# INFLATION TARGETING AND MONETARY ANALYSIS IN CHILE AND MEXICO

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## Abstract

This paper studies the role of monetary and open economy indicators in inflation targeting (IT) economies through the analysis of a nested Phillips curve/ P-star model for Chile and Mexico. For Chile a real money gap and a money growth indicator are found to be relevant in predicting deviations of observed from target inflation. In contrast, for Mexico a real exchange rate gap, a standard measure of the real exchange rate, and a money growth indicator are consistently significant as predictors of deviations of actual from (i) expected (in the pre-IT period) and (ii) target inflation (in the post-IT span).

JEL classification numbers: E30; E40; E50; F41.

**Keywords:** Inflation targeting; monetary policy; P-star; Phillips curve; Chile; Mexico; Latin America.

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## INFLATION TARGETING AND MONETARY ANALYSIS IN CHILE AND MEXICO

## 1. Introduction

During the 1990s a substantial number of countries have adopted inflation targeting (IT) as a formal monetary policy strategy (See Bernanke et al, 1999; Mishkin and Schmidt-Hebbel, 2001; Amato and Gerlach, 2002). A by-product of this trend is that once such a policy is in place several momentous operational questions have to be addressed. For instance, is there a role for monetary and open economy indicators in the conduct of monetary policy in these economies? See, for instance, the related studies by Baltensperger et al (2001) on Switzerland; Batini and Nelson (2001) on the UK and the US; and Gerlach and Svensson (2001), Rudebusch and Svensson (2002), and Trecroci and Vega (2002) on the Euro area.

There are two ways in which money can be utilised by monetary policy makers: (1) as an information and/or (2) predictive variable. The first one critically depends on the stability of the money demand function, a topic widely studied in the literature. Basically, monetary aggregates might provide useful information if they are expected to help in the prediction of imperfectly observed variables that are of interest to the monetary authorities (e.g. aggregate output). In contrast, money's role as a predictor becomes relevant if the central bank's strategy (e.g. inflation targeting) implies the use of expectations of future variables that it can help to predict (see Svensson, 1997). The usefulness of money as a predictor is the main interest of the present investigation.

Particularly, this paper inquires into the predictive content of monetary indicators in the conduct of monetary policy in two Latin American emerging market economies: Chile and Mexico<sup>1</sup>. In so doing an augmented P-star model is employed. The P-star model was advanced by Hallman et al (1991), who investigated the usefulness of the M2 monetary aggregate in the US for inflation forecasting purposes. This inquiry considers as a starting point the augmented version of such a model analysed by Gerlach and Svensson (2001), and Trecroci and Vega (2002), but also incorporating open economy elements. Chiefly, the proposed specification considers real money, output, and exchange rate gaps within a nested Phillips curve/P-star framework.

The rest of the paper proceeds as follows. Section 2 provides an overview of monetary policy and inflation targeting in Chile and Mexico. A Phillips curve/P-star based model of inflation and money is expounded in section 3. The data and its univariate characteristics, as well as the estimation of the several gap measures are dealt with in section 4. The block Granger non-causality methodology is elucidated, and the corresponding analyses of the Philips curves are undertaken, in section 5. Concluding remarks are contained in section 6.

# 2. Monetary policy and inflation targeting in Chile and Mexico<sup>2</sup>

Inflation targeting (IT) is a monetary policy strategy that became popular in the 1990s, New Zealand being the first country to adopt it in 1990<sup>3</sup>. Brazil, Chile, Colombia,

<sup>&</sup>lt;sup>1</sup> It is worth pointing out that these countries have in the past provided a fertile ground for comparative economic analyses. E.g. Arrau and De Gregorio (1993); Edwards (1998).

<sup>&</sup>lt;sup>2</sup> A detailed explanation of IT in Chile and Mexico, and other Latin American economies, is furnished in the works of Mishkin and Savastano (2000), and Corbo and Schmidt-Hebbel (2001).

<sup>&</sup>lt;sup>3</sup> See Mishkin and Schmidt-Hebbel (2001) for a clear analysis of inflation targeting countries' experience with such a policy.

Mexico, and Peru are a group of Latin American countries that have adopted IT as their monetary policy strategy. Amongst this group, Chile and Mexico provide two interesting cases, not least because similar circumstances and corresponding policy responses have in the past been observed in these economies (e.g. Edwards, 1998)<sup>4</sup>. Remarkably in the context of the present investigation, the Central Bank of Chile adopted inflation targeting in order to achieve disinflation, whereas it can be argued that the Bank of Mexico did so having a relatively low and stable inflation. In what follows a closer look is taken at the developments in monetary policy making in these economies.

## 2.1. Chile

Chile formally granted independence to its central bank in 1989. Alongside this step came a mandate to have inflation as the monetary policy makers' primary objective. In 1990 a more formal monetary policy stance was signalled by the announcement of an inflation target to fall within a range of 15-20% for 1991.

It is worth noting that from the mid-1980s until August 1999 the Central Bank of Chile had an exchange rate band regime in place. Nevertheless, after the formal introduction of IT the authorities have always made clear that their primary objective is the pre-announced inflation target. The exchange rate band was supposed to help in keeping the real exchange rate at a level consistent with external equilibrium. However,

<sup>&</sup>lt;sup>4</sup> Focusing on Chile and Mexico within this group is also supported by the fact that Colombia and Peru cannot be unambiguously classified as inflation targeting countries, as noted by Mishkin and Schmidt-Hebbel (2001). On the other hand, Brazil experienced very high inflation at the beginning of the 1990s, which complicates an analysis of the type undertaken in this paper.

in 1998 the Chilean monetary authorities did not allow the exchange rate to depreciate in order to accommodate a negative terms of trade shock. Instead, their policy reaction was to raise interest rates and narrow the exchange rate band. This led to the IT being undershot in that year.

This isolated episode aside, and as can be seen in Table 1, and graphically in Figure 3, Chile's IT has been successfully reduced after its introduction, helping to bring down inflation from 29% (1990.Q4) to 3.8% (1999.Q1). From 1995 the central bank started to announce a *point* rather than a *range* IT. Moreover, in the light of this success in 2000 the Central Bank of Chile committed to a target of 2-4% for 2001 onwards. It is important to emphasise that the success of the IT strategy in Chile is also a product of fiscal surpluses, and a tight regulation and supervision of the financial system.

#### 2.2. Mexico

December 1987 is an important date in the recent history of the Mexican economy, since it was on this date that the 'Pacto' stabilisation programme was introduced. As a result, in the lapse between 1987 and 1991 monetary policy in Mexico was based on a pegged exchange rate regime. Also, under this strategy government spending saw a marked decrease. This prudent fiscal management helped in the reduction of the inflation rate from 132% (1987) to around 20% (1989), while output growth increased from 1.4% (1987) to 12.9% (1989). In fact, Cecchetti et al (2000) argue, and support empirically, that monetary policy in Mexico became more efficient, as measured by the

lower variability of inflation and output, from 1991 onwards, at least in part due to the policies undertaken in the late 1980s.

Accordingly, and in order to provide a more flexible exchange rate framework, the Bank of Mexico adopted exchange rate bands in 1991. However, this period came to an abrupt halt in December 1994, with the well-known Mexican crisis that coined the 'Tequila effect'. The economic and political developments that led to such a meltdown implied a substantial widening of the prevalent exchange rate band, and therefore a devaluation of the Peso.

At the beginning of 1995 the Mexican authorities aborted the pegged exchange rate regime, switching to a policy in which the monetary base played a mayor role (See Khamis and Leone, 2001). This new policy stance included the announcement of an annual inflation target of 19%, which was subsequently increased to 42% due to the Peso's instability. Although the announced target of 17% for the monetary base was achieved, the inflation rate was 10% higher than the announced goal.

The Bank of Mexico continued to announce targets for money and inflation in the next two years, but without success. In 1998 the inflation target was missed by 7%, even though the target for base money was undershot by 1.5%. Under these circumstances the monetary authorities tried to reassure the public that their main goal was to control inflation. With the more formal implementation of an IT monetary policy in 1999, Mexican monetary policy's effectiveness has consolidated. Notably, Mexico's inflation has experienced a reduction from 17.6% (1998.Q4) at IT adoption to 10.6% (2000.Q1).

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## 3. A model of inflation and money

The investigation is based on an augmented version of the  $P^*$  model of Hallman et al (1991). The  $P^*$  concept arises from a simple representation of the quantity theory equation

$$MV \equiv PY \tag{1}$$

where *M* is nominal money, *V* is the average velocity of circulation, *P* is the aggregate price level, and *Y* is aggregate nominal output. Re-arranging (1) and assuming long run values for  $V^*$  and  $Y^*$  produces

$$P^* \equiv \left(\frac{M}{Y^*}\right) V^* \quad . \tag{2}$$

In (2) M determines  $P^*$ . Substituting (2) in (1), and after further manipulations the following arises

$$p^* - p \equiv (y - y^*) - (v^* - v).$$
(3)

In (3) all the variables are expressed in logs. The key element in (3) is the price gap  $(p^*-p)$  component, which plays a prominent role in the determination of inflation in a Phillips curve equation of the form

$$\pi_t = \pi_{t-1}^e - \rho(p_{t-1} - p_{t-1}^*) + \xi_t, \tag{4}$$

where  $\rho \succ 0$ . As can be seen, the  $P^*$  model substitutes the output gap and velocity with the negative of the price gap as the main determinant of inflation. This is the 'baseline' inflation adjustment equation in Hallman et al (1991). The theoretical models by McCallum (1980) and Mussa (1981), for instance, also incorporate adjustment equations analogous to (4).

Gerlach and Svensson (1999), and Trecroci and Vega (2002) consider a nested version of the price gap and Phillips curve models for empirical purposes. The structure of this model is composed of the following elements,

$$\pi_{t} = \pi^{e}_{\langle t|t-1 \rangle} + \eta(y_{t-1} - y^{*}_{t-1}) + \varphi(\tilde{m}_{t-1} - \tilde{m}^{*}_{t-1}) + \xi_{t}$$
(5)

$$\pi^{e}_{\langle t|t-1\rangle} = \hat{\pi}_{t} \tag{6}$$

$$\tilde{m}_t = (m_t - p_t) = \beta_0 + \beta_y y_t + \beta_R R_t + v_t$$
(7)

$$p_t^* \equiv m_t - \beta_0 - \beta_y y_t^* - \beta_R R_t^*$$
(8)

$$\tilde{m}_{t} - \tilde{m}_{t}^{*} \equiv (m_{t} - p_{t}) - (m_{t} - p_{t}^{*}) \equiv -(p_{t} - p_{t}^{*}) = (m_{t} - p_{t}) - \beta_{0} - \beta_{y} y_{t}^{*} - \beta_{R} R_{t}^{*}$$
(9)

 Equation (5) is a mixture of a Phillips curve and a price gap model of inflation, where π, π<sup>e</sup>, y, y<sup>\*</sup>, m̃, and m̃<sup>\*</sup> are actual and expected inflation rates, output and potential output, and actual and long run real money balances, respectively. η and φ are parameters to be estimated empirically. ξ is expected to be a well-behaved disturbance term.

- Equation (6) specifies how inflation expectations (π<sup>e</sup>) are formed, where π̂ is the central bank's inflation objective. Note that this implies that the model focuses on explaining deviations of inflation from its target.
- Equation (7) is a long-run money demand function, which assumes a standard specification. In (7) *y* is real output and *R* is a measure of the opportunity cost of holding money. β<sub>0</sub>, β<sub>y</sub>, and β<sub>R</sub> are parameters to be estimated, and *v* is expected to be a well-behaved disturbance term.
- Equation (8) generates the equilibrium price level ( p<sup>\*</sup>), for a level of the money stock assuming that the other variables in the model are at their equilibrium levels, by inverting the long-run money demand given by equation (7).
- Equation (9) defines the real money gap as the negative of the price gap.

A further element that should be considered in the empirical assessment of the above model is the role of the exchange rate. Svensson (2000), for instance, provides three key reasons as to why this is particularly important for inflation targeting open economies. Firstly, the exchange rate explicitly allows for an additional channel through which monetary policy can be transmitted. Secondly, the exchange rate is *a forward-looking* variable, and therefore can provide valuable information in the design and implementation of monetary policy. Thirdly, foreign shocks mainly propagate thorough the exchange rate.

In addition to the above elements, in a thought provoking paper Calvo and Reinhart (2002, page 394) note that "...central bankers in emerging market economies appear to be extremely mindful of external factors in general and the foreign exchange value of their currency, in particular." Some of the reasons Calvo and Reinhart highlight for the important role played by the exchange rate in monetary policymaking, and which give rise to what they label *fear of floating*, even under an IT regime, are

- Liability dollarisation;
- Output costs associated with exchange rate fluctuations;
- Inelastic supply of funds at time of crises; and
- Lack of credibility and fear of loss of access to the international capital markets. In the light of these factors the present paper proposes a further augmentation of equation (5) as follows

$$\pi_{t} = \pi_{\langle t|t-1\rangle}^{e} + \eta(y_{t-1} - y_{t-1}^{*}) + \varphi(\tilde{m}_{t-1} - \tilde{m}_{t-1}^{*}) + \alpha(e_{t-1} - e_{t-1}^{*}) + \xi_{t}.$$
(10)

Equation (10) incorporates departures of actual (*e*) from equilibrium ( $e^*$ ) exchange rates, where  $\alpha$  is a parameter to be estimated.

## 4. Data

Data on money, real income, prices, interest rates, exchange rates, and inflation objectives are employed in the empirical analyses that follow. The data are monthly, and span from 1990.01 to 2001.06 for Chile, and from 1986.01 to 2001.06 Mexico<sup>5</sup>. In what follows all the time series are expressed in logs, with the exception of the interest

<sup>&</sup>lt;sup>5</sup> Note that for Chile the data covers only the IT regime, whereas for Mexico the statistical information encompasses several periods. Specifically, a pegged exchange rate policy (1987-1991), an exchange rate band (1991-1994), monetary targeting (1995-1998), and inflation targeting (1999-present).

rates, which are expressed in percentage points. The definitions and sources of the data are contained in Table A1.

In order to ascertain the order of integration of the economic time series under scrutiny the Augmented Dickey-Fuller test (Dickey and Fuller, 1979) is implemented. Panel A of Table 2 displays the results. All the series seem to contain a unit root in their levels, but become stationary after being differenced. The exceptions are e and  $p^{p}$ , and p for Chile, which appear I(2) and I(0), respectively. Nevertheless, in assessing the above results the reader should bear in mind the low power of these tests in rejecting the null of a unit root. The univariate properties of the data under scrutiny can be further examined by glancing at Figures 1 and 2, which present data for Chile and Mexico, respectively.

## 4.1. Inflation objectives

The variable  $\hat{\pi}$  is given by the annual inflation target announced by the Central Bank of Chile and the Bank of Mexico (Table 1). However, in the light of the fact that the Bank of Mexico adopted a 'formal' IT from January 1999 (Mishkin and Schmidt-Hebbel, 2001) expected inflation ( $\pi^e$ ) up to that point in time is used in estimating the structural model. Such a variable is proxied by the *filtered* series derived from  $\pi$  by applying a univariate structural time series model and the *Kalman filter* (Harvey, 1989; Koopman et al, 2000). This variable intends to reproduce the economic agents' expected inflation by using a technically compelling technique. Details on this estimation are provided in Appendix 2. Figures 3 and 4 display actual and target inflation for Chile; and actual, expected, and target inflation for Mexico, respectively.

#### 4.2. Output gaps

The output gap series,  $(y - y^*)$ , are expressed as deviations of log output from its potential. Potential real output,  $y^*$ , is the *smoothed* series estimated from y by applying a basic structural model (BSM) and the *Kalman filter* (See Koopman et al, 2000).

## 4.3. Exchange rate gaps

In estimating the exchange rate gaps *bivariate (stage three) cointegration tests of PPP* were applied (See Froot and Rogoff, 1995; Edwards and Savastano, 1999), after experimenting with alternative specifications. In equation form

$$e_t = \lambda (p_t - p_t^*) + \zeta_t \qquad , \tag{11}$$

where *e* is the nominal exchange rate measured in units of home currency per unit of foreign currency, while *p* and *p*<sup>\*</sup> are the domestic and foreign price levels, respectively. Equation (11) implies that the exchange rate between the currencies of two countries should equal the ratio of their price levels. Note that in (11)  $\lambda$  is expected to be around one.

Panel B of Table 2 exhibits the long run solutions to the autoregressive distributed lag (*ADL*) (Hendry, Pagan, and Sargan, 1984) PPP cointegrating relations<sup>6</sup>. The coefficients are statistically significant and economically interpretable, with values

of 0.92 and 0.88 for Chile and Mexico, respectively. Furthermore, the *ADF* test applied to the residuals of such equations rejects non-stationarity. Henceforth, PPP cointegrating relations hold for Chile and Mexico (See Froot and Rogoff, 1995; and Edwards and Savastano, 1999, for related empirical evidence). These results are employed in the computation of the real exchange rate gaps for both economies.

## 4.4. Real money gaps

A simple, textbook, money demand relationship relating real monetary balances to a scale variable and a measure of the opportunity cost of holding money, such as equation (7), is used to estimate the real money gaps. Panel B of Table 2 displays the long run solutions to the corresponding *ADL* equations. All the estimated coefficients are statistically significant and have economically sensible coefficients. Notably, the estimated income elasticities, 1.72 for Chile and 1.38 for Mexico, are similar in magnitude to those obtained by Edwards (1998, Table A2, page 700).

Furthermore, these elasticities could be seen as endorsing the findings of Arrau and De Gregorio (1993). These authors explicitly consider, and find a role for, financial innovation in the modelling of the demand for  $M_1$  in Chile and Mexico. One simple way of rationalising the income elasticities greater than unity displayed in Table 2 is by hypothesising that money has increased faster than output in the period under analysis, and therefore the velocity of circulation has decreased in this time span. This conjecture is validated by Figure 5, which shows a decreasing velocity of circulation of  $M_2$  for

<sup>6</sup> Note that for Chile (11) is an *unbalanced regression*, given that  $e \square I(2)$  and  $(p - p^*) \square I(1)$  (See Banerjee et al, 1993). The *ADL* specification used should, however,

Chile and Mexico during the period under scrutiny. In the light of the fact that this paper analyses a broad monetary aggregate, the above patterns could be arising, for instance, as a result of more advanced financial services, which should allow consumers to hold more of their money in the non-money component of  $M_2$  without compromising their liquidity. However, other factors, e.g. generated by policy, could also be playing a role.

Finally, the *ADF* test applied to the residuals of such equations rejects nonstationarity. Consequently, money demand cointegrating relations exist for Chile and Mexico during the periods under investigation. These results are used in the calculation of the real money gaps for both economies.

At this point it is convenient to exhibit graphically the output, money, and exchange rate gaps calculated for Chile and Mexico, which are gathered in Figure 6.

# 5. Block Granger non-causality analyses

As highlighted by Stock and Watson (2001), due to the rich lag dynamics implied in the application of vector autoregressions (VARs) to time series data, statistics derived from such estimations, such as Granger causality tests, tend to be more informative than the usual regression coefficients. In the empirical assessment of the model outlined in section 3 this paper computes multivariate and bivariate block Granger non-causality tests for Chile and Mexico.

The econometric methodology behind the block Granger non-causality test can be illustrated by using an augmented vector autoregression of order n, VAR (n), such as

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help to address that issue.

$$y_{t} = \alpha_{0} + \lambda t + \sum_{i=1}^{n} \Omega_{i} y_{t-i} + \Psi x_{t} + \xi_{t}$$
(12)

where  $y_t$  is a  $m \times 1$  vector of jointly determined (endogenous) variables, t is a linear time trend,  $x_t$  is a  $p \times 1$  vector of exogenous variables, and  $\xi_t$  is a well-behaved disturbance term. If in (12)  $y_t$  is divided in two subsets  $y_{1t}$  and  $y_{2t}$ , which are  $m_1 \times 1$  and  $m_2 \times 1$  vectors, respectively, and  $m = m_1 + m_2$ , the following block decomposition can be written down

$$y_{1t} = \alpha_{10} + \lambda_{11}t + \sum_{i=1}^{n} \Omega_{i,11}y_{1,t-i} + \sum_{i=1}^{n} \Omega_{i,12}y_{2,t-i} + \Psi_{1}x_{t} + \xi_{1t}$$

$$y_{2t} = \alpha_{20} + \lambda_{21}t + \sum_{i=1}^{n} \Omega_{i,21}y_{1,t-i} + \sum_{i=1}^{n} \Omega_{i,22}y_{2,t-i} + \Psi_{2}x_{t} + \xi_{2t} .$$
(13)

The hypothesis that the subset  $y_{2t}$  is not Granger causal on  $y_{1t}$  is given by  $\Omega_{12} = 0$ , with  $\Omega_{12} = (\Omega_{1,12}, \Omega_{2,12}, \Omega_{3,12}, ..., \Omega_{n,12})$ . The log-likelihood ratio statistic that arises from testing  $\Omega_{12} = 0$  has  $m_1 m_2 n$  degrees of freedom, and is  $\chi^2$  asymptotically distributed<sup>7</sup>.

## 5.1. Chile

The study proceeds by analysing equation (10), that is, the joint impact of  $y - y^*$ ,  $\tilde{m} - \tilde{m}^*$ , and  $e - e^*$  on  $\pi - \hat{\pi}$ . The outcome of the multivariate block Granger non-

<sup>&</sup>lt;sup>7</sup> See Pesaran and Pesaran (1997) for further details on this methodology.

causality VAR tests are reported in Table 3. For Chile they support the joint significance of  $y - y^*$ ,  $\tilde{m} - \tilde{m}^*$ , and  $e - e^*$  on  $\pi - \hat{\pi}$ .

However, the corresponding bivariate tests show that the hypothesis of noncausality can only be rejected for the real money gap. Therefore, it can be said that for Chile the  $P^*$  model provides some rationale for using a real money gap indicator in the conduct of monetary policy under an IT regime. These outcomes are further analysed below.

## 5.2. Mexico

Following the results in Cecchetti et al (2000) the baseline estimation period adopted for Mexico is 1991.01-2001.06. As mentioned above, these authors find that monetary policy in Mexico became more effective from 1991. Additionally, given that in Mexico a formal IT strategy is dated as having initiated in 1999 the study analyses three periods: 1991.01-2001.06, 1991.01-1998.12, and 1999.01-2001.06. If the way in which money and the other relevant variables considered affect inflationary developments have changed due to the introduction of IT this should be reflected through the estimates for each period.

As for Chile, Mexico's outcomes reject the hypothesis of non-causality from  $y - y^*$ ,  $\tilde{m} - \tilde{m}^*$ , and  $e - e^*$  to  $\pi - \hat{\pi}$  for the three time spans under analysis. However, the bivariate results indicate that of these indicators only  $e - e^*$  is individually significant (at the 7%) during 1991.01-2001.06. Furthermore,  $e - e^*$  also seems to contain significant information (at the 1% level) on determining  $\pi - \hat{\pi}$  in the sub-periods 1991.01-1998.12 and 1999.01-2001.06. Table 3 also shows that  $y - y^*$  and

 $\tilde{m} - \tilde{m}^*$  help in predicting  $\pi - \hat{\pi}$ , but only for the post-IT regime ranging from 1999.01 to 2001.06.

### 5.3. Robustness check

In order to establish the robustness of the results contained in the previous sub-sections bivariate block Granger non-causality VAR tests are employed using alternative money, output, and exchange rate indicators. The output gap proxy is achieved by passing y through the Hodrick-Prescott filter (HPF) (Hodrick and Prescott, 1997) using a value of 129,600 for the HPF's smoothness parameter  $\lambda$ , as suggested by Ravn and Uhlig (2002) for the analysis of monthly data. Also, two money growth measures,  $\Delta \tilde{m}$  and  $\Delta \tilde{m} - \Delta y$ , and a standard real exchange rate indicator, *REXR*, are considered. Note that  $\Delta \tilde{m}$  and  $\Delta y$  are calculated as annual changes in the log of the money and output variables, respectively, whereas *REXR* is given by  $e - p + p^*$ .

The outcome of the bivariate block Granger non-causality tests are reported in Table 4. For Chile only the output gap and nominal money growth indicators are found to be significant. Similarly, Mexico's results endorse the previous findings: the indicator *REXR* contains statistically significant information on inflation in the three periods under scrutiny. So it can be argued that the exchange rate is a consistently useful predictor of inflation in Mexico during the last decade, and should be a closely monitored variable by the monetary authorities. These comes as no surprise, particularly in the time span under scrutiny, which encompasses the well-documented Mexican exchange rate crisis of 1995 [See, for instance, the November 1996, volume 41, issue of the *Journal of International Economics*.] Similarly, but in contrast to Mexico's case, it seems that in Chile the role played by the exchange rate was not statistically significant. This could be reflecting the effectiveness of the exchange rate band regime implemented by Chile during most of the sample period under study, referred to in section 9.2.1. Specifically, Reinhart and Rogoff's (2002) novel approach to the classification of exchange rate regimes catalogues Chile's exchange rate regimes during 1989-2001 as: pre-announced crawling band around the US Dollar (June 1, 1989-January 22, 1992), *de facto* announced crawling band around the US Dollar (January 22, 1992-June 25,1998), pre-announced crawling band to US Dollar (June 25, 1998-September 2, 1999), and managed floating (September 2, 1999-December 2001).

Additionally, for Mexico  $\Delta m$  is rejected as being non-causal on  $\pi - \hat{\pi}$  during both 1991.01-1998.12 and 1999.01-2001.06, whereas  $HPF(y-y^*)$  is so only for the post IT period. From this last findings it is natural to derive the preliminary conclusion that at the early stages of an IT regime the monetary authorities should be particularly careful in monitoring a wide array of variables.

### Interim summary of findings

The main findings of the paper so far, derived from multivariate and bivariate Granger non-causality tests, show that deviations of inflation from target are explained

- <u>In Chile and Mexico</u>: jointly by the real money, output, and exchange rate gaps;
- <u>In Chile</u>: individually by the real money gap, a real output gap calculated by applying the Hodrick-Prescott filter, and a real money growth indicator;

<u>In Mexico</u>: individually, and across pre and post-IT periods, by the real money, output, and exchange rate gaps, and a real output gap calculated by applying the Hodrick-Prescott filter, real money growth and real exchange rate indicators.

# 5.3. Generalised impulse responses

The research proceeds by calculating the generalised impulse response functions (GIRs) of  $\pi - \hat{\pi}$  from shocks to the multivariate and bivariate VARs containing the variables found to be statistically relevant in the previous sections. Why are GIRs useful in the analysis of a VAR model? Impulse responses aid in visually determining the impact through time of a one-off shock to a given variable on a system, other variables, or itself. Koop et al (1996), and Pesaran and Shin (1998) show that GIRs are more convenient than the widely used orthogonalised approach to impulse responses championed by Sims (1980) because, in contrast to the orthogonalised impulse responses, the results obtained from the generalised ones are invariant to the ordering of the variables in the VAR. The reader should note, however, that the generalised and orthogonalised impulse responses are the same for the bivariate VARs.

Figures 7 and 8 show the outcomes of these exercises for Chile and Mexico, respectively. Figure 7's panel (a) exhibits the GIR of  $\pi - \hat{\pi}$  to a one standard error shock to the equation for the real money gap in a VAR containing  $y - y^*$ ,  $e - e^*$  and  $\pi - \hat{\pi}$ . Notably, the impact of that shock on inflation lasts up to the 24<sup>th</sup> month. This is in harmony with the well known long and variable lags in the transmission of monetary impulses postulated by Milton Friedman (1961) (See also Batini and Nelson, 2001, for a related study). Figure 7's panel (b) displays the positive impact on  $\pi - \hat{\pi}$  of a one standard error shock to the equation for the  $HPF(y-y^*)$  in a VAR containing

$$HPF(y-y^*), \ \tilde{m}-\tilde{m}^*, \ e-e^*, \ \text{and} \ \pi-\hat{\pi}^{-8}.$$

The variables found to be most robust (i.e. significant in the various sub-periods considered) in explaining inflation in Mexico, namely  $e - e^*$ , *REXR*, and  $\Delta \tilde{m}$ , are examined for the full sample period (1991.01-2001.06) by studying the GIRs arising from shocks to the VAR equations explaining their behaviour. In a VAR containing  $y - y^*$ ,  $\tilde{m} - m^*$ ,  $e - e^*$  and  $\pi - \hat{\pi}$  a positive impact on  $\pi - \hat{\pi}$  from a disturbance to the equation for  $e - e^*$  is shown in Figure 8's panel (a), lasting up to twelve months.

The effect of ruffling the equation for *REXR*, displayed in panel (b), generates an analogous pattern. These two graphs visually demonstrate the forefront role played by the exchange rate in determining Mexico's inflationary dynamics. Finally, panel (c) portrays that shocks to  $\Delta \tilde{m}$  can generate significant swings on  $\pi - \hat{\pi}$ , lasting up to two years. Henceforth, the role of money should also be carefully assessed in the conduct of monetary policy in Mexico.

# 6. Conclusion

This paper analyses the role of monetary and open economy indicators in the conduct of monetary policy in two inflation targeting Latin American economies: Chile and Mexico. A nested P-star/Phillips curve model is developed and estimated. The main findings of the analyses reveal that for Chile both the real money gap and real money growth indicators contain significant information on deviations of inflation from the inflation target.

<sup>&</sup>lt;sup>8</sup> This VAR is significant, as shown by the corresponding test  $LR = 98.37(0.00^{**})$ .

In contrast, for Mexico different measures of the real exchange rate are found to be consistently relevant in the pre and post-IT (1999) periods. This is also the case for a real money growth indicator. These results are of considerable importance to policymakers. Remarkably, they convey that neglecting the role that monetary and open economy indicators play in monetary policy making within an IT framework could be detrimental to the successful operation of such a strategy.

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Country	Date introduced	Target price index	Target	width
Chile	January 1991	Headline CPI	1991 1992 1993 1994 1995 1996 1997	$\begin{array}{c} 15\text{-}20\% \\ 13\text{-}16\% \\ 10\text{-}12\% \\ 9\text{-}11\% \\ \pm 8\% \\ \pm 6.5\% \\ \pm 5.5\% \end{array}$
			1998 1999 2000 2001	$\pm 4.5\%$ $\pm 4.3\%$ $\pm 3.5\%$ 2-4%
Mexico	January 1999	Headline CPI	1999 2000 2001	13% <10% 6.5%

# Table 1Chile and MexicoInflation targeting: Adoption dates and targets width

Source. Mishkin and Schmidt-Hebbel (2001), Table 2. Note: CPI = consumer price index.

A. ADF unit root tests							
Variables	Le	vels	First differe	nces ( $\Delta 12$ )			
	Chile	Mexico	Chile	Mexico			
	(1991.02-	(1987.02-	(1992.02-	(1988.02-			
	2001.06)	2001.06)	2001.06)	2001.06)			
$\tilde{m}$	-2.368	-0.6335	-2.937*	-5.194**			
У	-2.422	-0.7214	-3.006*	-4.365**			
R	-2.710	-2.519	-3.483*	-3.926**			
e	0.336	-2.488	-1.665	-3.815**			
$p^{D}$	-1.773	-1.763	-1.812	-7.265**			
р	-6.586**	-2.042	-2.927*	-6.783**			
$p^*$	0.022	-1.685	-3.870**	-3.748**			
$p^{D}-p^{*}$	-0.782	-1.949	-3.554**	-7.228**			
	B. Long r	un solutions to AL	DL equations				
Variables	Chile (1991	.01-2001.07)	Mexico (1988.01-2001.06)				
	Money demand	PPP	Money demand	PPP			
	Equation (1)	Equation (2)	Equation (3)	Equation (4)			
Cons	-1.21 (0.448)**	6.24 (0.025)**	9.03 (0.693)**	1.53 (0.071)**			
$\beta_y$	1.72 (0.078)**	-	1.38 (0.143)**	-			
$\beta_{R}$	-0.02 (0.011)*	-	-0.003 (0.001)**	-			
λ	-	0.92 (0.093)**	-	0.88 (0.087)**			
ADF test	-8.052 (-4.40)	-7.908 (-3.98)	-9.465 (-4.37)	-8.904 (-3.96)			
$WALD - \chi^2$	493.74 (2)**	97.84 (1)**	262.72 (2)**	101.367 (1)**			

# Table 2Chile and MexicoUnit root and cointegration tests

<u>Notes</u>. - <u>Part A</u>: the augmented Dickey-Fuller test (ADF) is based on a regression of the form  $\Delta y_t = \alpha + \phi y_{t-1} + \sum_{i=1}^T \Theta \Delta y_{t-i} + \delta t + \varepsilon_t$ , where  $\varepsilon_t$  is a random error term, and  $\alpha$  and t

are a constant and time trend, respectively. The ADF test corresponds to the value of the t-ratio of the coefficient  $\phi$ . The null hypothesis of the ADF test is that  $y_t$  is a nonstationary series, which is rejected when  $\phi$  is significantly negative. Only a constant was added to the tests. \*\* and \* denote rejection of the unit root hypothesis at the 1% and 5% level, respectively. Part B: (1) Coefficients' standard errors are inside parentheses. is a test of the null that all long-run coefficients are zero, with  $WALD - \chi^2$  $\chi^2(\cdot)$  distribution (2) ADL = autoregressive distributed lag. Critical values (1%) for the ADF test applied to the residuals of the cointegrating relations are from MacKinnon (1991), and are shown in parentheses next to the corresponding ADF statistic. A significant test means rejection of the hypothesis of non-stationarity, i.e. a cointegrating relationship exists between the variables under analysis. (3) \*\* and \* denote a coefficient/test is significant at the 1% and 5% level, respectively. (4) The ADL equations initially allowed for twelve lags of each of the variables considered, reducing the corresponding initial systems using a general to specific modelling strategy. Further details on these procedures can be obtained from the author upon request.

Table 3Chile and MexicoMultivariate and bivariate block Granger non-causality VAR tests of the augmented Phillips curve/P-star model

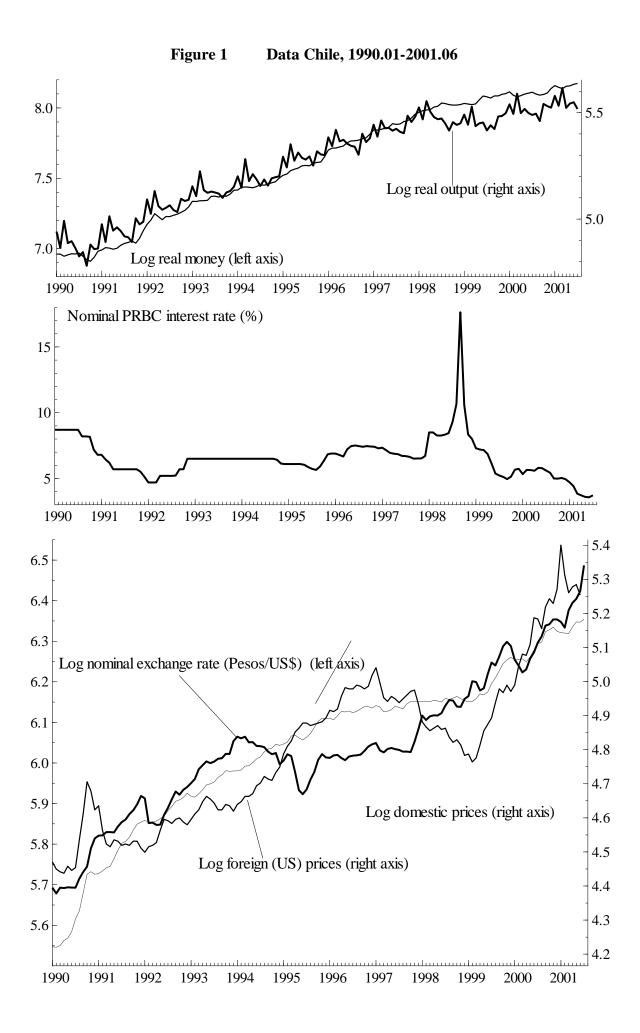
Nul	l hypotheses	LR test statistic (probability)							
	Chile		Mexico						
		1992.09-	Lags	1991.01-	Lags	1991.01-	Lags	1999.01-	Lags
		2001.06		2001.06		1998.12		2001.06	
$y - y^*$	Non causal on	76.909	10	51.302	12	98.816	12	33.483	5
$\tilde{m}-\tilde{m}^*$	$\pi - \hat{\pi}$	(0.000)**		(0.047)*		(0.000)**		(0.004)**	
$e-e^*$									
y - y*	Non causal on	0.776	2	5.752	7	5.664	8	13.763	5
	$\pi - \hat{\pi}$	(0.678)		(0.569)		(0.685)		(0.017)*	
$\tilde{m} - \tilde{m}^*$	Non causal on	26.648	12	11.081	12	11.244	11	24.733	11
	$\pi - \hat{\pi}$	(0.009)**		(0.522)		(0.423)		(0.010)**	
$e-e^*$	Non causal on	8.242	12	20.257	12	47.948	12	37.452	12
	$\pi - \hat{\pi}$	(0.766)		(0.062)†		(0.000)**		(0.000)**	

<u>Notes</u>: the block Granger non-causality statistic is calculated through a LR test, and has a  $\chi^2$  distribution. The test statistics are displayed with corresponding probability values inside parentheses. For both countries an unrestricted vector autoregression (VAR) of an initial lag order of 12 was used. For Mexico, the multivariate estimations for 1999.01-2001.01 start with 6 lags due to data limitations. The lag lengths were determined through the Schwarz Bayesian Criteria (SBC).  $\dagger$ , \* and \*\* denote rejection of the null at the 10%, 5%, and 1%, respectively.

Table 4Chile and MexicoBivariate block Granger non-causality VAR tests of the robustness of the augmented Phillips curve/P-star model

Null	hypotheses	LR test statistic (probability)							
		Chile		Mexico					
		1992.09-	Lags	1991.01-	Lags	1991.01-	Lags	1999.01-	Lags
		2001.06		2001.06		1998.12		2001.06	
HPF	Non causal on	21.572	9	10.050	11	15.811	11	21.291	9
( <i>y</i> – <i>y</i> *)	$\pi - \hat{\pi}$	(0.010)**		(0.526)		(0.148)		(0.011)*	
$\Delta \tilde{m}$	Non causal on	19.995	12	15.807	12	25.757	12	26.012	10
	$\pi - \hat{\pi}$	(0.067)†		(0.200)		(0.012)*		(0.004)**	
$\Delta \tilde{m} - \Delta y$	Non causal on	10.376	11	5.625	11	13.724	11	0.476	1
	$\pi - \hat{\pi}$	(0.497)		(0.897)		(0.249)		(0.490)	
REXR	Non causal on	12.042	12	22.624	12	60.235	12	23.539	10
	$\pi - \hat{\pi}$	(0.442)		(0.031)*		(0.000)**		(0.009)**	

<u>Notes</u>: the block Granger non-causality statistic is calculated through a LR test, and has a  $\chi^2$  distribution. The test statistics are displayed with corresponding probability values inside parentheses. For both countries an unrestricted vector autoregression (VAR) of an initial lag order of 12 was used. The lag lengths were determined through the Schwarz Bayesian Criteria (SBC).  $\dagger$ , \* and \*\* denote rejection of the null at the 10%, 5%, and 1%, respectively.



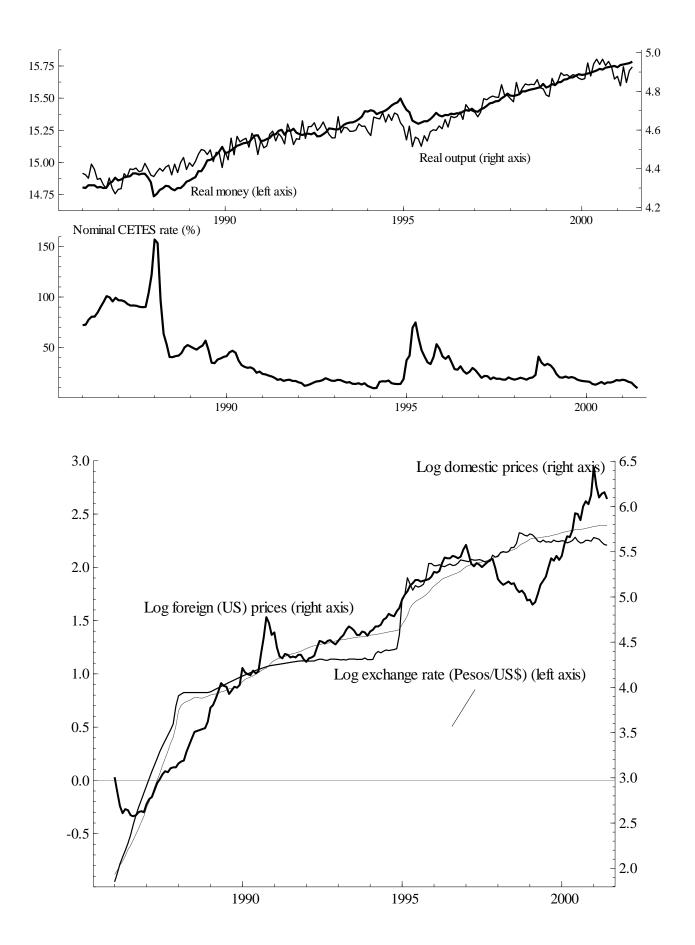
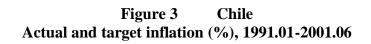
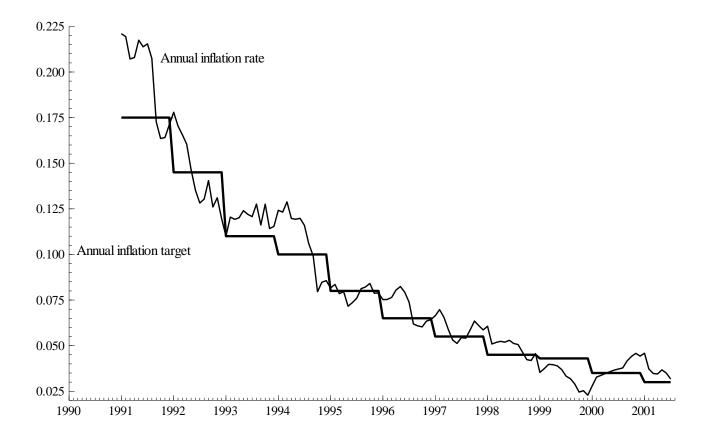


Figure 2 Data Mexico, 1986.01-2001.06





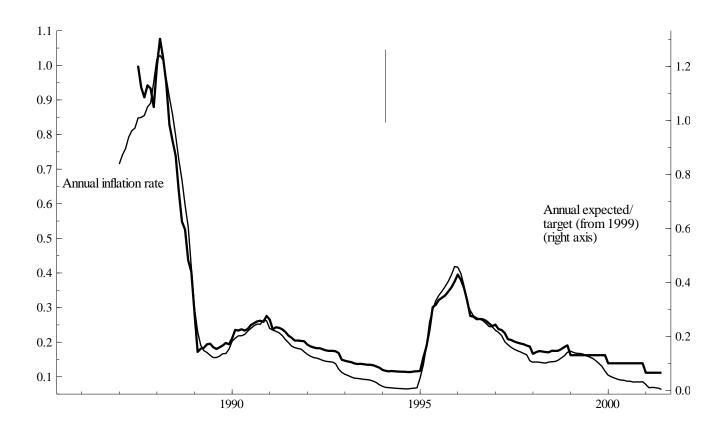
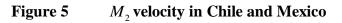
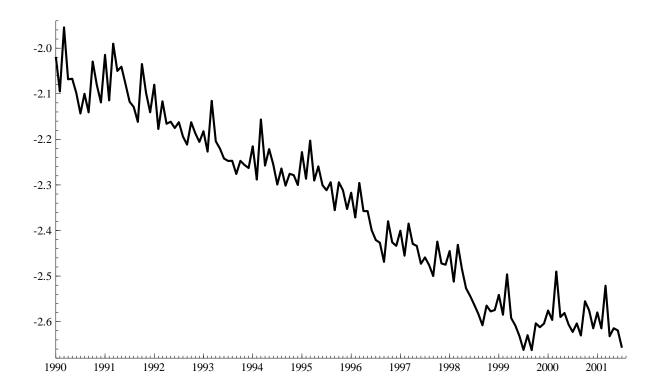
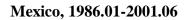


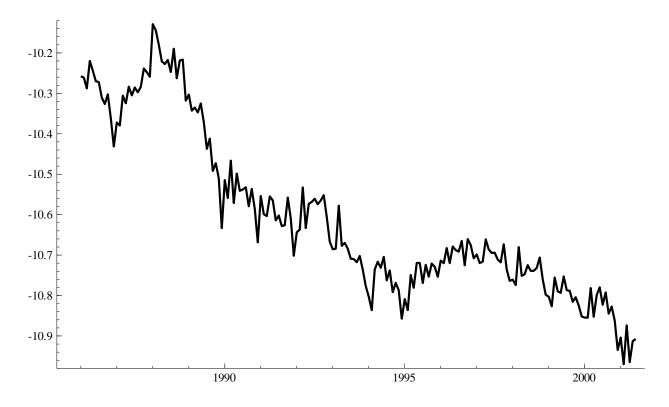
Figure 4MexicoActual and expected/target inflation (%), 1987.07-2001.06











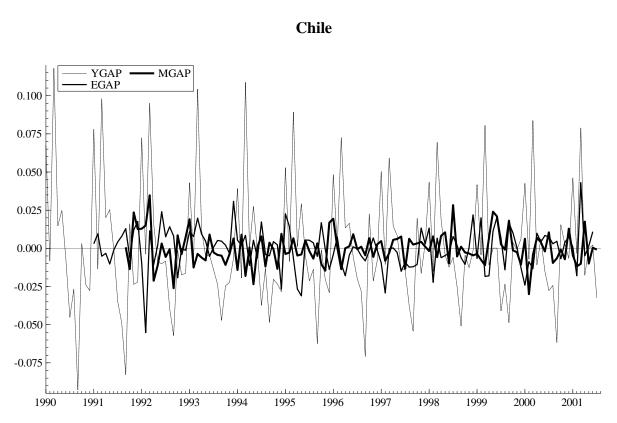
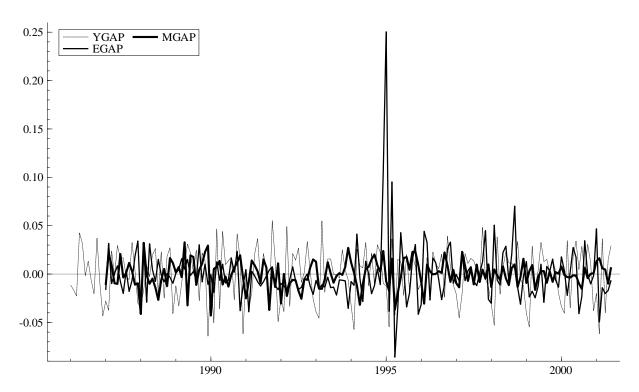


Figure 6 Output, money, and exchange rate gaps

Mexico



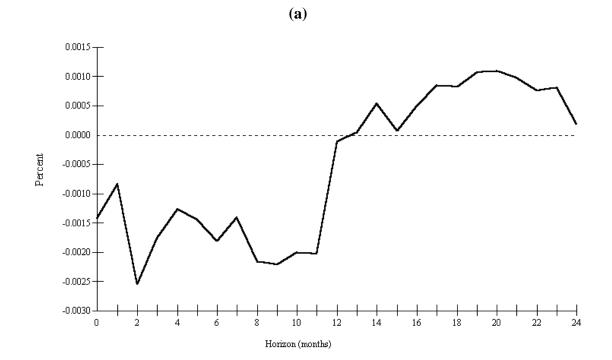
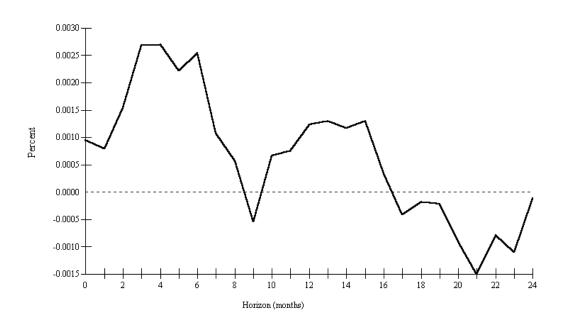


Figure 7 GIRs of  $\pi - \pi^*$  to one S.E. shock in the equation for (a)  $(m - m^*)$  and (b)  $HPF(y - y^*)$ , Chile





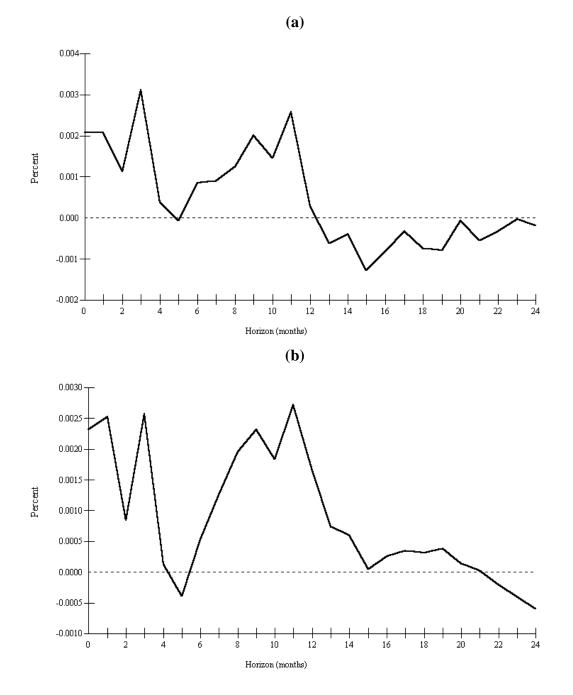
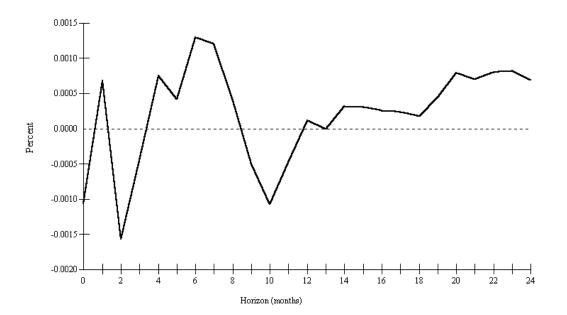


Figure 8 GIRs of  $\pi - \pi^*$  to one S.E. shock in the equation for the (a)  $(e - e^*)$ , (b) *REXR*, and (c)  $\Delta m$ , Mexico







Variables	Chile	Mexico			
М	$M_2A$ , real private money, monthly	$M_2 = M_1$ + internal financial assets in the			
	averages in thousands of millions of	hands of residents, thousands of Mexican			
	1986 Chilean Pesos.	Pesos.			
Y	Monthly indicator of economic activity,	Industrial production index, 1993 = base.			
	IMAE, 1986 = 100.				
R	PRBC, 90 days in annual percentage	Interest rate of 28 days CETES, in annual			
	points.	percentage points.			
E	Observed monthly exchange rate,	Exchange rate, Mexican Pesos per United			
	Chilean Pesos per United States Dollar.	States Dollar, monthly average.			
$P^{D}$	Producer price index, June $1992 = 100$ .	Wholesale price index, excluding oil,			
		1994 = base.			
Р	Consumer price index,	Consumer price index, $1994 = base$ .			
	December 1998 = 100.				
$P^{*}$	Foreign (US) producer price index, 1982 = base, from the FRED of the Federal				
	Reserve Bank of St Louis (http://www.stls.frb.org/fred/data/ppi/ppiaco).				

Table A1Chile and MexicoData definitions and sources

Sources.

<u>Chile</u>: Unless otherwise indicated, Central Bank of Chile, <u>http://www.bcentral.cl/Indicadores/eindicadores.htm</u>. <u>Mexico</u>: Unless otherwise indicated, Central Bank of Mexico, <u>http://www.banxico.org.mx/eInfoFinanciera/FSinfoFinanciera.html</u>.

# Appendix 2 Econometric estimation of the inflation target for Mexico, 1987.04-2001.06

In the light of the fact that Mexico adopted a formal inflation target from January 1999, a proxy for an 'implicit' inflation target ( $\hat{\pi}$ ) is estimated using a *univariate structural time series model* (Harvey, 1989). Details on the specification and related statistics are displayed in Table A2.

$\frac{\text{Model}}{\hat{\pi}_t} = \text{Fixed level} + \text{cycle} + \text{irregular} + \text{interventions}$ <b>Statistics</b>				
Level (final state vector)	T-ratio = 5.0692			
= 0.0052	RMSE = 0.00103			
Standard error of equation	0.0084			
RPEV	0.000071			
$R_d^2$	0.8726			
$R_s^2$	0.8724			

Table A2Univariate time series structural time series model<br/>for inflation in Mexico, 1987.04-2001.06

<u>Notes</u>: RMSE = root mean square error. RPEV = residuals prediction error variance.  $R_d^2$  and  $R_s^2$  are goodness of fit statistics that compare the results with a random walk plus drift and a random walk with fixed seasonal dummies, respectively. See Koopman et al (2000) for details on these tests.

1. The baseline specification employed for the case at hand is based on a 'local

linear trend' model that can be written as

$$\hat{\pi}_t = \lambda_t + \zeta_{t,}$$

$$\zeta_t \square NID(0, \sigma_{\zeta}^2).$$
(A1)

In this application, the model search process led to an equation in which the level ( $\lambda$ ) is fixed, i.e. does not contains a stochastic element.

Furthermore

- 2. Two lags of the dependent variable are used in fitting (A1).
- 3. Cycle and irregular components are also added to equation (A1). Further details on the rationality of these elements, notably the first, can be found in Koopman et al (2000).
- 4. Interventions affecting  $\lambda$  in (A1) are included in the periods 1987.12, 1988.01, and 1988.02 to account for the impact of 'The Pacto' economic program.
- 5. All the elements outlined in points 2. to 4. are statistically significant at the 1% level, with the exception of the intervention for 1988.01 which is so at the 2% level.
- 6. The fitted model is used to calculate the *filtered* estimate of the trend in (A1) at all points in the sample, (remarkably) using only data available up to the previous period, i.e.  $\hat{\pi}_{\langle t|t-1\rangle}$ . Therefore, the 'econometric inflation target' intends to reproduce the economic agents' expected central bank inflation objective. Empirically, this is achieved by using a technically compelling updating technology, the *Kalman filter*, to estimate the state ( $\lambda$ ).
- Note that the λ implied by the final state vector, reported in Table A2, is
   0.0052, which amounts to an annual inflation rate of 0.0624 (6.24%) at the end of the sample period.

Further details on the above exercise can be obtained from the author upon request.