Genetic Algorithm is applied as well, and the results of a 5-WEC array are compared to with those presented by [13].

In the present study, the Search Group Algorithm is applied on the wind farm examples presented by [11] and [12], and the 5-WEC array presented by [13]. The objective is to compare the obtained WT and WEC farm performance results to the studies reported in the above mentioned literature.

In Section I of the present manuscript, an introduction is given on wind and wave farm effects important for farm design, as well as a very short presentation of the current state-of-the-art. In Section II the principles of the Search Group Algorithm (SGA) are presented. SGA is here used to optimise either an offshore wind farm (Sections III and IV) or a wave farm (Sections V and VI). Finally conclusions are presented in Section VII, as well as future work.

II. The Search Group Algorithm

The Search Group Algorithm (SGA) was originally used for the optimization of truss structures. This section briefly explains how the SGA works. For a more detailed explanation of the SGA, reference is made to [20]. It is a metaheuristic algorithm and hence must have two capabilities, exploration and exploitation, in order to be able to find reasonable solutions. Exploration may be described as the ability of the algorithm to find promising regions on the design domain, i.e. regions in which the optimal solution may be located. Exploitation is the ability of the algorithm to refine the solution on these promising regions, i.e. to pursue a local search on them. An adequate balance between the exploration and exploitation tendencies is important in order to be competitive in terms of robustness and performance [21]. In order to find designs which are closer to the optimal one, the proposed algorithm aims at having a good balance between the exploration and exploitation of the design domain. In fact, the manner in which a new individual is generated, is what makes it possible for the SGA to achieve this goal. The basic idea is that in the first iterations of the optimization process the SGA tries to find promising regions on the domain (exploration), and as the iterations pass by, the SGA refines the best design in each of these promising regions (exploitation).

Also, a mutation operator is employed to generate new designs away from the ones of the current search group. Moreover, the generation of new individuals is pursued only by a few members of the population, which are named here the search group. Thus, the SGA is comprised by five steps: 1) the initial population, 2) the initial search group selection, 3) the mutation of the search group, 4) the generation of the families, and 5) the selection of the new search group.

The initial population, P, is generated randomly on the search domain depending on the number of design variables, their lower and upper bounds etc.. Each row of P represents an individual of the population and each column represents a design variable. After the initial population, P, is generated, the objective function of each individual is evaluated. After that, the initial search group, R, is constructed by selecting n_g individuals from P. A standard tournament selection is applied to pursue this step of the algorithm. Each row of R represents an individual, i.e. R_i represents the i^{th} row of R and consequently the i^{th} member of the search group. The members of the search group are ranked after each iteration of the algorithm, i.e. R1 is always the best design and Rn_g is always the worst design amongst the search group members.

In order to increase the global search ability of the proposed algorithm, the search group, R, is mutated at each iteration. This mutation strategy consists in replacing n_{mut} individuals from R by new individuals, generated based on the statistics of the current search group. The idea here is to include in the search group individuals away from the current position of the current members, exploring new regions of the search domain. The probability of a member to be replaced, depends on its rank in the current search group, i.e. the worse the design is the more likely it is to be replaced.

A family is the set comprised by each member of the search group and the individuals that it has generated. Thus once the search group is determined, each one of its members generates a family by the perturbation depending on a perturbation parameter. In the first iterations of the algorithm any individual generated by a given search group member is allowed to visit any point in the design domain, at least in a probabilistic sense. That is, the individuals generated by a given search group member are not necessarily in its neighbourhood. The better a member of the search group is ranked, the more individuals it generates. That is, the number of individuals that each member of the search group generates, depends on the quality of its objective function. After this, a new search group can be selected. The new search group is formed by selecting the best member of each family. When the iteration number is higher than the global maximal iterations, the selection scheme is modified: the new search group is formed by the best n_g individuals amongst all the families.

The parameters of the algorithm may vary according to the characteristics of the problem to be solved. For example, for more difficult problems, usually the algorithm needs to increase its exploration capability in order to avoid local minima. The parameters set the ratio between the exploration and exploitation of the algorithm. In Fig. 1, a flow chart of the proposed Search Group Algorithm for Wind and/or Wave Farm Layout Optimization is presented. Here the SGA parameters and their purposes in the optimization process, are listed:

- $Alpha_{Min}$ = 0.01: Minimum value which perturbation constant Alpha, that controls the exploration and exploitation procedure, may assume for the generation of families;
- $Alpha_{Initial}$ = 2.00: Initial value of Alpha for the generation of families;
- it_{max}=300: Maximum number of iterations within the algorithm;
- GlobalIterationsRatio = 0.30: Percentage of it_{max} dedicated to global phase selection scheme;
- PopulationSize = 40; Number of individuals in the population, n_{pop};
- SearchGroupRatio=0.10; Percentage of n_g that forms the search group (0.10 = 10% of 40);
- n_{mut} = 1; Number of mutated individuals of the search group;
- PlotFamily = false; Defines if the value of families will be plotted or not.

III. Wind Farm Layout Optimization

In the previously mentioned benchmark problem [11], a number of basic assumptions and simplifications have been made for their WFLO problem, as listed below:

- The number of wind turbines, N, is predefined. Hence the capacity of the power plant is already determined before its construction;

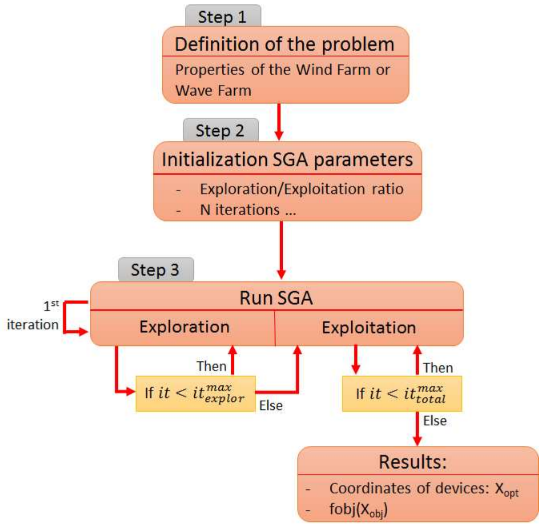

Fig. 1 Flow chart of the proposed Search Group Algorithm for Wind and/or Wave Farm Layout Optimization.

- The wind farm is installed at a flat terrain, hence the farm layout can be described with a 2D Cartesian coordinate system with x_i and y_i the coordinates of WTi;
- Only one WT type is used, and therefore all turbines have the same diameter, D;
- For a specific location, height, and direction wind speed follows a two-parameter Weibull distribution described by:

$$p_v(v,k,c) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{(-\frac{v}{c})^k}$$

  where c and k are the scale and shape parameter of this distribution, respectively, p_v is its probability density function, and v is the wind velocity;
- Wind velocity v is a continuous function of the wind direction, θ, i.e. $k = k(\theta), c = c(\theta), 0° \leq \theta < 360°$ and consequently, the wind velocity at a given direction θ follows the Weibull distribution with parameters $c(\theta)$ and $k(\theta)$ at any location of the wind farm. Finally, θ follows a known probability distribution, $p(\theta)$;
- The minimum distance between two WTs must be 4 times its diameter D in order to avoid hazardous loads due to the turbulent flow downstream, in the wake of the WTs;
- The shape of the wind farms is assumed to be circular with a 500 m radius and any WT may be installed at any position within this domain. This radius has been selected based on that used in [11] and [12] which is here used for comparison reasons;

- The objective function to be maximized is the total output power (W) of the WT considering the wake losses.

Based on the assumptions above, the resulting WFLO problem can be described as:

Maximize:

$$W(d) = \sum_{i=1}^{N_t} E_i(d)$$

$$G1 = {x_i}^2 + {y_i}^2 \leq 500^2, i = 1, \dots, N$$
$$G2 = (x_i + x_j)^2 + (y_i + y_j)^2 \geq 16D^2$$
$$i, j = 1, \dots, N, i \neq j$$

With:

- d , the design vector consisting of the coordinates of the N wind turbines;
- E_i , the power generated by WTi;
- G1 , the constraint regarding that all WTs must be positioned within a 500 m radius circular area;
- G2 , the constraint defining the minimum distance between the WTs.

IV. Numerical Analysis WFLO

The specific optimization problem explained in Section III consists of a circular wind farm with a radius of 500 m which accommodates a pre-defined number of WTs. The power output of the entire farms calculated using the SGA are compared to results of the Evolutionary Algorithm (EA) by [11] and the Imperialist Competitive Algorithm (ICA) by [12].

Three different wind scenarios are solved for farms composed of 2 up to 8 WTs with their specifications shown in Table 1. For each wind scenario, knowing the cut-in wind speed and the rated wind speed, wind speed is divided at Nv = 20 intervals of 0.5 (m) each. Similarly, the wind direction is divided at N_θ = 23 intervals of 15° each.

Wind scenario 1

The data of the first wind scenario ("*Wind scenario 1*") are presented in Table 2. The wind direction is divided in 23 intervals (*l-1*) of 15° each, from angle θ_{l-1} to angle θ_l. Table 2 can be read as follows: when wind direction is between 0° and 15°, the wind speed follows a Weibull distribution with shape parameter k =2, scale parameter c = 13; the probability for wind blowing between this interval from 0° to 15° (ω_{l-1}) is zero. Similarly, when wind direction is between 90° and 105°, the wind speed follows a Weibull distribution with k = 2, c = 13; the probability for wind blowing between 90° and 105° is 0.6. Table 2 shows that wind blows predominantly from 75° to 105° with a probability of 0.8.

The stopping criterion is 12,000 objective function evaluations (NIterations x PopulationSize= 300 x 40) This is the same number of evaluations as applied in [11] and [12].

The results of the SGA vary depending on the randomly generated initial population. Hence not only the best design found through applying these different methods is compared, but also their statistics over several runs. As there is no standard procedure in literature to compare those algorithms, the results of 100 runs of the SGA for each wind scenario are presented. The optimal results of the energy production are presented together with the average values and *Coefficients of Variation* (CoV).

TABLE 1
Wind Turbine Specifications

WT parameter name and symbol	Units
Rotor radius, R	38.5 (m)
Cut-in speed, $v_{cut\text{-}in}$	3.5 (m / s)
Rated speed, v_{Rated}	14 (m / s)
Rated power, P_{Rated}	1500 (kW)
Slope parameter, λ	140.86 (-)
Intercept parameter, η	500 (-)
Thrust coefficient, C_T	0.8 (-)
Spreading constant, κ	0.075 (-)

Table 3 shows that the SGA is able to improve the designs of the EA and ICA for all cases. The best found scenario of a 100 runs of the algorithms, i.e. the design with the highest energy production, is compared to the ideal power scenario of standalone turbines without any wake effects. The ratio of these gives the efficiency of a particular farm layout. In case of 4 WTs the SGA was able to find a 100% efficient design where the most efficient design so far was only 99.85%. The SGA is also able to design wind farms up to 8 wind turbines, respecting constraints on the minimum distance between the WTs, were the EA and ICA failed to do so.

Table 4 shows the statistics of 100 independent runs for "Wind scenario 1". As the number of WTs increases, the CoV slightly increases as well. This is due to the increasing complexity of the problem as there are more design variables and constraints if the number of WTs increases. However, Table 4 shows that the SGA is a very robust method as even for the most complex case with 8 turbines, the CoV is still very low (0.006%) which means that the dispersion of the results is very low as well. The geometrical layouts for the optimal designs of wind farms composed of 2 to 8 WTs for "Wind scenario 1" (Table 2) and after applying SGA, are illustrated in Fig. 2.

TABLE 2
Main parameters for "Wind Scenario 1"

l-1	θ_{l-1} (degrees, °)	θ_l (degrees, °)	k	c	ω_{l-1}
0	0	15	2	13	0.00
1	15	30	2	13	0.01
2	30	45	2	13	0.01
3	45	60	2	13	0.01
4	60	75	2	13	0.01
5	75	90	2	13	0.20
6	90	105	2	13	0.60
7	105	120	2	13	0.01
8	120	135	2	13	0.01

TABLE 2 (Continues)

l-1	θ_{l-1} (degrees, °)	θ_l (degrees, °)	k	c	ω_{l-1}
9	135	150	2	13	0.01
10	150	165	2	13	0.01
11	165	180	2	13	0.01
12	180	195	2	13	0.01
13	195	210	2	13	0.01
14	210	225	2	13	0.01
15	225	240	2	13	0.01
16	240	255	2	13	0.01
17	255	270	2	13	0.01
18	270	285	2	13	0.01
19	285	300	2	13	0.01
20	300	315	2	13	0.01
21	315	330	2	13	0.01
22	330	345	2	13	0.01
23	345	360	2	13	0.00

TABLE 4
RESULTS (STATISTICS) OF THE OPTIMAL DESIGNS FOR "WIND SCENARIO 1"

Number of wind turbines, N	Mean design results in terms of total output power, P (kW)	Worst design in terms of total output power, P (kW)	Coefficient of Variation, CoV (-)
2	28091.47	28091.47	0.000
3	42132.72	42100.00	0.000
4	56114.22	56071.93	0.000
5	70037.85	69981.65	0.000
6	83902.09	83687.74	0.001
7	97543.42	96968.55	0.002
8	110718.89	108865.19	0.005

TABLE 3
RESULTS OF THE OPTIMAL DESIGNS FOR "WIND SCENARIO 1"

Number of wind turbines, N	Ideal Power, W_{ideal}	Evolutionary Algorithm (EA) [11]		Imperialist Competitive Algorithm, (ICA) [12]		Search Group Algorithm, (SGA)	
	(kW)	Best design in terms of total output power, P (kW)	Efficiency (%)	Best design in terms of total output power, P (kW)	Efficiency (%)	Best design in terms of total output power, P (kW)	Efficiency (%)
2	28091.47	28083.42	99.97	28091.47	100.00	28091.47	100.00
3	42137.21	42101.06	99.91	42137.21	100.00	42137.21	100.00
4	56182.95	56057.77	99.78	56097.37	99.85	56182.95	100.00
5	70228.69	69922.97	99.56	69954.02	99.61	70084.88	99.80
6	84274.42	83758.79	99.39	83647.75	99.26	83989.20	99.66
7	98320.16	-	-	-	-	97854.98	99.53
8	112365.90	-	-	-	-	111670.24	99.36

TABLE 6 . RESULTS OF THE OPTIMAL DESIGNS FOR "WIND SCENARIO 2"

Number of wind turbines, N	Ideal Power, W_{ideal}	Evolutionary Algorithm (EA) [11]		Imperialist Competitive Algorithm, (ICA) [12]		Search Group Algorithm, (SGA)	
	(kW)	Best design in terms of total output power, P (kW)	Efficiency (%)	Best design in terms of total output power, P (kW)	Efficiency (%)	Best design in terms of total output power, P (kW)	Efficiency (%)
2	28091.47	14631.21	100.00	14631.37	100.00	14630.76	100.00
3	42137.21	21925.16	99.90	21947.07	100.00	21946.14	100.00
4	56182.95	29113.71	99.49	29211.87	99.83	29225.41	99.87
5	70228.69	36316.23	99.28	36320.66	99.30	36460.39	99.68
6	84274.42	43195.84	98.41	42594.56	97.04	43149.24	98.30
7	98320.16	-	-	-	-	49871.84	85.21
8	112365.90	-	-	-	-	56291.07	76.95

Wind scenario 2

In the second wind scenario ("Wind scenario 2") the wind blows mainly from the direction section 120° to 225°. However, the parameters of the Weibull distribution are not constant for every wind direction this time, as can be seen in Table 5. The obtained solutions are again compared in Table 6 with results obtained by [11] and [12].

The geometrical layouts for the optimal designs of wind farms composed of 2 to 8 WTs for "Wind scenario 2" (Table 6) found by the SGA, are illustrated in Fig. 3.

In most of the cases for "Wind scenario 2" the SGA finds the best farm layouts, however for a 4-WT farm the EA was able to calculate a slightly better layout.

The statistics over 100 runs for "Wind scenario 2" are displayed in Table 7. Just as in the previous scenario, the CoV increases with the increasing complexity of the problem, but is still considered low for the most complex cases. Also for "Wind scenario 2" the SGA proves to be a robust method.

TABLE 5
MAIN PARAMETERS FOR "WIND SCENARIO 2"

l-1	θ_{l-1} (degrees, °)	θ_l (degrees, °)	k	c	ω_{l-1}
0	0	15	2	7.0	0.0002
1	15	30	2	5.0	0.0227
2	30	45	2	5.0	0.0242
3	45	60	2	5.0	0.0225
4	60	75	2	5.0	0.0339
5	75	90	2	4.0	0.0423
6	90	105	2	5.0	0.0290
7	105	120	2	6.0	0.0617
8	120	135	2	7.0	0.0813
9	135	150	2	7.0	0.0994
10	150	165	2	8.0	0.1394
11	165	180	2	9.5	0.1839
12	180	195	2	10	0.1115
13	195	210	2	8.5	0.0765
14	210	225	2	8.5	0.0080
15	225	240	2	6.5	0.0510
16	240	255	2	4.6	0.0019
17	255	270	2	2.6	0.0012
18	270	285	2	8.0	0.0010
19	285	300	2	5.0	0.0017
20	300	315	2	6.4	0.0031
21	315	330	2	5.2	0.0097
22	330	345	2	4.5	0.0100
23	345	360	2	3.9	0.0317

Wind scenario 3

The data of "Wind scenario 3" are presented in Table 8. The Weibull parameters are equal to those of "Wind scenario 1", but in the case of "Wind scenario 3" each wind direction has the same probability. The results of "Wind scenario 3" are presented in Table 9 along with the "Ideal power" i.e. the sum of the power production of the separate WTs without any wake losses. They are only compared to those of [12] since [11] did not address this wind scenario.

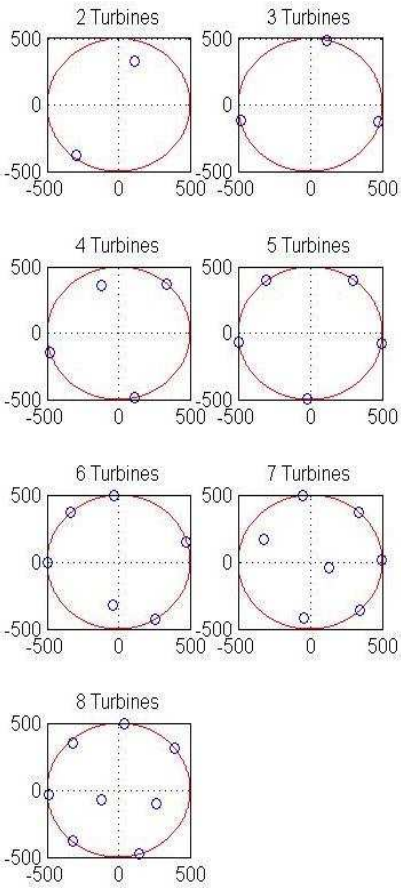

Fig. 2 Geometrical layouts for optimal designs of wind farms composed of 2 to 8 WTs for "Wind scenario 1" (Table 2), found by the SGA. The red circle of 500 m radius denotes the available area for installing the WTs of the wind farm. The smaller blue circles indicate the WT locations.

TABLE 7
RESULTS (STATISTICS) OF THE OPTIMAL DESIGNS FOR WIND SCENARIO 2

Number of wind turbines, N	Mean design results in terms of total output power, P (kW)	Worst design in terms of total output power, P (kW)	Coefficient of Variation, CoV (-)
2	14630.76	14630.76	0.000
3	21938.01	21899.91	0.001
4	29135.82	29058.87	0.001
5	36215.18	36047.97	0.002

TABLE 7 (Continues)

Number of wind turbines, N	Mean design results in terms of total output power, P (kW)	Worst design in terms of total output power, P (kW)	Coefficient of Variation, CoV (-)
6	42867.27	42582.53	0.003
7	49366.27	48830.31	0.004
8	55765.05	55176.05	0.004

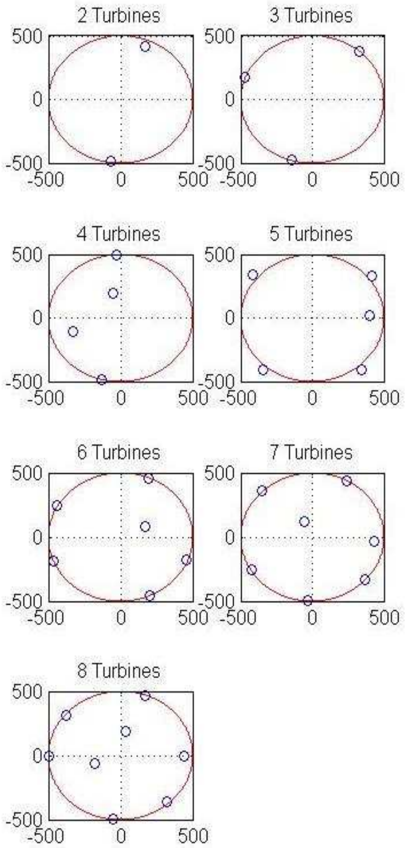

Fig. 3 Geometrical layouts for optimal designs of wind farms composed of 2 to 8 turbines WTs for "Wind scenario 2" (Table 5), found by the SGA. The red circle of 500 m radius denotes the available area for installing the WTs of the wind farm. The smaller blue circles indicate the WT locations.

TABLE 8
MAIN PARAMETERS FOR "WIND SCENARIO 3"

l-1	θ_{l-1} (degrees, °)	θ_l (degrees, °)	k	c	ω_{l-1}
0	0	15	2	13	0.041667
1	15	30	2	13	0.041667
2	30	45	2	13	0.041667
3	45	60	2	13	0.041667
4	60	75	2	13	0.041667
5	75	90	2	13	0.041667
6	90	105	2	13	0.041667

TABLE 8 (Continues)

l-1	θ_{l-1} (degrees, °)	θ_l (degrees, °)	k	c	ω_{l-1}
9	135	150	2	13	0.01
7	105	120	2	13	0.041667
8	120	135	2	13	0.041667
9	135	150	2	13	0.041667
10	150	165	2	13	0.041667
11	165	180	2	13	0.041667
12	180	195	2	13	0.041667
13	195	210	2	13	0.041667
14	210	225	2	13	0.041667
15	225	240	2	13	0.041667
16	240	255	2	13	0.041667
17	255	270	2	13	0.041667
18	270	285	2	13	0.041667
19	285	300	2	13	0.041667
20	300	315	2	13	0.041667
21	315	330	2	13	0.041667
22	330	345	2	13	0.041667
23	345	360	2	13	0.041667

TABLE 9
RESULTS OF THE OPTIMAL DESIGNS FOR "WIND SCENARIO 3"

N	Ideal Power, W_{ideal}	Imperialist Competitive Algorithm, (ICA) [12]		Search Group Algorithm, (SGA)	
	(kW)	Best design in terms of total output power, P (kW)	Efficiency (%)	Best design in terms of total output power, P (kW)	Efficiency (%)
2	28091.47	28091.70	100.00	28091.70	100.00
3	42137.21	42137.55	100.00	42137.55	100.00
4	56182.95	56183.40	100.00	56183.40	100.00
5	70228.69	68628.64	97.72	69740.32	99.30
6	84274.42	81611.79	96.84	83146.66	98.66
7	98320.16	-	-	96268.43	97.91
8	112365.90	-	-	109244.6	97.22

For the wind farms composed of up to 4 WTs, both methods find a solution with 100 % efficiency. For the subsequent cases, the SGA provides the best designs. An 8-WT farm found by the SGA would generate 34 % more energy compared to a 6-WT farm.

Generally, over all 3 wind scenarios the SGA accomplishes better designs compared to the EA and the ICA.

Fig. 4 illustrates the geometrical layouts for the optimal designs of wind farms composed of 2 to 8 WTs for "Wind scenario 3" (Table 8), found by the SGA.

The statistics over 100 independent runs for the third wind scenario are displayed in Table 10. Also for this scenario the CoV stays low so confirms that the SGA is adequate for WFLO problems.

TABLE 10
RESULTS (STATISTICS) OF THE OPTIMAL DESIGNS FOR "WIND SCENARIO 3"

Number of wind turbines, N	Mean design results in terms of total output power, P (kW)	Worst design in terms of total output power, P (kW)	Coefficient of Variation, CoV (-)
2	28091.70	28091.70	0.000
3	42137.26	42125.13	0.000
4	55982.77	55795.06	0.002
5	69575.02	69341.17	0.001
6	82741.21	82438.94	0.002
7	95920.50	95582.99	0.002
8	108910.39	108404.32	0.002

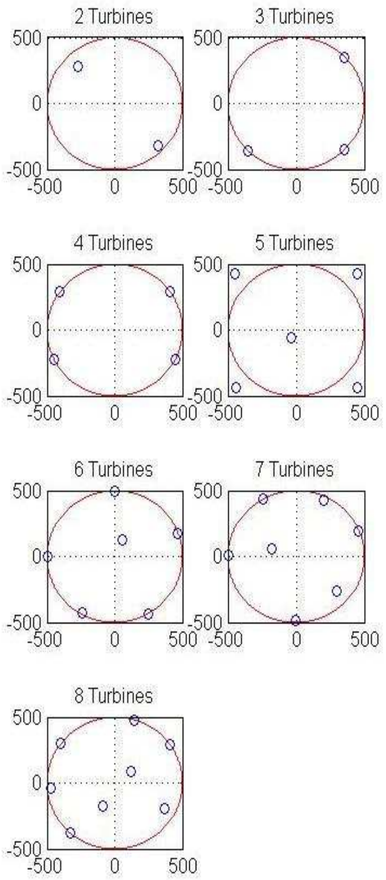

Fig. 4 Geometrical layouts for optimal designs of wind farms composed of 2 to 8 turbines WTs for "Wind scenario 3" (Table 8), found by the SGA. The red circle of 500 m radius denotes the available area for installing the WTs of the wind farm. The smaller blue circles indicate the WT locations.

V. WEC ARRAY LAYOUT OPTIMIZATION (WALO)

To determine the optimal positions for each WEC in a wave farm, the total power of the farm is again used as the objective function. This power output is calculated using the method based on the analytical interaction theory described by [22], taking into account the interactions between the WECs. Specifically, the recent study performed by [22] focuses on a novel method for deriving the diffraction transfer matrix and its application to multi-body interactions in water waves. The method consists of computing the diffraction transfer matrix by probing a body with plane incident waves. The method is straight-forward and can be performed with results from most standard software or experiments as long as linearity is assumed or given. A novel operator called the force transfer matrix is introduced. The force transfer matrix transforms a vector of incident partial cylindrical wave coefficients into forces on the body. It is used in both the diffraction and radiation problems, and is computed in a manner similar to that of the diffraction transfer matrix. With the inclusion of the force transfer matrix, the interaction problem becomes purely algebraic and programming it is relatively uncomplicated.

A metric value used to quantify the effect of these interactions on the power absorption of the WEC farm, and therefore a measure to describe the efficiency of the geometrical configuration of the WEC farm, is the interaction factor, $\bar{q}$-factor. As mentioned in the introductory part, the interaction factor is the ratio of the total power from the entire WEC farm, P_{farm}, to the power sum of the same number of WECs in isolation:

$$\bar{q} = \frac{P_{farm}}{N * P_{isolated}}$$

With:

- N: number of WECs
- P_{farm}: power produced by the WEC farm
- $P_{isolated}$: power produced by one isolated WEC

When WECs are placed close to each other, strong interactions occur between the devices which affect the power output of the entire farm. On the contrary to wind farms, these farm effects do not necessarily lead to an interaction factor lower than one. The radiated and scattered waves caused by WECs can be either constructive or destructive, and thus result into a $\bar{q}$-factor greater or lower than unity, respectively. However, the geometrical layout of the devices within a farm is not the only parameter influencing the interaction factor. Other parameters, such as the number of WECs, the distance between them, the characteristics of the WEC and its Power Take-off (the WEC part through which wave energy is captured and is abbreviated as PTO), the characteristics of the installation site, the wave direction and the wave climate have to be taken into account too.

In previous research found in the literature, pre-determined WEC farm layouts were often used. However, with the approach presented here, the optimal position of each WEC can be determined within a farm, with a varying number of WECs.

VI. Numerical Analysis WALO

For the numerical analysis, a WEC characterized by a truncated floating cylinder with a diameter of 2.0 m and a draft of 1.0 m is used. The same numerical modelling approach is followed as in [13] and [19] in order to allow a comparison of our results with these previous studies. The WEC cylinder is constrained in heave (vertical motion) and located in constant water depth of 8 m. These parameters represent a scaled WEC farm of devices with a diameter of 10 m installed at a location where the water depth is 40 m.

A single floating cylinder is hydrodynamically modelled using a Boundary Element Method solver (e.g. WAMIT) to compute its hydrodynamic properties. These results are used to estimate the interactions between the WECs and the power produced by the entire farm. A Bretschneider wave spectrum with a significant wave height of 2 m, a modal frequency of 0.2 Hz, and periods in 0.5 s increments between 4 and 8 s, is used to represent the incoming irregular long-crested waves.

The SGA with the WEC interaction method of [22] implemented, subsequently calculates the optimal WEC positions for a maximal power production when the devices are placed in a farm. These calculations respect a minimum distance between the WECs of the farm of 3 m (or 1.5 times the WEC diameter) e.g. for facilitating maintenance activities and to avoid collisions.

In order to compare the performance of the SGA, an example of a 5-WEC farm is employed. Since determining the power output of a wave farm is much more calculation intensive compared to that of a wind farm, and hence more time consuming, some adjustments have to be made in the SGA to keep the calculation time reasonable.

Since the SGA proved a robust method with very low CoV values, in this case it is ran only once instead of 100 times, but the parameters are altered so the stopping criterion is 30 000 objective function evaluations instead of 12 000.

More specific the SGA parameters consist of:

- PopulationSize = 100: Number of individuals in the population;
- it_{max} =300: Maximum number of iterations;

All other parameters remain the same as in Section II.

The comparison of the values of the interaction $\bar{q}$-factor for a 5-WEC farm of floating cylinders with a 3 m minimum spacing, with the interaction results by [13] and [19], is provided in Table 11. The minimum distance of 3.0 m between the WECs has been selected based on that used in [13] and [19] which are here used for comparison reasons.

Using less iterations, the SGA finds a slightly better WEC farm layout compared to [19], though it is noted that the difference in the values of the interaction $\bar{q}$-factor is small. However, less iterations might indicate that less computational time is needed which becomes a crucial element, once larger WEC farms have to be calculated (e.g. farms composed of hundreds of WECs). Regarding the comparison between the SGA results and those by [13] the SGA calculates again a WEC farm layout which results in better interaction factor. Also in this case, the difference in interaction factors is small.

TABLE 11
INTERACTION FACTORS

Results' Source	Interaction factor, $\bar{q}$ (-)	Number of iterations (-)
[13] Child et al.	1.019	Unknown
[19] Sharp & DuPont	1.0252	37 690
Present study using SGA	1.0277	30 000

Fig. 5 shows the optimal WEC positions for this 5-WEC farm as calculated using the SGA. Also the disturbance coefficient K_d is presented (ratio between the local wave height and the wave height at the wave generation boundary, H_s/H_{s0}) which represents the resulting wave field due to the interaction of the WECs with the incoming waves, but also due to the interactions between the WECs of the farm. The incoming waves propagate from the left side of Figure 4 to the right side. The shadow zones (or "wake effects") in the lee of the WECs are visible (areas of K_d values lower than 0.95 indicated by dark blue colour). The larger 'shadow zone' in terms of extents is observed in the lee of 2 WECs which are arranged in a column configuration with regard to the direction of the incident waves. In front of the WECs K_d values higher than 1.00 are observed as a result of the waves reflected by the devices (areas indicated by light yellow colour). The optimal WEC positions result in WEC farm extents of approximately 60 m (width direction perpendicular to the wave propagation direction) by 10 m (length direction parallel to the wave propagation direction).

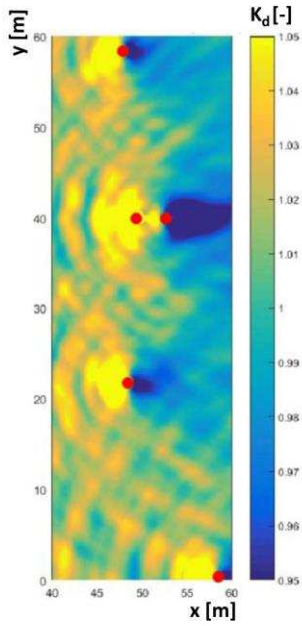

Fig. 5 Geometrical layout for the optimal design of a 5-WEC farm, calculated using the SGA. The small solid red circles indicate the positions of

the WECs. Also the contour plot of the K_d coefficients [-] is presented. Note that the waves are propagating from the left to right side of the figure.

VII. Conclusions

In general the SGA outperforms the other algorithms for both wind and wave farms. It provides improved solutions for the best designs reported in literature regarding the presented studies, used here as benchmarking cases.

However for complex wind farms with a large number of wind turbines in a restricted area, the efficiency rates of the optimal farm layouts decrease strongly.

Regarding wave farms, the here proposed combination of the WEC interaction method presented by [22] with the Search Group Algorithm allows to determine the optimal WEC positions in large farms at a reasonable computational cost.

Part of the research is the further implementation of a cost function in order to investigate the influence of the farm layout on capital and maintenance costs. With these insights the optimal power-cost layout can be determined and a comparison can be made between a wind and a wave farm. Other applications of the SGA can focus on farms of floating wind turbines, or co-located wind-wave farms.

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