DEMAND EFFECTS IN PRODUCTIVITY AND EFFICIENCY ANALYSIS

A Dissertation

by

CHIA-YEN LEE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2012

Major Subject: Industrial Engineering

Demand Effects in Productivity and Efficiency Analysis

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Approved by:

Chair of Committee,	Andrew L. Johnson
Committee Members,	Wilbert E. Wilhelm
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	Steven L. Puller
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ABSTRACT

Demand Effects in Productivity and Efficiency Analysis. (May 2012)

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Demand fluctuations will bias the measurement of productivity and efficiency. This dissertation described three ways to characterize the effect of demand fluctuations.

First, a two-dimensional efficiency decomposition (2DED) of profitability is proposed for manufacturing, service, or hybrid production systems to account for the demand effect. The first dimension identifies four components of efficiency: capacity design, demand generation, operations, and demand consumption, using Network Data Envelopment Analysis (Network DEA). The second dimension decomposes the efficiency measures and integrates them into a profitability efficiency framework. Thus, each component's profitability change can be analyzed based on technical efficiency change, scale efficiency change and allocative efficiency change.

Second, this study proposes a proactive DEA model to account for demand fluctuations and proposes input or output adjustments to maximize *effective* production. Demand fluctuations lead to variations in the output levels affecting measures of technical efficiency. In the short-run, firms can adjust their variable resources to address the demand fluctuates and perform more efficiently. Proactive DEA is a short-run

capacity planning method, proposed to provide decision support to a firm interested in improving the effectiveness of a production system under demand uncertainty using a stochastic programming DEA (SPDEA) approach. This method improves the decision making related to short-run capacity expansion and estimates the expected value of effectiveness given demand.

In the third part of the dissertation, a Nash-Cournot equilibrium is identified for an oligopolistic market. The standard assumption in the efficiency literature that firms desire to produce on the production frontier may not hold in an oligopolistic market where the production decisions of all firms will determine the market price, i.e. an increase in a firm’s output level leads to a lower market clearing price and potentially-lower profits. Models for both the production possibility set and the inverse demand function are used to identify a Nash-Cournot equilibrium and improvement targets which may not be on the strongly efficient production frontier. This behavior is referred to as rational inefficiency because the firm reduces its productivity levels in order to increase profits.

DEDICATION

To my parents Pai-Hung Lee and Yen-Chieh Feng, my wife Yu-Chun Chiu, and my children Tiffany and Henry for their patience, love and great support.

ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Andrew Johnson, from the bottom of my heart for his great guidance and patience, and my committee members, Dr. Wilbert Wilhelm, Dr. Sergiy Butenko, Dr. Steven Puller, and Dr. Robin Sickles, for their guidance and support throughout the course of this research.

Thanks to my colleagues Maethee Mekaroonreung, Brandon Pope, Taeho Yoon, Zeynep Ertem, and Abhilasha Katariya for a great research discussion. Thanks to my friends Meng-Kun Liu, Ya-Jen Yu, Yu-Pin Hsu, Yu-Ping Kuo, and Shih-Yi Hsiung for making my time at Texas A&M University a great experience. I would also like to thank Judy Meeks for being helpful at all times.

Finally, thanks to my mother and father for their encouragement, to my wife for her patience and love, and to my children for bringing happiness to me.

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CHAPTER I

INTRODUCTION

1.1 Background and Motivation

Productivity and efficiency analysis measures the performance of firms, which transform input resources into output products or services (Coelli *et al*., 2005). The efficient frontier (or production function) can be constructed to characterize a benchmark to measure how efficiently production processes use inputs to generate outputs; given the same level input resources, inefficiency is indicated by lower levels of system output. In a competitive market, if the firm is far from the production function and operates inefficiently, either productivity should increase to make the firm competitive or the firms would likely go out of business. In practice, a true production function is not observed and must be estimated. Production theory provides a useful framework to estimate the production function and efficiency levels using the parametric functional forms (e.g. Stochastic Frontier Analysis, SFA) (Aigner *et al*., 1977; Meeusen and van den Broeck, 1977) or nonparametric benchmarking technique (e.g. Data Envelopment Analysis, DEA) (Charnes *et al*., 1978; Banker *et al*., 1984). Using productivity and efficiency analysis the firm can identifies inefficient performance, then develop productivity improvement strategy and reallocate resources.

This dissertation follows the style of European Journal of Operational Research.

Due to rapid development in information technology, global logistics, and electronic commerce, business environments change swiftly and are increasingly uncertain. Environment uncertainty challenges business operations. In particular, demand uncertainty creates critical shocks to business environments. Short product life cycle, product customization, and price competition contribute to variability in demand and lead to changes in service requirements. Demand uncertainty will affect capacity installation, product pricing, product-mix, on-time delivery, vendor selection, and order allocation. Demand uncertainty pushes the firm to build a more flexible business model focusing on core competence using concepts such as flexible manufacturing system (Vokurka and O'Leary-Kelly, 2000), lean manufacturing (Shah and Ward, 2003), Just-In-Time (JIT) and Kanban production system (Ohno, 1988a; 1988b), build-to-order (Holweg and Pil, 2001), assemble-to-order, etc. Other firms address demand uncertainty by developing common platforms of information transparency and sharing information across the supply chain using information technology (Simchi-Levi *et al*., 2007). Electronic Data Interchange (EDI) and Point-of-Sale (PoS) systems significantly shorten the lead time of data collection and improve vertical integration (Monteverde and Teece, 1982; Premkumar *et al*., 1994). In addition, firms develop strategic alliances and build relationships that alternate between competition and cooperation such as Vendor-Managed Inventory (VMI) (Waller et al., 1999) and risk pooling which aggregates demand to reduce demand variation (Simchi-Levi et al., 2007). There is not doubt that demand fluctuations also change the rules of thumb for typical measures in productivity analysis.

This study addresses three issues related to demand fluctuations when performing productivity and efficiency analysis. The first issue is that demand fluctuation bias the estimates of efficiency. The second issue is demand fluctuation create a gap between demand and output level as a surplus or shortage of capacity. The third issue is firms that increase output to become technically efficient may actually reduce overall profits because increasing the overall quantity in the market will lead to lower prices.

First, demand fluctuations lead to biased estimates of efficiency. A decrease in actual output can be the result of insufficient demand. If actual output is reduced by changes in demand, then the efficiency is underestimated relative to what the firm could have produced. Demand fluctuations can create bias in two ways: 1) if forecasted demand is underestimated, capital resources might be under used to avoid creating excess inventories. In this case higher output could have been achieved, but due to forecasting error and production lead times, the firm achieves lower productivity. 2) if units sold is used as the output measure, insufficient realized demand will cause measured output to be lower. Similarly, in a panel data analyses, technical regress is often attributed to production issues, when in reality it may be a result of a reduction in demand. Thus, productivity analysis attributes changes in demand to production (Lee and Johnson, 2011; 2012). In chapter II, a network DEA model to decompose the efficiency of production system is described. This method separates the demand and production process in efficiency analysis.

Second, demand fluctuations cause a surplus or shortage of capacity. Capacity surplus occurs when the demand realized is less than the supply that can be produced by the facility; or alternatively, capacity shortage occurs where the demand for a product exceeds the capacity of the facility. In this study, "effective" output is defined as the output products or services produced and consumed. Over or under production causes profit loss. Even though demand fluctuations make demand forecasting and capacity installation decisions challenging, in the short run, firms can change variable input resources to adjust output levels and partly address demand uncertainty. This sort of capacity flexibility is critical to achieving cost savings in demand downturns or to increase profits when demand is unexpectedly high (Alp and Tan, 2008). In chapter III the proactive DEA model is described to recommend adjustments in variable input levels to match demand and maximize the effectiveness (i.e., the difference between demand and production output level).

Third, typically productivity analysts do not consider demand and assume all firms want to be as productive as possible. However, microeconomic theory tells us firms in less than perfectly competitive markets can reduce production levels and increase the market price for a product, in some cases increasing the firm’s profits. In oligopoly markets a particular firm's output level along with the output level of all other firms jointly determine the price, whereas in a monopoly market a firm has absolute control over price by selecting its output level. In these situations, a firm that changes the quantity supplied will affect the clearing price. From a revenue efficiency perspective, an inefficient firm that increases output to become technically efficient may actually reduce overall profits by increasing the market quantity and causing the market price to fall (Johnson and Ruggiero, 2011). Thus in this case a firm is said to be rationally inefficient in the sense that a firm is maximizing its profit by intentionally operating at lower productivity levels. In chapter IV this study, an inverse demand function is used to identify a Nash-Cournot equilibrium and the profit maximizing output level.

1.2 Research Aims and Methodologies

This dissertation describes the integration demand effects into a productivity analysis framework. Figure 1.1 proposes the functional position of productivity and efficiency analysis (PEA) within the production planning framework. Currently there is not a link between PEA and the demand management function, thus this dissertation develops this link shown with the dashed arrow. PEA is a tactical-level decision and part of mid-term production planning. There are two services which PEA provides - performance benchmarking and production guidance. The former, PEA can provide an ex-post analysis estimating efficiency from the dataset of multiple inputs and multiple outputs; alternatively, PEA can be used in an ex ante analysis to suggest guidelines of resource allocation. However, currently PEA is used on production data and ignores demand information which may bias efficiency measures intended to characterize operational performance.

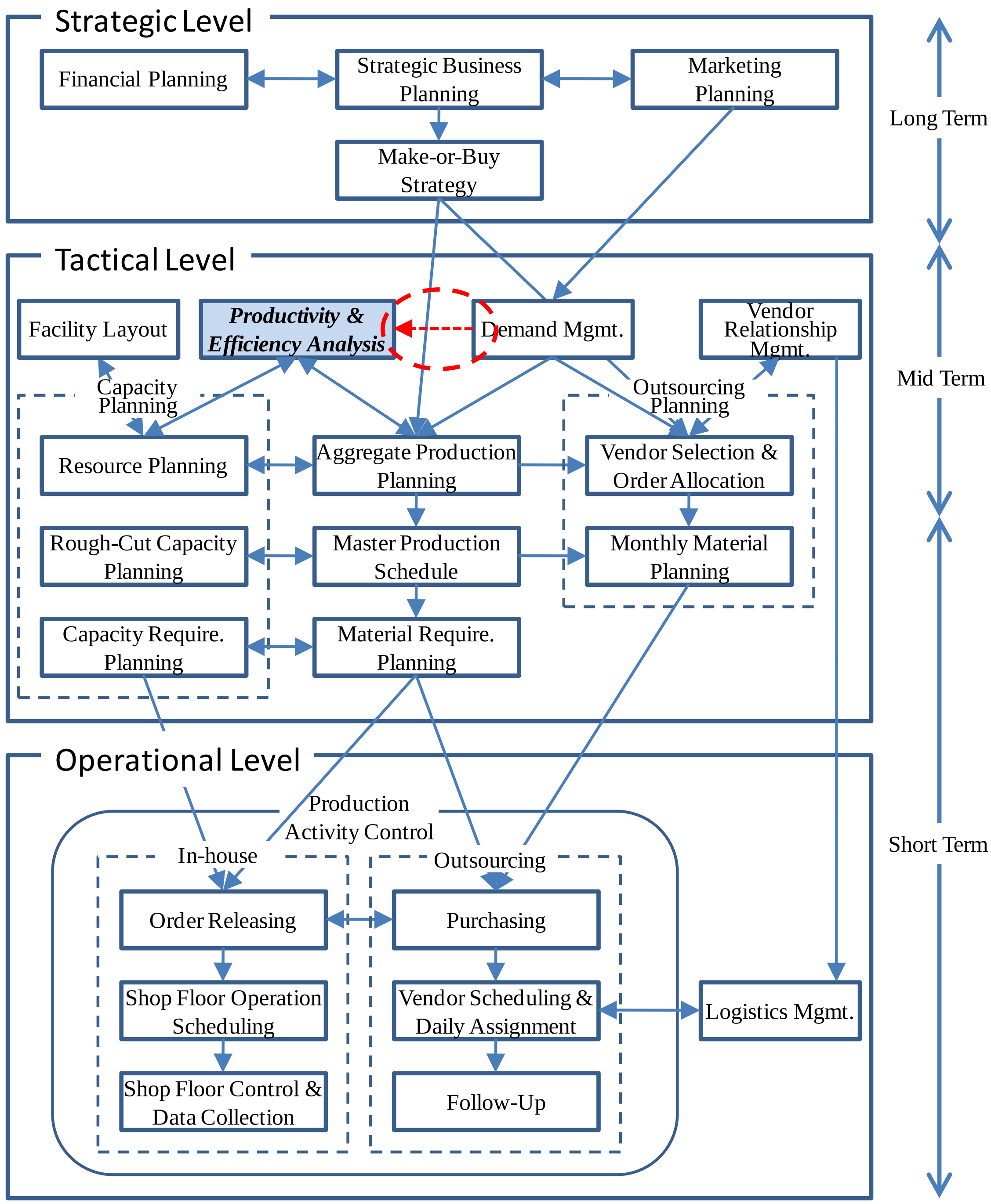

Figure 1.1 Research position in production planning and control (revised from Lin, 2006)

1.3 Significance and Objectives

The *objectives of the research* is to develop mathematical models to account for demand effects and the related uncertainty in productivity and efficiency analysis. The current productivity and efficiency analysis literature introduces a series of methodologies to assess production performance. However, in practice demand fluctuations will affect the production output level and bias the efficiency estimates. Output levels are partially decided based on expected demand. Thus, efficiency measures may capture not only production performance but also demand effects and customer relationships. If the production function is strictly defined as the relationship between input resource and output level and will be used to measure production performance, demand must be modeled. Insufficient demand levels cause overproduction and excess inventory; a higher demand level leads to underproduction and limited profits. The typical efficiency analysis does not model demand. Characterizing the effects of demand is critical to improving benchmarking techniques in a variety of applications.

Three tasks are executed to accomplishing the objective

1. Develop a Network DEA model to *decompose the production system* and decompose profitability efficiency change. The results of this model are used to identify improvement strategies. This model *separates the production process from the demand process* to allow efficiency estimates of both processes.
2. Develop a Proactive DEA model *to identify ex ante operational strategies and maximize effectiveness*. The firm adjusts variable inputs to change the output level and match demand levels.

3. Develop a Nash equilibrium model to ***identify the firms' profit maximizing production strategy***. This is important in an oligopolistic market in which price is a function of the output level of all firms.

1.4 Overview of This Study

This dissertation is organized as follows. Chapter I provides a background, significance and motivation of the demand effect in productivity and efficiency analysis, and describes the research aims. Chapter II describes a Network DEA model characterizing a manufacturing system or service system or hybrid of the two. This model addresses the issue of biased efficiency estimates caused by ignoring the effects of demand. An empirical study of the U.S. airlines industry is presented to demonstrate the proposed model. To deal with demand fluctuations in the short run, chapter III proposes a Proactive DEA model to measure "effectiveness". An application to Japanese Convenience Stores (CVS) is presented. Chapter IV demonstrates a Mix Complementary Problem (MCP) to find the Nash-Cournot equilibrium in an oligopoly market. Chapter V concludes with remarks and further research directions for considering demand issues in productivity analysis.

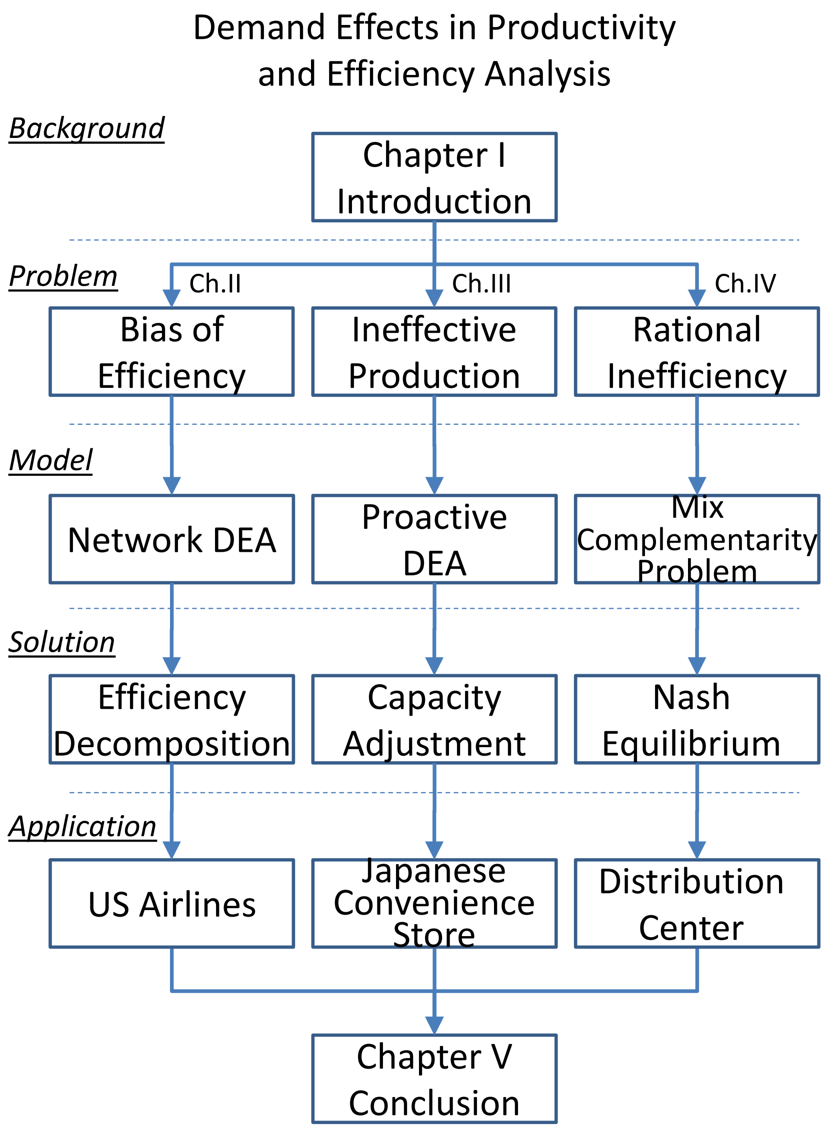

Figure 1.2 Overview of this study

CHAPTER II

EFFICIENCY DECOMPOSITION BY NETWORK DEA*

2.1 Introduction

Analysts of production systems use a variety of techniques to assess performance and search for improvement alternatives. Singh *et al.* (2000) claim three main categories of performance measurement techniques: index measurement, linear programming, and econometric models. The first includes the concept of total factor productivity or financial ratio, while the latter two categories are based on production function. The production economics approach, Hackman (2008), can be used to estimate the frontier production function and characterize how efficient production processes use inputs to generate outputs. Consequently, given the same input resource, a system is termed inefficient when its outputs levels are lower than other potential production processes. However, the reduced actual output can be caused by insufficient demand, i.e. demand fluctuations can bias productivity measures and lead to a decrease in measured efficiency. Similarly, in panel data analysis, the Malmquist productivity index quantifies efficiency change and technology change over time. Technical regress is often attributed to production issues when in actuality it may result from lack of demand. This study incorporates demand into the analysis and attributes some changes in production to demand.

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The literature on the demand effect in productivity analysis divides into two streams. One stream uses parametric equilibrium models to measure total factor productivity (TFP) change. Nakiri and Schankerman (1981) discuss the reasons for productivity slowdown observed between 1965 and 1978. The authors propose a model for decomposing changes in TFP that identifies the contributions of factor-price effect, demand effect, R&D effect, and technical change. They conclude the productivity slowdown of American manufacturing was mainly due to the deceleration in demand growth. Appelbaum and Berechman (1991) provide a market equilibrium model that considers supply (cost), demand, and government regulatory conditions. The model builds an output demand function to represent the relationship between the supply-side provision of firms and the demand-side consumption of customers, calculates the cost growth rate, and decomposes it into changes in outputs scale, factor prices and technical efficiency. It also calculates the growth rate of cost efficiency to clarify the effects on demand and regulatory conditions. Good *et al.* (1999) further describe static and dynamic factor demand models to measure TFP growth and its decomposition.

The second literature stream models demand generation or consumption as a component of a production system. In other words, a firm uses its marketing or sales departments to change its demand level. Studying the performance evaluation of a transportation system, Fielding *et al.* (1985) distinguish between the production process and the consumption process, arguing that output consumption is substantially different from output production since transportation services cannot be stored. They propose three performance indicators for a transit system: cost efficiency, service effectiveness, and cost effectiveness. More specifically, they define service effectiveness as service consumption normalized by the service output. However, their study only considers a single factor productivity ratio which assumes that other resources are unlimited and other outputs are unrelated. Chen and McGinnis (2007) discuss the limitations of focusing on a single productivity indicator rather than attempting to model all important factors in a production system. Lan and Lin (2005) and Yu and Lin (2008) study similar transportation systems and use Data Envelopment Analysis (DEA) and Network DEA models to characterize a consumption process. Ertay and Ruan (2005) present a methodology with one efficiency measure to identify the most efficient number of operators and the efficient assignment of labor in a cellular manufacturing system. DEA is used to measure efficiency and a simulation model is used to model capacity design and demand generation. Ertay *et al.* (2006) present a DEA approach to evaluate a facility layout with both quantitative and qualitative metrics. They apply an analytic hierarchy process (AHP) to aggregate the qualitative data such as flexibility in volume and variety and quality, and quantitative criteria such as material handling cost, adjacency score, shape ratio, and material handling vehicle utilization. Although these and similar studies integrate demand factors and the variable levels of demand, they do not considered the network structure of production or a dynamic productivity analysis.

Noting that the demand and the production system characterized in service production systems typically differ from those in manufacturing due to the types of demand described below, this study models the demand generation process (i.e. a marketing department) explicitly as a component of the production system. Figure 2.1

describes a manufacturing production system in terms of a serial model where the input resource is transformed into actual output product (Lee and Johnson, 2011). The first component, capacity design process, identifies the maximal output level as the peak output or the historical best performance. The demand generation process attempts to generate sufficient demand to support maximal output without idle capacity. The operations process transforms raw material into final product. Finally, the demand consumption process measures realized demand – the amount of final product consumed by customers. Manufacturers tend to receive demand based on the contractual agreements made between a manufacturer and a customer with defined sales quantities and prices prior to production. Thus, the manufacturer commonly develops an internal demand-generating process due to long production lead times (unlike service production systems which tend to rely mainly on non-contract demand requested informally by customers after production). The result is an external demand consumption process. Since services are typically non-storable commodities which must be immediately consumed by customers once transformed from inputs, i.e. demand consumption can be inefficient in that many service opportunities go unused.

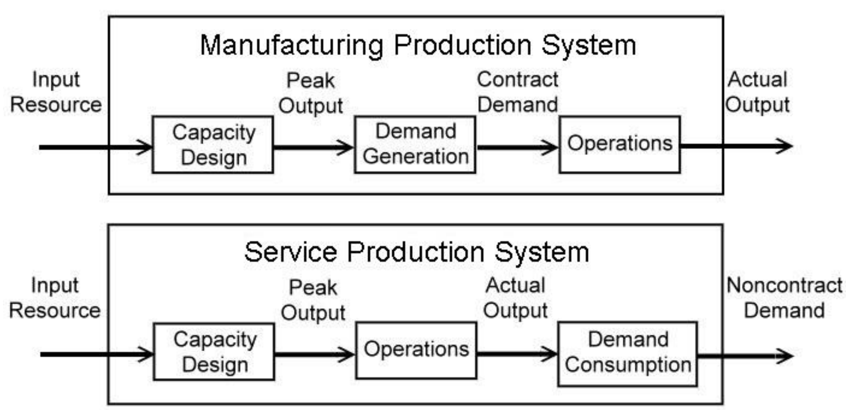

Figure 2.1 Manufacturing vs. service production systems

In addition, changes in demand can also have an effect on the measurement of productivity or profitability changes over time as estimated through frontier shifts indicating either technical progress or regress. Nishimizu and Page (1982) decompose total factor productivity into technology progress and change in efficiency. Färe *et al.* (1992, 1994) develop the explicit measurement of productivity change based on the Malmquist productivity index (MPI) proposed by Cave *et al.* (1982), which uses Shephard's input distance function (Shephard, 1953) to estimate inefficiency nonparametrically. The productivity change estimated via MPI can also be decomposed into two components: change in technology and change in efficiency. Färe *et al.* (1994) develop an additional component, change in scale. Alternatively, Ray and Mukherjee (1996) use the Fisher productivity index and propose a decomposition into efficiency change, technical change described by the cost function index, change in scale efficiency captured by the average cost index, change in allocative efficiency, and an adjustment index which captures the pure effect of a change in the attributes on the measured productivity index. However, their decomposition is restricted to the single-output technology and mixed-period measures, making interpretation difficult. Zofio and Prieto (2006) present a decomposition of the Fisher index into the MPI and an economic component consisting of allocative efficiency and a residual allocative term based on a generalized distance function which employs a relative weight of the input- and output-oriented projection paths to the frontier. Their decomposition also has some limitations, because the residual terms with mixed-period measures are difficult to interpret and weighting the projections is debatable. Recently, Kuosmanen and Sipiläinen (2009) propose an exact decomposition of the Fisher productivity index into five components: change in efficiency, technical change, change in scale efficiency, change in allocative efficiency, and price effect. Their decomposition reveals that the change in profitability efficiency is the product of only three components (change in efficiency, change in scale efficiency, and change in allocative efficiency) and is invariant to both technical change and price effect. Note that the price effect begins to integrate demand-side effects into the productivity analysis. This study extends Kuosmanen and Sipiläinen work to make the effects of demand more explicit.

2.2 Literature Review of Productivity in the Airline Industry

An airline's production system is a hybrid of the manufacturing and service systems described above. Any individual airline's production process is characterized by transforming capital, labor, energy, and materials into passenger and cargo services. The sources of uncertainty are capital utilization rates, changing technology, labor-intensive services, and demand diversity. Obviously, an airline operates under enormous pressure to maintain the high service rates that give it a competitive edge. The existing academic literature discusses the productivity change in the global airline industry in light of price changes in crude oil and jet fuel, the introduction of ecommerce, rising interest rates, deregulation, etc.

Sickles *et al.* (1986) consider the passage of the US federal Air Deregulation Act of 1978 (ADA) in improving the ability of price adjustment and competition capability and identify the effect of a rapid increase in the price of jet fuel. The result of analyzing allocative inefficiency from 1970 to 1981 supports the common perception that deregulation reduces inefficiency and the total cost of distortions from cost-minimizing allocation. However, Sickles et al. attribute the largest benefits to administrative reforms in the early 1970s, including multiple route authorizations and show-cause proceedings to reduce cost and time in obtaining certificates, rather than ADA itself. Good *et al.* (1993a) investigate differences in productivity growth between European and US carriers during the period 1976–1986. Using a Cobb-Douglas stochastic frontier production model, potential efficiency gains of European liberalization are identified; however, while Great Britain favors liberalization, France and Italy oppose it since their airlines benefit from high levels of subsidies to cover operating losses. Ray and Mukherjee (1996) employ an efficiency decomposition of the Fisher productivity index in the US airline industry in 1983–1984 and quantify the productivity growth in each component. The comprehensive decomposition provides more detailed benchmarking information for productivity improvement.

Semenick Alam and Sickles (2000) use DEA and MPI to estimate the productivity growth of US airlines between 1970 and 1990 and employ second-stage regression with contextual variables to capture the efficiency difference caused by firm-specific characteristics. They use cointegration analysis to examine the existence of a stationary relationship between non-stationary variables over time and indicate that efficiency estimates of firms within the industry should be co-integrated since one firm's efficiency-enhancing technology should be adopted by other firms, else all will be driven out of the industry. Semenick Alam and Sickles identify a narrowing of the differences in efficiency over time between the top performers and the other firms. Färe *et al.* (2007) employ MPI to estimate productivity growth after deregulation from 1979 to 1992 and show that service quality, such as direct routings and arriving on time indeed affects industry productivity. Nevertheless, slow productivity growth indicates a decline in the quality of service post-deregulation. The additional research regarding dynamic efficiency or deregulation issue in airline industry, see Good *et al.* (1993b, 1995), Sickles (1985), Sickles *et al.* (2002).

The method proposed in this study provides an integrated decomposition of a production system and decomposes profitability change. The decomposition of a production system can characterize a typical manufacturing system where demand is realized and products are built-to-order, a typical service production system with spot demand, or a hybrid of the two.

Some previous studies neglect demand fluctuations which can have a significant impact on productivity. To address this omission, we apply 2DED to an empirical study of the US airline industry from 2006 to 2008. We decompose the production system into capacity design, demand generation, operations, and demand consumption, while characterizing potential frontier shifts over time by decomposing profitability efficiency change into technical efficiency change, scale efficiency change, and allocative efficiency change.

This section is organized as follows. Section 2.3 describes the modeling framework, illustrates the decomposition of the production system, and explicitly quantifies the role of demand in efficiency analysis. The 2DED model is presented for the purpose of productivity diagnosis and improvement. Section 2.4 describes a method to estimate production capacity via a sequential model, and then introduces a Network DEA model for efficiency decomposition of the production system. Section 2.5 focuses on profitability change and reviews both Shephard's distance function and the Malmquist productivity index, while integrating demand into a decomposition of change in profitability efficiency. Section 2.6 discusses the results of the case study and Section 2.7 concludes.

2.3 Model Description

2.3.1 Production System Decomposition

Our goal is to help a firm allocate its resources and efforts more effectively to improve system performance. Figure 2.2 illustrates an integrated production model of a hybrid of a manufacturing and a service production system. In order to identify the sources of inefficiency, we decompose the system efficiency into four components: capacity design, demand generation, operations, and demand consumption. In general, we note that a typical firm's industrial engineering division is responsible for capacity design (capacity planning). The marketing division is responsible for pricing and demand generation. The manufacturing and general maintenance divisions are responsible for operations. The sales, marketing and public relation divisions are responsible for demand consumption. Thus, depending on the source of inefficiency management is likely to have to work with different departments. To describe the decomposition comprehensively, we develop a Network DEA model and define the linking variables below.

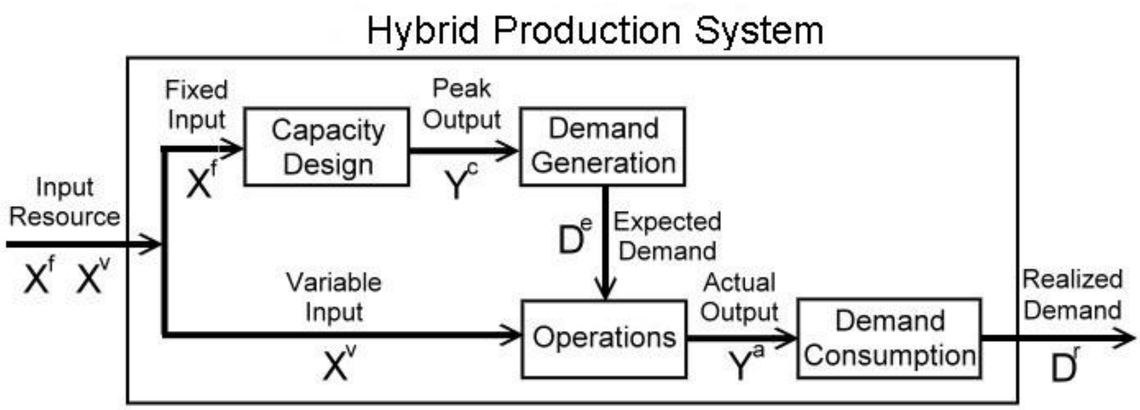

Figure 2.2 Process decomposition of a hybrid production system

2.3.1.1 Capacity Design

The first component is the capacity design process which defines the physical capacity of the production system and represents a limitation on long-term system performance. Poor capacity design would include purchasing capital that is incompatible with existing or other purchased capital, selecting outdated technologies, etc. The inputs to this phase, Fixed input, are the resources used to generate the infrastructure of the production system and support the operations of the production process. Peak output is the maximal output level the firm can achieve; it characterizes the production system's physical capability. Section 2.4.1 explains how to estimate peak output.

The efficiency of the capacity design component is defined as the ratio of the fixed input resources used to the production capacity. A critical assumption at this stage is sufficient demand exists to use the firm's current inputs completely. The design phase has a long-term impact on production performance.

2.3.1.2 Demand Generation

The second component is the demand generation process in which the sales group attempts to generate enough demand to completely utilize the built-in production capacity. The output of this stage is Expected demand, which is the sum of contracted demand and expected spot demand. The firm might generate more actual output as a buffer to capitalize on potential spot demand. For the purposes of simplification the expected spot demand is characterized as a proportional expansion of the contract demand or an expected value calculated from a historical distribution. Section 2.6.1 explains expected demand and Scheduled demand as they apply to the US airline industry. Typically contract demand is tractable and fulfilled more easily than spot demand. It is based on an agreement between the firm and a customer and has a specific sales quantity and price associated with it. However, in some situation such as the airline industry, the number of passengers flown is highly stochastic. Passengers might change their flight routes or cancel the itineraries just before the flight takes off. This uncertainty leads to differences between the contracted demand and the realized demand. This issue will be discussed in Section 2.6.3 in terms of contextual variables.

The efficiency of the demand generation component is defined as the ratio of expected demand to peak output. Typical productivity analysis assumes all deviations from the efficient frontier are attributed to inefficiency in the production system. Under these standard assumptions, insufficient demand may bias productivity measures.

2.3.1.3. Operations

The third component is the operational process in which raw materials are transformed into final goods or services. Thus, Actual output is the number of final products generated from the production process. In the airline industry it is characterized by Available output, the number of passenger-miles and freight-ton-miles generated.

The efficiency of this component is defined as the ratio of actual output to a weighted aggregation of expected output and variable input. In general, observed output may be reduced by scheduling inefficiencies, machine breakdown, inconsistent operational performance, etc.

2.3.1.4. Demand Consumption

The fourth component is the demand consumption process in which the sales group tries to sell any production beyond the contracted demand to maximize profit. Realized demand is the realized quantity of product or output customers actually consume at the market price after production. It is the sum of contract demand and realized spot demand. Our empirical study of the airline industry considers contract demand and spot demand as scheduled demand and non-scheduled demand, respectively.

The efficiency of this component is defined as the ratio of realized demand to actual output. This study focuses on the scenario in which realized demand (contract demand plus spot demand) is less than actual output. If the realized demand exceeds actual output, some customer requests will be off-loaded to other providers, substituted with a similar but different product, filled from inventories, produced using overtime, renegotiated for delivery to a subset of customers, etc. For the empirical study the linkage between the four components of efficiency and airlines service context is shown in Appendix A, where table A1 indicates the subcomponent and its corresponding factors mapping to flight factors in application.

2.3.2 Two-dimensional Efficiency Decomposition (2DED)

As mentioned, our 2DED model is a tool for productivity diagnosis and improvement. The two-dimensions for decomposition are the network structure of the firm in each cross-section of time (described in detail in Section 2.4) and profit efficiency change between periods over time (described in detail in Section 2.5). In the empirical study we collect panel data with the necessary variables to analyze the four components of the hybrid production systems defined above (see Section 2.6.1 and Appendix A for an explicit definition). The panel data will be analyzed as a series of cross-sectional analyses and then an index number approach will be used to investigate the change in profitability efficiency and its components. An index number is a metric to quantify productivity growth. If the index number is larger than 1, there is productivity growth, otherwise, productivity is constant or regresses; the details are described in Section 2.5.1. Decomposing an index number identifies the components of profitability change and can aid in identifying strategies a firm may use to improve.

Detail components of profitability allow us to scrutinize each airline firm's technical innovation, scale of production, and resource allocation. Figure 2.3 shows that we initially collect the crosssectional data in period t_0 and add the new data collected in period t_1 to the data set. In this way we use Diewert's sequential reference set method (Diewert, 1992) to estimate efficiency. Using Fisher's index (Kuosmanen and Sipiläinen, 2009) allows us to estimate the profitability change between the two periods. In other words, for one dimension, the components of the production system are identified in a cross-section; in the second dimension, panel data provides a dynamic efficiency analysis of profitability change. Table 2.1 illustrates that efficiency change can be decomposed into 12 components to help managers further identify improvement strategies.

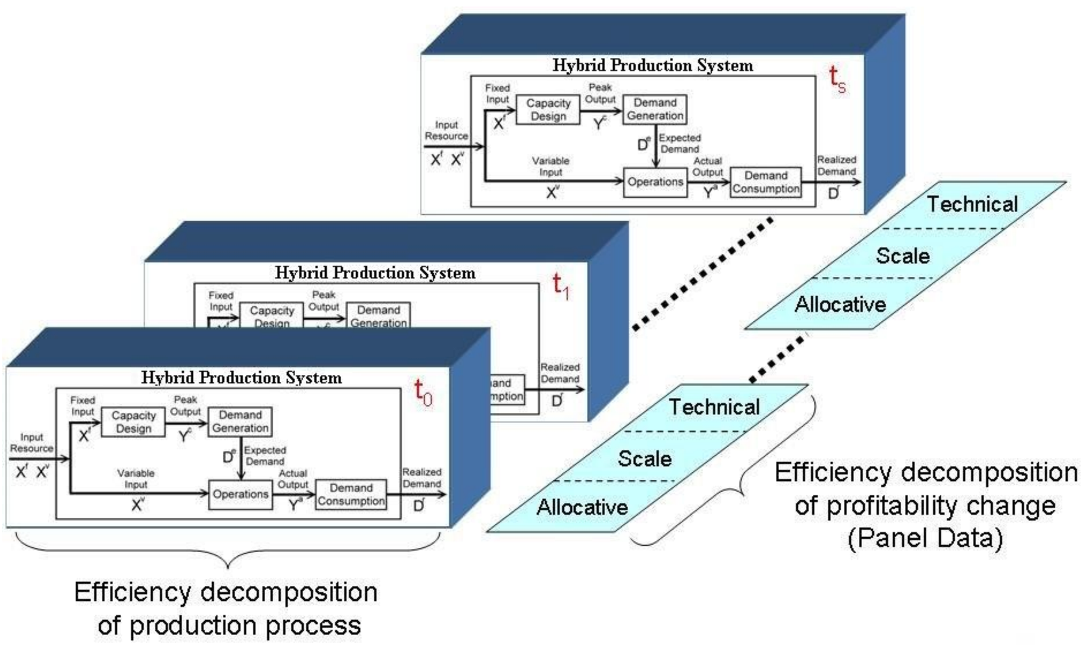

Figure 2.3 Two-dimensional efficiency decomposition (2DED)

Table 2.1 2DED illustration

		Hybrid production system			
		Capacity design	Demand generation	Operations	Demand consumption
Profitability	Technical Scale Allocative	Efficiency change estimates			

2.4 Efficiency Decomposition of Production Process

2.4.1 Capacity Estimation

To construct our Network DEA model, we first need to estimate the capacity level if no capacity data can be collected directly. For the purposes of this study we use Johansen's (1968) definition of physical capacity, which is the maximum amount that can be produced with existing plant and equipment (fixed inputs) given an unlimited availability of variable factors. Eilon and Soesan (1976) extend the concept from a single output to a multiple output case and propose a measure involving the radial expansion of the output vector given current technology and a fixed input vector. Based on Eilon and Soesan's definition, Färe *et al.* (1989) employ a nonparametric approach to obtain the capacity measure with a cross-sectional dataset.

The capacity is not directly observable, thus we will estimate the peak observed output as a proxy for capacity. In order to estimate the peak observed output, we need to identify a reference set to which we compare each observation in each period of time. Diewert (1980, 1992) describes a sequential method which constructs the production reference set by adding new observations to augment each previous period's reference set. The method assumes that a production process can be compared to any previously observed production process. Therefore, our empirical study uses Diewert's sequential method to estimate the firm specific capacity via output-oriented variable returns to scale (VRS) data envelopment analysis (DEA) and the reference set constructed from all previous period's observations of the firm's production. X_{ikt}^{f} is the ith fixed input resource of kth firm in tth period, Y_{qkt}^{a} is the amount of actual output for the qth product of kth firm in tth period, and λ_{kt} is the DEA envelopment multiplier variable of the kth firm in tth period. θ_{rs} is the efficiency estimate of firm r in the current period s. For firm r, the linear programming formulation is:

$$
\begin{aligned}
Max \quad & \theta_{rs} \\
s.t. \quad & \sum_{k,t\in\{0,\dots,s\}} \lambda_{kt} X_{ikt}^{f} \le X_{irs}^{f},\ \forall i \\
& \sum_{k,t\in\{0,\dots,s\}} \lambda_{kt} Y_{qkt}^{a} \ge \theta_{rs} Y_{qrs}^{a},\ \forall q \\
& \sum_{k,t\in\{0,\dots,s\}} \lambda_{kt} = 1 \\
& \lambda_{kt} \ge 0,\ \forall k, \forall t
\end{aligned}
\tag{2.1}
$$

If the efficiency is equal to 1, the physical capacity is equal to the number of actual outputs in period s , otherwise, the physical capacity is equal to the actual output multiplied by the efficiency estimate θ_{rs} , that is, $Y_{qrs}^{c} = Y_{qrs}^{a} \times \theta_{rs}$. Then, the time shifts to the next period and the new observation is added into the reference set, and the process repeats.

2.4.2 Efficiency Measurement by Network DEA

We use Kao's (2009) Network DEA model[1] for efficiency decomposition because it accounts for the interrelationship of the components of the production system rather than estimating efficiencies independently. Kao's model was developed under the constant return to scale (CRS) assumption. However, we relax this assumption and estimate the model assuming VRS. Let X_{ikt}^{f} and X_{jkt}^{v} be the i^{th} fixed and j^{th} variable input resource, and Y_{qkt}^{c}, D_{qkt}^{e}, Y_{qkt}^{a}, and D_{qkt}^{r} be the capacity, expected demand, actual output, and realized demand of the q^{th} product of the k^{th} firm in t^{th} period respectively. v_i^f, v_j^v, z_q^c, u_q^e, z_q^a and u_q^r are the associated multiplier variables respectively. z_0^c, u_0^e, z_0^a and u_0^r are the intercept variables. We estimate the input-oriented efficiency E_{rs}^{P} of the production system of firm r in period s with a sequential reference set using the following formulation.

The proposed VRS model estimates technical efficiency and provides scale efficiency estimation by means of Kao's CRS model. The formulation (2.2) adds the variables z_0^c, u_0^e, z_0^a and u_0^r. These variables characterize the intercept and relax the condition that the production function must pass through the origin.

[1] An important property of Kao's network DEA model is that the whole system is efficient only when all components are efficient in contrast to the traditional network DEA (Färe and Grosskopf, 2000).

$$E_{rs}^{P} = Max \sum_{q\in Q} u_q^r D_{qrs}^r - u_0^r$$

$$s.t. \ \sum_{i\in I} v_i^f X_{irs}^f + \sum_{j\in J} v_j^v X_{jrs}^v = 1$$

$$\sum_{q\in Q} z_q^c Y_{qkt}^c - \sum_{i\in I} v_i^f X_{ikt}^f - z_0^c \le 0, \ \forall k, \forall t \in \{1,\ldots,s\} \tag{2.2}$$

$$\sum_{q\in Q} u_q^e D_{qkt}^e - (\sum_{q\in Q} z_q^c Y_{qkt}^c - z_0^c) - u_0^e \le 0, \ \forall k, \forall t \in \{1,\ldots,s\}$$

$$\sum_{q\in Q} z_q^a Y_{qkt}^a - \left(\sum_{q\in Q} u_q^e D_{qkt}^e - u_0^e + \sum_{j\in J} v_j^v X_{jkt}^v \right) - z_0^a \le 0, \ \forall k, \forall t \in \{1,\ldots,s\}$$

$$\sum_{q\in Q} u_q^r D_{qkt}^r - (\sum_{q\in Q} z_q^a Y_{qkt}^a - z_0^a) - u_0^r \le 0, \ \forall k, \forall t \in \{1,\ldots,s\}$$

$$v_i^f, v_j^v, z_q^c, u_q^e, z_q^a, u_q^r \ge 0, \ \forall i, \forall j, \forall q$$

By solving this optimization model, the optimal multipliers, v_i^{f*}, v_j^{v*}, z_q^{c*}, u_q^{e*}, z_q^{a*} and u_q^{r*} are obtained and efficiency can be decomposed. Recall that estimating the efficiency of each component allows the firm to identify which component will give the largest system productivity gain if improved. Let E_{rs}^{D}, E_{rs}^{S}, E_{rs}^{O} and E_{rs}^{C} denote efficiency of production design, efficiency of demand generation, efficiency of operations and efficiency of demand consumption respectively:

$$E_{rs}^{D} = (\sum_{q\in Q} z_q^{c*} Y_{qrs}^c - z_0^{c*}) / (\sum_{i\in I} v_i^{f*} X_{irs}^f) \tag{2.3}$$

$$E_{rs}^{G} = (\sum_{q\in Q} u_q^{e*} D_{qrs}^e - u_0^{e*}) / (\sum_{q\in Q} z_q^{c*} Y_{qrs}^c - z_0^{c*}) \tag{2.4}$$

$$E_{rs}^{O} = (\sum_{q\in Q} z_q^{a*} Y_{qrs}^a - z_0^{a*}) / (\sum_{q\in Q} u_q^{e*} D_{qrs}^e - u_0^{e*} + \sum_{j\in J} v_j^{v*} X_{jrs}^v) \tag{2.5}$$

$$E_{rs}^{C} = (\sum_{q\in Q} u_q^{r*} D_{qrs}^r - u_0^{r*}) / (\sum_{q\in Q} z_q^{a*} Y_{qrs}^a - z_0^{a*}) \tag{2.6}$$

2.5 Efficiency Decomposition of Profitability Change

As mentioned, Kuosmanen and Sipiläinen (2009) propose to decompose the change in profitability efficiency into change in technical efficiency, change in scale efficiency, and change in allocative efficiency. More interestingly, change in profitability efficiency is invariant to technical change and change in price effect, i.e. competition and price fluctuation would not affect the change in profitability efficiency in their decomposition. Kuosmanen and Sipiläinen also assume that demand is beyond a firm's influence, however this assumption may not hold in many industries.

2.5.1 Decomposition of Profitability Efficiency Change

Now, we describe how the network decomposition of the production process can provide additional information for a decomposition of profitability efficiency characterizing a broader set of production processes and including firms that influence their demand levels through sales, advertising, etc. Let $x^t \in R_+^{i+j}$ denote an input factor of the production system in period t, and $y^t \in R_+^q$ denote an output factor of the production system in period t. $T^t = \{(x,y): x\ can\ produce\ y\ in\ period\ t\}$ is the technology defining the production possibility set at period t by a piece-wise linear convex function enveloping all observations. Model (2.7) has a dual formulation with dual multipliers, α_{kt}, β_{kt}, γ_{kt} and δ_{kt}, and illustrates the feasible region of production possibility set T^t and the multipliers associated with the four components of the production system decomposition:

$$\widetilde{T}^t = \{(x,y): \sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\alpha_{kt}Y^c_{qkt} - \sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\beta_{kt}Y^c_{qkt} \geq 0,\ \forall q$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\beta_{kt}D^e_{qkt} - \sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt}D^e_{qkt} \geq 0,\ \forall q$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt}Y^a_{qkt} - \sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt}Y^a_{qkt} \geq 0,\ \forall q \tag{2.7}$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt}D^r_{qkt} \geq D^r_{qt},\ \forall q$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\alpha_{kt}X^f_{ikt} \leq X^f_{it},\ \forall i$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt}X^v_{jkt} \leq X^v_{jt},\ \forall j$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\alpha_{kt} = 1,\ \sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\beta_{kt} = 1,$$
$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt} = 1,\ \sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt} = 1,$$
$$\alpha_{kt},\beta_{kt},\gamma_{kt},\delta_{kt} \geq 0,\ \forall k,\forall t = \{1,\ldots,s\}\ \}.$$

Note that VRS is allowed through this characterization of the production possibility set. Defining $D^t_x(x,y)$ generated directly from the above model as the inverse of Shephard's input-oriented distance function allows us to measure the production efficiency of an observation at period t relative to the production possibility set at period t. In other words, the input-oriented technical efficiency (ITE) is defined as $D^t_x(x,y) = \inf\{\theta \mid (\theta x, y) \in \widetilde{T}^t\}$.[2] Similarly, the output-oriented technical efficiency (OTE) is defined as $D^t_y(x,y) = \inf\{\theta \mid (x, y/\theta) \in \widetilde{T}^t\}$.

---

[2] We set $D^t_x(x,y)$ as the inverse of Shephard's input-oriented distance function allowing $D^t_x(x,y)$ to be obtained directly from the proposed model. This change allows a more natural intuition regarding productivity change, i.e. $D^{t+1}_x(x,y)/D^t_x(x,y) > 1$ represents technical progress between periods t and $t+1$, vice versa $D^{t+1}_x(x,y)/D^t_x(x,y) < 1$ shows technical regress.

Now, we obtain the cost function and revenue function as $C^t(w,y)=\min_x\{w\cdot x\,|\,(x,y)\in\tilde{T}^t\}$ and $R^t(x,p)=\max_y\{p\cdot y\,|\,(x,y)\in\tilde{T}^t\}$, given input price w and output price p respectively. Then, the profitability function $\rho^t(w,p)=\max_{x,y}\{\frac{p\cdot y}{w\cdot x}\,|\,(x,y)\in\tilde{T}^t\}$ presents the maximal return to Dollars achievable with the given input and output price. We define the profitability efficiency (ρE) as the ratio of the profitability of an observation and the maximum profitability given the specific input and output price $\rho E^t(w^t,p^t;x^t,y^t)=\dfrac{p^t\cdot y^t/w^t\cdot x^t}{\rho^t(w^t,p^t)}$.

While such envelopment models allow us to easily calculate the efficiency of the component, the use of dual multiplier models facilitates the analysis of the cost, revenue, and profitability functions. For example given a firm r in period s, we can calculate the profitability function of production system by the following formulation, where p^r_{qrs}, w^f_{irs} and w^v_{jrs} are the unit prices for the q^{th} product with realized demand D^r_{qrs}, the i^{th} fixed input X^f_{irs}, and the j^{th} variable input X^v_{jrs} respectively. The formulation below is similar for the cost and revenue function except that we replace the objective function by minimizing $\sum_i w^f_{irs}X^f_{irs}+\sum_j w^v_{jrs}X^v_{jrs}$ and maximizing $\sum_q p^r_{qrs}D^r_{qrs}$ respectively:

$$Max\ \frac{\sum_q p^r_{qrs}D^r_{qrs}}{\sum_i w^f_{irs}X^f_{irs}+\sum_j w^v_{jrs}X^v_{jrs}} \tag{2.8.1}$$

$$s.t.\ \sum_{k\in K}\sum_{t\in\{1,\dots,s\}}\alpha_{kt}Y^c_{qkt}-\sum_{k\in K}\sum_{t\in\{1,\dots,s\}}\beta_{kt}Y^c_{qkt}\geq 0,\ \forall q \tag{2.8.2}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \beta_{kt} D^{e}_{qkt} - \sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \gamma_{kt} D^{e}_{qkt} \geq 0, \ \forall q \tag{2.8.3}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \gamma_{kt} Y^{a}_{qkt} - \sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \delta_{kt} Y^{a}_{qkt} \geq 0, \ \forall q \tag{2.8.4}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \delta_{kt} D^{r}_{qkt} \geq D^{r}_{qrs}, \ \forall q \tag{2.8.5}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \alpha_{kt} X^{f}_{ikt} \leq X^{f}_{irs}, \ \forall i \tag{2.8.6}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \gamma_{kt} X^{v}_{jkt} \leq X^{v}_{jrs}, \ \forall j \tag{2.8.7}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} (\alpha_{kt} - \beta_{kt}) = 0 \tag{2.8.8}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} (\beta_{kt} - \gamma_{kt}) = 0 \tag{2.8.9}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} (\gamma_{kt} - \delta_{kt}) = 0 \tag{2.8.10}$$

$$\sum_{k \in K} \sum_{t \in \{1,\ldots,s\}} \delta_{kt} = 1 \tag{2.8.11}$$

$$\alpha_{kt}, \beta_{kt}, \gamma_{kt}, \delta_{kt} \geq 0, \ \forall k, \forall t \in \{1,\ldots,s\} \tag{2.8.12}$$

Note that we can augment or replace the constraints in the dual model (2.8.1)-(2.8.12) with different equations and can adjust the objective function to estimate the profitability function of each component. Equations (2.8.13)-(2.8.18) for the profitability function of capacity design, (2.8.19)-(2.8.27) for the demand generation, (2.8.28)-(2.8.35) for operations, and (2.8.36)-(2.8.40) for the demand consumption appear in Appendix A.

As mentioned, Kuosmanen and Sipiläinen (2009) also propose an exact decomposition of the Fisher ideal TFP index. The Fisher ideal TFP is the product of the change in the components of technical efficiency (ΔTE), technical change ($\Delta Tech$), change in scale efficiency (ΔSE), change in allocative efficiency (ΔAE), and change in price effect (ΔPE). Interestingly, Kuosmanen and Sipiläinen show that the change in profitability efficiency ($\Delta \rho E$) is invariant to $\Delta Tech$ and ΔPE, i.e. $\Delta \rho E$ has three parts: ΔTE, ΔSE, and ΔAE. $\Delta \rho E$ already captures technical change and price change through the target point and the price change is characterized through the identification of the allocatively efficient benchmark. The formulation of change in profitability efficiency is shown in appendix A.

2.5.2 Profitability Efficiency and Financial Performance Index

This section identifies a connection between profitability efficiency and financial performance indices in order to motivate the relevance of the less widely used metric profitability efficiency. Return on investment (ROI) is considered a crucial indicator of a firm's financial performance. The Dupont ROI formula (Brown, 1927) decomposes this index into two ratios. The first is the ratio of return on sales (ROS) which measures a firm's ability to generate profit related to its sales revenue. The second is the ratio of investment turnover which measures how effectively a firm can generate revenue using investments. The Dupont ROI formula is:

$$\begin{aligned} \mathrm{ROI} &= \mathrm{profits} / \mathrm{investment} = \frac{\mathrm{profits}}{\mathrm{revenue}} \times \frac{\mathrm{revenue}}{\mathrm{investment}} \\ &= \mathrm{ROS} \times \mathrm{InvestmentTurnover} \end{aligned} \tag{2.9}$$

The ROS component reveals the profitability ratio which measures the revenue to cost (Banker *et al.*, 1993; 1996):

$$\mathrm{ROS} = \frac{\mathrm{profits}}{\mathrm{revenue}} = \frac{\mathrm{revenue} - \mathrm{cost}}{\mathrm{revenue}} = 1 - \frac{1}{\mathrm{profitability}} \tag{2.10}$$

Under a fixed investment turnover rate the higher the profitability the higher the ROI. This illustrates a strong relationship between profitability and ROI.

There are three reasons for employing a profitability efficiency index to assess a firm's productivity performance. First, profitability is a more reasonable index to assess productivity than a profit index, because the profitability function is homogenous of degree zero in prices. Namely, while the price doubles, the profit doubles, but the profitability does not change. This unscaled nature of profitability is similar to productivity and represents the input-to-output performance. Second, profitability efficiency is a benchmarking technique that builds on the concept of the production possibility set and clearly identifies the frontier and facilitates for comparisons, in contrast to profitability or profit indices. Third, simple output-input ratios do not reflect all of the critical factors in performance evaluation (Chen and McGinnis, 2007), because partial productivity ratios relating a single output to a single input postulate that all other resources are always adequate and the production of any other outputs are irrelevant. Therefore, we select profitability efficiency and change in profitability efficiency as our indices.

2.6 Empirical Study

Our empirical case study analyzes the US airline industry from 2006 to 2008 using a data set of 15 firms. The data was gathered from Air Carrier Financial Statistics and Air Carrier Traffic Statistics published by the Bureau of Transportation Statistics within the Research and Innovative Technology Administration (RITA, 2009). Each observation is one airline firm in a given year. The data definitions of input and output factors for the productivity analysis are described in Section 2.6.1. Section 2.6.2 gives a detailed analysis of each firm's production process employing a Network DEA model for process decomposition. Further profitability efficiency change is quantified for each component and the production system as a whole. Section 2.6.3 summarizes the efficiency differences between civil airlines and cargo airlines using a contextual variable approach.

2.6.1 Data Description

We characterize the resources used in the production system as: aircraft fleet size as a fixed input, fuel and employees as variable inputs, and capacity, scheduled demand, and available output as intermediate factors with the two dimensions, passenger and freight; realized demand is the final output (see Appendix A for the raw data). We estimate capacity peak output by fixed input and scheduled demand data via the sequential method in Section 2.4.1. The following describes the resources.

2.6.1.1 Inputs

Aircraft fleet size (FS) is the average number of aircraft employed in a firm over a particular year. However, a firm may own different models of airplanes purchased in different years, giving rise to a vintage issue, Johansen (1968). To address the heterogeneity of capital issue, we transform the data based on number of seats per model type so that each fleet is measured in Boeing-737 equivalent units. In general, since a firm's fleet is the most significant component of capital and is difficult to change in the short-term, we model the capital as a fixed input. We obtain firm-specific prices by dividing the flight equipment capital reported in the firms' balance sheets by the average number of equivalent Boeing- 737 aircraft.

Fuel (FU) is the number of gallons consumed annually, estimated by fuel expenses over the average jet fuel cost per gallon. Note that FU is a variable input because its usage can be controlled on a day-to-day basis.

Employee (EP) is defined as the number of employees during the year, which includes flight shipping staff, pilots, flight attendants, and managers but not ground shipping drivers. Average prices are calculated by salaries and benefits expenses over number of employees. EP is modeled as a variable input since firms can partially adjust this variable in the short-term.

2.6.1.2 Demand and Output Levels

Scheduled passenger demand (SPD) is the scheduled revenue passenger-miles for a particular year. We measure passenger service using revenue passenger-miles, the number of revenue-paying passengers aboard the airplane multiplied by the distance traveled measured in miles. The average price per passenger mile for SPD is calculated as the scheduled passenger revenue divided by passenger-miles.

Scheduled freight demand (SFD) is defined as the demand of scheduled revenue freight-ton-miles for a particular year. We measure freight service using revenue freight-ton-miles, the weight of freight and mail measured in tons multiplied by the distance flown measured in miles. The average price for SFD is calculated as the scheduled freight and mail revenue divided by ton-miles.

Available passenger output (APO) is the actual output of available seat-miles during the year. Available seat-miles is calculated as the number of seats including first class and economy on an airplane multiplied by the distance traveled measured in miles. The average price for APO is equivalent to the price used in scheduled passenger demand.

Available freight output (AFO) is the actual output of available freight-ton-miles during the year. Available freight-ton-miles is calculated as the number of available tons of freight and mail multiplied by the distance flown measured in miles. Note that it is calculated by subtracting revenue passenger-ton-miles from total available ton-miles. The average price for AFO is equivalent to the price employed in scheduled freight demand.

Realized passenger demand (RPD) is the realized demand of scheduled and nonscheduled revenue passenger-miles during the year. The realized demand is calculated as the sum of scheduled and nonscheduled revenue passenger-miles. The average price for RPD is calculated by total passenger revenue over scheduled and nonscheduled passenger-miles.

Realized freight demand (RFD) is the realized demand of scheduled and nonscheduled revenue freight-ton-miles during the year. The realized demand is calculated as the sum of scheduled and nonscheduled revenue freight-ton-miles. The average price for RFD is calculated by total freight and mail revenue over scheduled and nonscheduled ton-miles.

2.6.1.3 Capacity Estimation

Peak passenger output (PPO) is the maximal output level of revenue passenger-miles during the year. We estimate it using the sequential frontier method described in Section 2.4.1 with aircraft fleet size as fixed input and available passengers as the output. The average price for PPO is equivalent to the price in scheduled passenger demand.

Peak freight output (FPO) is defined as the maximal output level of revenue freight-ton-miles during the year. We estimate it using the sequential frontier method with aircraft fleet size as fixed input and available freight as the output. The average price for FPO is equivalent to the price employed in scheduled freight demand.

The flight data is ordered according to capacity design, demand generation, operations, and demand consumption. Table 2.2 shows the factor mapping table of the production process and the data set.

Table 2.2 Factor mapping table of production process and airline data set

Components	Factor (ref. Figure 2)	Flight Factor
Capacity Design	Fixed Input	FS
	Peak Output	PPO, PFO
Demand Generation	Peak Output	PPO, PFO
	Expected Demand	SPD, SFD
Operations	Variable Input	FU, EP
	Expected Demand	SPD, SFD
	Actual Output	APO, AFO
Demand	Actual Output	APO, AFO
Consumption	Realized Demand	RPD, RFD

2.6.2 Productivity Change Analysis

Table 2.3 presents the results of our efficiency decomposition analysis based on network structure for a partial set of the firms in 2006 (the entire table appears in Table A2 of Appendix A). Note that the efficiency estimates and thus the decomposition are based on the production possibility set of all previous periods because Diewert's sequential method is used.

Consider Alaska Airlines with an input-oriented technical efficiency (ITEff) of 0.89. Further investigation of the components of efficiency reveals that it is not an issue of poor capacity design or operational inefficiency, but rather that the system inefficiency is mainly caused by insufficient demand generation and consumption (both efficiencies are 0.94). We conclude that management should focus on raising demand rather than making operational changes, perhaps by asking sales and marketing to address the productivity concerns. In contrast, Continental Airlines' system efficiency of 0.81 is largely due to poor capacity design and unfavorable operation process (both efficiencies are 0.90). We conclude that management should engage in capacity redesign and investigate operation behavior to improve overall productivity.

Table 2.4 and table 2.5 show how 2DED provides process and dynamic efficiency analysis. Recall that we separate the efficiency decomposition of profitability change into changes in technical efficiency, change in scale efficiency, and change in allocative efficiency and decompose them into our four components. Note that ΔTE, ΔSE , and ΔAE are not mutually independent, but have different strategic interpretations. ΔTE characterizes the firm's change in efficiency and productivity, which is largely driven by process improvement. ΔSE measures a firm's ability to adjust scale size in the long-term. ΔAE indicates a firm's ability to allocate input and output resource to achieve maximal profitability with respect to a specific price.

Table 2.4 shows the weighted average profitability efficiency change for the production system and for each component. Within each component performance is decomposed into technical, scale, and allocative effect for the 15 airlines. The average is weighted by the dollar measure of peak output. Observe the overall progress in the average profitability change from 2006 to 2008, where the average profitability change of production system is 1.015. The capacity design component is 1.02, demand generation is 0.99, operational component is 1.00, and demand consumption is 0.99. Considering each component individually, the 2% improvement on average of capacity design efficiency over the time horizon indicates that the airlines have been proactive in improving their capacity installation. Note also that the profitability efficiency changes in demand generation and consumption components (around 0.99) indicate that some airlines are failing to generate sufficient demand and to stimulate product consumption.[3] The operations component represents no significant average change in profitability. Further investigating the yearly effect, profitability regresses in 2007–2008 and nine firms experience profitability decline (67% regress in the design component, 89% in demand generation, 33% in operation, and 78% in demand consumption). These results indicate that most firms could improve productivity through stimulating demand and improved marketing. Table 2.4 also shows that the variation of capacity design is larger

[3] The demand-related components have statistically significant differences from the design component by t-test with α value equal to 0.1.

than the other three components in 2006–2008. We conclude that the design process is a significant component and will influence profitability.

Table 2.3 Technical, scale, allocative, and profitability efficiency decomposition

Firm	Production System				Design				Generation				Operations				Consumption			
	ITE	ISE	IAE	ρE	ITE	ISE	IAE	ρE	ITE	ISE	IAE	ρE	ITE	ISE	IAE	ρE	ITE	ISE	IAE	ρE
AirTran Airways	0.97	0.71	0.79	0.55	0.94	0.73	1.00	0.68	0.90	0.91	1.00	0.82	1.00	0.91	0.95	0.86	0.90	0.91	1.00	0.82
Alaska Airlines	0.89	0.65	0.96	0.55	1.00	0.74	1.00	0.74	0.94	0.86	0.92	0.74	1.00	0.90	0.94	0.84	0.94	0.86	0.92	0.74
American Airlines	1.00	0.71	1.00	0.71	1.00	0.87	1.00	0.87	1.00	0.89	1.00	0.89	1.00	0.86	1.00	0.86	1.00	0.89	1.00	0.89
American Eagle	0.92	0.42	0.84	0.32	0.92	0.51	1.00	0.46	1.00	0.80	1.00	0.80	1.00	0.74	0.86	0.64	1.00	0.80	1.00	0.80
Continental	0.81	0.96	0.94	0.73	0.90	0.89	1.00	0.79	1.00	1.00	1.00	1.00	0.90	0.92	0.93	0.77	1.00	1.00	1.00	1.00

Table 2.4 2DED of US airline industry

Overall Components	2006-2007				2007-2008				2006-2008 (GeoMean)			
	$\Delta\rho E$	ΔTE	ΔSE	ΔAE	$\Delta\rho E$	ΔTE	ΔSE	ΔAE	$\Delta\rho E$	ΔTE	ΔSE	ΔAE
Production	1.035	0.999	1.024	1.012	0.996	1.015	0.994	0.987	1.015	1.007	1.009	0.999
Design	1.052	1.039	1.014	0.998	0.989	0.990	0.999	1.001	1.020	1.014	1.007	0.999
Generation	0.995	0.998	1.003	0.994	0.985	0.992	0.998	0.994	0.990	0.995	1.000	0.994
Operations	0.990	0.993	0.999	0.999	1.009	0.998	1.014	0.997	1.000	0.995	1.007	0.998
Consumption	0.996	0.999	1.003	0.994	0.988	0.993	0.998	0.997	0.992	0.996	1.001	0.995

Table 2.5 shows the detailed 2DED for a partial set of the airlines in 2006-2008; the full table which includes the average (Geometric Mean, GM) change of production system is summarized in appendix A. The figure shown in appendix A maps the average ΔTE and ΔAE of the production system by each airline on a two-dimensional coordinate (Figure A1), and figure A2 uses ΔSE and ΔAE to construct a similar figure. Thus, the four quadrants reveal the strategy of productivity improvement. Using

SkyWest Airlines (point K) as an example, observe a high performance in profitability change $\Delta\rho E$ of 1.091, an above-average ΔTE of 1.023 and an ΔSE of 1.071, and a relatively poor ΔAE of 0.996. Further drilling down into ΔAE via efficiency decomposition reveals an ΔAE value of capacity design of 1.00, demand generation of 0.98, operations of 1.00, and demand consumption of 0.98. Thus, SkyWest Airlines should strive to improve its resource allocation in demand generation and consumption process to catch up with its competitors.

Table 2.5 2DED of US airline firms

			Production				Design				Generation				Operations				Consumption			
Firm	#	Year	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA
AirTran Airways	A	06->07	1.12	0.97	1.10	1.04	1.08	1.00	1.07	1.01	1.06	1.05	1.01	1.00	0.96	1.00	0.97	0.99	1.06	1.05	1.01	1.00
		07->08	1.07	1.07	1.03	0.97	1.03	1.02	1.01	1.00	1.05	1.05	0.99	1.01	0.96	1.00	0.99	0.97	1.05	1.05	0.99	1.01
		GM	1.09	1.02	1.06	1.00	1.05	1.01	1.04	1.00	1.05	1.05	1.00	1.00	0.96	1.00	0.98	0.98	1.05	1.05	1.00	1.00
Alaska Airlines	B	06->07	0.99	1.00	1.01	0.99	1.01	1.00	1.01	1.00	0.97	0.99	1.00	0.98	1.02	1.00	1.00	1.02	0.97	0.99	1.00	0.98
		07->08	0.92	0.98	0.96	0.97	0.97	0.97	1.00	1.00	0.98	1.01	0.99	0.98	0.95	1.00	1.00	0.95	0.98	1.01	0.99	0.98
		GM	0.95	0.99	0.99	0.98	0.99	0.99	1.00	1.00	0.97	1.00	0.99	0.98	0.99	1.00	1.00	0.99	0.97	1.00	0.99	0.98
American Airlines	C	06->07	0.98	1.00	1.00	0.98	0.97	0.97	1.01	1.00	1.01	1.00	1.01	1.00	0.97	1.00	0.98	0.99	1.01	1.00	1.01	1.00
		07->08	0.99	0.98	1.02	0.99	1.00	0.98	1.02	1.00	0.98	0.99	1.00	1.00	1.01	1.00	1.00	1.00	0.98	0.99	1.00	1.00
		GM	0.99	0.99	1.01	0.98	0.99	0.98	1.01	1.00	1.00	0.99	1.01	1.00	0.99	1.00	0.99	1.00	1.00	0.99	1.01	1.00

Appendix A includes airlines that are largely cargo service carriers as indicated by triangle points N and H. Observation H is below the average of productivity growth. However, this result may seem counter-intuitive because it performs well in terms of the profitability efficiency levels shown in Table A2. Note that firms with high levels of efficiency initial tend to have small productivity changes in the future. In general, this phenomenon is the result of the public good nature of technology that leads to spillover effects from leaders to followers as the laggards learn from the innovators and play catch-up (Semenick Alam and Sickles, 2000). Here, the distinct nature of civil and cargo service may weaken the catch-up effect between these two types of providers. We discuss the efficiency differences below.

2.6.3 Contextual Variables

Service type plays an important role and significantly affects the earning structure of airline firms. As previously mentioned, there are two business strategies, one of which focuses primarily on civil and the other on cargo services. A hypothesis test (Banker, 1993) is commonly used to assess which group is more efficient.

We use three hypothesis tests: two F-tests assume inefficiency follows exponential distribution and half-normal distribution respectively; the Kolmogorov–Smirnov test imposes no assumption on the distribution of inefficiency. All three tests have p-values less than 0.05, indicating that the distribution of profitability inefficiency differs significantly between civil and cargo carriers.

We also use the two-stage approach proposed by Ray (1988, 1991) to evaluate how the business models affect the profitability efficiency of the production systems. This model has received considerable attention in the recent literature (Banker and Natarajan, 2008; Simar and Wilson, 2012). See Johnson and Kuosmanen (2009, 2012) for alternative models and an insightful discussion to this debate. In the two-stage approach, a dummy variable equal to 1 represents a cargo airline service, while 0 represents a civil airline service. Table 2.6 shows the results of a second stage least square regression model, in which efficiency is regressed against the dummy variable. Note that the profitability efficiency of the overall production system in cargo service is

21% more efficient than civil service, and that efficiency is significantly affected by the capacity design component. Two main reasons support these results: the distinct nature of the passenger shipping network structure, and the more consistent demand of the cargo shipping structure.

Most passengers prefer direct flights and are generally unwilling to endure long travel times. In contrast packages may use a variety of routes to arrive at their final destination. Thus, fewer routing constraints and the possibility of consolidation at hub locations benefit cargo-shipping airlines. Often, passengers are flexible, choosing which airline to fly with, and even substituting driving or postponing travel by air. Thus, traveler's uncertainty can significantly reduce civil carriers' profitability. In contrast, the package delivery industry has fewer firms and substitutes for their services. Nevertheless, the slopes of the other three components show a less significant difference between civil and cargo services because both airline types rely on the performance of marketing forecasts, operations control, and sales effort rather than capacity design with respect to earning structure.

Table 2.6 Profitability efficiency difference shown by second stage regression

Regression	Production	Design	Generation	Operations	Consumption
Intercept	0.59	0.68	0.85	0.81	0.85
Slope	0.21	0.31	0.10	0.02	0.10

2.7 Concluding Remarks

This study has proposed a two-dimensional efficiency decomposition (2DED) model as a diagnostic tool for identifying the sources of production system and profitability efficiency change. A typical production system consists of four components: capacity design, demand generation, operations, and demand consumption. Efficiency was decomposed via a rational Network DEA model, and the profitability efficiency change was decomposed into technical efficiency change, scale efficiency change, and allocative efficiency change. An empirical study of profitability change in the US airline industry from 2006 to 2008 illustrated and validated the proposed method.

We found that the regress of productivity was mainly caused by demand fluctuation in 2007–2008 rather than technical regression in production capabilities. Furthermore, our contextual variable analysis suggests that the profitability efficiency of the overall production system in cargo service was 21% more efficient than civil service and that the capacity design component significantly affected efficiency.

We believe that the proposed model can be generalized and applied to other production systems for which a network structure can be identified and decomposed. For example, a supply chain system is usually defined by its materials suppliers, manufacturers, distribution centers, and retailers, hence, Network DEA efficiencies could properly estimate these entities. We suggest that the use of such decomposition enhances the rapid identification of sources of inefficiency as well as providing support for managerial troubleshooting.

CHAPTER III

EFFECTIVE PRODUCTION BY PROACTIVE DEA

3.1 Introduction

Data envelopment analysis (DEA) is a deterministic mathematical programming approach to productive efficiency analysis. Given the same input resources, a production unit is called efficient if its outputs levels are higher than other production processes. However, in practice, efficiency measure may be affected by demand fluctuations rather than the capability of production system. Reduced actual output can be caused by insufficient demand. In other words, demand fluctuations can bias productivity measures. A typical DEA study cannot model these demand effects. Thus, this research develops a productivity and "*effectiveness*" analysis to distinguish demand effects from productive efficiency using stochastic programming techniques in the short-run capacity expansion problem. The literature regarding the demand effect in productivity and efficiency analysis is limited. Recently, Lee and Johnson (2011) decompose a production process into capacity design, demand generation and operations components, and measure the productivity change of each component. They distinguish the production process from the demand generation/consumption process. The results indicate technical regress can be caused by demand fluctuations rather than production capabilities. Further the capacity design component generally has a significant effect on long-term productivity. To measure the demand effect in the short-run capacity planning problem, this study proposes the "*truncated production function*" and estimates "*effectiveness*" so as to distinguish from efficient production where the output levels are not limited by the customers' demands.

Lots of studies investigate the capacity expansion problem but limited literature on the short-run capacity expansion problem. Short-run capacity expansion addresses different issues from typical capacity expansion problem. The typical capacity expansion is a well-known economic and optimization problem (Manne, 1961; Luss, 1982). To define it rigorously, let decision variable $\mathbf{x}^t$ be number of working hour of machine needed at period t, $\mathbf{y}^t$ be the number of products generated, $\mathbf{D}^t$ be the demand quantity, $\mathbf{A}$ be the required machine hour per unit, $\mathbf{C}$ be the cost per machine hour and $\mathbf{P}$ be the selling price of product. The firm would like to maximize profit and finally solution shows the requirement of working hour. The optimization model can be formulated as equation (3.1).

$$\begin{aligned} \text{Max} \quad & \mathbf{P}\mathbf{y}^t - \mathbf{C}\mathbf{x}^t \\ s.t. \quad & \mathbf{A}\mathbf{y}^t \le \mathbf{x}^t \\ & \mathbf{y}^t = \mathbf{D}^t \\ & \mathbf{x}^t \ge \mathbf{0},\ \mathbf{y}^t \ge \mathbf{0} \end{aligned} \tag{3.1}$$

The primary issues in the capacity expansion problem are determining the expansion sizes, expansion times, and expansion locations (or capacity types) and the objective function is to minimize the discounted costs with respect to expansion process (Luss, 1982). In general, all the factors of production can be adjusted without limits of time period. The capacity expansion problem is a component of long-run production analyses. However, in the short-run, the stocks of appliances capital of production are practically fixed, but employment varies with demand (Marshall, 1920). In other words, the plant size and location might be fixed but variable factors such as material could be adjusted to control the production output level and satisfy the demand. Moreover, since the quantities of fixed factors are held constant in the short run, Stigler (1939) argues that the quantitative variations of output can be described in terms of the law of diminishing returns and marginal productivity theory while all but one of productive factors keep constant in quantity, remaining one adjusting in quantity. Wilson and Eckstein (1964) claim long run and short run economic analysis represents different productivity behaviors. They focus on the long-run tend and short-run cyclical behavior of productivity, and provide an interpretation of cyclical changes to an analysis of unit labor costs and price movements. They conclude the long-run cost curve forms an envelope of short-run cost curve when plant capital is fixed in short run but variable in long run. Thus, after distinguishing the characteristics of long run and short run, this study measures the marginal product of variables inputs through a DEA frontier and allows the adjustment of resource to influence output levels to handle demand fluctuations in short-run.

The capacity expansion problem with demand fluctuation is popularly discussed in the late 1990s. The internet shock and commodity customization caused the manufacturing industry to transition from a traditional manufacturing model to a service-oriented business model, emphasizing customer satisfaction, decreasing lead-times and characterized by a more uncertain environment. New issues such as product diversity, larger demand fluctuations, and shorter product life cycles came to the forefront.

Considering the stochastic nature of demand, the typical capacity expansion problem can be extended to uncertain demand and formulated as (3.2) (Birge and Louveaux, 1997). Let $\tilde{\mathbf{D}}^t$ be a random variable of demand and the firm would like to maximize expected profit.

$$
\begin{aligned}
&\text{Max} \quad E[f(\mathbf{x}^t, \tilde{\mathbf{D}}^t)] - \mathbf{C}\mathbf{x}^t \\
&s.t. \quad \mathbf{x}^t \geq \mathbf{0} \\
&\text{where for an realization } \mathbf{d}^t \in \tilde{\mathbf{D}}^t \\
&f(\mathbf{x}^t, \mathbf{d}^t) = \text{Max} \quad \mathbf{P}\mathbf{y}^t \\
&\qquad\qquad s.t. \quad \mathbf{A}\mathbf{y}^t \leq \mathbf{x}^t \\
&\qquad\qquad\qquad \mathbf{y}^t = \mathbf{d}^t \\
&\qquad\qquad\qquad \mathbf{y}^t \geq \mathbf{0}
\end{aligned}
\tag{3.2}
$$

Robust optimization (RO) is proposed in Mulvey *et al.* (1995) to handle noisy and uncertain demand with respect to large number of scenarios in the capacity expansion problem. RO is a general stochastic programming (SP) formulation which identifies a solution as robust if the solution remains “close” to optimal for any realization of scenarios and identifies a model as robust if solution is “almost” feasible for all scenarios by introducing error variables to measure infeasibility. Thus, the objective function can represent a tradeoff between solution and model robustness. Zhang *et al.* (2004) considers a capacity expansion problem involving multi-product, multi-machine and nonstationary stochastic demand, and solves efficiently an equivalent minimum-cut problem via a network structure. In addition, flexible manufacturing systems (FMS) have become well-known and popular to allow manufacturers to quickly respond to variability in both the items and the quantity demanded (Fine and Freund, 1990). Fine and Freund (1990) propose a cost-flexibility model formulated as two-stage stochastic programming with recourse to support product-flexible manufacturing capacity investment. The first stage determines the investment level in either dedicated capacity or the more costly flexible capacity, and then after demand is realized, a second stage analysis specifies the production levels given the first-stage investments.

This chapter discusses the convenience stores. The high uncertainty of customer demand and logistics environment is like the nature in semiconductor manufacturing. In additions, high-tech industries are characterized by capital intensity, high costs related to capital expansion, complicated processes and long production lead times. In these industries, short-run capacity adjustments are critical to profit margins and long-run financial well-being. Benavides *et al.* (1999) discussed the long-run optimal scale and timing of wafer fab construction using a Brownian motion model of demand and suggests a conservative deployment policy. Namely better late than early deployment of capacity, rather than a sequential deployment of capacity, is suggested to avoid idle resources and to maximize profit. Hood *et al.* (2003) considers the network flow associated with multiple products, operations, and tool groups, and develops a multi-period stochastic programming model with discrete demand scenarios to determine the allocation of tool sets that is robust to demand uncertainty and change in product mix. Karabuk and Wu (2003) introduce strategic capacity planning under demand and capacity uncertainty, while considering the distinct perspectives of marketing and manufacturing where product managers in marketing would like to pursue order fulfillment but manufacturing managers prefer to pursue minimizing operating costs.

The authors formulate a multi-stage stochastic program and compare centralized and decentralized planning strategies.

This chapter's contribution is distinguished from previous capacity expansion studies. First, we employ DEA to estimate production performance allowing for short-run capacity expansion decisions developing a proactive DEA model. The objective of the traditional capacity expansion problem attempts to minimize operational costs, maximize revenues or fulfill orders focusing on a specific firm. Thus, these models are normative and highly dependent on the abstraction of the production process. Applying traditional capacity expansion models, the decisions are completely dependent on the accuracy of the normative model. However, using a production function estimated from observed production processes assures the feasibility of the recommended short-run capacity adjustment. Second, effective production, defined as the product generated from production system to be consumed by realized demand, complements the typical efficiency analysis. The demand effect biasing productivity analyses can be identified via efficiency and effectiveness estimates to clarify the source of poor performance. Third, this study considers diminishing marginal benefits of inputs and estimates the marginal product which is typical ignored in the short-run capacity expansion problem. By ignoring the diminishing marginal rate of return when estimating responses to growing demand, typical capacity expansion methods assume a constant marginal product and expand resource to meet demand. However, by considering diminishing marginal return, capacity expansion decisions to increase resources may severally underestimate the resources necessary to be able to produce the demand required and result in cost ineffective outcomes. In other words, in some cases, it may not be cost effective to fill demand, thus reducing resources to decrease output levels and obtain better effectiveness may be preferred.

Mulvey *et al.* (1995) validates the benefits of the proactive approach and the benefits of SP which allows for resource adjustments after forecasting demand. For this reason, the present study introduces a proactive DEA model using stochastic programming (SP) techniques to estimate effectiveness under demand uncertainty. The definition and properties of the truncated production function are discussed from viewpoint of production economics. Then, we illustrate the relationship between efficiency and effectiveness. The marginal product estimation and SP model are developed to support short-run capacity expansion decision in a stochastic environment focusing on the quantity analysis without price information. Finally the numerical example and application to Japanese convenience store (CVS) data illustrates the interesting interpretation and insights regarding the proposed model.

This study is organized as follows. Section 3.2 defines a truncated production function and illustrates the relationship between efficiency and effectiveness. Section 3.3 describes capacity adjustment in terms of the marginal product of inputs, and shows single-output marginal products can be estimated via differential characteristics of DEA frontier. Stochastic programming models introduced in section 3.4 involve scenario-based approach and two-stage recourse approach. Criteria for assessing quality of solution in light of expected value of perfect information (EVPI) and value of stochastic solution (VSS) are described and developed. The model is formulated as geometric programming problem; section 3.5 introduces a technique to convexify the problem providing a solvable formulation which approximates global optimum solution. Then, section 3.6 gives a numerical example without capacity adjustment and section 3.7 illustrates the method with an empirical study of Japanese convenience store to validate the SPDEA model. Finally, section 3.8 concludes the section.

3.2 Effective Production

3.2.1 Truncated Production Function

The production function defines the maximum output that can be produced given the quantities of input resources. Let X^F be the fixed input resources, X^V be the variable input resources, and Y be the single-output level generated from production system. A standard production function with a single output can be shown as equation (3.3) and satisfies the properties of nonegativity, weak essentiality, monotonicity, and concavity (Coelli *et al.*, 2005).

$$Y = f(X^F, X^V) \tag{3.3}$$

In this study, *effective output* is defined as the output product or service generated by the production system to be consumed via customer demand. Furthermore, we can define the *truncated production function* as the maximum demand for a product or service that can be fulfilled given the quantities of the input resources. A firm is achieving *effective production* if the effective output level is generated by the truncated production function.

The truncated production functions are defined based on the demand level. To maintain generality, the demand is firm-specific, each firm can have the different demand levels, and the truncated production function is defined as production function truncated by the demand of the specific firm. Let D be the potential realized demand and Y^E be the effective output which is the smaller of the two variables: actual output Y and realized demand D. The truncated production function with output level Y^E is formulated as equation (3.4).

$$Y^E = \min(Y, D) = \min(f(X^F, X^V), D) \tag{3.4}$$

In a short-run analysis, the fixed input levels cannot be adjusted, so the production function is a function of variable input. Figure 3.1 illustrates the short-run truncated production function and its properties. The point A presents a supply-demand equilibrium where $D = Y_A^E = Y_A = f(X_A^F, X_A^V)$. That is, a firm can produce the optimal output level without unfulfilled demand or excessive inventory. In addition, it is straight-forward to validate the properties- nonegativity, weak essentiality, monotonicity, and concavity of truncated production function since the minimum function of a production function and constant, demand, is a convex polyhedral.

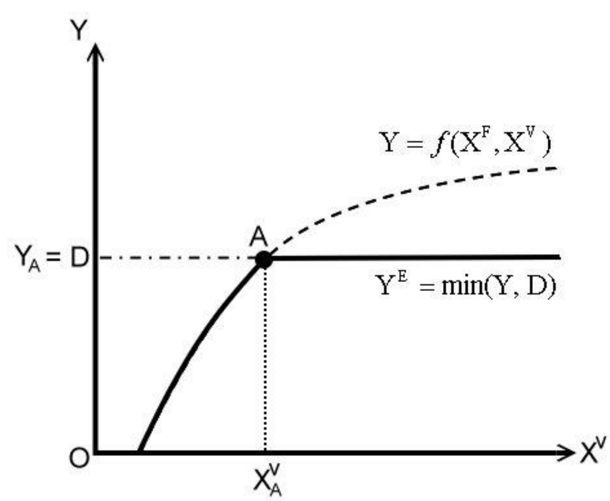

Figure 3.1 Truncated production function with firm-fixed demand

Proposition 3.1: *The truncated production function with firm-specific demand defined as* $Y^E = \min(f(X^F, X^V), D)$ *satisfies the underlying properties of nonegativity, weak essentiality, monotonicity, and concavity.*

The definition of truncated production function implies some notable issues. Given this definition, if actual output exceeds demand, then inventories are built and the inventory created characterize an ineffective gap; vice versa, if demand exceeds production capacity, the shortage products become an ineffective gap with respect to the demand level. Thus, the truncated production function is suitable for characterizing push production system with perishable goods, make-to-order production systems (pull systems), or service systems where services or inventories cannot be stored. The proposed model is applied to Japanese convenience store, in section 3.7. The convenience store industry is a business with high turn-over commodities and high product substitution. Each shop typically has a limited space for storing inventory. The portion of daily-supplied foods is over 30% (Japan Franchise Association, 2010). The high ratio of perishable goods and inability to hold significant inventories justify the use of the truncated production function. The detail will be discussed in section 3.7.

3.2.2 Efficiency vs. Effectiveness

Let $x \in R_+^{I+J}$ denote the inputs and $y \in R_+^Q$ denote outputs of the production system. The production possibility set is defined as $T = \{(x, y) : x \text{ can produce } y\}$ and is estimated by a piece-wise linear convex function enveloping all observations shown in (3.5). X_{ik}^{F} is the i^{th} fixed input resource, X_{jk}^{V} is the j^{th} variable input resource, Y_{qk} is the amount of the q^{th} production output, and λ_k is the multiplier of k^{th} firm. The equations defines the feasible region of the production possibility set $\tilde{T}$. Then, the efficiency θ can be measured using the DEA estimator. The input-oriented technical efficiency (ITE) can be defined as distance function $D_x(x,y) = \inf\{\theta \mid (\theta x, y) \in \tilde{T}\}$ and the output-oriented technical efficiency (OTE) is defined as $D_y(x,y) = \sup\{\theta \mid (x, \theta^{-1} y) \in \tilde{T}\}$ respectively.

$$\begin{aligned} \tilde{T} = \{(x,y):\ & \sum_k \lambda_k Y_{qk} \geq Y_q\ ,\ \forall q \\ & \sum_k \lambda_k X_{ik}^{F} \leq X_i^{F}\ ,\ \forall i \\ & \sum_k \lambda_k X_{jk}^{V} \leq X_j^{V}\ ,\ \forall j \\ & \sum_k \lambda_k = 1 \\ & \lambda_k \geq 0\ ,\ \forall k\ \}. \end{aligned} \tag{3.5}$$

Similarly, let $y^{E} \in R_{+}^{Q}$ denote an output vector produced and consumed. The $T^{E} = \{(x, y^{E}): x\ can\ produce\ y^{E}\ that\ will\ be\ consumed\ in\ the\ current\ period\}$ is called effective production possibility set which can be estimated by piece-wise linear convex function envelopment truncated by demand level shown as model (3.6). Y_{qk}^{E} is the amount of the q^{th} output produced and consumed with respect to demand D_q. The model illustrates the feasible region of the effective production possibility set T^{E}. Then, we can measure effectiveness θ^{E} using DEA estimator. If $Y_q \leq D_q$, then set $Y_q^{E} = Y_q$;

otherwise $Y_q^E = D_q - \min(Y_q - D_q, D_q)$ while $Y_q > D_q$ and capacity surplus $Y_q - D_q$ represents a penalty. The output-oriented production effectiveness (OPE), θ^E, is defined as $D_y(x, y^E) = \sup\{\mu^E \mid (x, \mu^E y^E) \in \tilde{T}^E\}$ and $\theta^E = 1/\mu^E$.

$$\begin{aligned} \tilde{T}^E = \{(x, y^E): \; & \sum_k \lambda_k Y_{qk}^E \geq Y_q^E, \; \forall q \\ & \sum_k \lambda_k X_{ik}^F \leq X_i^F, \; \forall i \\ & \sum_k \lambda_k X_{jk}^V \leq X_j^V, \; \forall j \\ & \sum_k \lambda_k = 1 \\ & \lambda_k \geq 0, \; \forall k \}. \end{aligned} \tag{3.6}$$

Since the number of discrete observations of k firms is finite, model (3.6) causes a bias of effectiveness measure due to a lack of firms in the shaded area in figure 3.2.

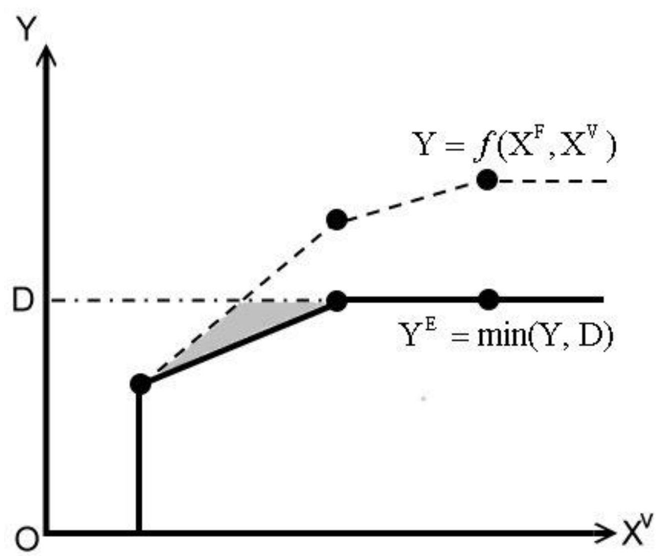

Figure 3.2 Bias of effectiveness measure

To correct this issue, model (3.7) and theorem 3.1 is proposed.

$$\tilde{T}^{E} = \{(x, y^{E}): \begin{array}{l} \sum_{k} \lambda_k Y_{qk} \geq Y_q^E, \ \forall q \\ D_q \geq Y_q^E, \ \forall q \\ \sum_{k} \lambda_k X_{ik}^F \leq X_i^F, \ \forall i \\ \sum_{k} \lambda_k X_{jk}^V \leq X_j^V, \ \forall j \\ \sum_{k} \lambda_k = 1 \\ \lambda_k \geq 0, \ \forall k \ \}. \end{array} \tag{3.7}$$

Theorem 3.1: *The truncated production possibility set described in model (3.7) is consistent to model (3.6) constructed with infinite observations. That is,* $\lim_{k \to \infty} \theta_{[k]}^E = \theta^E$, where $\theta_{[k]}^E$ *is effectiveness measure with* k *-observations truncated production function.*

Efficiency and effectiveness complement each other and are not mutually independent, but have different strategic interpretations. Efficiency measures the relative return on inputs used while effectiveness indicates the ability to match demand given an existing production technology. High effectiveness generates revenues by providing products and services to customers; low effectiveness implies insufficient or shortage demand generation and consumption. Figure 3.3 illustrates a two-dimensional strategic position between efficiency and effectiveness. If both efficiency and effectiveness are low, the firm is labeled a "*Follower*" who adopts other's superior strategy and attempts to catch-up before they will be driven out of the industry. If a firm performs well in terms of efficiency and bad in terms of effectiveness, the firm is labeled "*Superior Technology*", indicating the firm is leading the industry in terms of making the best use of their input resources and technology. If a firm is performs poorly in terms of efficiency and well in terms of effectiveness, the firm is labeled "*Superior Market*" indicating a market-oriented strategy focuses on generating demand and maintaining or expanding market share. Finally, if the firm is both efficient and effective, the firm is labeled "*Leader*" indicating it is developing new markets while also innovating to keep a competitive advantage.

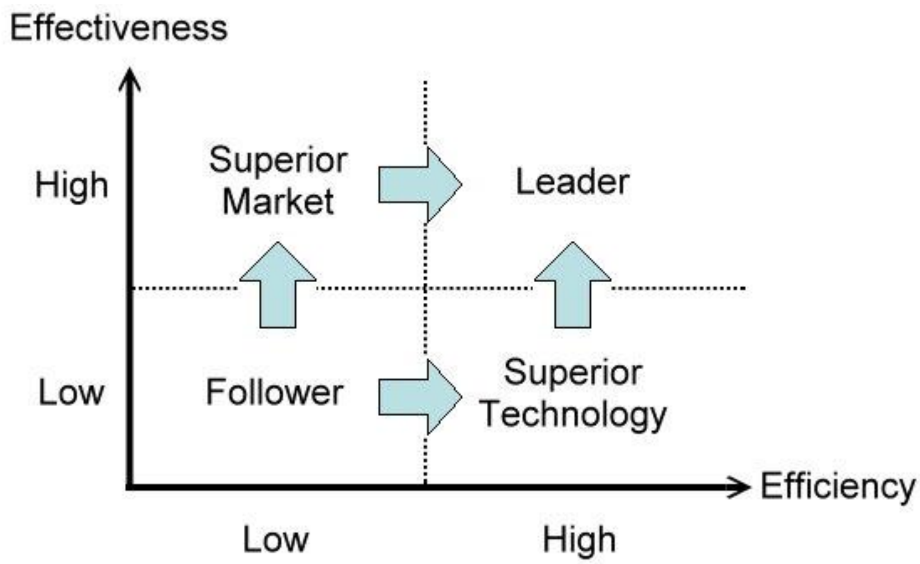

Figure 3.3 Strategic position

As technologies and markets evolve over time, new paradigms of competition can emerge. Product design, machinery development and demand diversity can shock an industry and push firms to enhance core competence. "*Paradigm shift*" is a term to describe a hybrid of progress in marketing and in technology leading to a new competitive setting as shown in figure 3.4. The Malmquist productivity index is a popular tool to measure the productivity change (Cave *et al.*, 1982; Färe *et al.*, 1992;

1994). Similarly, it can be used to measure market evolution, technical change, and identify paradigm shifts.

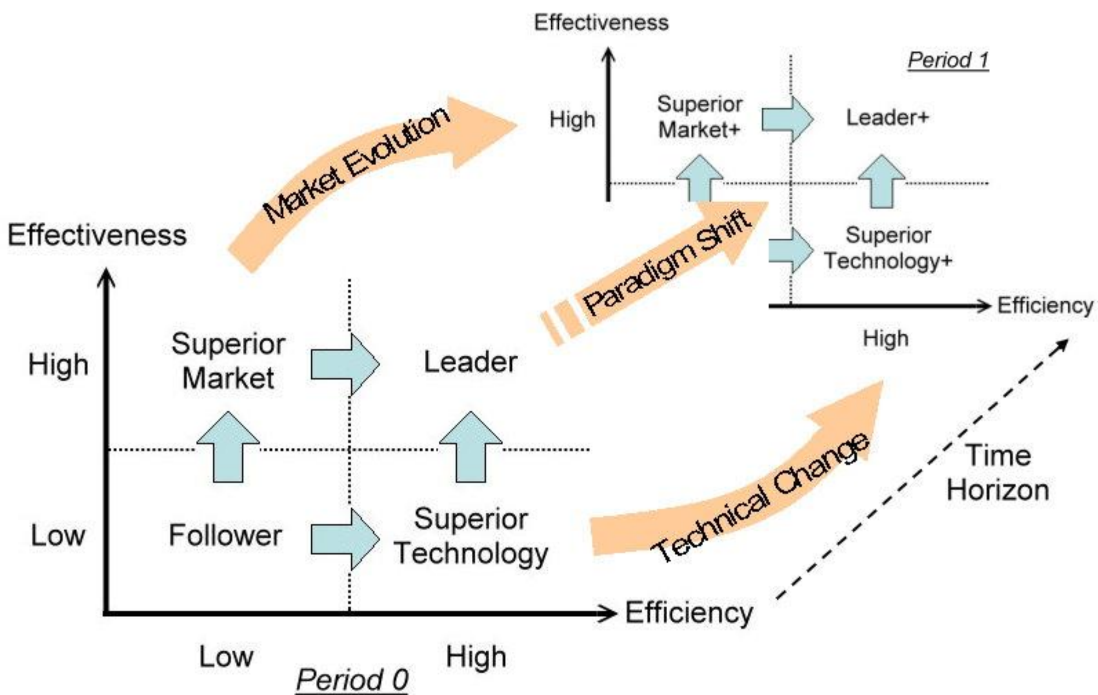

Figure 3.4 Paradigm Shift

3.3 Capacity Expansion

3.3.1 Variable Input Adjustment and Marginal Product

Capacity can be defined as the maximal output level of a production process. The output is a result of the total productive capability of all the resources including workforce, machinery, and utilities; while capacity adjustment is the ability to adjust output levels to deal with uncertainty by controlling variable resources in the short run (Alp and Tan, 2008). Note that production resources can be separated into fixed and variable inputs. The fixed inputs such as building and facilities are not easily changed in a short period of time due to high costs and long lead-times of installation; while variable inputs such as labor and material can be adjusted and released into production. In particular, Johansen's (1968) defines the capacity as maximum amount that can be produced with existing plant and equipment given an unlimited availability of variable factors. Eilon and Soesan (1976) extend the concept from single output to multiple output case and Färe *et al.* (1989) employ a nonparametric approach to obtain a capacity measure. In addition, in the short run, capacity expansion is a way to adjust variable inputs to control output levels while pursuing maximal profit and efficient production in an uncertain environment. In production theory, capacity adjustment can be interpreted as the marginal product (MP) of production function. That is, the extra output generated by one more unit of an input. Figure 3.5 shows the marginal product and production frontier. Y, X^F and X^V denotes output, fixed input and variable input respectively. The production function can be formulated as $Y = f(X^F, X^V)$ and marginal product of point B is $\mathrm{MP_B} = \left.\frac{\partial f(X^F, X^V)}{\partial X^V}\right|_{X_B^V}$.

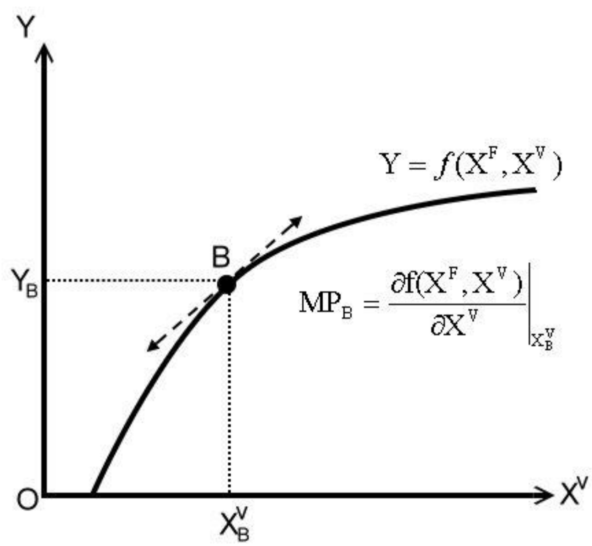

Figure 3.5 Marginal product of production function

Changing the input level is a natural way to cope with demand fluctuations. Maximal capacity is defined by the fixed resource level and is thus unchanged in the short-run; demand requirements can be met by adjusting variable inputs. For example temporary workers can be hired to raise outputs when demand increases or workers can be laid off when demand decreases. However, the adjusted range of variable input is restricted due to the difficulty in hiring qualified workers and training them for the job or firing too many workers in a short period of time may hurt morale or put the company at risk of not being able to meet future demand. In this situation, firm-specific marginal rates of output expansion and of output contraction are assumed to be limited to a small adjusted range. Note that in general overtime production might be feasible to create additional capacity rather than temporary workers because of skill requirements. In fact in some cases overtime may be preferred to hiring temporary workers because of the learning curve associated with new workers. Nevertheless, in the empirical study of convenience store, the service-oriented and high-turnover nature leads to a higher ratio of part-time employees. In order to reduce employee cost, stores prefer to hire part-time employees because of lower skill requirement and interchangeability. For these reasons, adjusting employees to increase capacity in convenience store industry is feasible and encouraged.

3.3.2 Marginal Product Estimation

The marginal effect on output of an increase or decrease in variable input can play an important role in operations. In practice, these effects can be estimated via the marginal product in a set of data. There are two typical ways to estimate marginal product. One is to use stochastic frontier analysis (SFA) to estimate the production function with a given functional form. Take a simple case of a linear function estimated by ordinary least squares (OLS), the coefficients associated with the independent factors provide estimates of the marginal product. When employing DEA to construct piece-wise linear production function approximate to a true production function, the shadow prices of input and output characterize the relationship between inputs and outputs, i.e., marginal product. However, there are drawbacks to estimating marginal products using these methods. SFA requires defining a functional form and risks potential misspecification. While observations on the production frontier in DEA do not have unique shadow prices and shadow price values of zero are common.

Podinovski and Førsund (2010) proposed using directional derivative technique to assess the marginal rate of a nondifferential efficient frontier constructed by DEA estimator. Their approach can characterize without additional simplifying conditions the polyhedral production sets such as measures of the scale elasticity and marginal rates of substitution between factors. The concept of directional derivative is described in Shapiro (1979). Let X_{ik}^{F} be the i^{th} fixed input resource, X_{jk}^{V} be the j^{th} variable input resource, Y_{qk} be the q^{th} output level of k^{th} firm. Let v_i^F, v_j^V and u_q be the multipliers of factors respectively. Since marginal rate is a characteristic of the frontier, for one specific efficient firm r, the following revised formulation is proposed to calculate the marginal rate $\beta^{V+}_{j^*q^*r}$ approaching from the right side with respect to one particular variable input j^* to one output q^*.

$$\frac{\partial^+ Y_{q^*r}}{\partial X^V_{j^*r}} = \beta^{V+}_{q^*j^*r} = \text{Min } v^V_{j^*}$$
$$s.t. \quad \sum_i v^F_i X^F_{ir} + \sum_j v^V_j X^V_{jr} - \sum_q u_q Y_{qr} + u_0 = 0$$
$$\sum_i v^F_i X^F_{ik} + \sum_j v^V_j X^V_{jk} - \sum_q u_q Y_{qk} + u_0 \geq 0 \tag{3.8}$$
$$u_{q^*} = 1$$
$$v^F_i, v^V_j, u_q \geq 0, \quad u_0 \text{ is free}$$

For measuring the marginal rate approaching from the left side, the objective function is replaced by following equations.

$$\frac{\partial^- Y_{q^*r}}{\partial X^V_{j^*r}} = \beta^{V-}_{q^*j^*r} = \text{Max } v^V_{j^*} \tag{3.9}$$

That is, $\beta^{V+}_k = \beta^{V+}_{j^*q^*k}$ and $\beta^{V-}_k = \beta^{V-}_{j^*q^*k}$ denote the simplified notations of marginal products of short-run capacity expansion and contraction due to single variable input and single output discussed in this study. Figure 3.6 illustrates the marginal product β^{V+}_k or β^{V-}_k in terms of expansion or contraction in the short run. Note for inefficient firms operating inside of the production frontier, the marginal product is not defined. However, to estimate how the consumed output expands with an increase of variable input, we assume the marginal increase in output is the same as the marginal products of the reference firm on frontier via output-oriented expansion.

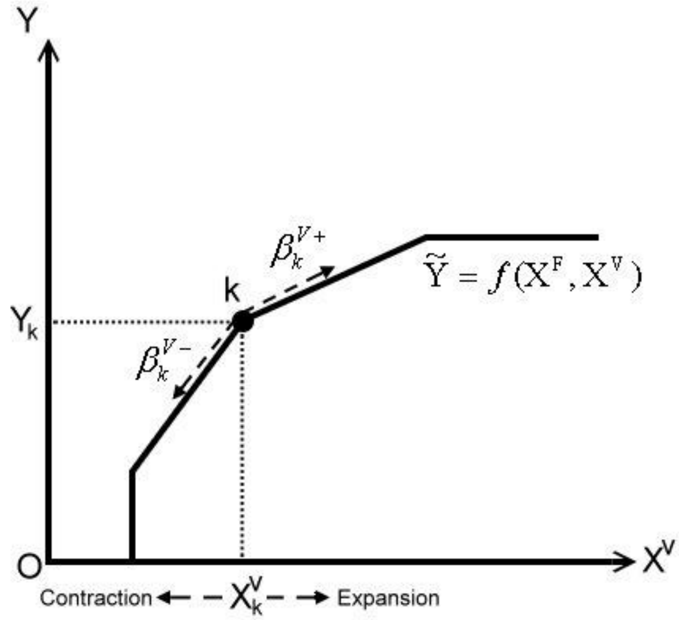

Figure 3.6 Marginal product regarding to short-run capacity expansion or contraction

3.4 Stochastic Programming Model

The fundamental stochastic programming (SP) models are described in Birge and Louveaux (1997). This section proposes a stochastic programming DEA (SPDEA) model which characterizes flexibility in the capacity level through changes to single variable input levels to adjust for uncertain demand. A specific firm can adjust variable input to change output levels when facing demand fluctuations. In section 3.4.1 a scenario-based approach provides solutions to individual scenario and expected scenario. A two-stage recourse stochastic programming model described in section 3.4.2 provides robust solutions to a variety of scenarios using a probabilistic characterization. Finally, section 3.4.3 describes how the solution varies based on the criteria (i.e. expected value of perfect information (EVPI) and value of the stochastic solution (VSS)).

3.4.1 Scenario-based Approach

A scenario-based approach (SB) is a deterministic programming model solved once for each scenario. A scenario is defined by a realization of the demand level. Given different scenarios, the model suggests the suitable decision regarding short-run capacity expansion or contraction via adjustments in variable input to maximize efficiency under demand fluctuations. Specifically, additional output produced and sold is estimated as the marginal product multiplied by the increment of variable input, or vice versa reduced output produced and sold via reducing the input level. By limiting the range of adjustment, a constant marginal rate of change in the output level is a reasonable approximation. Accounting for the effects of demand, the effective output level is determined considering the current production level, short-run capacity adjustments, and realized demand. Thus, for perishable goods, effective output is the minimum of actual output and realized demand. The production possibility set is estimate using the observed production data and is unchanged regardless of the assumptions made about short-run capacity expansion.

Assuming fixed inputs cannot be adjusted in the short-run, the output-oriented variable returns to scale (VRS) DEA formulation with single output is developed as a revised dual form of model (3.7). Let Y_{ks}^{E} be the effective output and D_{ks} be the realized demand of k^{th} firm in s^{th} scenario, β_{jr}^{V} be the marginal product characterized by β_{jr}^{V+} and β_{jr}^{V-} with respect to j^{th} variable input of firm r, and R_{jr} be the parameter of adjustable range of j^{th} variable input of firm r. Then the decision variables u_s, w_s,

v_{is}^{F}, v_{js}^{V}, v_{0s} are the associated multipliers, d_{jrs} is the additional adjustment of variable input characterized by d_{jrs}^{+} and d_{jrs}^{-}. y_{rs} and y_{rs}^{E} are actual and effective output respectively, and $\theta_{rs}^{E} = 1/\mu_{rs}^{E}$ measures production effectiveness. Note that in order to fix an identical production possibility set after capacity expansion or contraction of firm r, index k include the dummy firm r' which is firm r before capacity expansion.

$$\text{Min} \quad M\mu_{rs}^{E} + \sum_{j}(d_{jrs}^{+} + d_{jrs}^{-}) \tag{3.10.1}$$

$$s.t. \quad \mu_{rs}^{E} = \sum_{i} v_{is}^{F} X_{ir}^{F} + \sum_{j} v_{js}^{V}(X_{jr}^{V} + d_{jrs}) + w_{s} D_{rs} + v_{0s} \tag{3.10.2}$$

$$u_{s}(y_{rs}^{E} + \varepsilon) + w_{s}(y_{rs}^{E} + \varepsilon) = 1 \tag{3.10.3}$$

$$\sum_{i} v_{is}^{F} X_{ik}^{F} + \sum_{j} v_{js}^{V} X_{jk}^{V} - u_{s} Y_{ks} + v_{0s} \geq 0, \ \forall k \setminus r \tag{3.10.4}$$

$$\sum_{i} v_{is}^{F} X_{ir}^{F} + \sum_{j} v_{js}^{V}(X_{jr}^{V} + d_{jrs}) - u_{s}(y_{rs}^{E} + \varepsilon) + v_{0s} \geq 0 \tag{3.10.5}$$

$$y_{rs}^{E} = y_{rs}(1 - z1_{rs}) + \left[D_{rs} - \min(y_{rs} - D_{rs}, D_{rs})\right] z1_{rs} \tag{3.10.6}$$

$$y_{rs} - D_{rs} < Mz1_{rs} \tag{3.10.7}$$

$$y_{rs} - D_{rs} \geq -M(1 - z1_{rs}) \tag{3.10.8}$$

$$y_{rs} = Y_{r} + \sum_{j} \beta_{jr}^{V} d_{jrs} \tag{3.10.9}$$

$$\beta_{jr}^{V} = \beta_{jr}^{V+} z2_{jrs} + \beta_{jr}^{V-}(1 - z2_{jrs}), \ \forall j \tag{3.10.10}$$

$$d_{jrs} < Mz2_{jrs}, \ \forall j \tag{3.10.11}$$

$$d_{jrs} \geq -M(1 - z2_{jrs}), \ \forall j \tag{3.10.12}$$

$$d_{jrs} = d_{jrs}^{+} - d_{jrs}^{-}, \ \forall j \tag{3.10.13}$$

$$-R_{jr} X_{jr}^{V} \leq d_{jrs} \leq R_{jr} X_{jr}^{V}, \ \forall j \tag{3.10.14}$$

$$z1_{rs}, z2_{jrs} \in \{0,1\}, \ \forall j \tag{3.10.15}$$

$$y_{rs}^{E}, y_{rs}, d_{jrs}^{+}, d_{jrs}^{-}, u_{s}, w_{s}, v_{is}^{F}, v_{js}^{V} \geq 0, \ \forall i, \forall j \tag{3.10.16}$$

The objective function equation (3.10.1) maximizes the product of the estimated effectiveness $\theta_{rs}^{E} = 1/\mu_{rs}^{E}$ and a large number M with a secondary objective of minimizing the variation in input adjustment. Equations (3.10.2)-(3.10.5) are the envelope constraints to build the production possibility set. ε is a small number to maintain feasibility when $y_{rs}^{E} = 0$. Constraints (3.10.6)-(3.10.8) calculate the effective output level for two cases $Y_r \le D_{rs}$ or $Y_r > D_{rs}$. Equation (3.10.9) determines the actual output level of firm r through capacity expansion. Constraints (3.10.10)-(3.10.12) calculates the marginal output for short-run capacity expansion, i.e., $d_{jrs} \ge 0$, then $\beta_{jr}^{V} = \beta_{jr}^{V+}$; otherwise $\beta_{jr}^{V} = \beta_{jr}^{V-}$. Constraint (3.10.13) shows adjustment of variable input via goal programming, $d_{jrs} \ge 0$ if and only if capacity expansion with $d_{jrs}^{+} \ge 0$; otherwise capacity contracts with $d_{jrs} \le 0$ if and only if $d_{jrs}^{-} \ge 0$. The adjustment range is restricted in equation (3.10.14). Then equation (3.10.15) defines $z1_{rs}$ and $z2_{jrs}$ as binary variables and nonnegative constraints are including with equation (3.10.16). Note that for the case with multiple variable inputs, when two or more variable inputs are expanded simultaneously, the estimation of the increase in output is conservative. The marginal production of each variable input is estimated separately and then the dot product of the marginal product vector and the vector of change in variable input is taken. If there is a synergistic effect between the different variable inputs, this is not captured. However, because the production frontier limits the output level, the benefits of increasing multiple variables inputs leads to a resulting production vector within the production possibility set.

In order to maintain feasibility which means a firm remains in original production possibility set after taking action, the effectiveness and resource adjustments are calculated using proposed algorithm shown as follows.

Proposed Algorithm

1. Start from specific firm $r = 1$
2. For $r = 1$ to number of DMU

 2.1 Set step $t = 0$, $X_{jrt}^{V} = X_{jr}^{V}$ and $Y_{rt} = Y_{r}$

 2.2 Calculate marginal product β_{jrt}^{V+} and β_{jrt}^{V-}

 2.3 Run scenario-based approach to calculate $X_{jrt}^{VT} = X_{jrt}^{V} + d_{jrt}$, $\forall j$ and

 $Y_{rt}^{T} = Y_{rt} + \sum_{j} \beta_{jrt}^{V} d_{jrt}$

 2.4 Run output-oriented DEA estimator to calculate efficiency θ_{rt}^{DEA}

 2.5 If $\theta_{rt}^{DEA} \geq 1$, then get θ_{rt}^{E}, $d_{jr} = \sum_{t} d_{jrt}$, $\forall j$ and $y_{r} = Y_{rt}^{T}$. Go to step 3

 Else if $\theta_{rt}^{DEA} < 1$ and $d_{jr} = \sum_{t} d_{jrt} \leq 0$, then run (3.11) to find anchor point

$$
\begin{aligned}
\text{Min} \quad & \sum_{j} (X_{jrt}^{V} + d_{jrt}) \\
s.t. \quad & \sum_{k} \lambda_{k} Y_{k} = Y_{rt} + \sum_{j} \beta_{jrt}^{V-} d_{jrt} \\
& \sum_{k} \lambda_{k} X_{ik}^{F} \leq X_{ir}^{F}, \ \forall i \\
& \sum_{k} \lambda_{k} X_{jk}^{V} \leq X_{jrt}^{V} + d_{jrt}, \ \forall j \\
& \sum_{k} \lambda_{k} = 1 \\
& -R_{jr} X_{jr}^{V} \leq (X_{jrt}^{V} - X_{jr}^{V}) + d_{jrt} \\
& \lambda_{k} \geq 0, \ \forall k
\end{aligned}
\tag{3.11}
$$

Else run the model (3.12)

$$\begin{aligned}
\text{Max} \quad & \sum_j (X^V_{jrt} + d_{jrt}) \\
s.t. \quad & \sum_k \lambda_k Y_k = Y_{rt} + \sum_j \beta^{V+}_{jrt} d_{jrt} \\
& \sum_k \lambda_k X^F_{ik} \le X^F_{ir}, \ \forall i \\
& \sum_k \lambda_k X^V_{jk} \le X^V_{jrt} + d_{jrt}, \ \forall j \\
& \sum_k \lambda_k = 1 \\
& R_{jr} X^V_{jr} \ge (X^V_{jrt} - X^V_{jr}) + d_{jrt} \\
& \lambda_k \ge 0, \ \forall k
\end{aligned} \qquad (3.12)$$

End

2.6 Set $X^V_{jrt} = X^V_{jrt} + d_{jrt}, \ \forall j$ and $Y_{rt} = Y_{rt} + \sum_j \beta^V_{jrt} d_{jrt}$

2.7 Set $t = t + 1$, then go to Step 2.1

3. Set $r = r + 1$ and go to step 2

Proposition 3.2: *A firm that expands or contracts short-run capacity via proposed algorithm with marginal products* β^{V+}_{jk} *or* β^{V-}_{jk} *remain feasible in the original production possibility set.*

In addition, if demand is low, a significant gap between efficiency and effectiveness exists; however, if demand is high, efficiency and effectiveness are identical measures. This result shows the measure of effectiveness is particularly important during economic down-turns.

Proposition 3.3: *The effectiveness estimate converges to an efficiency estimate as demand increases.*

The advantage of this approach is simple and leads itself to scenario analysis. It is useful to suggest a solution to "*If…Then…*" situation, that is, what's the best capacity adjustment if some demand scenario is realized. There are two kinds of solutions suggested: scenario analysis and an expected value (EV) solution. Given a demand level defines an event, scenario analysis provides solutions, an adjustment to variable input, to each event respectively; however, the expected value solution is obtained by solving model with the expected value of variables representing uncertain events. In this study demand is the uncertain variables, thus $\overline{D}_r = \sum_s p_s D_{rs}$ where p_s represents the probability of s^{th} scenario occurring. However, the scenario-based approach does not consider a robust solution for all of scenarios; this is introduced in section 3.4.2.

3.4.2 Two-stage Recourse Approach

The two-stage recourse approach is a typical stochastic programming model and provides robust solution to all scenarios. The two-stage recourse model shows a two-stage decision process including "here-and-now" and "wait-and-see" decisions by considering the expected recourse function. The two-stage decision process is shown as figure 3.7. The first-stage decision is referred to as the here-and-now decision, and we make an ex-ante decision based on forecasts of the uncertain event. After the event occurs, the ex-post decision made in second stage adjusts to account for the new information. If the event does not occur, no decision is made and we just wait and see what happens. That second-stage decision is referred to as the wait-and-see decision. For example, in a make-to-order production model, the firm needs to make a decision to determine the amount of material to order, but demand is realized in the future due to lead times. The material order is the here-and-now decision. Then, a firm realizes demand, and accordingly decides how many products to produce to maximize profit. However, until the customer order is realized, the second-stage decision is pending. Similarly, for maximizing effectiveness, firm r needs to decide the short-run capacity expansion or contraction strategy before realizing demand. Then, the firm measures the technical effectiveness after demand is observed. Figure 3.7 illustrates this two-stage decision process.

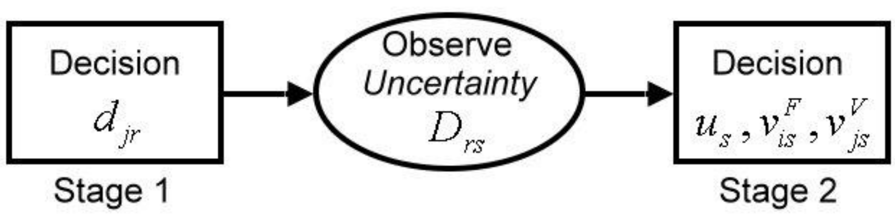

Figure 3.7 Two-stage decision process

The two-stage recourse approach introduces an *expected recourse function* using a divide-and-conquer strategy. The expected recourse function characterizes the performance of second-stage decision, namely the expected utility is estimated based on the individual decision for each specific scenario respectively. This divide-and-conquer strategy generates robust solutions for the first-stage decision in all scenarios. The two-stage recourse model of DEA with capacity adjustment and uncertain demand is shown in the following formulation.

$$\text{Max} \quad M\sum_s p_s\left[\frac{1}{\mu_{rs}^E+\varepsilon}\right]-\sum_j (d_{jr}^+ + d_{jr}^-) \qquad (3.13.1)$$

$$s.t. \quad \mu_{rs}^E = \sum_i v_{is}^F X_{ir}^F + \sum_j v_{js}^V (X_{jr}^V + d_{jr}) + w_s D_{rs} + v_{0s},\ \forall s \qquad (3.13.2)$$

$$u_s (y_{rs}^E + \varepsilon) + w_s (y_{rs}^E + \varepsilon) = 1,\ \forall s \qquad (3.13.3)$$

$$\sum_i v_{is}^F X_{ik}^F + \sum_j v_{js}^V X_{jk}^V - u_s Y_{ks} + v_{0s} \geq 0,\ \forall k \setminus r, \forall s \qquad (3.13.4)$$

$$\sum_i v_{is}^F X_{ir}^F + \sum_j v_{js}^V (X_{jr}^V + d_{jr}) - u_s (y_{rs}^E + \varepsilon) + v_{0s} \geq 0,\ \forall s \qquad (3.13.5)$$

$$y_{rs}^E = y_r (1 - z1_{rs}) + \left[D_{rs} - \min(y_r - D_{rs}, D_{rs})\right] z1_{rs},\ \forall s \qquad (3.13.6)$$

$$y_r - D_{rs} < Mz1_{rs},\ \forall s \qquad (3.13.7)$$

$$y_r - D_{rs} \geq -M(1 - z1_{rs}),\ \forall s \qquad (3.13.8)$$

$$y_r = Y_r + \sum_j \beta_{jr}^V d_{jr} \qquad (3.13.9)$$

$$\beta_{jr}^V = \beta_{jr}^{V+} z2_{jr} + \beta_{jr}^{V-} (1 - z2_{jr}),\ \forall j \qquad (3.13.10)$$

$$d_{jr} < Mz2_{jr},\ \forall j \qquad (3.13.11)$$

$$d_{jr} \geq -M(1 - z2_{jr}),\ \forall j \qquad (3.13.12)$$

$$d_{jr} = d_{jr}^+ - d_{jr}^-,\ \forall j \qquad (3.13.13)$$

$$-R_{jr} X_{jr}^V \leq d_{jr} \leq R_{jr} X_{jr}^V,\ \forall j \qquad (3.13.14)$$

$$z2_{jr} \in \{0,1\},\ \forall j \qquad (3.13.15)$$

$$z1_{rs} \in \{0,1\},\ \forall s \qquad (3.13.16)$$

$$y_{rs}^E, y_r, d_{jr}^+, d_{jr}^-, u_s, w_s, v_{is}^F, v_{js}^V \geq 0,\ \forall i, \forall j, \forall s \qquad (3.13.17)$$

The term objective function (3.13.1) indicates the expected recourse function of effectiveness estimate θ_{rs}^E of firm r with probability measures p_s for the s^{th} scenario. The two-stage recourse problem (RP) provides an adjustment to the variable input level d_{jr} in first-stage and attempts to maximize expected effectiveness by adjusting the variables u_s, v_{is}^F, v_{js}^V, and v_{0s} in second-stage after the demand is realized. A recourse function is defined as $g(d_{jr}, D_{ks}) = \max_{u,v \geq 0} \left\{ \theta_{rs}^{E} \mid (3.13.2) - (3.13.8), (3.13.16) - (3.13.17) \right\}$ substituting first-stage decision d_{jr} and realized outcome D_{ks} of random variable $\tilde{D}_{ks}$ into the formulation. The expected recourse function can be formulated as $E\left[g(d_{jr}, \tilde{D}_{ks})\right] = \sum_{s} p_s \theta_{rs}$ under discrete scenarios. However, an excess of scenarios will result in a large number of decision variables and constraints and increase the computational burden. Thus, confidence interval for the effectiveness estimate by means of simulation approach described in Birge and Louveaux (1997) provides insight into the level of certainty of the estimates.

3.4.3 Value of Information and Stochastic Solution

Given the solutions generated from SP models, it is interesting to investigate the quality of the solutions. There are two approaches commonly adopted: the expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) (Birge and Louveaux, 1997). The expected value of perfect information (EVPI) measures the maximum amount a decision maker is willing to pay in return for complete information about the future. Define the effectiveness measures from the wait-and-see (WS) problem and the recourse problem (RP) as $\text{WS} = E_{\tilde{D}}\left[\text{Max } g(d, \tilde{D})\right]$ and $\text{RP} = \text{Max } E_{\tilde{D}}\left[g(d, \tilde{D})\right]$, then EVPI is the difference between WS and RP described in equation (3.14).

$$\text{EVPI} = \text{WS} - \text{RP} = E_{\tilde{D}}\left[\text{Max } g(d, \tilde{D})\right] - \text{Max } E_{\tilde{D}}\left[g(d, \tilde{D})\right] \tag{3.14}$$

The value of the stochastic solution (VSS) is a measure of the quality of the expected value (EV) decision in terms of the recourse problem. Namely, it gives the cost of ignoring uncertainty. Let $\bar{d}(\bar{D})$ be a EV solution and define the *expected result of using the EV solution* (EEV) as $\text{EEV} = E_{\tilde{D}}\left[g(\bar{d}(\bar{D}), \tilde{D})\right]$. The VSS is defined as the difference between EEV and RP described in equation (3.15).

$$\text{VSS} = RP - \text{EEV} = \text{Max } E_{\tilde{D}}\left[g(d, \tilde{D})\right] - E_{\tilde{D}}\left[g(\bar{d}(\bar{D}), \tilde{D})\right] \quad (3.15)$$

3.5 Model Convexification

The above model, in particular, equations (3.10.1)-(3.10.17) is a nonlinear programming with a concave feasible region. A global optimum cannot be guaranteed. This section provides an equivalent geometric programming formulated with difference of two exponential-based convex functions through variable alteration and additional variables and constraints (Maranas and Floudas, 1997). For a minimization problem, this decomposition identifies a lower bound by solving a convex relaxation programming problem. To linearize some geometric terms, the equivalent formulation requires additional binary variables and continuous variables. Using this model an approximate solution can be obtained through a branch-and-bound algorithm (Li and Tsai, 2005).

First, constraint (3.10.15) can be replaced by constraint (3.16.1) and (3.16.2).

$$d_{jrs} = -R_{jr} X_{jr}^{V} + t_{jrs} 2R_{jr} X_{jr}^{V}, \ \forall j \quad (3.16.1)$$

$$0 \le t_{jrs} \le 1, \ \forall j \quad (3.16.2)$$

The minimum function $y_{rs}^{c} = \min(y_{rs} - D_{rs}, D_{rs})$ in constraint (3.10.6) can be transformed into (3.16.3)-(3.16.6).

$$y_{rs}^{c} = (y_{rs} - D_{rs})z3_{rs} + D_{rs}(1 - z3_{rs}) \tag{3.16.3}$$

$$D_{rs} - (y_{rs} - D_{rs}) < Mz3_{rs} \tag{3.16.4}$$

$$D_{rs} - (y_{rs} - D_{rs}) \geq -M(1 - z3_{rs}) \tag{3.16.5}$$

$$z3_{rs} \in \{0,1\} \tag{3.16.6}$$

Finally, the model (3.10.1)-(3.10.16) can be reformulated as an equivalent geometric programming with exponential-based convex functions in appendix B. A convex relaxation of this equivalent model can be solved to approximate optimal solution (Maranas and Floudas, 1997). Similar adjustments can be made to the two-stage recourse approach with a fractional objective function released by Charnes and Cooper (1962).

3.6 Example Illustration

This section gives a numerical example without capacity expansion to illustrate the proposed model. Table 3.1 shows the data for 12 decision-making units (DMUs). The data includes a fixed input, variable input, actual output, and three demand forecasts-pessimistic (PE), most-likely (ML), and optimistic (OP) situations. This numerical study postulates the probability of realizing each of the demand scenarios as 1/6 for the PE, 4/6 for the ML and 1/6 for the OP case. Then the efficiency and effectiveness measures are shown in table 3.2. Note that PE, ML, OP, and EV are deterministic (and use the scenario-based approach) while SP is stochastic. Specifically, the difference between the expected value (EV) solution and the stochastic programming (SP) solution is that former calculates effectiveness using expected demand $\overline{D}_k$, while the latter estimates the expected value of effectiveness of the three demand scenarios.

Table 3.1 Data of numerical example

DMU	Fix Input	Var. Input	Actual Output	Pessimistic Demand	Most-likely Demand	Optimistic Demand
A	9	5	10	6	9	12
B	4	7	8	5	6	9
C	4	9	11	6	8	13
D	5	9	9	7	8	10
E	7	7	10	7	9	13
F	6	7	7	4	6	9
G	10	8	10	7	8	11
H	8	6	7	7	8	9
I	5	6	11	6	7	12
J	4.5	10	10	8	10	12
K	4	8	12	7	8	12
L	10	7	5	3	5	8

Table 3.2 Efficiency, effectiveness and EVPI

DMU	Efficiency	Effectiveness						
		PE	ML	OP	EV	SP (RP)	WS	EVPI
A	1.000	0.333	0.889	1.000	0.889	0.815	0.815	0.000
B	1.000	0.400	0.667	1.000	0.737	0.678	0.678	0.000
C	0.917	0.167	0.625	0.917	0.706	0.597	0.597	0.000
D	0.750	0.714	0.875	0.900	0.898	0.852	0.852	0.000
E	0.870	0.571	0.889	0.870	0.929	0.833	0.833	0.000
F	0.609	0.250	0.833	0.778	0.865	0.727	0.727	0.000
G	0.833	0.571	0.750	0.909	0.800	0.747	0.747	0.000
H	0.636	1.000	0.875	0.778	0.875	0.880	0.880	0.000
I	1.000	0.167	0.429	1.000	0.565	0.480	0.480	0.000
J	0.833	0.750	1.000	0.833	1.000	0.931	0.931	0.000
K	1.000	0.286	0.500	1.000	0.588	0.548	0.548	0.000
L	0.435	0.333	1.000	0.625	0.968	0.826	0.826	0.000
Avg.	0.855	0.449	0.755	0.903	0.802	0.729	0.729	

Figure 3.8 maps the efficiency and the effectiveness level with different demand scenarios on a two-dimensional coordinate graph, and the four quadrants indicate the

strategic position. The intersection of two axes describes the performance of industry level which is the average of 12 DMUs weighted by actual output. For the PE-demand case, DMU F and L are the followers (using the terminology introduced in figure 3.3), DMU D, G, H and J belong to superior market, DMU A, B, C, I and K are attributed to superior technology, and DMU E is the leader. This strategic position provides guidelines of productivity improvement. Second, when demand is high, effectiveness is closely correlated with efficiency and they tend to a diagonal line. This is because demand does not limit the production possibility set. This result is consistent with proposition 3.3. Figure 3.8 shows a convergence process from pessimistic to optimistic demand situation. That is, effectiveness provides additional information beyond an efficiency measure during economic down-turns. Third, the result shows firms will prefer to underproduce rather than overproduce since inventory implies ineffective product. For example DMU C indicates totally ineffective in PE and ML cases but production function forms a demand-supporting limitation in OP case. It concludes DMU C would like to reduce output level to achieve the goal of effective production. Finally, the results using EV and SP are very similar as shown in figure 3.9, however the interpretation is different. The expected demand situation (EV) can not be realized in the future, but expected effectiveness (SP) can be estimated to justify the decision making. Table 3.2 demonstrates that SP identifies the infeasible DMU with effectiveness score 0.929, that is DMU E, since in three scenarios DMU E never shows scores higher than 0.929. In additions, all EVPI equal to zero point out no need to pay for perfect information. This is because capacity adjustment is not considered in this example and effectiveness identical under a WS (wait-and-see) model and a SP excluding short-run capacity expansion decision.

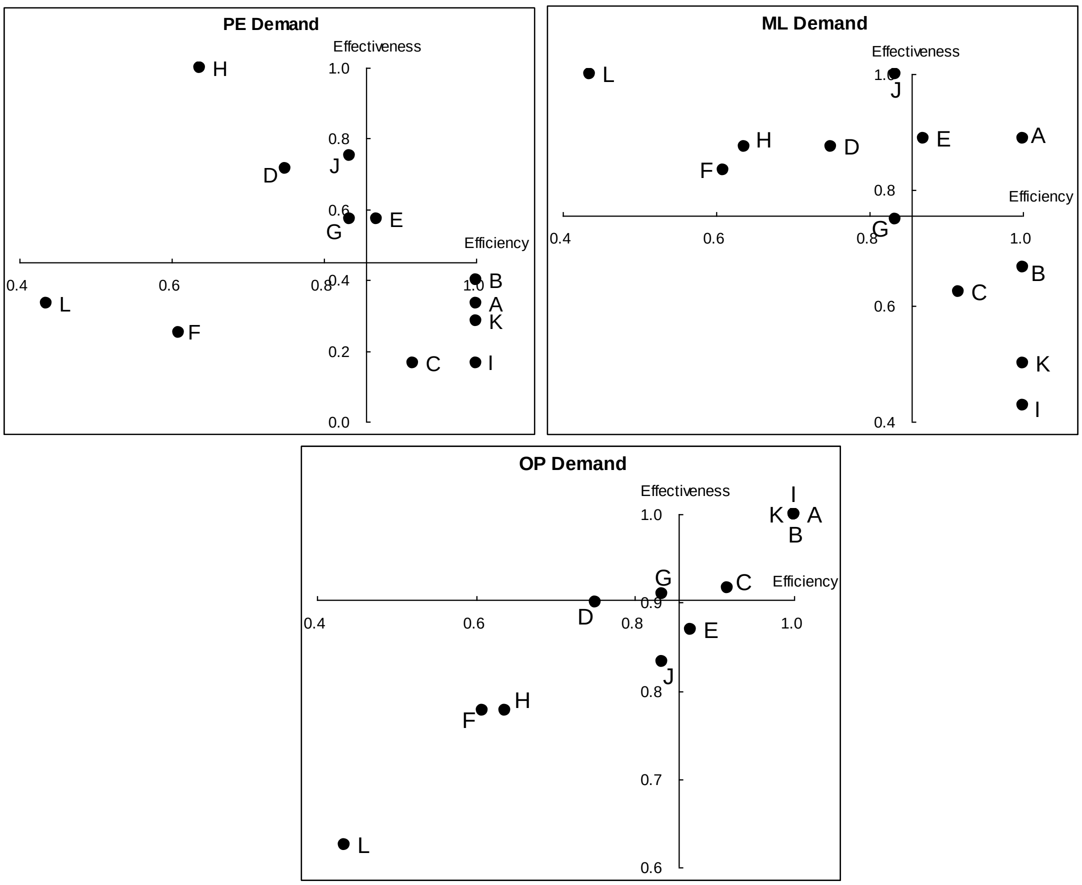

Figure 3.8 Efficiency vs. effectiveness

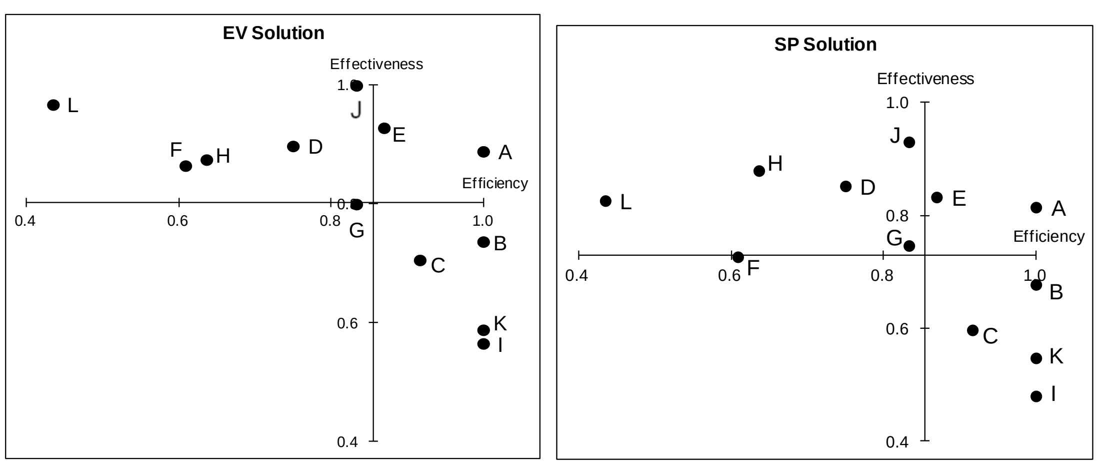

Figure 3.9 Expected value solution vs. stochastic programming

3.7 Empirical Study

An empirical study of Japanese convenience stores (CVS) over the first half of 2003 is presented. The first convenience store in Japan opened in 1969. These stores have become very popular over the past three decades. Now there are around 42,889 convenience stores (CVS) in Japan with 1.1 billions customers per year (Japan Franchise Association, 2010). Most CVSs are built with a floor area of 100 square meters and about 3000 types of product. Even though prices in a convenience store are typically higher than at a supermarket, they remain popular because of new service such as 24hr operating, courier and postal service, touch screen monitor for finding job and ordering ticket, telephone and utility bills payment, automated teller machine (ATM), online shopping, etc and of course convenience (Nipponia, 2001). Thus, CVS plays an important role and affects the lives of many Japanese people. This study evaluates the performance of 25 convenience store chains under demand uncertainty in the short run. Section 3.7.1 provides a description of the data. The effectiveness is estimated using scenario-based approach and the two-stage recourse approach in section 3.7.2.

3.7.1 Data Description

In the convenience store industry, the production process can be characterized by three input resources- capital, branch size (fixed inputs), and employee (variable input); forecasted demand and actual output are measured as goods. Efficiency is estimated using actual output. While effectiveness is estimated using truncated production function. The data primarily comes from Sueyoshi (2003) and this study adds additional data regarding to capacity adjustment shown in descriptive statistics of table 3.3. The following describes the details of data set.

Capital is defined as the net worth of equipment used to create goods or service in a production system. Branch size and employee are defined as the number of branches and manpower headcount the convenience store chain had during the first half year of 2003. Actual output is estimated by realized revenue divided by the average price per item[4]. Demand is firm-specific and defined as the estimated number of goods sold and characterized by a pessimistic, most-likely, and optimistic estimate which were provided by managers and chief executive officers, see Sueyoshi (2003) for more details. Similarly, demand is calculated by estimating revenue divided by average price per item. The marginal product is defined in section 3.3 and estimated by Podinovski and Førsund model (2010). Note that, in the short run, the variable input (employee) can be adjusted to change the capacity level while the fix inputs (capital and branch) remain unchanged. In other words, convenience store can adjust the stores capacity over some limited range by hiring or laying off employees. This study limits the positive and negative adjustment in manpower resources to 15%. This parameter is arbitrary and other recommended values were tested however then lead to similar results.

[4] Data from the Japan Franchise Association (2010).

Table 3.3 Descriptive statistics of raw data

Stat.	Fixed Input		Var. Input	Output	Demand		
	Capital	Branch	Employee	Goods	Pessimistic	Most-likely	Optimistic
	Yen (10^6)	Unit	Headcount	Unit (10^6)			
Average	3452.5	1421.4	683.5	794.4	795.5	857.9	920.4
Std. Dev.	7465.8	2291.5	1068.3	1330.2	1492.9	1558.6	1625.9
Max	30876.0	7780.0	4126.0	6191.3	6358.8	6586.6	6814.4
Min	3.0	22.0	8.0	12.1	11.7	13.1	14.7

3.7.2 Efficiency and Effectiveness Analysis

This section illustrates efficiency and effectiveness estimation in a demand-dependent context. A demand-dependent context means all firms consistently realize the same single demand state (such as pessimistic (PE), most-likely (ML), or optimistic (OP)). This empirical study postulates demand $\tilde{D}_k$ follows beta distribution. The expected value can be estimated as $\overline{D}_k = (PE_k + 4ML_k + OP_k)/6$. The mean of beta distribution is widely used in project management with PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method) to plan activity times and scheduling (Hillier and Lieberman, 2002). Table 3.4 presents the efficiency, effectiveness, EVPI and VSS estimations. "N" means estimation without short-run capacity expansion are allowed. Similarly, "Y" indicates results where capacity adjustment and manpower expansion decision. "Exp." column indicates the change in variable input where positive values indicate short-run capacity expansion and negative values indicate contraction.

The empirical study is consistent with the numerical study, in that higher demand increases the correlation between effectiveness and efficiency. The correlation coefficients are 0.26 and 0.76 in the pessimistic and the optimistic demand cases without capacity expansion, respectively. In addition, EVPI and VSS in most of CVS chains are close to or equal to zero. This leads to an inconspicuous performance difference between the two-stage recourse approach (RP) and expected value of demand solution (EV). This result is driven by the limited demand fluctuations in the CVS industry during the first-half of 2003. However, we suggest RP solution to DMUs which represent VSS value larger than 0.

Considering ex-ante and ex-post analysis of short-run capacity expansion, figure 3.10 and 3.11 map the efficiency and effectiveness under pessimistic demand on a two-dimensional coordinate, and the four quadrants map to the strategies for productivity improvement (similar to figure 3.3). The intersection of two coordinate axes indicates the industry performance weighted average. Nevertheless, in an ex-post analysis, the efficiency is calculated after applying the expansion of variable input recommended by the effectiveness maximization problem (3.10). This might leads an efficiency loss to achieve higher effectiveness. Thus, there exists a tradeoff between efficiency and effectiveness in the capacity expansion decision.

Table 3.4 Efficiency, effectiveness, EVPI and VSS

CVS	DMU	Efficiency			Effectiveness																		
					PE			ML			OP			EV			RP			WS	EVPI	EEV	VSS
		N	Y	Exp.	N	Y	Exp.	N	Y	Exp.	N	Y	Exp.	N	Y	Exp.	N	Y	Exp.				
am/pm	A	0.905	0.905	0.0	0.968	0.968	0.0	0.905	0.905	0.0	0.905	0.905	0.0	0.905	0.905	0.0	0.916	0.916	0.0	0.916	0.000	0.916	0.000
Heart in	B	0.821	0.821	0.0	0.970	0.970	0.0	0.951	0.951	0.0	0.883	0.883	0.0	0.951	0.951	0.0	0.943	0.943	0.0	0.943	0.000	0.943	0.000
HOTSPAR	C	0.541	0.552	-48.3	0.805	0.805	0.0	0.958	0.958	0.0	0.800	0.800	0.0	0.958	0.958	0.0	0.906	0.906	0.0	0.906	0.000	0.906	0.000
Apple Mart	D	1.000	1.000	0.0	0.891	0.891	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.982	0.982	0.0	0.982	0.000	0.982	0.000
Everyone	E	0.851	0.851	0.0	0.816	0.816	0.0	0.851	0.851	0.0	0.851	0.851	0.0	0.851	0.851	0.0	0.845	0.845	0.0	0.845	0.000	0.845	0.000
Caramel Mart	F	0.718	0.718	0.0	0.707	0.707	0.0	0.893	0.893	0.0	0.718	0.718	0.0	0.893	0.893	0.0	0.833	0.833	0.0	0.833	0.000	0.833	0.000
Coco Store	G	1.000	1.000	0.0	0.950	1.000	-12.8	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.992	1.000	-12.8	1.000	0.000	0.992	0.008
Community Store	H	1.000	1.000	0.0	0.866	0.920	-23.3	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.978	0.987	-23.3	0.987	0.000	0.978	0.009
Circle K	I	0.774	0.832	133.7	0.959	1.000	32.7	0.900	1.000	84.0	0.848	0.998	133.7	0.900	1.000	84.0	0.901	0.980	84.0	1.000	0.020	0.899	0.081
Sunkus	J	0.759	0.777	-141.3	0.956	1.000	66.9	0.924	1.000	120.6	0.893	0.980	141.3	0.924	1.000	120.5	0.924	0.989	120.6	0.997	0.008	0.923	0.065
Shop and Life	K	0.613	0.645	4.9	0.858	0.858	0.0	0.975	1.000	-1.5	0.938	1.000	4.0	0.973	1.000	-1.6	0.949	0.967	-1.5	0.976	0.009	0.949	0.018
Spar	L	0.905	0.905	0.0	0.868	0.868	0.0	0.923	0.923	0.0	0.905	0.905	0.0	0.921	0.921	0.0	0.911	0.911	0.0	0.911	0.000	0.911	0.000
Three F	M	0.760	0.760	0.0	0.954	0.954	0.0	0.914	0.914	0.0	0.878	0.878	0.0	0.914	0.914	0.0	0.915	0.915	0.0	0.915	0.000	0.915	0.000
Seikatsu Train	N	0.938	0.942	0.0	0.984	0.984	0.0	0.938	0.938	0.0	0.938	0.938	0.0	0.938	0.938	0.0	0.946	0.946	0.0	0.946	0.000	0.946	0.000
Seicomart	O	1.000	1.000	0.0	0.849	0.922	-40.4	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.975	0.987	-40.4	0.987	0.000	0.975	0.012
Seven Eleven	P	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	0.000	1.000	0.000
Daily Yamazaki	Q	1.000	1.000	0.0	0.863	0.997	-152.4	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.977	1.000	-152.4	1.000	0.000	0.977	0.022
Hiromaru Chain	R	0.470	0.470	0.0	0.787	0.787	0.0	0.996	0.996	0.0	0.844	0.844	0.0	0.996	0.996	0.0	0.936	0.936	0.0	0.936	0.000	0.936	0.000
Family Mart	S	0.757	0.757	0.0	0.970	1.000	43.5	0.903	1.000	151.5	0.845	1.000	259.5	0.903	1.000	151.5	0.904	0.977	151.5	1.000	0.023	0.901	0.076
My Shop Chain	T	1.000	1.000	0.0	0.696	0.696	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.949	0.949	0.0	0.949	0.000	0.949	0.000
Monpellie	U	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	0.000	1.000	0.000
Mon Mart	V	1.000	1.000	0.0	0.912	0.912	0.0	0.984	0.984	0.0	1.000	1.000	0.0	0.984	0.984	0.0	0.974	0.974	0.0	0.974	0.000	0.974	0.000
Lics	W	1.000	1.000	0.0	0.971	0.971	0.0	1.000	1.000	0.0	1.000	1.000	0.0	1.000	1.000	0.0	0.995	0.995	0.0	0.995	0.000	0.995	0.000
Little Star	X	0.743	0.743	0.0	0.825	0.825	0.0	0.893	0.893	0.0	0.743	0.743	0.0	0.893	0.893	0.0	0.857	0.857	0.0	0.857	0.000	0.857	0.000
Lawson	Y	0.694	0.694	0.0	0.980	0.980	0.0	0.924	0.924	0.0	0.874	0.874	0.0	0.924	0.924	0.0	0.925	0.925	0.0	0.925	0.000	0.925	0.000
Avg.		0.857	0.863		0.960	0.983		0.955	0.980		0.927	0.965		0.955	0.980		0.951	0.972					

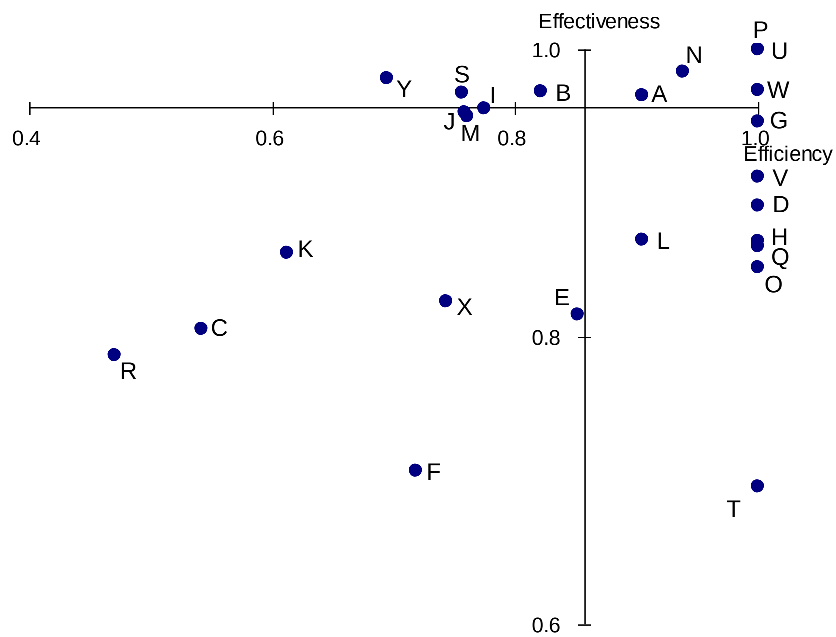

Figure 3.10 Efficiency vs. effectiveness with pessimistic demand before expansion

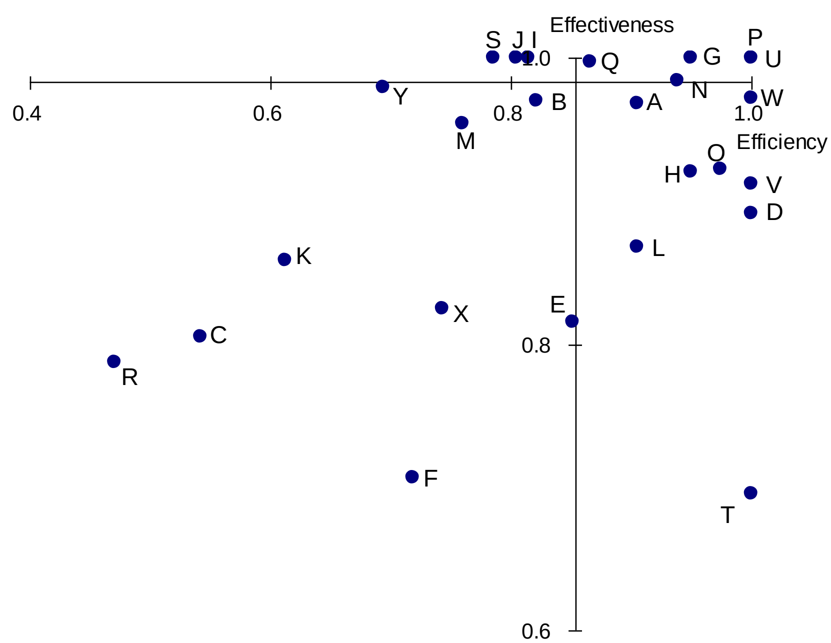

Figure 3.11 Efficiency vs. effectiveness with pessimistic demand after expansion

3.8 Concluding Remarks

This study proposes an original model for the short-run capacity expansion decision with uncertain demand. Proactive DEA embedded with stochastic programming enhances the decision making tools to consider performance benchmarking in short-run capacity planning. This study improves previous studies by considering varying marginal product rates for expansion and contraction and includes limitations due to diminishing returns. In addition, efficiency and effectiveness estimates identify the influence of demand on productivity analysis. An empirical study of Japanese convenience store illustrates the SPDEA model. In future studies, the development of multi-output model with price information would be a valuable contribution. Moreover, the panel data and a dynamic analysis will be valuable to support a sequential control of resource.

CHAPTER IV

RATIONAL INEFFICIENCY BY NASH EQUILIBRIUM

4.1 Introduction

Standard productivity and efficiency analysis assumes perfectly competitive markets and exogenous prices (Cherchye *et al.*, 2002). Basic microeconomic theory states that firms operating in less than perfectly competitive markets can reduce production levels and increase a product's market price when they face a downward sloping demand curve. Considering an oligopolistic market, Hicks (1935) proposes "the quiet life hypothesis (QLH)" and argues that, due to management's subjective cost of reaching optimal profits, firms use their market power to allow inefficient allocation of resources. Johnson and Ruggiero (2011) demonstrate from a revenue efficiency perspective that a firm that increases output to become technically efficient may actually reduce its overall profits by increasing the market quantity, which in turn reduces the market price. Figures 4.1 and 4.2 illustrate the endogenous prices of an oligopolistic market for a single product produced using a single input. The production frontier in figure 4.1 represents technically efficient production. Firms *A* and *B* would like to expand their output levels[5] to increase their productivity, yet increasing the output levels will lead to a change in the market output quantity from Y to Y' (shown in figure 4.2) and the market price will fall from P to P'. This change in price may reduce the profits of both firms. Thus, the

[5]Firms will either expand their outputs, contract their inputs, or both, depending on the cost/price structure of inputs/outputs and adjustment costs associated with changing input levels. For now we will assume input adjustment costs are very large and consider only output adjustment consistent with an output oriented efficiency analysis in the efficiency literature, (Fare and Primont, 1995). This assumption is relaxed in section 4.4.

standard assumption in the efficiency literature that all firms desire to produce on the production frontier may not hold in an oligopolistic market (Cherchye *et al.*, 2002; Johnson and Ruggiero, 2011). A firm is said to be rationally inefficient when it tries to maximize revenues or profits, or alternatively, minimize costs by intentionally operating at lower productivity levels. This study considers an oligopolistic market to estimate a firm's target production plans that may not be on the production frontier, but that maximize revenues or profits, or alternatively, minimize costs. The set of all firms' benchmark production plans is a Nash-Cournot oligopolistic market equilibrium.

Most of the efficiency and productivity literature adapts the work developed by Farrell (1957) and articulated by Leibenstein (1966) as concepts X-efficiency which assumes that deviations from a production frontier are due to managerial inefficiency, lack of motivation, and lack of knowledge (Leibenstein and Maital, 1994). However, Stigler (1976) argues that firms and individuals are rational, meaning that what is observed as inefficiency is actually the difference between individual employees of the firm maximizing their individual value functions and the firm's value function. Following Stigler, Bogetoft and Hougaard (2003) suggest that if the inefficiency is due to lack of motivation, performance may be improved by redesigning the incentive structure to stimulate employees to save inputs and expand outputs.

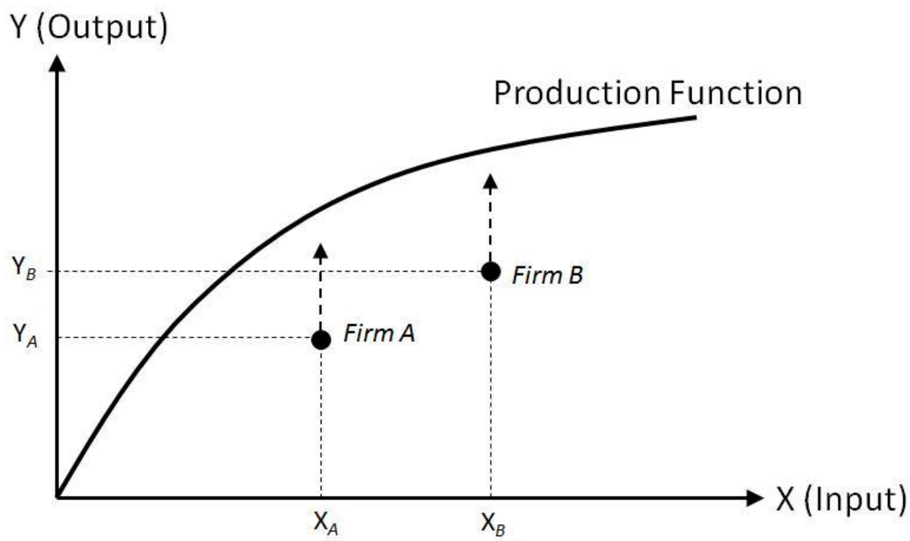

Figure 4.1 Economic efficiency and production frontier

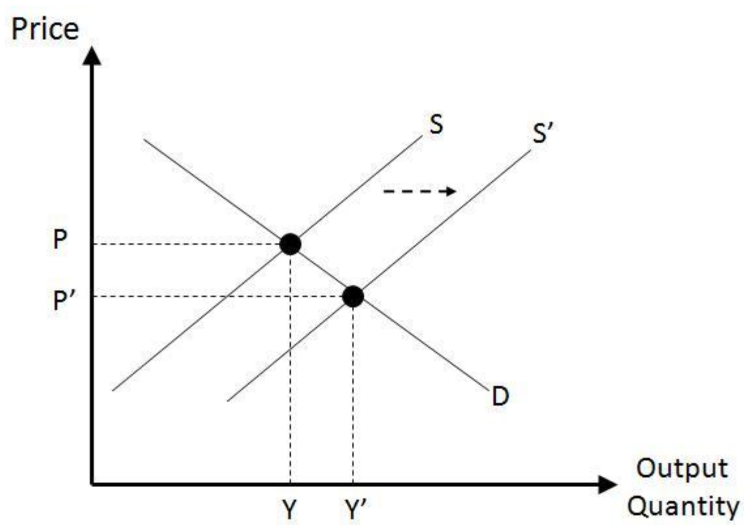

Figure 4.2 Change in supply and equilibrium price

In the case that these incentives or enforcement cost are higher than the cost of the inefficiency, it is rational for the firm to allow inefficient operations. Modeling the firm's intention as maximizing profits and the employees' intention as maximizing slack, the authors show the trade-offs between the consumption of different types of slack. Alternatively, Wibe (2008) considers a firm that does not scrap older equipment as new models become available. His dynamic production model demonstrates that a considerable proportion of cross-sectional technical inefficiency can be rational economic behavior in terms of capital acquisition, i.e. he shows the role of capital (fixed inputs) in rational inefficiency.

In this chapter we propose that rational inefficiency may in fact be a result of endogenous prices and the effect of output production on price – and profits. Cournot (1838), the first to consider endogenous prices, assumes a homogeneous product with an inverse demand function known to all firms which then independently select output levels; in this market characterized by imperfect competition, price is treated as an endogenous parameter. Nash (1950a, 1950b, 1951, 1953) considers more general non-cooperative games and defines a self-countering n-tuple as an equilibrium point in n-person games, i.e. for an equilibrium point, no firm can increase its objective function by unilaterally changing the quantity or price to any other feasible point. These games are consistent with the oligopolies described by Cournot where each firm maximizes its own profits and the output decisions affect the price faced by all of the firms. Rosen (1965) proves that a finite non-cooperative game always has at least one equilibrium point when the strategy space of each player is restricted to a simplex and the payoff functions are a bilinear function of the strategies. Further, for a constrained n-person game, he proves the existence and uniqueness of an equilibrium point with a strictly concave payoff function. A systematic discussion applying equilibrium concepts to economic systems is developed in Arrow and Debreu (1954). Discussing different classes of non-cooperative games, Milgrom and Roberts (1990) argue that all have identical bounds on the rationalizable strategies. In this study we consider production strategies bounded by the production possibility set.

Murphy *et al.* (1982) introduce a mathematical programming approach for finding Nash equilibria in oligopolistic markets. They show that if the revenue function is concave and the cost function is convex and continuously differentiable, and the inverse demand function is strictly decreasing and continuously differentiable, then a Nash equilibrium solution exists if and only if a solution to the Karush–Kuhn–Tucker (KKT) conditions exist. Based on their study, Harker (1984) presents a variational inequality (VI) approach to find a Nash equilibrium using an iterative procedure called the diagonalization algorithm. Bonanno (1990) gives a comprehensive survey on equilibrium theory with imperfect competition.

We use a variational inequality approach to identify Nash equilibria when production is limited by a production frontier. We focus on an oligopolistic market with endogenous prices and firms maximizing profits. We identify a Nash equilibrium in which each firm cannot improve its profit by changing production levels within the production possibility set. We find that, contrary to previous productivity and efficiency studies, under certain conditions some firms choose not to produce on the production frontier, and we interpret the behavior as rational inefficiency (choosing to be less productive in order to increase profits).

The remainder of this study is organized as follows. Section 4.2 shows the equivalence between a Nash equilibrium and the two approaches, variational inequalities and the complementarity problem, when production is restricted to the production possibility set. Section 4.3 examines revenue maximization with fixed input levels. Both a single output case and a multiple output case are presented. Section 4.4 introduces a generalized profit model in which a firm maximizes profits by adjusting both input and output levels. The existence and uniqueness of a Nash equilibrium identified through the complementarity problem is proven, and the relationship between the benchmark frontier and scale properties is discussed. Based on moving towards allocative efficient production, the direction for improvement used in the directional distance function is identified using the results of the Nash equilibrium analysis. Section 4.5 presents our conclusions.

4.2 Identify a Nash Equilibrium in Production Possibility Set

This section considers a general profit function and a production function with multiple inputs and multiple outputs, describes the conditions under which a Nash equilibrium solution exists, and how to identify it. We discuss the equivalence between the general concept of a Nash equilibrium and a set of variational inequalities and the complementary problem (CP) when production is limited to the production possibility set.

Let $x \in R_{+}^{I}$ denote the inputs and $y \in R_{+}^{Q}$ denote the outputs of a production system. $Q = 1$ in the single output case. The production possibility set defined as $T = \{(x, y): x\ can\ produce\ y\}$ is estimated by a piece-wise linear convex function enveloping all observations (Farrell, 1957; Boles, 1967; Afriat, 1972; Charnes *et al.*, 1978). For firm k, X_{ki} is the i^{th} input resource, Y_{kq} is the amount of the q^{th} production output, and λ_k is the multiplier for the convex combination. Equation (4.1) uses a dataset characterizing firms to estimate the smallest set that imposes monotonicity and convexity on the production function, the boundary of the production possibility set $\tilde{T}$.

$$\tilde{T} = \left\{ (x,y) \middle| \begin{array}{c} \sum_k \lambda_k Y_{kq} \geq Y_q \quad \forall q; \\ \sum_k \lambda_k X_{ki} \leq X_i \quad \forall i; \\ \sum_k \lambda_k = 1; \\ \lambda_k \geq 0 \quad \forall k; \end{array} \right\} \tag{4.1}$$

To identify a Nash equilibrium, the generalized profit function should be concave, the inverse demand function should be nonincreasing and continuously differentiable, and the inverse supply function should be nondecreasing and continuously differentiable. The variational inequality approach and mixed complementary problem (MCP) are proven to be alternative methods to calculating a Nash equilibrium in production possibility set in section 4.4.

To discuss the equilibria in oligopolistic markets characterized by imperfect competition, we define a Nash equilibrium problem (NEP) with respect to production possibility set as:

Definition 4.1: Let K be a finite number of players, θ_k a utility (profit) function, T_k a strategy set (production possibility set) for player $k = 1, \dots, K$, and $(\boldsymbol{x}_k, \boldsymbol{y}_k) = (x_{k1}, \dots, x_{kI}, y_{k1}, \dots, y_{kQ}) \in T_k$ an observed production vector; then a vector $(\boldsymbol{x}^*, \boldsymbol{y}^*) = ((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*), \dots, (\boldsymbol{x}_K^*, \boldsymbol{y}_K^*)) \in T_1 \times T_2 \times \dots \times T_K = T$ is called a Nash equilibrium and is a solution to the NEP if

$$\theta(\boldsymbol{x}^*, \boldsymbol{y}^*) \geq \theta(\boldsymbol{x}_k, \boldsymbol{x}^*_{(-k)}, \boldsymbol{y}_k, \boldsymbol{y}^*_{(-k)}), \forall(\boldsymbol{x}_k, \boldsymbol{y}_k) \in T_k,$$

where $\boldsymbol{x}^*_{(-r)} = (\boldsymbol{x}^*_1, \dots, \boldsymbol{x}^*_{k-1}, \boldsymbol{x}^*_{k+1}, \dots, \boldsymbol{x}^*_K)$ and $\boldsymbol{y}^*_{(-r)} = (\boldsymbol{y}^*_1, \dots, \boldsymbol{y}^*_{k-1}, \boldsymbol{y}^*_{k+1}, \dots, \boldsymbol{y}^*_K)$ holds for all $k = 1, \dots, K$.

Considering an NEP, Facchinei and Pang (2003) build a rigorous relationship among Nash equilibria, a set of variational inequalities (VI), and the complementarity problem (CP). We restate their results for the scenario in which a production function bounds the production possibility set, and consider a profit function as a specific utility function.

Lemma 4.1: Let output levels be decision variables denoted by y_{rq} as output q of firm r and $y_{rq} \geq 0$; further, let input levels be decision variables denoted by x_{ri} as input i of firm r, $x_{ri} \geq 0$, and $(x_{ri}, y_{rq}) \in \tilde{T}$. Then define $P^Y_q(y_{rq})y_{rq}$ as a concave function of y_{rq} and assume that either the inverse demand function $P^Y_q(y_{rq})$ is a non-increasing or a convex function of y_{rq}. Thus, for each $Y_{(-r)q} > 0$, where $Y_{(-r)q} = \sum_{k \neq r} y_{kq}$, $P^Y_q(y_{rq} + Y_{(-r)q})y_{rq}$ is a concave function of y_{rq} for $y_{rq} \geq 0$. Similarly, let $P^X_i(x_{ri} + X_{(-r)i})x_{ri}$ be a convex function of x_{ri} for $x_{ri} \geq 0$, where $X_{(-r)i} = \sum_{k \neq r} x_{ki}$ and $P^X_i(x_{ri})$ is an inverse supply function. Further, if either $P^Y_q(y_{rq})$ is strictly decreasing or is strictly convex, then $P^Y_q(y_{rq} + Y_{(-r)q})y_{rq}$ is a strictly concave function on the nonnegative $y_{rq} \geq 0$ and $\sum_q P^Y_q(y_{rq} + Y_{(-r)q})y_{rq} - \sum_i P^X_i(x_{ri} + X_{(-r)i})x_{ri}$ is a concave function on $(x_{ri}, y_{rq}) \in \tilde{T}$.

Lemma 4.1 is important because it attests that a global Nash equilibrium solution exists when the profit function $\sum_q P_q^Y\left(y_{rq} + Y_{(-r)q}\right)y_{rq} - \sum_i P_i^X(x_{ri} + X_{(-r)i})x_{ri}$ is concave and production is limited to a convex production possibility set. Generally, input markets are assumed to be competitive, in which case $P_i^X(x_{ri} + X_{(-r)i})$ is a constant, but in this case the lemma and the related results shown in theorems 4.1 and 4.2 and proven in section 4.4 still hold.

Gabay and Moulin (1980) propose that a Nash equilibrium will satisfy a set of VI. We reformulate the VI set with respect to the production possibility set:

Theorem 4.1: If the profit function of firm r, $\theta_r(x_{ri}, y_{rq}) = \sum_q P_q^Y\left(Y_q\right)y_{rq} - \sum_i P_i^X(X_i)x_{ri}$ is concave with respect to (x_{ri}, y_{rq}) and continuously differentiable, where $Y_q = \sum_k y_{kq}$ and $X_i = \sum_k x_{ki}$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*) \in \tilde{T}$ is a Nash-Cournot oligopolistic market equilibrium if and only if it satisfies the set of VI $\langle F\left((\boldsymbol{x}^*, \boldsymbol{y}^*)\right), (\boldsymbol{x}, \boldsymbol{y}) - (\boldsymbol{x}^*, \boldsymbol{y}^*)\rangle \geq 0$, $\forall(\boldsymbol{x}, \boldsymbol{y}) \in \tilde{T}$. That is,

$$\sum_k F_k\left((\boldsymbol{x}^*, \boldsymbol{y}^*)\right)((\boldsymbol{x}_k, \boldsymbol{y}_k) - (\boldsymbol{x}_k^*, \boldsymbol{y}_k^*)) \geq 0 \quad \forall(\boldsymbol{x}_k, \boldsymbol{y}_k) \in \tilde{T},$$

where

$F_k\left((\boldsymbol{x}, \boldsymbol{y})\right) = (-\nabla_{x_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}), -\nabla_{y_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}))$, $\nabla_{x_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}) = \left(\frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial x_{k1}}, \dots, \frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial x_{kI}}\right)$ and $\nabla_{y_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}) = \left(\frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial y_{k1}}, \dots, \frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial y_{kQ}}\right)$.

Karamardian (1971) proves that each generalized complementary problem, i.e. KKT condition, corresponds to a set of VI. We extend this result and give the relationship between the complementary problem and the set of VI for the case when production is limited by the production possibility set as:

Theorem 4.2: Consider an oligopoly with K firms, an inverse demand function $P^Y(\cdot)$ that is strictly decreasing and continuously differentiable in y, and an inverse supply function $P^X(\cdot)$ that is strictly increasing and continuously differentiable in x. Since lemma 1 shows that the profit function $\theta_k(x_k, y_k)$ is concave and $x_k, y_k \geq 0$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*) = ((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*), \ldots, (\boldsymbol{x}_K^*, \boldsymbol{y}_K^*))$ is a Nash equilibrium solution if and only if

$$\nabla_{x_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq 0 \text{ and } \nabla_{y_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq 0 \quad \forall k;$$

$$\boldsymbol{x}_k^*\left[\nabla_{x_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*)\right] = 0 \text{ and } \boldsymbol{y}_k^*\left[\nabla_{y_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*)\right] = 0 \quad \forall k,$$

where $(\boldsymbol{x}_k^*, \boldsymbol{y}_k^*) \in \tilde{T}$.

Note that theorem 4.2 develops a relationship between a Nash equilibrium solution and the KKT conditions. Having established the relationship, we use the results to estimate revenue or profit maximizing benchmark frontiers as described below.

4.3 Revenue Maximization Model

Consider a firm with fixed input levels wanting to maximize revenues by adjusting its output level.[6] We employ a production process with a vector of inputs to generate a single output and then generalize it to the multiple-output production process. We illustrate both cases with an example from the productivity literature.

[6] This is consistent with an output-oriented efficiency analysis in the productivity literature.

4.3.1 Single-output Model

We estimate a production function with a single output and identify a Nash equilibrium solution using the MCP. Each firm adjusts its output level y_r to maximize the revenue function R_r .[7] Formulation (4.2) represents the revenue maximization model. To endogenously determine the price level, we define the inverse demand function as $P(Y)$. In general this demand function need only be strictly decreasing in Y. Since the market price in our model is affected by the total supply quantity $Y = \sum_{k \neq r} y_k + y_r$, we obtain the optimal output level as $y_r^* = arg_{y_r} R_r^*$. The model is feasible while $P(Y) \geq 0$ and $y_r \geq 0$ (Farahat and Perakis, 2010) and can be estimated as follows:

$$R^* = \max_{y_r} \left\{ \sum_r P(Y) y_r \middle| \begin{array}{c} \sum_k \lambda_{rk} Y_k \geq y_r \quad \forall r; \\ \sum_k \lambda_{rk} X_{ki} \leq X_{ri} \quad \forall i, r; \\ \sum_k \lambda_{rk} = 1 \quad \forall r; \\ \lambda_{rk} \geq 0 \quad \forall k, r; \end{array} \right\} \tag{4.2}$$

Defining $\mathcal{Y}_i$ as a random variable of quantity supplied in the market, we need a generalized form for the price function, $P(\mathcal{Y}_i)$, to estimate the inverse demand function. If the inverse demand function is strictly decreasing and continuously differentiable, then the revenue function is concave and continuously differentiable, and a Nash equilibrium solution exists (Murphy *et al.*, 1982). For illustrative purposes, we assume a linear inverse demand function which satisfies these properties, i.e. $P(Y) = P^0 - \alpha Y$, where P^0 is a positive intercept and α indicates the nonnegative price sensitivity with respect to Y. (See appendix C for a detailed discussion of the inverse demand function and the use of instrumental variables.) If $\alpha = 0$, then the price is constant regardless of

[7] This is consistent with a profit maximization model, given fixed input prices and levels.

the output level consistent with the standard analysis of allocative efficiency in the productivity and efficiency literature (Fried *et al.*, 2008), i.e. the price is exogenous as in the case of perfect competition.

In the single-output revenue model (4.2) with a linear inverse demand function, we use the CP to find the Nash equilibrium solution. We define the Lagrangian function as:

$$L_r(y_r, \lambda_{rk}, \mu 1_r, \mu 2_{ri}, \mu 3_r) =$$
$$\sum_r P(Y) y_r - \sum_r \mu 1_r (y_r - \sum_k \lambda_{rk} Y_k) - \sum_r \sum_i \mu 2_{ri} (\sum_k \lambda_{rk} X_{ki} - X_{ri}) -$$
$$\sum_r \mu 3_r (\sum_k \lambda_{rk} - 1).$$

The MCP is:

$$0 = \frac{\partial L_r}{\partial y_r} = (P(Y) - \alpha y_r - \mu 1_r) \perp y_r \quad \forall r$$

$$0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = (\mu 1_r Y_k - \sum_i \mu 2_{ri} X_{ki} - \mu 3_r) \perp \lambda_{rk} \geq 0 \quad \forall r, k$$

$$0 \geq (y_r - \sum_k \lambda_{rk} Y_k) \perp \mu 1_r \geq 0 \quad \forall r \tag{4.3}$$

$$0 \geq (\sum_k \lambda_{rk} X_{ki} - X_{ri}) \perp \mu 2_{ri} \geq 0 \quad \forall r, i$$

$$0 = (\sum_k \lambda_{rk} - 1) \quad \forall r$$

If the MCP gives the solutions $P(Y) < 0$, or $y_r < 0$, i.e. the inverse demand function returns a negative value, or the production output level is less than zero, this Nash equilibrium solution is inconsistent with production theory. Clearly, the sales price of a product cannot be negative. Similarly, if production will cause a profit loss, a firm's best strategy is to shut down, i.e. the output level will be zero. Thus, we show that a Nash solutions satisfy these two properties.

Lemma 4.2: A Nash solution to MCP problem (4.3) will satisfy $y_r \geq 0$ and $P(Y) \geq 0$.

Given $P^0 > 0$ and $\alpha \geq 0$, a small α means that a change in quantity of output will not affect the price significantly, but a large α *will greatly affect the price.* If the industry output level changes, the price will drop significantly and the revenues for all firms will likely decrease. Therefore, the firms have an incentive to restrict production to keep the price – and revenues – high. The same output level chosen by all firms is characterized by a common output level $\bar{y}_r$. The revenue maximizing benchmarks constitute a Nash equilibrium. Figure 4.3 illustrates the relationship between a Nash equilibrium and single-input-single-output production function, given parameter α.

Theorem 4.3: If $P(Y) = P^0 - \alpha Y \geq 0$ and α is a small enough positive parameter, the Nash equilibrium solution is for all firms to produce on the production frontier.

Theorem 4.4: If $P(Y) = P^0 - \alpha Y \geq 0$ and α is a large enough positive parameter, the MCP will lead to a benchmark output level with $y_r = \overline{y_r}$ close to zero, where $\overline{y_r}$ defines a truncated output level.

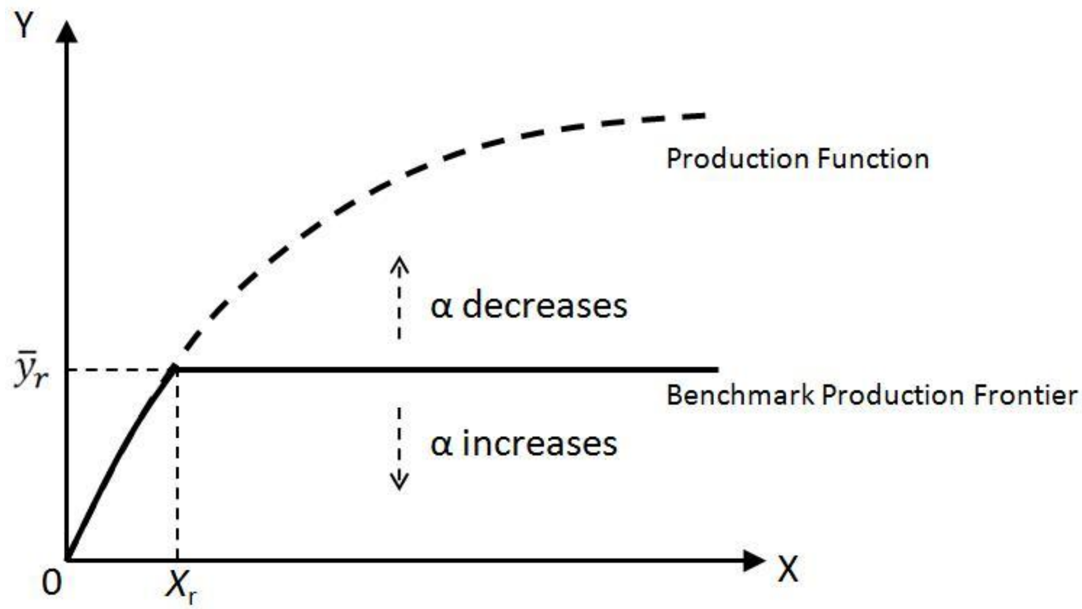

Figure 4.3 Nash equilibrium and α parameter adjustment

We select a dataset from Dyson *et al.* (1990) describing a set of distribution centers for a large supermarket organization to illustrate the single-output NEP. The two inputs are stocks and wages. The outputs correspond to the activities of the distribution center (DC). The three output variables available are: 1) number of issues representing deliveries to supermarkets, 2) number of receipts in bulk from suppliers, and 3) number of requisitions to suppliers. In this illustrative example, we only use the number of issues as a single output variable and assume a simple inverse demand function $P(Y) = 100 - \alpha Y$. Table 4.1 shows the best strategy for output expansion or contraction, given different price sensitivity values, α. As discussed, a firm's best strategy is to produce on the production frontier if the α value is small; alternatively, as α increases the benchmark function becomes truncated. Note that regardless of the value of α, the price and output quantity are always larger than zero as stated in lemma 4.2.

Table 4.1 Nash equilibrium in single-output production

Firms	Price sensitivity parameter α							
	0	0.05	0.1	0.3	0.5	1	10	100
DC 1	53.33	53.33	50.89	15.87	9.52	4.76	0.48	0.048
DC 2	49.17	49.17	49.17	15.87	9.52	4.76	0.48	0.048
DC 3	61.67	61.67	50.89	15.87	9.52	4.76	0.48	0.048
DC 4	70.00	70.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 5	32.50	32.50	32.50	15.87	9.52	4.76	0.48	0.048
DC 6	61.67	61.67	50.89	15.87	9.52	4.76	0.48	0.048
DC 7	80.00	80.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 8	65.00	65.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 9	53.33	53.33	50.89	15.87	9.52	4.76	0.48	0.048
DC 10	70.00	70.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 11	70.00	70.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 12	45.00	45.00	45.00	15.87	9.52	4.76	0.48	0.048
DC 13	70.00	70.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 14	45.00	45.00	45.00	15.87	9.52	4.76	0.48	0.048
DC 15	20.00	20.00	20.00	15.87	9.52	4.76	0.48	0.048
DC 16	53.33	53.33	50.89	15.87	9.52	4.76	0.48	0.048
DC 17	80.00	80.00	50.89	15.87	9.52	4.76	0.48	0.048
DC 18	61.67	61.67	50.89	15.87	9.52	4.76	0.48	0.048
DC 19	45.00	45.00	45.00	15.87	9.52	4.76	0.48	0.048
DC 20	61.67	61.67	50.89	15.87	9.52	4.76	0.48	0.048
SUM	1148.33	1148.33	949.11	317.46	190.48	95.24	9.52	0.96
PRICE	100.00	42.58	5.09	4.76	4.76	4.76	4.80	4.00

4.3.2 Multiple-output Model

To build a demand function for multiple differentiated substitutable products, we use the affine demand function proposed by Farahat and Perakis (2010) and define it as $Y_q\left(p_q, \boldsymbol{p}_{(-q)}\right) = Y_q^0 - \gamma_{qq} p_q + \sum_{h \neq q} \gamma_{qh} p_h$ for all q, where $Y_q \geq 0$ and $\boldsymbol{p}_{(-q)} \equiv (p_1, \dots, p_{q-1}, p_{q+1}, \dots, p_Q)$. For our purposes we define an inverse affine demand function, $Y_q(\cdot)^{-1}$, which exists if the condition of *diagonal dominance* of $\boldsymbol{\gamma}$ matrix is satisfied (Bernstein and Federgruen, 2004), i.e. $\gamma_{qq} > \sum_{h \neq q} \gamma_{qh}$. Specifically, we consider a linear inverse (indirect) affine demand function as $P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) = P_q^0 - \alpha_{qq}Y_q - \sum_{h \neq q} \alpha_{qh} Y_h$ for all q, where $P_q \geq 0$, $Y_q = \sum_k y_{kq}$, $\boldsymbol{Y}_{(-q)} \equiv (Y_1, \dots, Y_{q-1}, Y_{q+1}, \dots, Y_Q)$, and α_{qq} is the diagonal element of output q in the price sensitivity matrix $\boldsymbol{\alpha}$. In particular, $P_q \geq 0$ is not a prerequisite constraint in the revenue maximization problem and can be relaxed. Below we define a set of properties and the conditions for relaxing $P_q \geq 0$.

Four important properties of the price sensitivity matrix $\boldsymbol{\alpha}$ are:[8]

1) Weak diagonal dominance (WDD): if matrix $\boldsymbol{\alpha}$ satisfies diagonal dominance then the revenue function is strictly concave as discussed above.
2) Moderate diagonal dominance (MDD): if matrix $\boldsymbol{\alpha}$ satisfies $\alpha_{qq} \gg \sum_{h \neq q} \alpha_{qh}$ for all q. This property holds for product q if the main effect α_{qq} caused by the same product is more intense than the minor effect α_{qh} created by another substitute product.
3) Symmetric matrix: a symmetric matrix $\boldsymbol{\alpha}$ implies an equivalent bidirectional effect between any two substitute products.
4) Strong diagonal dominance (SDD): $\alpha_{qq} \gg sum(\boldsymbol{\alpha}) - tr(\boldsymbol{\alpha})$ for all q, where $sum(\boldsymbol{\alpha})$ denotes the sum of all elements in matrix $\boldsymbol{\alpha}$ and $tr(\boldsymbol{\alpha})$ denotes the trace which represents the sum of the elements on the diagonal of matrix $\boldsymbol{\alpha}$. SDD means that each product's quantity level generates a powerful main effect on the product's price.[9]

[8] Note that all output variables need to be normalized in data pre-processing to eliminate unit dependence.
[9] For a discussion of the relationship among these properties see the weak, moderate, and strong

The WDD property is likely to be true because, in general, the price of product *A* is more likely to be affected by the quantity produced of *A* than by the quantity produced of the substitute product *B*. We use the following formulation (4.4) to identify the optimal output levels:

$$R^* = \max_{y_{rq}} \left\{ \sum_r \sum_q P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) y_{rq} \;\middle|\; \begin{array}{c} \sum_k \lambda_{rk} Y_{kq} \geq y_{rq} \quad \forall q, r; \\ \sum_k \lambda_{rk} X_{ki} \leq X_{ri} \quad \forall i, r; \\ \sum_k \lambda_{rk} = 1 \quad \forall r; \\ \lambda_{rk} \geq 0 \quad \forall k, r; \end{array} \right\}. \tag{4.4}$$

Note that to identify a Nash equilibrium, the objective function has to be a strictly concave function in all arguments. Let $R = \sum_r \sum_q P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) y_{rq}$, giving $\frac{\partial R}{\partial y_{rq}} = P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh}$, $\forall q$, and $\frac{\partial^2 R}{\partial y_{rq} \partial y_{rh}} = -\alpha_{qh} - \alpha_{hq}$, $\forall q, h$. A negative definite Hessian matrix will imply a strictly concave revenue function. Thus, the necessary and sufficient conditions are $\alpha_{qh} > 0$ and the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies the WDD property, namely, $\alpha_{jj} > \sum_{h \neq j} \alpha_{jh}$ for all j.

To solve the Nash equilibrium of formulation (4.4), we construct the complementary problem and define the Lagrangian function as:

$$L_r\big(y_{rq}, \lambda_{rk}, \mu1_{rq}, \mu2_{ri}, \mu3_r\big) = \sum_r \sum_q P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) y_{rq} - \sum_r \sum_q \mu1_{rq} \big(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\big) - \sum_r \sum_i \mu2_{ri} \left(\sum_k \lambda_{rk} X_{ki} - X_{ri}\right) - \sum_r \mu3_r \left(\sum_k \lambda_{rk} - 1\right).$$

The MCP is:

dominance section in the appendix.

$$0 = \frac{\partial L_r}{\partial y_{rq}} = \left(P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}\right) \perp y_{rq} \quad \forall r, q$$

$$0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = \left(\sum_q \mu 1_{rq} Y_{kq} - \sum_i \mu 2_{ri} X_{ki} - \mu 3_r\right) \perp \lambda_{rk} \geq 0 \quad \forall r, k$$

$$0 \geq \left(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\right) \perp \mu 1_{rq} \geq 0 \quad \forall r, q \qquad (4.5)$$ [10]

$$0 \geq \left(\sum_k \lambda_{rk} X_{ki} - X_{ri}\right) \perp \mu 2_{ri} \geq 0 \quad \forall r, i$$

$$0 = \left(\sum_k \lambda_{rk} - 1\right) \quad \forall r$$

Similar results can now be developed for the multiple output case in theorem 4.5.

Theorem 4.5: If the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies WDD but is not necessarily symmetric, then the MCP (4.6) generates $(X_{ri}, y_{rq}) \in \tilde{T}$ where y_{rq} will approach the efficient frontier for small enough values of α_{qq}; $y_{rq} = \bar{y}_{rq}$ is the truncated benchmark output level that approaches zero as α_{qq} approaches infinity.

Corollary 4.1: If the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies the MDD property and $\alpha_{qq} \gg \alpha_{hh}, q \neq h$, then the solution to the MCP (4.6) will satisfy $y_{rq} < y_{rh}\ \forall r, q$.

---

[10] If matrix $\boldsymbol{\alpha}$ does not satisfy the SDD property, the resulting Nash equilibrium solution may include $y_{rq} < 0$. In this case MCP (4.5) is changed in the first inequality to state $0 \geq \frac{\partial L_r}{\partial y_{rq}}$, and $y_{rq} \geq 0$:

$$0 \geq \frac{\partial L_r}{\partial y_{rq}} = \left(P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}\right) \perp y_{rq} \geq 0 \quad \forall r, q$$

$$0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = \left(\sum_q \mu 1_{rq} Y_{kq} - \sum_i \mu 2_{ri} X_{ki} - \mu 3_r\right) \perp \lambda_{rk} \geq 0 \quad \forall r, k$$

$$0 \geq \left(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\right) \perp \mu 1_{rq} \geq 0 \quad \forall r, q \qquad (4.6)$$

$$0 \geq \left(\sum_k \lambda_{rk} X_{ki} - X_{ri}\right) \perp \mu 2_{ri} \geq 0 \quad \forall r, i$$

$$0 = \left(\sum_k \lambda_{rk} - 1\right) \quad \forall r$$

Theorem 4.5 is important because the relationship between price sensitivity matrix $\boldsymbol{\alpha}$ and the Nash equilibrium solution that can be identified from the characteristic of matrix $\boldsymbol{\alpha}$ gives insights into the Nash equilibrium regarding the elements in matrix $\boldsymbol{\alpha}$. The more price sensitive the product the more likely a firm will hold back production in order to increase its revenue.

Even if a large α_{qq} results in a truncated benchmark production level, it does not necessarily result in a common output value for all firms, because some firms may be limited by the production frontier. Referring to figure 4.3, X_r is the smallest input value to generate the truncated benchmark output level. Note that the production processes using an input quantity between 0 and X_r will identify a benchmark on the production frontier. Without loss of generality and $y_{rq} > 0$ from MCP (4.6), we have $P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} \geq 0$; therefore:

$$0 < y_{rq} \leq \frac{P_q^0 - \alpha_{qq} \sum_{k \neq r} y_{kq} - \sum_{h \neq q} \alpha_{qh} Y_h - \sum_{h \neq q} \alpha_{hq} y_{rh}}{2\alpha_{qq}} \qquad (4.7)$$

If for product q of firm r the efficient output level y_{rq} is lower than the truncated level $\bar{y}_q$, that is, the production frontier limits output level y_{rq}, then y_{rh} can exceed the truncated benchmark level $\bar{y}_h$ for some product h, because y_{rq} is smaller than the truncation level $\bar{y}_q$, and $\frac{\alpha_{hq}}{\alpha_{hh}}$ and $\frac{\alpha_{qh}}{\alpha_{hh}}$ do not go to zero in the inequality show in equation (4.7).[11] Simply stated, firms will adjust their mix in output space to maximize revenues

[11] Note the exchange of q and h.

and generally some variation from the truncated benchmark production level may exist.[12]

Again, we use our two-output illustrative example from the dataset described in section 4.3.1. The two output variables are the number of issues and the number of receipts, and the two inputs are stocks and wages. The inverse demand functions for issues and receipts are $P_{q_1}(Y_{q_1}, \boldsymbol{Y}_{(-q_1)}) = 100 - \alpha_{q_1q_1}Y_{q_1} - \alpha_{q_1q_2}Y_{q_2}$ and $P_{q_2}(Y_{q_2}, \boldsymbol{Y}_{(-q_2)}) = 50 - \alpha_{q_2q_2}Y_{q_2} - \alpha_{q_2q_1}Y_{q_1}$ respectively. Table 4.2 reports the Nash equilibrium solution to the MCP (4.6) for different price sensitivity matrix $\boldsymbol{\alpha}$, all of which satisfy the WDD property. Once more a firm's best strategy is to produce as close to the efficient frontier as possible for products with an insensitive inverse demand function implied by smaller values in the diagonal components of the $\boldsymbol{\alpha}$ matrix shown in case 1. As α_{qq} becomes larger the benchmark output level is truncated and approaches zero with respect to product q. In cases 2 the parameter $\alpha_{q_1q_1}$ is larger than case 1, the output q_1 decreases and output q_2 increases to maximize revenue. Similar in case 3, $\alpha_{q_2q_2}$ is increased relative to case 1 and the output q_2 decreases. In case 4 the parameter $\alpha_{q_2q_2}$ increases with respect to case 2, the solution shows output q_2 decreases to the truncated benchmark level. Increasing $\alpha_{q_1q_1}$ in cases 5 and 6, output q_1 approaches zero even though the $\boldsymbol{\alpha}$ matrices do not satisfy the symmetric condition. In cases 7 and 8 let $\alpha_{q_1q_1} = 2\alpha_{q_2q_2}$ and the results indicate that the ratio of output levels q_1 and q_2 are influence not only by the ratio of $\alpha_{q_1q_1}$ to $\alpha_{q_2q_2}$, but also by their absolute levels.

[12] This result is illustrated in table 4.2, case 2, DC 5.

Table 4.2 Nash equilibrium in two-output production

Case	1		2		3		4		5		6		7		8	
Output	q_1	q_2	q_1	q_2	q_1	q_2	q_1	q_2	q_1	q_2	q_1	q_2	q_1	q_2	q_1	q_2
$\boldsymbol{\alpha}$	0.05	0.02	1	0.02	0.05	0.02	1	0.02	1.2	0.08	10	5	20	0.02	0.1	0.02
	0.02	0.04	0.02	0.04	0.02	1	0.02	0.1	0.02	0.04	0.02	0.04	0.02	10	0.02	0.05
DC1	53.3	32.9	3.6	58.7	53.3	1.3	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC2	49.2	34.9	3.6	58.5	49.2	1.4	4.3	22.9	0.2	58.5	0.0	58.5	0.2	0.2	43.0	30.4
DC3	61.7	28.7	3.6	58.7	61.7	1.1	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC4	70.0	24.5	3.6	58.7	70.0	1.0	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC5	32.5	43.3	3.7	55.1	32.5	1.7	4.3	22.9	0.2	55.1	0.0	55.1	0.2	0.2	32.5	34.6
DC6	61.7	28.7	3.6	58.7	61.7	1.1	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC7	80.0	19.5	3.6	58.7	80.0	0.8	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC8	65.0	27.0	3.6	58.7	65.0	1.1	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC9	53.3	32.9	3.6	58.7	53.3	1.3	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC10	70.0	24.5	3.6	58.7	70.0	1.0	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC11	70.0	24.5	3.6	58.7	70.0	1.0	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC12	45.0	37.0	3.8	50.0	45.0	1.5	4.3	22.9	0.3	50.0	0.0	50.0	0.2	0.2	43.0	30.4
DC13	70.0	24.5	3.6	58.7	70.0	1.0	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC14	45.0	37.0	3.6	58.7	45.0	1.5	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC15	20.0	49.5	3.8	50.0	20.0	2.0	4.3	22.9	0.3	50.0	0.0	50.0	0.2	0.2	20.0	39.6
DC16	53.3	32.9	3.6	58.7	53.3	1.3	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC17	80.0	19.5	3.6	58.7	80.0	0.8	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC18	61.7	28.7	3.6	58.7	61.7	1.1	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC19	45.0	37.0	3.6	58.7	45.0	1.5	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
DC20	61.7	28.7	3.6	58.7	61.7	1.1	4.3	22.9	0.1	60.9	0.0	61.0	0.2	0.2	43.0	30.4
SUM	1148	616	72.2	1153	1148	24.7	86.1	459	3.0	1187	0.0	1189	4.8	4.8	826	621
Price	30.3	2.4	4.8	2.4	42.1	2.4	4.8	2.4	1.4	2.4	<0.0	2.4	4.7	2.3	4.9	2.4

Note that case 6 in Table 4.2, the price of product q_1 (issues) is less than 0, an unreasonable negative price, yet the revenue function is still equal to zero because $Y_{q_1} = 0$. Adding another constraint to restrict the price to be larger than zero will cause the quantity of product q_2 selected to drop, which gives a worse outcome.[13]

[13] The intuition for case 6 can be built using the single-output case considering only product q_2. The related problem of negative demand in demand function is modified using the price mappings (described in Shubik and Levitan (1980), Soon, *et al*. (2009), and Farahat and Perakis (2010)).

4.4 Generalized Profit Maximization Model

In our short-run profit models of oligopolistic output markets and a limited capacity input market, we only change the variable inputs, e.g., capital stock for production is fixed and employment or materials vary with demand (Marshall, 1920). Stigler (1939) argues that the quantitative variations of output can be described via the law of diminishing returns and marginal productivity theory when holding constant all but one of the productive factors and adjusting the quantity of the remaining factor. Thus, our generalized model treats fixed inputs and variable inputs separately.

This section also looks at the case of variable input markets with limited capacity and oligopolistic output markets, assuming that the inverse supply function of inputs and the inverse demand function of output are linear (see section 4.3 and the appendix C). We formulate our generalized profit maximization model as equation (4.8):

$$PF^* = \max_{y_{rq}, x_{rj}^V} \left\{ \begin{array}{c} \sum_r \sum_q P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) y_{rq} \\ -\sum_r \sum_j P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) x_{rj}^V \end{array} \middle| \begin{array}{c} \sum_k \lambda_{rk} Y_{kq} \geq y_{rq} \quad \forall q, r; \\ \sum_k \lambda_{rk} X_{ki}^F \leq X_{ri}^F \quad \forall i, r; \\ \sum_k \lambda_{rk} X_{kj}^V \leq x_{rj}^V \quad \forall j, r; \\ \sum_k \lambda_{rk} = 1 \quad \forall r; \\ \lambda_{rk} \geq 0 \quad \forall k, r; \end{array} \right\} \qquad (4.8)$$

where X_{ri}^F is the data for fixed input i and x_{rj}^V is the variable for variable input j of firm r . $Y_q = \sum_{k \neq r} y_{kq} + y_{rq} \quad \forall q$ and $P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) = P_q^{Y_0} - \alpha_{qq} Y_q - \sum_{h \neq q} \alpha_{qh} Y_h \quad \forall q$ indicate the overall quantity and price of the inverse demand function of output product q in the market. Similarly, for variable input j the overall quantity $X_j^V = \sum_{k \neq r} x_{kj}^V + x_{rj}^V \quad \forall j$ and the inverse supply function $P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) = P_j^{X_0^V} + \beta_{jj} X_j^V +$

$\sum_{l \neq j} \beta_{jl} X_l^V \quad \forall j$. Note that the objective function ignores the fixed input cost $\sum_i P_i^{X^F}\left(X_i^F, \boldsymbol{X}_{(-i)}^F\right) X_{ri}^F$ since it is a constant sunk cost.

To verify the existence and uniqueness of a solution, the profit function should be strictly concave. Let $PF = \sum_r \sum_q P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) y_{rq} - \sum_r \sum_j P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) x_{rj}^V$ be the profit function. That is, the revenue function $\sum_r \sum_q P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) y_{rq}$ should be strictly concave and the variable cost function $\sum_r \sum_j P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) x_{rj}^V$ strictly convex. We have $\frac{\partial PF_r}{\partial y_{rq}} = P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh}$, $\forall q$, and $\frac{\partial^2 PF_r}{\partial y_{rq} \partial y_{rh}} = -\alpha_{qh} - \alpha_{hq}$, $\forall q, h$. A negative definite Hessian matrix will imply a strictly concave revenue function. Thus, the necessary and sufficient conditions are $\alpha_{qh} > 0$ and the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies the WDD property, namely, $\alpha_{qq} > \sum_{h \neq q} \alpha_{qh}$ for all q. Also, we have $\frac{\partial PF_r}{\partial x_{rj}^V} = -P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) - \beta_{qq} x_{rj}^V - \sum_{l \neq j} \beta_{lj} x_{rl}^V$, $\forall j$, and $\frac{\partial^2 PF_r}{\partial x_{rj}^V \partial x_{rl}^V} = -\beta_{jl} - \beta_{lj}$, $\forall j, l$. A negative definite Hessian matrix will imply a strictly concave negative cost function. Similarly, the necessary and sufficient conditions are $\beta_{jl} > 0$ and the price sensitivity matrix $\boldsymbol{\beta}$ satisfies the WDD property.[14]

To solve for a Nash equilibrium associated with equation (4.8), the CP is built and the Lagrangian function defined as:

$L_r\left(y_{rq}, x_{rj}^V, \lambda_{rk}, \mu 1_{rq}, \mu 2_{ri}, \mu 3_{rj}, \mu 4_r\right)$

[14] In a special case in which input markets are perfectly competitive $\beta_{jl} = 0$, the inverse supply function will be constant and the cost function becomes a linear function. This does not affect the optimality condition, i.e. the profit function is still a strictly concave function if the revenue function is strictly concave.

$= \sum_r \sum_q P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) y_{rq} - \sum_r \sum_j P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) x_{rj}^V - \sum_r \sum_q \mu 1_{rq} \left(y_{rq} -\right.$

$\left.\sum_k \lambda_{rk} Y_{kq}\right) - \sum_r \sum_i \mu 2_{ri} \left(\sum_k \lambda_{rk} X_{ki}^F - X_{ri}^F\right) - \sum_r \sum_j \mu 3_{rj} \left(\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V\right) -$

$\sum_r \mu 4_r \left(\sum_k \lambda_{rk} - 1\right).$

The MCP is:

$$0 \geq \frac{\partial L_r}{\partial y_{rq}} = \left(P_q^Y\left(Y_q, \boldsymbol{Y}_{(-q)}\right) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}\right) \perp y_{rq} \geq 0 \quad \forall r, q$$

$$0 \geq \frac{\partial L_r}{\partial x_{rj}^V} = \left(-P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) - \beta_{jj} x_{rj}^V - \sum_{l \neq j} \beta_{lj} x_{rl}^V + \mu 3_{rj}\right) \perp x_{rj}^V \geq 0 \quad \forall r, j$$

$$0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = \left(\sum_j \mu 1_{rq} Y_{kq} - \sum_i \mu 2_{ri} X_{ki}^F - \sum_j \mu 3_{rj} X_{kj}^V - \mu 4_r\right) \perp \lambda_{rk} \geq 0 \quad \forall r, k$$

$$0 \geq \left(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\right) \perp \mu 1_{rq} \geq 0 \quad \forall r, q \qquad (4.9)$$

$$0 \geq \left(\sum_k \lambda_{rk} X_{ki}^F - X_{ri}^F\right) \perp \mu 2_{ri} \geq 0 \quad \forall r, i$$

$$0 \geq \left(\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V\right) \perp \mu 3_{rj} \geq 0 \quad \forall r, j$$

$$0 = \left(\sum_k \lambda_{rk} - 1\right) \quad \forall r$$

The Nash equilibrium solution generated from MCP (4.9) exists and is unique when the price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ satisfy the WDD property. See section 4.4.1 for a similar proof.

In a perfectly competitive market, the profit efficient firms, i.e. achieving maximum profits (Farrell, 1957), must be allocatively efficient by using the least cost mix of inputs to produce the maximum revenue mix of outputs, and technically efficient by generating the most outputs with their level of inputs, (Färe, *et al*., 1994). In oligopolistic markets, however, profit maximization can be achieved without technical efficiency, i.e. rational inefficiency. We will continue to refer to the profit maximizing production possibility as allocatively efficient because it is not possible to change either the input mix or the output mix to increase profits. MCP (4.9) generates an allocatively efficient Nash solution.

Theorem 4.6: Given arbitrary price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ that satisfy WDD, MCP (4.9) generates all allocatively efficient Nash solutions $(X_{ri}^{F}, x_{rj}^{V*}, y_{rq}^{*}) \in \tilde{T}$. These solutions are on the frontier including the weakly efficient frontier[15], but excluding the portion of the frontier associated with positive slacks and dual variables equal to zero on the input constraints.

Theorem 4.6 implies that the Nash equilibrium benchmark generated from MCP (4.9) exists on the production frontier using the same or fewer inputs than at least one anchor point, (Bougnol and Dulá, 2009).[16] Based on theorem 4.5, if α_{qq} becomes large, the production level will approach zero with respect to q and the Nash solution will be located on the weakly efficient portion of the production frontier which uses minimal input levels. In other words, if the price sensitivity to output is large enough, the Nash equilibrium benchmark suggests that a firm should operate on the weakly efficient portion of the frontier where more output can be generated using the same level of

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[15] Weakly efficient frontier is defined as the portion of the input (output) isoquant along which one of the inputs (outputs) can be reduced (expanded) while holding all other netputs constant and remaining on the isoquant; see Färe and Lovell (1978) for more details.

[16] Bougnol and Dulá (2009) propose a procedure to identify *anchor points* and show that if a point is an anchor point, then increasing an input or decreasing an output generates a new point on the free-disposability portion of the production possibility set.

inputs. In this case, note that the profits are maximized by operating inefficiently, motivating the connection to rational inefficiency.

The illustrative example of the generalized profit model also uses the dataset in section 4.3. The two output variables are the number of issues and the number of receipts, and the two variable inputs are stocks and wages. One fixed input is randomly generated from a Uniform [3,10] distribution. The inverse demand functions for issues and receipts are $P_{q_1}^{Y}\left(Y_{q_1}, \boldsymbol{Y}_{(-q_1)}\right) = 100 - \alpha_{q_1 q_1} Y_{q_1} - \alpha_{q_1 q_2} Y_{q_2}$ and $P_{q_2}^{Y}\left(Y_{q_2}, \boldsymbol{Y}_{(-q_2)}\right) = 50 - \alpha_{q_2 q_2} Y_{q_2} - \alpha_{q_2 q_1} Y_{q_1}$ respectively. The inverse supply functions for stocks and wages are $P_{j_1}^{X^V}\left(X_{j_1}^V, \boldsymbol{X}_{(-j_1)}^V\right) = 50 + \beta_{j_1 j_1} X_{j_1}^V + \beta_{j_1 j_2} X_{j_2}^V$ and $P_{j_2}^{X^V}\left(X_{j_2}^V, \boldsymbol{X}_{(-j_2)}^V\right) = 30 + \beta_{j_2 j_2} X_{j_2}^V + \beta_{j_2 j_1} X_{j_1}^V$ respectively. Table 4.3 reports the Nash equilibrium solution to MCP (4.9) for the price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, all of which satisfy the WDD property. Again, for outputs with insensitive inverse demand functions implied by smaller values in the diagonal components of the $\boldsymbol{\alpha}$ matrix, a firm's best strategy is to produce near the efficient frontier; as α_{qq} becomes large, the production approaches zero with respect to q as shown in case 4. Similarly on the input side, for inputs with sensitive inverse supply functions implied by larger values in the diagonal components of the $\boldsymbol{\beta}$ matrix, the best strategy is to use smaller input levels to produce on the weakly efficient frontier; as β_{jj} becomes smaller, the input level of the Nash equilibrium solution grows larger. However, the input level of the solution is always limited to the range of lower and upper bounds identified in theorem 6. Furthermore, the price sensitivity value $\boldsymbol{\beta}$ will affect the price of Nash solution significantly, cost will increase quickly and profits will drop. Cases 1, 2 and 3 show that as $\beta_{j_2 j_2}$ increases, costs also increase and producers have less incentive to produce. Cases 4 and 5 decrease the output level due to changes in the $\boldsymbol{\alpha}$ matrix; in particular, case 5 illustrates rational inefficiency because firms hold back producing additional output in order to maximize profits.

4.4.1 Existence and Uniqueness

If a Nash equilibrium does not exist, there is no purpose in talking about its properties, identification, etc. Further, if multiple equilibria exist, it is not clear which might result in any particular case. In this section we prove the existence and uniqueness of the Nash equilibrium solution identified by the MCP.

Theorem 4.7: MCP (4.9) generates a Nash equilibrium solution $(\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*) \in \tilde{T}$.

To get a unique Nash equilibrium, a strictly concave profit function is assumed. Given a convex production possibility set, theorem 4.8 states the uniqueness of the Nash equilibrium.

Table 4.3 Nash equilibrium in two-output two-variable-input production

Case	1				2				3				4				5			
Variable	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2
$\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$	0.05	0.02	0.05	0.02	0.05	0.02	0.05	0.02	0.05	0.02	0.05	0.02	1	0.02	0.05	0.02	1	0.02	0.05	0.02
	0.02	0.04	0.02	0.04	0.02	0.04	0.02	1	0.02	0.04	0.02	10	0.02	0.04	0.02	0.04	0.02	1	0.02	0.04
DC1	70.0	24.5	5.0	7.0	70.0	24.5	5.0	7.0	45.0	37.0	3.0	4.0	3.4	67.0	3.0	4.0	4.7	2.3	3.0	4.0
DC2	70.0	24.5	5.0	7.0	55.0	32.0	3.2	5.2	45.0	37.0	2.4	4.0	3.8	50.0	2.2	3.3	4.7	2.3	2.2	3.3
DC3	55.0	32.0	4.0	6.0	55.0	32.0	4.0	6.0	55.0	32.0	4.0	6.0	3.9	45.0	4.0	6.0	4.7	2.3	4.0	6.0
DC4	70.0	24.5	5.0	7.0	65.0	27.0	4.4	6.4	45.0	37.0	2.8	4.0	3.6	58.5	2.7	3.8	4.7	2.3	2.7	3.8
DC5	70.0	24.5	5.0	7.0	60.0	29.5	3.8	5.8	45.0	37.0	2.6	4.0	3.8	50.0	2.3	3.5	4.7	2.3	2.3	3.5
DC6	70.0	24.5	5.0	7.0	70.0	24.5	5.0	7.0	45.0	37.0	3.0	4.0	3.4	67.0	3.0	4.0	4.7	2.3	3.0	4.0
DC7	70.0	24.5	5.0	7.0	48.8	35.1	2.5	4.5	45.0	37.0	2.0	4.0	3.8	50.0	2.0	3.0	4.7	2.3	2.0	3.0
DC8	70.0	24.5	5.0	7.0	65.0	27.0	4.4	6.4	45.0	37.0	2.8	4.0	3.6	58.5	2.7	3.8	4.7	2.3	2.7	3.8
DC9	70.0	24.5	5.0	7.0	65.0	27.0	4.4	6.4	45.0	37.0	2.8	4.0	3.6	58.5	2.7	3.8	4.7	2.3	2.7	3.8
DC10	70.0	24.5	5.0	7.0	70.0	24.5	5.0	7.0	45.0	37.0	3.0	4.0	3.4	67.0	3.0	4.0	4.7	2.3	3.0	4.0
DC11	70.0	24.5	5.0	7.0	60.0	29.5	3.8	5.8	45.0	37.0	2.6	4.0	3.8	50.0	2.3	3.5	4.7	2.3	2.3	3.5
DC12	70.0	24.5	5.0	7.0	48.8	35.1	2.5	4.5	45.0	37.0	2.0	4.0	3.8	50.0	2.0	3.0	4.7	2.3	2.0	3.0
DC13	70.0	24.5	5.0	7.0	55.0	32.0	3.2	5.2	45.0	37.0	2.4	4.0	3.8	50.0	2.2	3.3	4.7	2.3	2.2	3.3
DC14	70.0	24.5	5.0	7.0	70.0	24.5	5.0	7.0	45.0	37.0	3.0	4.0	3.4	67.0	3.0	4.0	4.7	2.3	3.0	4.0
DC15	70.0	24.5	5.0	7.0	48.8	35.1	2.5	4.5	45.0	37.0	2.0	4.0	3.8	50.0	2.0	3.0	4.7	2.3	2.0	3.0
DC16	70.0	24.5	5.0	7.0	65.0	27.0	4.4	6.4	45.0	37.0	2.8	4.0	3.6	58.5	2.7	3.8	4.7	2.3	2.7	3.8
DC17	70.0	24.5	5.0	7.0	60.0	29.5	3.8	5.8	45.0	37.0	2.6	4.0	3.8	50.0	2.3	3.5	4.7	2.3	2.3	3.5
DC18	70.0	24.5	5.0	7.0	48.8	35.1	2.5	4.5	45.0	37.0	2.0	4.0	3.8	50.0	2.0	3.0	4.7	2.3	2.0	3.0
DC19	70.0	24.5	5.0	7.0	70.0	24.5	5.0	7.0	45.0	37.0	3.0	4.0	3.4	67.0	3.0	4.0	4.7	2.3	3.0	4.0
DC20	70.0	24.5	5.0	7.0	48.8	35.1	2.5	4.5	45.0	37.0	2.0	4.0	3.8	50.0	2.0	3.0	4.7	2.3	2.0	3.0
SUM	1385	498	99	139	1199	591	77	117	910	735	53	82	73	1114	51	73	94	46	51	73
Price	20.8	2.4	57.7	37.5	28.2	2.4	56.2	148.2	39.8	2.4	54.3	851.1	4.8	4.0	54.0	33.9	4.8	2.4	54.0	33.9
Revenue	29980				35257				37960				4782				558			
Cost	10933				21599				72653				5233				5233			
Profit	19047				13658				-34692				-451				-4675			

Theorem 4.8: If the profit function is a strictly concave function on $(\boldsymbol{X}^F, \boldsymbol{x}^V, \boldsymbol{y}) \in \tilde{T}$ that is continuous and differentiable and the price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ satisfy the WDD property, then the Nash equilibrium solution found using MCP (4.9) is unique if a solution exists for the maximization problem.

4.4.2 Price Sensitivity and Returns to Scale

It is necessary to understand the relationship between the price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ and the returns to scale (RTS) properties of the Nash equilibrium benchmarks. To address RTS properties, we must first identify the "Most Productive Scale Size (MPSS)". The production frontier is characterized by three regions: constant returns to scale (CRS), increasing returns to scale (IRS), and decreasing returns to scale (DRS). The MPSS can be identified for firm r's input and output mix using the input-oriented CRS DEA technique formulated in (4.10). If the sum of $\sum_k \lambda_{rk}^{CRS} = 1$ in the input-oriented CRS DEA[17], we can identify such observations as operating at the MPSS (Banker, 1984):

$$\min_{\theta_r, \lambda_{rk}^{CRS}} \left\{ \sum_r \theta_r \middle| \begin{array}{c} \sum_k \lambda_{rk}^{CRS} Y_{kq} \geq Y_{rq} \quad \forall q, r; \\ \sum_k \lambda_{rk}^{CRS} X_{ki}^F \leq X_{ri}^F \quad \forall i, r; \\ \sum_k \lambda_{rk}^{CRS} X_{kj}^V \leq \theta_r X_{rj}^V \quad \forall j, r; \\ \lambda_{rk}^{CRS} \geq 0 \quad \forall k, r; \end{array} \right\} \tag{4.10}$$

Let k^{MPSS} denote the set of observations having the MPSS property, one for each firm r in the dataset, and let y_{rq}^* and x_{rj}^{V*} be the Nash equilibrium solutions obtained from MCP (4.9). Using these additional observations as the reference set, optimization problem (4.11) can be used to identify the returns to scale property for each production plan in the Nash solution.

$$\min_{\theta_r, \lambda_{rk^{MPSS}}^{CRS}} \left\{ \sum_r \theta_r \middle| \begin{array}{c} \sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS} Y_{k^{MPSS}q} \geq y_{rq}^* \quad \forall q, r; \\ \sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS} X_{k^{MPSS}i}^F \leq X_{ri}^F \quad \forall i, r; \\ \sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS} X_{k^{MPSS}j}^V \leq \theta_r x_{rj}^{V*} \quad \forall j, r; \\ \lambda_{rk^{MPSS}}^{CRS} \geq 0 \quad \forall k, r; \end{array} \right\}. \tag{4.11}$$

[17] Note there are potential multiple optimal solutions. See Zhu (2000) for additional details.

For each Nash solution of firm r, if $\sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS*} < 1$, firm r operates under increasing returns to scale; if $\sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS*} > 1$, firm r operates under decreasing returns to scale; or if $\sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS*} = 1$, firm r operates under constant returns to scale.[18] The equation $\sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS*}$ is termed the RTS index (RTSI).

For a one-input one-output production process, figure 4.4 depicts the true production function as a solid curve, the CRS estimated frontier as a straight dashed line, the VRS estimated frontier as a piece-wise linear bold dashed line, and the MPSS as Point B. In particular, based on theorem 4.6 the Nash equilibrium generated from MCP (4.9) should be located on the bold dashed lines. X_A and X_E are the upper and lower bounds for the variable input level as discussed below theorem 4.6.

Corollary 4.2: Assume all input and output variables are normalized to eliminate unit dependence, and the price of outputs dominates the price of inputs to ensure a positive marginal profit. Given a production frontier including three portions: IRS, CRS, and DRS, the MCP (4.9) generates a Nash equilibrium solution that is characterized by DRS when the inverse demand and supply functions are less sensitive, or the Nash equilibrium is characterized by IRS when the inverse demand and supply functions are more sensitive.

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[18] In (4.9) note that both inputs and outputs are defined as adjustable; thus, all Nash equilibria are located on the production frontier. If this is not the case, for example if there are adjustment costs when changing input levels (Choi et al., 2006), then some equilibria may not be on the frontier as shown in figure 4.3. For these equilibria RTS are not defined because RTS is a frontier property. In (4.11) if θ_r is not equal to 1, then RTS may not defined for that production possibility; see for example Seiford and Zhu (1999) or Ray (2010).

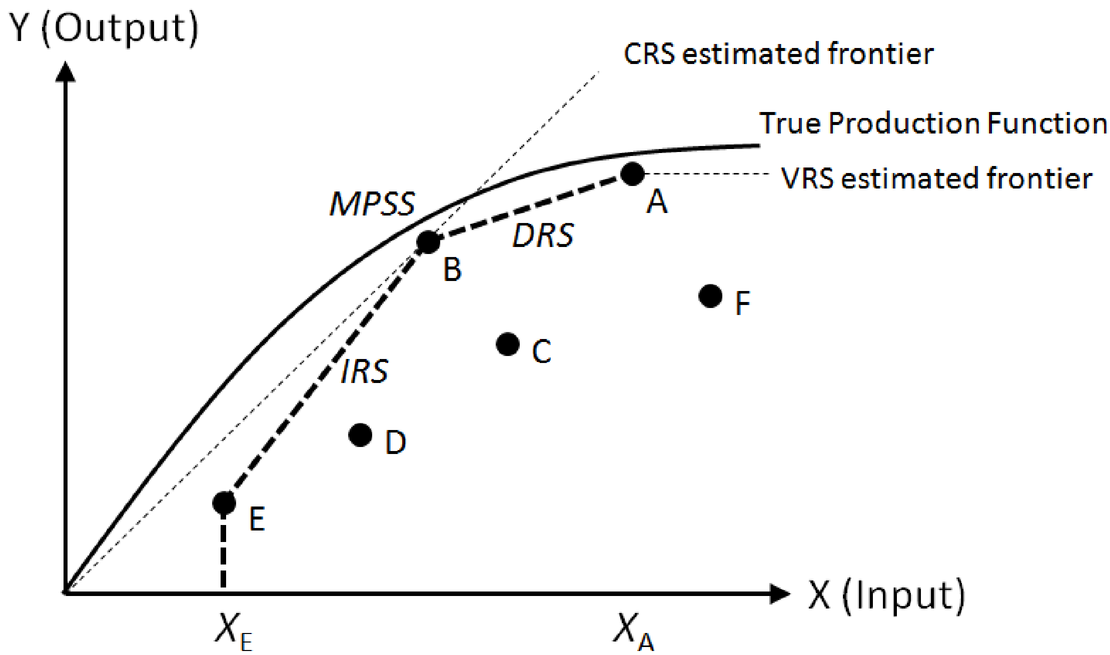

Figure 4.4 Nash equilibrium on bold dashed lines

Extending the illustrative example in section 4.2, we use formulation (4.11) to identify the RTS property of the Nash equilibrium solution shown in table 4.3; DCs 3, 10, 12, 15, and 19 are identified as operating at MPSS. Table 4.4 shows the RTS associated with the Nash solutions for cases 1 through 5 in table 4.3. Based on corollary 4.2, the sensitivity of output and sensitivity of input are the two oppositional forces in terms of scale. Case 1 represents a baseline and the Nash solutions present CRS or DRS properties. The sensitivity parameter of the supply function in case 2 increases relative to case 1, which encourages firms to hold back on the consumption of inputs, i.e. more DCs operate at MPSS in case 2. If we further increase the sensitivity parameter of the supply function, all DCs operate at MPSS in case 3. Case 4 results in all firms operating at MPSS or IRS, by increasing the sensitivity parameter of the demand function and leaving the sensitivity parameter of the supply function parameter the same as in case 1.

Case 5 shows that all firms operate at IRS and on the weakly efficient portion of the frontier. This demonstrates the concept of rational inefficiency.

Table 4.4 Returns to scale of Nash equilibrium

Case	1				2				3				4				5			
	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2	q_1	q_2	j_1	j_2
$\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$	0.05	0.02	0.05	0.02	0.05	0.02	0.05	0.02	0.05	0.02	0.05	0.02	1	0.02	0.05	0.02	1	0.02	0.05	0.02
	0.02	0.04	0.02	0.04	0.02	0.04	0.02	1	0.02	0.04	0.02	10	0.02	0.04	0.02	0.04	0.02	1	0.02	0.04
Returns to scale	RTSI		RTS		RTSI		RTS		RTSI		RTS		RTSI		RTS		RTSI		RTS	
DC1	1		C		1		C		1		C		1		C		0.104		I	
DC2	1.5		D		1		C		1		C		0.916		I		0.104		I	
DC3	1		C		1		C		1		C		0.685		I		0.104		I	
DC4	1.167		D		1		C		1		C		0.937		I		0.104		I	
DC5	1.333		D		1		C		1		C		0.873		I		0.104		I	
DC6	1		C		1		C		1		C		1		C		0.104		I	
DC7	1.556		D		1.084		D		1		C		1		C		0.104		I	
DC8	1.167		D		1		C		1		C		0.937		I		0.104		I	
DC9	1.167		D		1		C		1		C		0.937		I		0.104		I	
DC10	1		C		1		C		1		C		1		C		0.104		I	
DC11	1.333		D		1		C		1		C		0.873		I		0.104		I	
DC12	1.556		D		1.084		D		1		C		1		C		0.104		I	
DC13	1.5		D		1		C		1		C		0.916		I		0.104		I	
DC14	1		C		1		C		1		C		1		C		0.104		I	
DC15	1.556		D		1.084		D		1		C		1		C		0.104		I	
DC16	1.167		D		1		C		1		C		0.937		I		0.104		I	
DC17	1.333		D		1		C		1		C		0.873		I		0.104		I	
DC18	1.556		D		1.084		D		1		C		1		C		0.104		I	
DC19	1		C		1		C		1		C		1		C		0.104		I	
DC20	1.556		D		1.084		D		1		C		1		C		0.104		I	
Returns to Scale	CRS or DRS				CRS or DRS				CRS				CRS or IRS				IRS			

* C, D and I indicate constant, decreasing, and increasing returns to scale respectively.

4.4.3 Allocative Efficiency and Directional Distance Function

The Nash equilibrium identified by using (4.9) is an allocatively efficient solution as shown in theorem 4.6. Zofio and Prieto (2006) suggest choosing the direction in the direction distance function (DDF) to move towards the allocatively efficient point. We extend their suggestion to the case of oligopolistic markets and suggest that each firm should select the direction for improvement in the DDF to move towards its Nash equilibrium benchmark.

The DDF as defined by Chambers *et al.* (1996; 1998) is the simultaneous contraction of inputs and expansion of outputs:

$$\vec{D}_T\left(\boldsymbol{X}^F, \boldsymbol{X}^V, \boldsymbol{Y}; \boldsymbol{g}^{x^V}, \boldsymbol{g}^y\right) = \max\{\delta \in \mathcal{R}: (\boldsymbol{X}^F, \boldsymbol{X}^V + \delta \boldsymbol{g}^{x^V}, \boldsymbol{Y} + \delta \boldsymbol{g}^y) \in T\} \qquad (4.12)$$

where δ is the distance measure and $\boldsymbol{g}^{x^V}, \boldsymbol{g}^y$ are the direction vectors for variable inputs and outputs respectively. Recall that since we do not change the fixed inputs in the short run, no direction is associated with them. We estimate the DDF for firm r as:

$$\vec{D}_{\tilde{T}}\left(\boldsymbol{X}_r^F, \boldsymbol{X}_r^V, \boldsymbol{Y}_r; \boldsymbol{g}^{x^V}, \boldsymbol{g}^y\right) = \max_{\delta_r, \lambda_k} \left\{ \delta_r \left| \begin{array}{c} \sum_k \lambda_k Y_{kq} \geq Y_{rq} + \delta_r g_q^y \quad \forall q; \\ \sum_k \lambda_k X_{ki}^F \leq X_{ri}^F \quad \forall i; \\ \sum_k \lambda_k X_{kj}^V \leq X_{rj}^V + \delta_r g_j^{x^V} \quad \forall j; \\ \sum_k \lambda_k = 1; \\ \lambda_k \geq 0 \quad \forall k; \end{array} \right. \right\}. \qquad (4.13)$$

Because the method for selecting a direction $\left(\boldsymbol{g}^{x^V}, \boldsymbol{g}^y\right)$ is an open issue, the direction $(\boldsymbol{-1}, \boldsymbol{1})$ is usually chosen for simplicity. Alternatively, Frei and Harker (1999) determine the least-norm projection from an inefficient firm to the frontier, but this direction is non-proportional and is not unit-invariant. Färe *et al.* (2011) estimate an endogenous direction, but it is void of economic meaning. Therefore, we propose that firms' direction for improvement move towards the allocatively efficient benchmark identified by the Nash equilibrium. Thus, the direction is firm-specific and can be calculated by following the equation for firm r:

$$\left(\boldsymbol{g}_r^{x^V}, \boldsymbol{g}_r^{y}\right) = \frac{(\boldsymbol{x}_r^{V*} - \boldsymbol{X}_r^V, \boldsymbol{y}_r^* - \boldsymbol{Y}_r)}{\|(\boldsymbol{x}_r^{V*} - \boldsymbol{X}_r^V, \boldsymbol{y}_r^* - \boldsymbol{Y}_r)\|} \tag{4.14}$$

where $\boldsymbol{x}_r^{V*}$ and $\boldsymbol{y}_r^*$ are the benchmarks determined by the Nash equilibrium, $\boldsymbol{X}_r^V$ and $\boldsymbol{Y}_r$ are the vectors of the current variable input and output production, and $\|\cdot\|$ is the Euclidean norm. This ratio imposes $\left(\boldsymbol{g}^{x^V}, \boldsymbol{g}^{y}\right)$ is a unit vector.[19]

Extending the example in table 4.3, case 1, we calculate the direction of improvement associated with this example as shown in table 4.5. The results indicate that when trying to maximize overall economic efficiency[20] using formulation (4.8) it is not necessary to contract the variable inputs and expand the outputs. To maintain higher price and profit maximization, firm r may achieve economic efficiency by changing its mix to become allocatively efficient. However, no firm takes a direction which increases all variable inputs and decreases all output levels as this would lead to a loss in profit.

[19] The length of the directional vector influences the efficiency estimates in the DDF; the use of a unit vector has also been used in Fare *et al.* (2011).

[20] Economic efficiency is the product of allocative efficiency and technical efficiency, see for example Fried *et al.* (2008).

Table 4.5 Direction determination

Case 1	Direction $\left(\boldsymbol{g}^{x^V}, \boldsymbol{g}^{y}\right)$			
	j_1	j_2	q_1	q_2
DC1	0.0467	0.0467	0.7000	-0.7111
DC2	0.0697	0.0697	0.6970	-0.7103
DC3	0.0000	0.0000	0.0000	-1.0000
DC4	-0.0445	0.0000	0.9785	0.2012
DC5	0.0547	0.0710	0.8516	-0.5165
DC6	0.0445	0.0222	0.9783	0.2012
DC7	-0.0478	-0.0717	-0.2390	-0.9672
DC8	0.0118	0.0118	0.8865	-0.4625
DC9	0.0427	0.0427	0.5341	-0.8433
DC10	0.0000	0.0000	0.0000	-1.0000
DC11	0.0000	0.0000	0.5255	-0.8508
DC12	0.1010	0.1010	0.8416	-0.5210
DC13	0.0000	0.0000	0.9955	-0.0948
DC14	0.0305	0.0914	0.9753	0.1988
DC15	0.0532	0.0710	0.8875	-0.4522
DC16	0.0617	0.0309	0.9878	0.1396
DC17	-0.0503	-0.1006	0.0503	-0.9924
DC18	0.0213	0.0213	0.9575	-0.2867
DC19	0.0405	0.0607	0.5059	-0.8595
DC20	0.0000	0.0265	0.3440	-0.9386

4.5 Concluding Remarks

This study analyzes endogenous prices in productivity analysis. Given inverse demand and supply functions, a Nash equilibrium solution corresponding to profit maximization production plan within the production possibility set is identified using a mixed complementary problem (MCP). When the inverse demand and supply functions are constant functions, the standard analysis of efficiency assuming perfect competition and exogenous prices follows. For markets in which demand is heavily influence by the total supply quantity, firms seek to decrease their output levels and maintain higher product prices to maximize profits. The proposed MCP model integrates oligopolistic market equilibrium and productivity analysis. We find that the resulting Nash equilibrium is an example of rational inefficiency.

Deviating from standard economic analysis, we consider the production limitations estimated from observed data and interpret the Nash equilibrium as the benchmark, or the production plans each of the firms should work towards for more profitable production. Our work extends the efficiency literature on demand functions by considering multiple output production and allowing both outputs and variable inputs to be adjusted by the firm. Prior work primarily focused on individual firms decisions without consideration for the other firms in the market.

The identification of a unique Nash equilibrium allows further insights to operational improvement strategies. We show the relationship between price sensitivity and returns to scale in the Nash equilibrium. Based on the concept of allocative efficiency, we conclude that the Nash equilibrium is a useful guide for determining direction in the directional distance function.

CHAPTER V

CONCLUSIONS

5.1 Summary

The strategic position of productivity analysis in production planning is identified and a set of the new models to assess the effect of demand on productivity and efficiency measurement is investigated.

In chapter I, the functional position of productivity and efficiency analysis (PEA) within the production planning framework is defined. Performance benchmarking and production guidance are two services which PEA can provide. Furthermore, this study builds a bridge between PEA and demand management.

In chapter II, a two-dimensional efficiency decomposition (2DED) of profitability for a production system to account for the demand effect observed in productivity analysis is described. The first dimension identifies four components of efficiency: capacity design, demand generation, operations, and demand consumption, using Network Data Envelopment Analysis (Network DEA). The second dimension decomposes the efficiency measures and integrates them into a profitability efficiency framework. Thus, each component's profitability change is analyzed based on technical efficiency change, scale efficiency change and allocative efficiency change. An empirical study based on data from 2006 to 2008 for the US airline industry finds that the regress of productivity is mainly caused by a demand fluctuation in 2007–2008 rather than technical regression in production capabilities.

In chapter III, a proactive DEA model to account for demand fluctuations while maximizing effective production is described. Demand fluctuations lead to variations in the output levels affecting technical efficiency measures. In the short-run, firms can adjust their variable resources to either increase production if demand increases or to reduce costs if demand decreases. The present study proposes a short-run capacity planning method, proactive DEA, that quantifies the effectiveness of the production system under demand uncertainty using a stochastic programming DEA (SPDEA) approach. This method estimates the expected value of effectiveness given the demand distribution. An empirical study of Japanese convenience stores is discussed to demonstrate the proposed model. The result shows that proactive SPDEA provides actionable advice regarding the level of variable inputs in uncertain demand environments.

In chapter IV, a Nash equilibrium in oligopolistic market is identified using an estimate of the production possibility set and an inverse demand function. The standard assumption in the efficiency literature that firms desire to produce on the production frontier may not hold in an oligopolistic market where the production decisions of all firms will determine the market price, i.e. an increase in a firm's output level leads to a lower market clearing price and potentially-lower profits. This study identifies a Nash-Cournot equilibrium and improvement targets which may not be on the production frontier. This behavior is referred to as rational inefficiency because the firm reduces its productivity levels in order to increase profits. For a general multiple input/output production process and allowing a firm to adjust its output levels and variable input levels, the existence and the uniqueness of the Nash-Cournot equilibrium is proven. The relationship between the benchmark frontier, scale properties and allocative efficiency is discussed. When changes in quantity have a significant influence on price, more benchmark production plans are on the increasing returns to scale portion of the frontier. Additionally, a direction for improvement towards the allocatively efficient production plan is estimated, thus providing a solution to the direction selection issue in a directional distance analysis.

5.2 Main Contributions

This dissertation provides following contributions.

1. Identifies the PEA function in production planning
2. Develops a connection between PEA and demand management
3. Proposes a hybrid system generalizing manufacturing and service systems
4. Defines a two-dimensional efficiency decomposition of productivity
5. Proposes a proactive DEA approach for addressing demand fluctuations
6. Defines the truncated benchmark production function for effectiveness measure
7. Identifies the source of rational inefficiency
8. Develops the relationship between the price sensitivity of demand and an allocatively efficient benchmarking production plan
9. Determine the direction in the directional distance function analysis
10. Conduct empirical studies in US airline, Japanese convenience store, and a set of distribution center

5.3 Recommendations and Further Research

In this research, a framework was constructed for productivity improvement and strategy identification. Some issues are recommended to improve this study.

In the network DEA study, in practice, the airline fleet is usually leased and thus a variable input, and employees are unionized are perhaps a fixed input. The truly fixed factors are the routes and the network structure. In addition, the airline study uses system-wide data and the demand effects in specific areas are not easy to interpreted. A study focusing on a specific route and city-pair would allow a detailed analysis of the area/market level inefficiency. This focus could clarify the sources of inefficiency.

In the proactive DEA study, a single-variable-input and a single-output could be generalized to the multiple-input and multiple-output setting. This requires estimating a "directional" marginal product and extends the work of Podinovski and Førsund (2010). The demand for the various outputs could each be truncated and thus effectiveness would be defined in term of an aggregation.

In the rational inefficiency study, already addresses scale; however extend the concept to include scope issues is potentially interesting. Multiple products in different market structures may lead to a hybrid effect. In addition, classical production theory hypothesizes an S-shaped production function. Developing methods to estimate the Nash equilibria is a potentially challenging and interesting problem. The nonconvex production function may lead to multiple profit maximizing points. The additional information and economic criteria will need to be applied to select among the equilibria.

Further research should address the following issue.

<u>Noise consideration in PEA</u>

DEA is a nonparametric technique to estimate a production frontier without consideration noise. To improve the robustness of the analysis, Convex Nonparametric Least Squares (CNLS) and Stochastic non-smooth envelopment of data (StoNED) are two approaches to estimate a production function imposing only monotonicity and convexity on the function. For detail, see Hildreth (1954), Kuosmanen (2008), Kuosmanen and Johnson (2010), Kuosmanen and Kortelainen (2011), Johnson and Kuosmanen (2012), Lee *et al.* (2012).

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APPENDIX A

SUPPORTING MATERIAL FOR CHAPTER II

Component profitability estimation by dual model

The profitability represents the ratio equal to revenue by cost. First, the output and input factors are identified in each component respectively. Thus, their monetary value can be calculated for revenue gain and cost expenditure. Second, based on profitability, the profitability efficiency can be estimated and decomposed. The detail of profitability in each component is described below.

Capacity Design Component

The revenue and cost are the monetary value of peak output and fixed input respectively.

$$Max \ \frac{\sum_{q} p^{c}_{qrs} Y^{c}_{qrs}}{\sum_{i} w^{f}_{irs} X^{f}_{irs}} \qquad \text{(2.8.13) replaces (2.8.1)}$$

$$\sum_{k\in K} \sum_{t\in\{1,\ldots,s\}} \alpha_{kt} Y^{c}_{qkt} - \sum_{k\in K} \sum_{t\in\{1,\ldots,s\}} \beta_{kt} Y^{c}_{qkt} \geq Y^{c}_{qrs}, \ \forall q \qquad \text{(2.8.14) replaces (2.8.2)}$$

$$\sum_{k\in K} \sum_{t\in\{1,\ldots,s\}} \delta_{kt} D^{r}_{qkt} \geq 0, \ \forall q \qquad \text{(2.8.15) replaces (2.8.5)}$$

$$\sum_{k\in K} \sum_{t\in\{1,\ldots,s\}} \gamma_{kt} X^{v}_{jkt} \leq 0, \ \forall j \qquad \text{(2.8.16) replaces (2.8.7)}$$

$$\sum_{k\in K} \sum_{t\in\{1,\ldots,s\}} (\alpha_{kt} - \beta_{kt}) = 1 \qquad \text{(2.8.17) replaces (2.8.8)}$$

$$\sum_{k\in K} \sum_{t\in\{1,\ldots,s\}} \delta_{kt} = 0 \qquad \text{(2.8.18) replaces (2.8.11)}$$

where p^{c}_{qrs} means the unit price of q^{th} capacity Y^{c}_{qrs}.

Demand Generation Component

The revenue and cost are the monetary value of expected demand and peak output respectively.

$$Max \ \frac{\sum_q p^e_{qrs} D^e_{qrs}}{\sum_q p^c_{qrs} Y^c_{qrs}} \qquad \text{(2.8.19) replaces (2.8.1)}$$

$$-\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \alpha_{kt} Y^c_{qkt} + \sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \beta_{kt} Y^c_{qkt} \le Y^c_{qrs},\ \forall q \qquad \text{(2.8.20) replaces (2.8.2)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \beta_{kt} D^e_{qkt} - \sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \gamma_{kt} D^e_{qkt} \ge D^e_{qrs},\ \forall q \qquad \text{(2.8.21) replaces (2.8.3)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \delta_{kt} D^r_{qkt} \ge 0,\ \forall q \qquad \text{(2.8.22) replaces (2.8.5)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \alpha_{kt} X^f_{ikt} \le 0,\ \forall i \qquad \text{(2.8.23) replaces (2.8.6)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \gamma_{kt} X^v_{jkt} \le 0,\ \forall j \qquad \text{(2.8.24) replaces (2.8.7)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} (\alpha_{kt} - \beta_{kt}) \le 0 \qquad \text{(2.8.25) replaces (2.8.8)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} (\beta_{kt} - \gamma_{kt}) = 1 \qquad \text{(2.8.26) replaces (2.8.9)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \delta_{kt} = 0 \qquad \text{(2.8.27) replaces (2.8.11)}$$

where p^e_{qrs} means the unit price of q^{th} expected demand D^e_{qrs}.

Operations Component

The revenue is the monetary value of actual output and the cost is the summation of the monetary value of variable input and expected demand respectively.

$$Max \ \frac{\sum_q p^a_{qrs} Y^a_{qrs}}{\sum_j w^v_{jrs} X^v_{jrs} + \sum_q p^e_{qrs} D^e_{qrs}} \qquad \text{(2.8.28) replaces (2.8.1)}$$

$$-\sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \beta_{kt} D^e_{qkt} + \sum_{k\in K}\sum_{t\in\{1,\dots,s\}} \gamma_{kt} D^e_{qkt} \le D^e_{qrs},\ \forall q \qquad \text{(2.8.29) replaces (2.8.3)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt}Y_{qkt}^{a}-\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt}Y_{qkt}^{a}\geq Y_{qrs}^{a},\ \forall q \qquad \text{(2.8.30) replaces (2.8.4)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt}D_{qkt}^{r}\geq 0,\ \forall q \qquad \text{(2.8.31) replaces (2.8.5)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\alpha_{kt}X_{ikt}^{f}\leq 0,\ \forall i \qquad \text{(2.8.32) replaces (2.8.6)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}(\beta_{kt}-\gamma_{kt})\leq 0 \qquad \text{(2.8.33) replaces (2.8.9)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}(\gamma_{kt}-\delta_{kt})=1 \qquad \text{(2.8.34) replaces (2.8.10)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt}=0 \qquad \text{(2.8.35) replaces (2.8.11)}$$

where p_{qrs}^{a} means the unit price of q^{th} actual output Y_{qrs}^{a}.

Demand Consumption Component

The revenue and cost are the monetary value of realized demand and actual output respectively.

$$Max\ \frac{\sum_{q}p_{qrs}^{r}D_{qrs}^{r}}{\sum_{q}p_{qrs}^{a}Y_{qrs}^{a}} \qquad \text{(2.8.36) replaces (2.8.1)}$$

$$-\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt}Y_{qkt}^{a}+\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\delta_{kt}Y_{qkt}^{a}\leq Y_{qrs}^{a},\ \forall q \qquad \text{(2.8.37) replaces (2.8.4)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\alpha_{kt}X_{ikt}^{f}\leq 0,\ \forall i \qquad \text{(2.8.38) replaces (2.8.6)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}\gamma_{kt}X_{jkt}^{v}\leq 0,\ \forall j \qquad \text{(2.8.39) replaces (2.8.7)}$$

$$\sum_{k\in K}\sum_{t\in\{1,\ldots,s\}}(\gamma_{kt}-\delta_{kt})\leq 0 \qquad \text{(2.8.40) replaces (2.8.10)}$$

Formulation of decomposition of profitability efficiency

Profitability efficiency can be estimated by equation (A1). In particular, equation (A2) represents technical efficiency change, equation (A3) indicates scale efficiency change, and (A4) represents allocative efficiency change. Each component takes the geometric mean of the input-oriented and output-oriented measures.

$$\begin{aligned}\Delta\rho E &= \rho E^1(w^1, p^1; x^1, y^1) / \rho E^0(w^0, p^0; x^0, y^0) \\ &= \Delta TE \cdot \Delta SE \cdot \Delta AE\end{aligned} \quad \text{(A1)}$$

where

$$\begin{aligned}\Delta TE &\equiv (\Delta ITE \cdot \Delta OTE)^{1/2} \\ \Delta ITE &\equiv D_x^{t+1}(x^{t+1}, y^{t+1}) / D_x^t(x^t, y^t) \\ \Delta OTE &\equiv D_y^{t+1}(x^{t+1}, y^{t+1}) / D_y^t(x^t, y^t)\end{aligned} \quad \text{(A2)}$$

$$\begin{aligned}\Delta SE &\equiv \frac{(ISE^1 \cdot OSE^1)^{1/2}}{(ISE^0 \cdot OSE^0)^{1/2}} \\ ISE^t &\equiv \left(\frac{p^t \cdot y^t}{C^t(w^t, y^t)}\right) \Big/ \rho^t(w^t, p^t) \\ OSE^t &\equiv \left(\frac{R^t(x^t, p^t)}{w^t \cdot x^t}\right) \Big/ \rho^t(w^t, p^t)\end{aligned} \quad \text{(A3)}$$

$$\begin{aligned}\Delta AE &\equiv \left(\frac{IAE^1}{IAE^0} \cdot \frac{OAE^1}{OAE^0}\right)^{1/2} \\ IAE^t &\equiv \frac{C^t(w^t, y^t)}{w^t \cdot (D_x^t(x^t, y^t) x^t)} \\ OAE^t &\equiv \frac{p^t \cdot (y^t / D_y^t(x^t, y^t))}{R^t(x^t, p^t)}\end{aligned} \quad \text{(A4)}$$

Here, the ΔTE and ΔAE follow the traditional definition of Fare *et al.* (1994) and Ray and Mukherjee (1996); however, the scale efficiency defined in Kuosmanen and Sipiläinen (2009) employs the dual perspective and characterizes the optimal scale size in terms of the profitability function. We use the dual approach because the traditional distance function will estimate different values under different assumptions regarding strong and weak disposability and congestion (McDonald, 1996).

<u>Definition and raw data in US airline industry</u>

The data set of 15 US airline firms from 2006 to 2008 was primarily gathered from Air Carrier Financial Statistics and Air Carrier Traffic Statistics published by the Bureau of Transportation Statistics at the Research and Innovative Technology Administration (RITA, 2009).

Sources for inputs, demand, and output levels

Aircraft fleet size (FS) is the average number of aircraft employed in a firm during a particular year. The make-up of the fleet for each firm is shown in AirSafe.com (2008).

Fuel (FU) is the number of gallons consumed annually, estimated by fuel expenses over the average jet fuel cost per gallon. The fuel expenses and average jet fuel cost per gallon are found in the Transportation Statistics Annual Report 2008 published by the Bureau of Transportation Statistics (BTS).

Employee (EP) is the number of employees during the year, which includes flight shipping staff, pilots, flight attendants, and managers but not ground shipping drivers. The data of number of employees and salaries and benefits expenses are collected from individual annual financial reports, BTS and the Securities and Exchange Commission (SEC), and the related summary report shown in AirlineFinancials.com (2009).[21]

Note that all demand and output measures come from Air Carrier Financial Statistics and Air Carrier Traffic Statistics.

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[21] The data of personnel structure of flight and ground shipping with respect to FedEx (Federal Express) and UPS (United Parcel Service) is estimated in FedEx official website (2010) and UPS official website (2010).

Table A1 Raw data of US airline industry 2006-2008

			Fixed input		Variable input				Peak Output				Scheduled Demand				Available Output				Realized Demand			
			Aircraft		Fuel		Employee		Passenger		Freight		Passenger		Freight		Passenger		Freight		Passenger		Freight	
Firm	No.	Year	Fleet Size	price (10^6)	Gals (10^6)	price	Units	Wages	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price
AirTran	A	2006	100	9.0	346	1.95	7415	52650	21363	0.13	959	0.54	13798	0.13	7	0.54	18984	0.13	852	0.54	13814	0.13	7	0.54
Alaska	B	2006	109	18.7	388	1.95	9307	82841	23263	0.13	1268	1.39	17814	0.13	64	1.39	23263	0.13	1268	1.39	17826	0.13	64	1.38
American	C	2006	800	24.2	2966	1.95	72757	85243	173940	0.13	13489	0.37	139392	0.13	2231	0.37	173940	0.13	13489	0.37	139451	0.13	2231	0.37
American Eagle	D	2006	85	40.8	317	1.95	13000	85243	15590	0.23	713	1.31	8420	0.23	0	1.31	11298	0.23	517	1.31	8420	0.23	0	1.31
Continental	E	2006	418	16.8	1556	1.95	39363	71895	104546	0.12	3568	0.39	76251	0.12	1006	0.39	93512	0.12	3191	0.39	76319	0.12	1006	0.39
Delta Air Lines	F	2006	1072	15.3	2215	1.95	45562	90602	173940	0.12	12984	0.39	98769	0.12	1239	0.39	125100	0.12	9338	0.39	98909	0.12	1239	0.39
ExpressJet	G	2006	78	2.9	116	1.95	6800	56864	13199	0.16	567	0.94	10296	0.16	1	0.94	13199	0.16	567	0.94	10298	0.16	1	0.94
Federal Express	H	2006	717	12.9	1670	1.95	122116	56709	0	N/A	16476	1.23	0	N/A	10426	1.23	0	N/A	16476	1.23	0	N/A	10543	1.22
JetBlue Airways	I	2006	116	26.4	403	1.95	9272	59642	28581	0.10	1112	0.49	23310	0.10	14	0.49	28581	0.10	1112	0.49	23310	0.10	14	0.49
Northwest	J	2006	523	19.9	1736	1.95	30729	86628	118763	0.12	10190	0.42	72588	0.12	2268	0.42	85582	0.12	7343	0.42	72690	0.12	2269	0.42
SkyWest	K	2006	132	10.3	518	1.95	8792	76656	31681	0.19	1539	1.11	9497	0.19	0	1.11	11954	0.19	581	1.11	9497	0.19	0	1.11
Southwest	L	2006	506	23.2	1096	1.95	32167	94880	126682	0.13	7870	0.78	67691	0.13	171	0.78	92662	0.13	5756	0.78	67782	0.13	171	0.78
United	M	2006	570	16.0	2474	1.95	55027	77507	142780	0.12	10969	0.37	117247	0.12	2048	0.37	142780	0.12	10969	0.37	117471	0.12	2048	0.37
UPS	N	2006	468	23.9	1362	1.95	176150	58683	0	N/A	10772	0.70	0	N/A	6262	0.70	0	N/A	10545	0.70	0	N/A	6270	0.70
US Airways	O	2006	386	4.2	1291	1.95	34462	60647	96497	0.14	6690	0.44	37357	0.14	279	0.44	47754	0.14	3311	0.44	37366	0.14	279	0.44
AirTran	A	2007	108	11.8	384	2.09	8304	54408	25343	0.13	992	0.55	17233	0.13	6	0.55	22680	0.13	888	0.55	17252	0.13	6	0.55
Alaska	B	2007	112	22.2	353	2.09	9680	79277	24197	0.13	1349	1.53	18446	0.13	58	1.53	24197	0.13	1349	1.53	18456	0.13	58	1.52
American	C	2007	798	23.9	2876	2.09	71818	85382	173669	0.13	13138	0.39	138417	0.13	2129	0.39	169856	0.13	12850	0.39	138448	0.13	2129	0.39
American Eagle	D	2007	86	40.5	315	2.09	13000	85382	15972	0.24	732	1.53	8340	0.24	0	1.53	11211	0.24	514	1.53	8340	0.24	0	1.53
Continental	E	2007	424	17.1	1605	2.09	40948	75046	106055	0.13	3650	0.43	81380	0.13	971	0.43	99061	0.13	3409	0.43	81428	0.13	972	0.43
Delta Air Lines	F	2007	912	9.8	2241	2.09	47286	88589	173940	0.12	12773	0.42	103279	0.12	1128	0.42	127323	0.12	9350	0.42	103450	0.12	1128	0.42
ExpressJet	G	2007	86	2.9	155	2.09	7500	58342	15885	0.16	741	1.02	10182	0.16	1	1.02	13729	0.16	640	1.02	10206	0.16	1	1.02
Federal Express	H	2007	715	14.6	1690	2.09	133258	56800	0	N/A	17189	1.23	0	N/A	10809	1.23	0	N/A	17189	1.23	0	N/A	10965	1.23
JetBlue Airways	I	2007	139	24.9	463	2.09	9713	66715	34367	0.10	1507	0.66	25722	0.10	16	0.66	32148	0.10	1410	0.66	25722	0.10	16	0.66
Northwest	J	2007	410	18.1	1616	2.09	29619	86701	94203	0.13	7747	0.41	72907	0.13	2059	0.41	86123	0.13	7082	0.41	73023	0.13	2067	0.41
SkyWest	K	2007	135	11.7	508	2.09	10249	70929	31507	0.18	1702	1.11	11564	0.18	0	1.11	14923	0.18	806	1.11	11564	0.18	0	1.11
Southwest	L	2007	523	24.8	1287	2.09	33680	95398	130958	0.12	7901	0.95	73493	0.12	136	0.95	103274	0.12	6231	0.95	73640	0.12	136	0.95
United	M	2007	581	16.4	2394	2.09	55160	77175	143368	0.13	11116	0.38	117376	0.13	2012	0.38	141838	0.13	10998	0.38	117399	0.13	2012	0.38
UPS	N	2007	489	24.4	1423	2.09	185300	74641	0	N/A	11539	0.70	0	N/A	6792	0.70	0	N/A	11539	0.70	0	N/A	6802	0.70
US Airways	O	2007	409	5.8	1141	2.09	34256	67200	102282	0.14	6368	0.44	43547	0.14	282	0.44	54427	0.14	3388	0.44	43567	0.14	282	0.44
AirTran	A	2008	109	11.7	390	3.06	8259	57501	25748	0.13	943	0.56	18745	0.13	6	0.56	23756	0.13	870	0.56	18789	0.13	6	0.56
Alaska	B	2008	115	25.0	380	3.06	9628	78781	25032	0.14	1407	1.76	18698	0.14	56	1.76	24183	0.14	1359	1.76	18715	0.14	57	1.76
American	C	2008	767	21.4	2665	3.06	70923	85219	169470	0.14	12905	0.43	131724	0.14	2014	0.43	163483	0.14	12449	0.43	131755	0.14	2014	0.43
American Eagle	D	2008	78	39.9	281	3.06	12000	85219	11564	0.30	567	1.98	7383	0.30	0	1.98	10370	0.30	509	1.98	7383	0.30	0	1.98
Continental	E	2008	461	18.6	1603	3.06	40630	70145	115362	0.14	4339	0.47	80428	0.14	950	0.47	99047	0.14	3726	0.47	80495	0.14	951	0.47
Delta Air Lines	F	2008	979	9.8	2068	3.06	47420	101265	173940	0.13	12460	0.48	105568	0.13	1217	0.48	128635	0.13	9214	0.48	105698	0.13	1217	0.48
ExpressJet	G	2008	77	2.9	75	3.06	6700	59335	11962	0.14	536	0.90	9088	0.14	1	0.90	11962	0.14	536	0.90	9144	0.14	1	0.90

Table A1 Raw data of US airline industry 2006-2008 (Cont.)

			Fixed input		Variable input				Peak Output				Scheduled Demand				Available Output				Realized Demand			
			Aircraft		Fuel		Employee		Passenger		Freight		Passenger		Freight		Passenger		Freight		Passenger		Freight	
Firm	No.	Year	Fleet Size	price (10^6)	Gals (10^6)	price	Units	Wages	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price	Pasgr -miles (10^6)	price	Ton -miles (10^6)	price
Federal Express	H	2008	723	15.0	1502	3.06	140000	55882	0	N/A	17189	1.37	0	N/A	10423	1.37	0	N/A	17152	1.37	0	N/A	10591	1.37
JetBlue Airways	I	2008	153	24.2	457	3.06	10177	68193	37888	0.12	1708	0.60	26069	0.12	26	0.60	32436	0.12	1462	0.60	26069	0.12	26	0.60
Northwest	J	2008	455	17.4	1716	3.06	29124	92776	113091	0.13	8545	0.46	71199	0.13	1636	0.46	83862	0.13	6337	0.46	71646	0.13	1643	0.46
SkyWest	K	2008	137	12.1	399	3.06	8987	80571	32196	0.18	1728	1.21	11156	0.18	0	1.21	14618	0.18	785	1.21	11156	0.18	0	1.21
Southwest	L	2008	547	25.2	1213	3.06	34680	96309	136995	0.14	9070	1.05	72320	0.14	138	1.05	99636	0.14	6597	1.05	72410	0.14	138	1.05
United	M	2008	560	15.5	2524	3.06	51536	83670	135480	0.13	10928	0.44	109804	0.13	1921	0.44	135480	0.13	10928	0.44	110062	0.13	1921	0.44
UPS	N	2008	500	25.3	1351	3.06	186000	61181	0	N/A	11966	0.82	0	N/A	6863	0.82	0	N/A	11966	0.82	0	N/A	6866	0.82
US Airways	O	2008	423	6.8	1299	3.06	32683	68262	105804	0.13	6080	0.48	60532	0.13	300	0.48	74106	0.13	4258	0.48	60567	0.13	300	0.48

Table A2 Technical, scale, allocative, profitability efficiency decomposition

			Production System							Design							Generation							Operations							Consumption						
Firm	Index	Year	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E
AirTran Airways	A	2006	0.97	0.90	0.71	0.63	0.79	0.98	0.55	0.94	0.89	0.73	0.78	1.00	0.98	0.68	0.90	0.90	0.91	0.93	1.00	0.98	0.82	1.00	1.00	0.91	0.86	0.95	1.00	0.86	0.90	0.90	0.91	0.93	1.00	0.98	0.82
Alaska Airlines	B	2006	0.89	0.82	0.65	0.72	0.96	0.94	0.55	1.00	1.00	0.74	0.74	1.00	1.00	0.74	0.94	0.94	0.86	0.90	0.92	0.88	0.74	1.00	1.00	0.90	0.85	0.94	0.99	0.84	0.94	0.94	0.86	0.90	0.92	0.88	0.74
American Airlines	C	2006	1.00	1.00	0.71	0.71	1.00	1.00	0.71	1.00	1.00	0.87	0.87	1.00	1.00	0.87	1.00	1.00	0.89	0.89	1.00	1.00	0.89	1.00	1.00	0.86	0.86	1.00	1.00	0.86	1.00	1.00	0.89	0.89	1.00	1.00	0.89
American Eagle	D	2006	0.92	0.60	0.42	0.56	0.84	0.96	0.32	0.92	0.72	0.51	0.65	1.00	0.99	0.46	1.00	1.00	0.80	0.80	1.00	1.00	0.80	1.00	1.00	0.74	0.64	0.86	1.00	0.64	1.00	1.00	0.80	0.80	1.00	1.00	0.80
Continental	E	2006	0.81	0.85	0.96	0.88	0.94	0.98	0.73	0.90	0.89	0.89	0.99	1.00	0.90	0.79	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90	0.91	0.92	0.92	0.93	0.91	0.77	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Delta Air Lines	F	2006	0.95	0.96	0.86	0.53	0.60	0.98	0.49	0.47	0.72	0.99	0.65	1.00	0.99	0.46	0.96	0.96	0.93	0.92	0.97	0.98	0.86	1.00	1.00	0.90	0.89	0.99	1.00	0.89	0.96	0.96	0.93	0.92	0.97	0.98	0.86
ExpressJet airlines	G	2006	1.00	1.00	0.66	0.66	1.00	1.00	0.66	1.00	1.00	0.58	0.58	1.00	1.00	0.58	1.00	1.00	0.85	0.85	1.00	1.00	0.85	1.00	1.00	0.87	0.87	1.00	1.00	0.87	1.00	1.00	0.85	0.85	1.00	1.00	0.85
Federal Express	H	2006	1.00	1.00	0.97	0.97	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
JetBlue Airways	I	2006	1.00	1.00	0.79	0.78	0.95	0.97	0.75	1.00	1.00	0.85	0.85	1.00	1.00	0.85	1.00	1.00	0.92	0.95	1.00	0.97	0.92	1.00	1.00	0.89	0.83	0.94	1.00	0.83	1.00	1.00	0.92	0.95	1.00	0.97	0.92
Northwest Airlines	J	2006	1.00	1.00	0.98	0.66	0.68	1.00	0.66	0.75	0.72	0.90	1.00	1.00	0.93	0.67	1.00	1.00	0.97	0.95	0.98	1.00	0.95	1.00	1.00	0.90	0.82	0.92	1.00	0.82	1.00	1.00	0.97	0.95	0.98	1.00	0.95
SkyWest Airlines	K	2006	0.77	0.45	0.54	0.64	0.66	0.96	0.27	0.59	0.38	0.54	0.86	1.00	0.99	0.32	1.00	1.00	0.85	0.85	1.00	1.00	0.85	1.00	1.00	0.79	0.62	0.78	1.00	0.62	1.00	1.00	0.85	0.85	1.00	1.00	0.85
Southwest Airline	L	2006	0.88	0.88	0.85	0.73	0.75	0.88	0.56	0.73	0.73	0.94	0.99	1.00	0.94	0.69	0.88	0.88	0.94	0.95	0.89	0.88	0.74	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88	0.88	0.94	0.95	0.89	0.88	0.74
United Airlines	M	2006	1.00	1.00	0.82	0.80	0.98	1.00	0.80	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.91	0.91	1.00	1.00	0.91	1.00	1.00	0.88	0.86	0.97	1.00	0.86	1.00	1.00	0.91	0.91	1.00	1.00	0.91
United Parcel Service	N	2006	0.88	0.86	0.99	0.73	0.72	1.00	0.63	1.00	1.00	0.98	0.98	1.00	1.00	0.98	1.00	1.00	0.94	0.94	1.00	1.00	0.94	1.00	1.00	0.65	0.65	1.00	1.00	0.65	1.00	1.00	0.93	0.93	1.00	1.00	0.93
US Airways	O	2006	0.45	0.46	0.92	0.91	0.97	0.96	0.40	0.53	0.49	0.91	0.99	1.00	0.99	0.48	0.94	0.94	0.98	0.97	0.94	0.94	0.86	0.97	0.97	0.95	0.76	0.78	0.97	0.72	0.94	0.93	0.98	0.97	0.94	0.94	0.85
AirTran Airways	A	2007	0.94	0.88	0.75	0.72	0.87	0.97	0.62	0.94	0.89	0.78	0.82	1.00	1.00	0.74	0.95	0.95	0.92	0.94	0.99	0.98	0.87	1.00	1.00	0.89	0.82	0.92	1.00	0.82	0.95	0.95	0.92	0.94	0.99	0.98	0.87
Alaska Airlines	B	2007	0.88	0.82	0.64	0.74	0.97	0.90	0.55	1.00	1.00	0.75	0.75	1.00	1.00	0.75	0.93	0.93	0.85	0.90	0.90	0.85	0.72	1.00	1.00	0.89	0.86	0.96	1.00	0.86	0.93	0.93	0.85	0.90	0.90	0.85	0.72
American Airlines	C	2007	1.00	1.00	0.73	0.70	0.96	1.00	0.70	0.96	0.98	0.88	0.87	1.00	1.00	0.85	1.00	1.00	0.90	0.90	1.00	1.00	0.90	1.00	1.00	0.85	0.84	0.99	1.00	0.84	1.00	1.00	0.90	0.90	1.00	1.00	0.90
American Eagle	D	2007	0.91	0.58	0.41	0.58	0.84	0.93	0.31	0.91	0.71	0.50	0.65	1.00	0.99	0.45	1.00	1.00	0.79	0.79	1.00	1.00	0.79	1.00	1.00	0.72	0.63	0.88	1.00	0.63	1.00	1.00	0.79	0.79	1.00	1.00	0.79
Continental	E	2007	0.78	0.84	0.96	0.90	0.99	0.98	0.74	0.93	0.93	0.89	0.99	1.00	0.89	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.87	0.89	0.93	0.93	0.94	0.91	0.76	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Delta Air Lines	F	2007	0.96	0.97	0.85	0.62	0.72	0.98	0.59	0.56	0.73	0.99	0.76	1.00	0.99	0.55	0.98	0.98	0.93	0.92	0.97	0.98	0.88	1.00	1.00	0.90	0.86	0.95	1.00	0.86	0.98	0.98	0.93	0.92	0.97	0.98	0.88
ExpressJet airlines	G	2007	0.91	0.80	0.65	0.72	0.95	0.98	0.56	0.94	0.86	0.59	0.65	1.00	0.99	0.56	0.93	0.93	0.84	0.86	1.00	0.99	0.79	1.00	1.00	0.86	0.86	1.00	1.00	0.86	0.94	0.93	0.84	0.86	1.00	0.99	0.79
Federal Express	H	2007	1.00	1.00	0.97	0.97	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	0.99	1.00	1.00	0.99	0.99	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
JetBlue Airways	I	2007	0.99	0.98	0.79	0.75	0.88	0.93	0.68	0.94	0.94	0.85	0.86	1.00	0.98	0.79	0.98	0.98	0.91	0.95	0.98	0.93	0.86	1.00	1.00	0.90	0.85	0.94	1.00	0.85	0.98	0.98	0.91	0.95	0.98	0.93	0.86
Northwest Airlines	J	2007	0.97	0.97	1.00	0.79	0.79	1.00	0.77	0.92	0.91	0.92	0.99	1.00	0.94	0.85	1.00	1.00	0.99	0.96	0.96	0.99	0.95	1.00	1.00	0.91	0.83	0.91	1.00	0.83	1.00	1.00	0.99	0.96	0.96	0.99	0.95
SkyWest Airlines	K	2007	0.68	0.42	0.60	0.81	0.76	0.92	0.31	0.65	0.47	0.61	0.86	1.00	0.98	0.40	0.96	0.96	0.85	0.87	0.96	0.95	0.79	1.00	1.00	0.85	0.67	0.78	1.00	0.67	0.96	0.96	0.85	0.87	0.97	0.95	0.79
Southwest Airline	L	2007	0.86	0.86	0.83	0.74	0.77	0.87	0.55	0.79	0.79	0.92	1.00	1.00	0.92	0.73	0.85	0.86	0.93	0.93	0.88	0.87	0.69	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.85	0.86	0.92	0.93	0.88	0.87	0.69
United Airlines	M	2007	1.00	1.00	0.83	0.79	0.95	1.00	0.78	0.98	0.99	0.99	0.99	1.00	1.00	0.98	1.00	1.00	0.92	0.91	1.00	1.00	0.91	1.00	1.00	0.87	0.84	0.97	1.00	0.84	1.00	1.00	0.92	0.91	1.00	1.00	0.91
United Parcel Service	N	2007	0.88	0.86	0.98	0.72	0.72	1.00	0.62	1.00	1.00	0.98	0.98	1.00	1.00	0.98	0.98	0.98	0.95	0.95	1.00	1.00	0.93	1.00	1.00	0.70	0.65	0.92	1.00	0.65	0.98	0.97	0.94	0.95	1.00	1.00	0.92
US Airways	O	2007	0.54	0.55	0.92	0.90	0.94	0.95	0.47	0.54	0.53	0.95	0.99	1.00	0.97	0.51	0.96	0.96	0.98	0.98	0.95	0.94	0.89	0.89	0.89	0.97	0.84	0.86	0.99	0.74	0.96	0.96	0.98	0.98	0.95	0.94	0.89
AirTran Airways	A	2008	0.98	0.97	0.81	0.70	0.82	0.97	0.66	0.95	0.92	0.79	0.83	1.00	0.99	0.76	1.00	1.00	0.92	0.91	0.99	1.00	0.91	1.00	1.00	0.91	0.79	0.87	1.00	0.79	1.00	1.00	0.92	0.91	0.99	1.00	0.91
Alaska Airlines	B	2008	0.87	0.79	0.62	0.71	0.93	0.89	0.50	0.98	0.97	0.74	0.75	1.00	1.00	0.73	0.94	0.94	0.84	0.90	0.89	0.83	0.70	1.00	1.00	0.93	0.83	0.88	0.99	0.82	0.94	0.94	0.84	0.90	0.89	0.84	0.70
American Airlines	C	2008	0.98	0.99	0.76	0.70	0.93	1.00	0.69	0.94	0.96	0.90	0.88	1.00	1.00	0.85	0.98	0.99	0.90	0.90	1.00	1.00	0.89	1.00	1.00	0.86	0.84	0.99	1.00	0.84	0.98	0.99	0.90	0.90	1.00	1.00	0.89
American Eagle	D	2008	0.99	0.65	0.38	0.48	0.79	0.96	0.30	0.99	0.90	0.47	0.58	1.00	0.90	0.47	1.00	1.00	0.74	0.74	1.00	1.00	0.74	1.00	1.00	0.78	0.67	0.86	1.00	0.67	1.00	1.00	0.74	0.74	1.00	1.00	0.74
Continental	E	2008	0.77	0.84	0.96	0.85	0.95	0.98	0.70	0.86	0.86	0.89	0.99	1.00	0.90	0.77	0.99	0.99	1.00	1.00	0.99	1.00	0.98	0.88	0.89	0.94	0.93	0.94	0.92	0.77	0.99	0.99	1.00	1.00	0.99	1.00	0.98

Table A2 Technical, scale, allocative, profitability efficiency decomposition (Cont.)

			Production System							Design							Generation							Operations							Consumption						
Firm	Index	Year	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E	ITE	OTE	ISE	OSE	IAE	OAE	ρ E
Delta Air Lines	F	2008	0.99	0.99	0.84	0.63	0.74	0.98	0.61	0.52	0.74	0.98	0.71	1.00	0.98	0.52	0.99	0.99	0.92	0.92	0.97	0.98	0.89	1.00	1.00	0.91	0.86	0.94	1.00	0.86	0.99	0.99	0.92	0.92	0.97	0.98	0.89
ExpressJet airlines	G	2008	1.00	1.00	0.72	0.72	1.00	1.00	0.72	1.00	1.00	0.53	0.53	1.00	1.00	0.53	1.00	1.00	0.82	0.81	0.99	1.00	0.81	1.00	1.00	0.91	0.91	1.00	1.00	0.91	1.00	1.00	0.82	0.82	1.00	1.00	0.82
Federal Express	H	2008	0.97	0.97	0.99	0.96	0.97	1.00	0.93	0.99	1.00	1.00	0.99	1.00	1.00	0.99	0.96	0.96	1.00	0.99	1.00	1.00	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.97	1.00	1.00	1.00	1.00	0.96
JetBlue Airways	I	2008	0.97	0.97	0.84	0.73	0.82	0.94	0.67	0.86	0.86	0.87	0.89	1.00	0.98	0.75	0.98	0.98	0.93	0.96	0.98	0.94	0.88	0.99	0.99	0.93	0.85	0.92	1.00	0.85	0.98	0.98	0.93	0.96	0.98	0.95	0.88
Northwest Airlines	J	2008	0.96	0.96	0.99	0.72	0.72	0.99	0.68	0.78	0.76	0.96	0.99	1.00	1.00	0.75	0.98	0.98	0.99	0.97	0.95	0.97	0.92	0.97	0.97	0.95	0.81	0.85	1.00	0.79	1.00	1.00	0.99	0.97	0.96	0.98	0.95
SkyWest Airlines	K	2008	0.77	0.49	0.64	0.71	0.67	0.93	0.33	0.64	0.46	0.61	0.85	1.00	0.99	0.39	0.93	0.93	0.85	0.87	0.97	0.95	0.76	1.00	1.00	0.90	0.69	0.77	1.00	0.69	0.95	0.95	0.85	0.87	0.97	0.95	0.78
Southwest Airline	L	2008	0.85	0.87	0.83	0.72	0.76	0.86	0.54	0.73	0.73	0.95	1.00	1.00	0.95	0.69	0.87	0.87	0.93	0.93	0.86	0.85	0.69	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.86	0.87	0.93	0.93	0.87	0.86	0.69
United Airlines	M	2008	0.96	0.98	0.84	0.75	0.90	1.00	0.73	1.00	1.00	0.98	1.00	1.00	0.98	0.98	0.98	0.98	0.92	0.91	0.98	0.99	0.88	1.00	1.00	0.86	0.84	0.97	1.00	0.84	0.98	0.98	0.92	0.91	0.98	0.99	0.88
United Parcel Service	N	2008	0.83	0.81	0.99	0.79	0.78	1.00	0.64	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95	0.95	0.95	0.96	1.00	1.00	0.91	1.00	1.00	0.75	0.72	0.96	1.00	0.72	0.95	0.94	0.94	0.95	1.00	1.00	0.90
US Airways	O	2008	0.71	0.72	0.93	0.90	0.93	0.95	0.62	0.70	0.70	0.94	0.99	1.00	0.95	0.66	0.98	0.98	0.97	0.99	0.95	0.93	0.90	0.87	0.87	0.99	0.93	0.92	0.98	0.79	0.97	0.97	0.97	0.99	0.95	0.94	0.90

Profitability efficiency change in airline firms

Table A3 2DED of airline firms

			Production				Design				Generation				Operations				Consumption			
Firm	#	Year	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA	$\Delta\rho$	ΔT	ΔS	ΔA
AirTran Airways	A	06-07	1.12	0.97	1.10	1.04	1.08	1.00	1.07	1.01	1.06	1.05	1.01	1.00	0.96	1.00	0.97	0.99	1.06	1.05	1.01	1.00
		07-08	1.07	1.07	1.03	0.97	1.03	1.02	1.01	1.00	1.05	1.05	0.99	1.01	0.96	1.00	0.99	0.97	1.05	1.05	0.99	1.01
		GM	1.09	1.02	1.06	1.00	1.05	1.01	1.04	1.00	1.05	1.05	1.00	1.00	0.96	1.00	0.98	0.98	1.05	1.05	1.00	1.00
Alaska Airlines	B	06-07	0.99	1.00	1.01	0.99	1.01	1.00	1.01	1.00	0.97	0.99	1.00	0.98	1.02	1.00	1.00	1.02	0.97	0.99	1.00	0.98
		07-08	0.92	0.98	0.96	0.97	0.97	0.97	1.00	1.00	0.98	1.01	0.99	0.98	0.95	1.00	1.00	0.95	0.98	1.01	0.99	0.98
		GM	0.95	0.99	0.99	0.98	0.99	0.99	1.00	1.00	0.97	1.00	0.99	0.98	0.99	1.00	1.00	0.99	0.97	1.00	0.99	0.98
American Airlines	C	06-07	0.98	1.00	1.00	0.98	0.97	0.97	1.01	1.00	1.01	1.00	1.01	1.00	0.97	1.00	0.98	0.99	1.01	1.00	1.01	1.00
		07-08	0.99	0.98	1.02	0.99	1.00	0.98	1.02	1.00	0.98	0.99	1.00	1.00	1.01	1.00	1.00	1.00	0.98	0.99	1.00	1.00
		GM	0.99	0.99	1.01	0.98	0.99	0.98	1.01	1.00	1.00	0.99	1.01	1.00	0.99	1.00	0.99	1.00	1.00	0.99	1.01	1.00
American Eagle Airlines	D	06-07	0.95	0.97	1.00	0.98	0.98	0.98	1.00	1.00	0.98	1.00	0.98	1.00	0.99	1.00	0.98	1.01	0.99	1.00	0.99	1.00
		07-08	0.96	1.11	0.88	0.99	1.03	1.18	0.91	0.95	0.94	1.00	0.94	1.00	1.06	1.00	1.07	0.99	0.94	1.00	0.94	1.00
		GM	0.96	1.04	0.94	0.99	1.00	1.07	0.95	0.98	0.96	1.00	0.96	1.00	1.02	1.00	1.03	1.00	0.96	1.00	0.96	1.00
Continental	E	06-07	1.02	0.98	1.01	1.03	1.04	1.04	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	1.01	1.01	1.00	1.00	1.00	1.00
		07-08	0.94	0.99	0.98	0.98	0.93	0.92	1.01	1.00	0.98	0.99	1.00	0.99	1.01	1.01	1.00	1.00	0.98	0.99	1.00	0.99
		GM	0.98	0.98	0.99	1.00	0.98	0.98	1.00	1.00	0.99	0.99	1.00	1.00	1.00	0.99	1.01	1.01	0.99	0.99	1.00	1.00
Delta Air Lines	F	06-07	1.19	1.01	1.08	1.09	1.19	1.10	1.08	1.00	1.02	1.02	1.00	1.00	0.96	1.00	0.98	0.98	1.02	1.02	1.00	0.99
		07-08	1.03	1.02	0.99	1.02	0.94	0.98	0.96	1.00	1.01	1.01	0.99	1.00	1.00	1.00	1.01	1.00	1.01	1.01	0.99	1.00
		GM	1.11	1.02	1.04	1.05	1.06	1.04	1.02	1.00	1.02	1.02	1.00	1.00	0.98	1.00	0.99	0.99	1.02	1.02	1.00	1.00
ExpressJet Airlines	G	06-07	0.84	0.85	1.03	0.96	0.95	0.90	1.06	1.00	0.92	0.93	1.00	1.00	1.00	1.00	1.00	1.00	0.93	0.93	1.00	1.00
		07-08	1.28	1.17	1.05	1.04	0.96	1.11	0.86	1.00	1.03	1.07	0.96	1.00	1.06	1.00	1.06	1.00	1.03	1.07	0.96	1.00
		GM	1.04	1.00	1.04	1.00	0.96	1.00	0.96	1.00	0.97	1.00	0.98	1.00	1.03	1.00	1.03	1.00	0.98	1.00	0.98	1.00
Federal Express	H	06-07	1.01	1.00	1.01	1.00	1.00	1.00	1.00	1.00	0.99	1.00	0.99	1.00	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00
		07-08	0.96	0.97	1.00	0.99	0.99	0.99	0.99	1.00	0.96	0.96	1.00	1.00	1.01	1.00	1.01	1.00	0.97	0.97	1.00	1.00
		GM	0.98	0.98	1.01	0.99	0.99	1.00	1.00	1.00	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.98	1.00	1.00
JetBlue Airways	I	06-07	0.90	0.98	0.98	0.94	0.94	0.94	1.01	0.99	0.94	0.98	0.99	0.97	1.02	1.00	1.01	1.00	0.94	0.98	0.99	0.97
		07-08	0.99	0.99	1.02	0.97	0.94	0.92	1.03	1.00	1.03	1.00	1.01	1.01	1.00	0.99	1.02	0.98	1.03	1.00	1.01	1.01
		GM	0.94	0.99	1.00	0.96	0.94	0.93	1.02	0.99	0.98	0.99	1.00	0.99	1.01	1.00	1.02	0.99	0.98	0.99	1.00	0.99
Northwest Airlines	J	06-07	1.16	0.97	1.10	1.08	1.27	1.25	1.01	1.01	1.00	1.00	1.02	0.98	1.00	1.00	1.01	1.00	1.00	1.00	1.01	0.98
		07-08	0.89	0.98	0.95	0.95	0.89	0.84	1.02	1.03	0.97	0.98	1.00	0.99	0.95	0.97	1.02	0.97	1.00	1.00	1.01	1.00
		GM	1.02	0.98	1.02	1.02	1.06	1.02	1.02	1.02	0.99	0.99	1.01	0.99	0.98	0.99	1.01	0.98	1.00	1.00	1.01	0.99
SkyWest Airlines	K	06-07	1.13	0.91	1.19	1.05	1.24	1.17	1.06	1.00	0.94	0.96	1.02	0.96	1.08	1.00	1.08	1.00	0.94	0.96	1.02	0.96
		07-08	1.05	1.16	0.96	0.95	0.98	0.98	1.00	1.00	0.96	0.97	0.99	1.00	1.04	1.00	1.04	0.99	0.98	0.98	0.99	1.00
		GM	1.09	1.02	1.07	1.00	1.10	1.07	1.03	1.00	0.95	0.96	1.01	0.98	1.06	1.00	1.06	1.00	0.96	0.97	1.01	0.98
Southwest Airline	L	06-07	0.98	0.97	0.99	1.01	1.06	1.08	0.99	0.99	0.94	0.97	0.98	0.98	1.00	1.00	1.00	1.00	0.94	0.97	0.98	0.98
		07-08	0.97	1.00	0.99	0.98	0.95	0.92	1.02	1.01	0.99	1.02	1.00	0.98	1.00	1.00	1.00	1.00	0.99	1.01	1.00	0.99
		GM	0.97	0.99	0.99	0.99	1.00	1.00	1.00	1.00	0.97	0.99	0.99	0.98	1.00	1.00	1.00	1.00	0.97	0.99	0.99	0.99
United Airlines	M	06-07	0.98	1.00	1.00	0.98	0.98	0.99	0.99	1.00	1.00	1.00	1.01	1.00	0.98	1.00	0.99	1.00	1.00	1.00	1.00	1.00
		07-08	0.93	0.97	0.99	0.98	1.00	1.01	0.99	0.99	0.97	0.98	1.00	0.99	0.99	1.00	0.99	1.00	0.97	0.98	1.00	0.99
		GM	0.96	0.98	0.99	0.98	0.99	1.00	0.99	0.99	0.98	0.99	1.00	0.99	0.99	1.00	0.99	1.00	0.98	0.99	1.00	0.99
United Parcel Service	N	06-07	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.01	1.00	0.99	1.00	1.03	0.96	0.99	0.98	1.02	1.00
		07-08	1.04	0.95	1.05	1.04	1.01	1.00	1.01	1.00	0.97	0.97	1.00	1.00	1.11	1.00	1.09	1.02	0.97	0.97	1.00	1.00
		GM	1.02	0.97	1.02	1.02	1.01	1.00	1.01	1.00	0.98	0.97	1.01	1.00	1.05	1.00	1.06	0.99	0.98	0.97	1.01	1.00
US Airways	O	06-07	1.18	1.21	1.00	0.98	1.06	1.04	1.02	0.99	1.04	1.03	1.00	1.01	1.04	0.92	1.06	1.06	1.04	1.03	1.00	1.01
		07-08	1.30	1.31	1.00	0.99	1.29	1.31	0.99	0.99	1.02	1.02	1.00	0.99	1.07	0.98	1.07	1.03	1.02	1.02	1.00	1.00
		GM	1.24	1.26	1.00	0.99	1.17	1.17	1.01	0.99	1.03	1.02	1.00	1.00	1.05	0.95	1.06	1.05	1.03	1.02	1.00	1.01
Airline Industry		06-07	1.035	0.999	1.024	1.012	1.052	1.039	1.014	0.998	0.995	0.998	1.003	0.994	0.990	0.993	0.999	0.999	0.996	0.999	1.003	0.994
		07-08	0.996	1.015	0.994	0.987	0.989	0.990	0.999	1.001	0.985	0.992	0.998	0.994	1.009	0.998	1.014	0.997	0.988	0.993	0.998	0.997
		GM	1.015	1.007	1.009	0.999	1.020	1.014	1.007	0.999	0.990	0.995	1.000	0.994	1.000	0.995	1.007	0.998	0.992	0.996	1.001	0.995

Table A4 2006-2008 change of production system in airline firms

Firm	Firm	$\Delta\rho E$	ΔTE	ΔSE	ΔAE
AirTran Airways	A	1.091	1.021	1.063	1.005
Alaska Airlines	B	0.953	0.986	0.986	0.980
American Airlines	C	0.985	0.991	1.013	0.982
American Eagle Airlines	D	0.959	1.037	0.938	0.985
Continental	E	0.980	0.985	0.992	1.003
Delta Air Lines	F	1.109	1.018	1.037	1.051
ExpressJet Airlines	G	1.041	1.000	1.041	1.000
Federal Express	H	0.982	0.983	1.005	0.993
JetBlue Airways	I	0.945	0.986	0.999	0.959
Northwest Airlines	J	1.017	0.978	1.024	1.016
SkyWest Airlines	K	1.091	1.023	1.071	0.996
Southwest Airline	L	0.974	0.987	0.992	0.995
United Airlines	M	0.957	0.984	0.992	0.980
United Parcel Service	N	1.015	0.973	1.022	1.021
US Airways	O	1.243	1.260	0.999	0.987
Industry (Avg)		1.015	1.007	1.009	0.999
Max		1.243	1.260	1.071	1.051
Min		0.945	0.973	0.938	0.959

Firm distribution of efficiency change

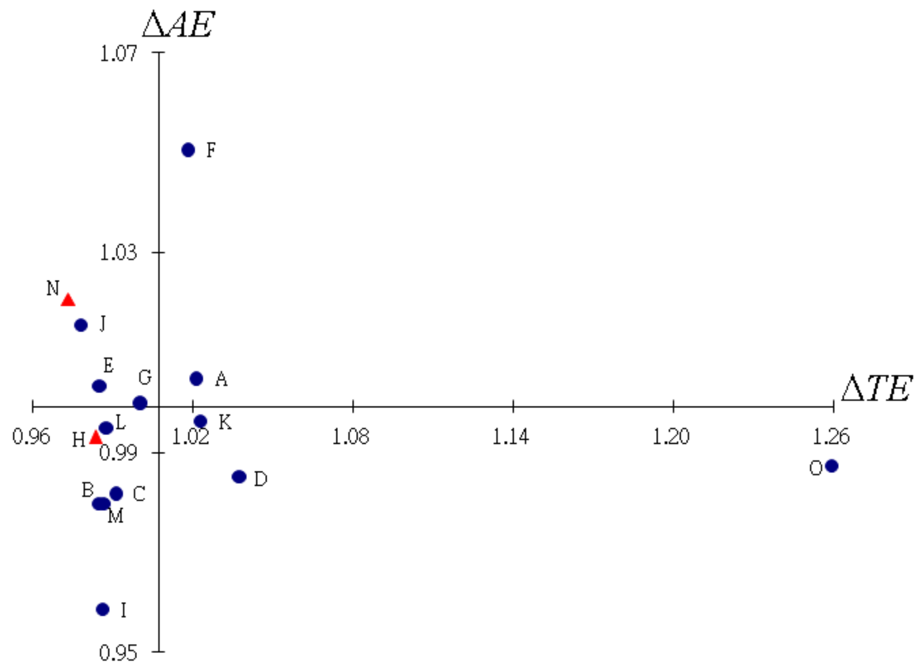

Figure A1 Firm distribution of ΔTE and ΔAE

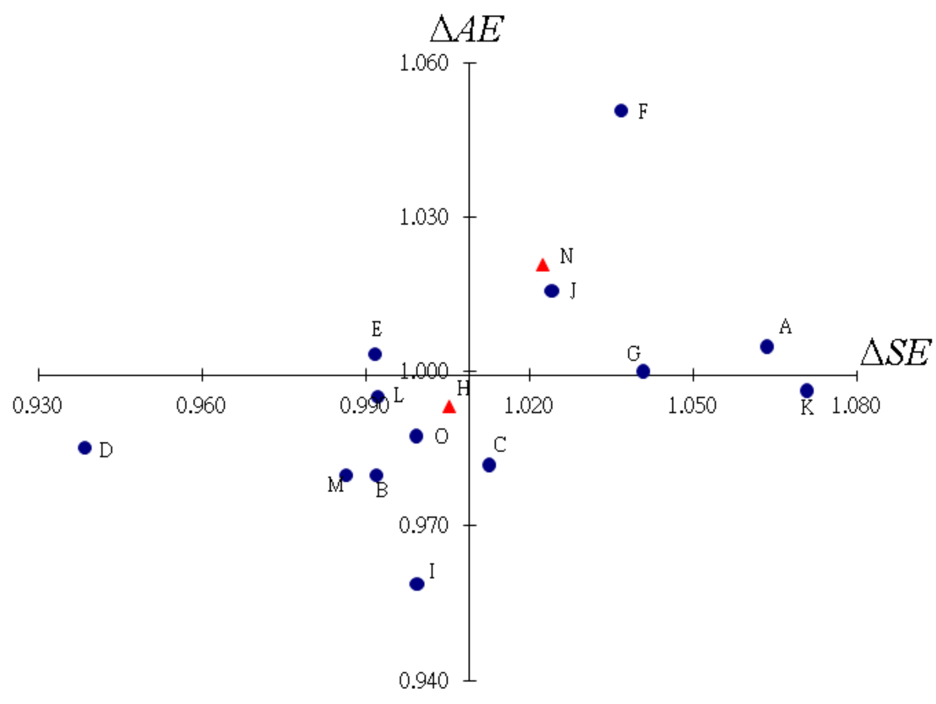

Figure A2 Firm distribution of ΔSE and ΔAE

APPENDIX B

MODEL CONVEXIFICATION FOR CHAPTER III

The terms u_{qs}, v_{js}^{V}, t_{jrs} and w_{qs} will be replaced by $e^{u'_{qs}}$, $e^{v_{js}^{V'}}$, $e^{t'_{jrs}}$ and $e^{w'_{qs}}$ respectively; specifically, $u'_{qs} = \ln u_{qs}$, $v_{js}^{V'} = \ln v_{js}^{V}$, $t'_{jrs} = \ln t_{jrs}$ and $w'_{qs} = \ln w_{qs}$, and constraints $\ln \varepsilon \le u'_{qs}$, $\ln \varepsilon \le v_{js}^{V'}$, $\ln \varepsilon \le t'_{jrs} \le 0$ and $\ln \varepsilon \le w'_{qs}$ are added to restrict the variables, where ε is a smaller positive value. Note in this study $q = 1$ for single output case.

Then, let the nonlinear term $e^{u'_{qs}} z1_{qrs} = b1_{qrs}$, it can be convexified using the following constraints.

$$e^{u'_{qs}} - M(1 - z1_{qrs}) \le b1_{qrs} \le e^{u'_{qs}} + M(1 - z1_{qrs}),\ \forall q \quad (3.18.1)$$

$$-Mz1_{qrs} \le b1_{qrs} \le Mz1_{qrs},\ \forall q \quad (3.18.2)$$

Similarly, other nonlinear terms can be transformed, $z1_{qrs} z2_{jrs} = a12_{qjrs}$, $z1_{qrs} z3_{qrs} = a13_{qrs}$, $z1_{qrs} z2_{jrs} z3_{qrs} = a123_{qjrs}$, $e^{u'_{qs}} z2_{jrs} = b2_{qjrs}$, $e^{u'_{qs}} z1_{qrs} z2_{jrs} = b12_{qjrs}$, $e^{u'_{qs}} z1_{qrs} z3_{qrs} = b13_{qrs}$, $e^{u'_{qs}} z1_{qrs} z2_{jrs} z3_{qrs} = b123_{qjrs}$, $e^{u'_{qs}+t'_{jrs}} z1_{qrs} = g1_{qjrs}$, $e^{u'_{qs}+t'_{jrs}} z2_{jrs} = g2_{qjrs}$, $e^{u'_{qs}+t'_{jrs}} z1_{qrs} z2_{jrs} = g12_{qjrs}$, $e^{u'_{qs}+t'_{jrs}} z1_{qrs} z3_{qrs} = g13_{qjrs}$, $e^{u'_{qs}+t'_{jrs}} z1_{qrs} z2_{jrs} z3_{qrs} = g123_{qjrs}$.

Finally, the model (3.9.1)-(3.9.17) can be reformulated as an solvable equivalent geometric programming with exponential-based convex functions (3.19.1)-(3.19.43).

$$\text{Min} \quad M\mu^E_{rs} + \sum_j (d^+_{jrs} + d^-_{jrs}) \tag{3.19.1}$$

$$\begin{aligned} s.t. \quad \mu^E_{rs} &= \sum_i v^F_{is} X^F_{ir} + \sum_j e^{v^{V'}_{js}} (X^V_{jr} + d_{jrs}) + \sum_q e^{w'_{qs}} D_{qrs} + v_{0s} \\ &= \sum_i v^F_{is} X^F_{ir} + \sum_j e^{v^{V'}_{js}} X^V_{jr} + \sum_j e^{v^{V'}_{js}} (-R_{jr} X^V_{jr} + e^{t'_{jrs}} 2R_{jr} X^V_{jr}) + \sum_q e^{w'_{qs}} D_{qrs} + v_{0s} \\ &= \sum_i v^F_{is} X^F_{ir} + \sum_j e^{v^{V'}_{js}} X^V_{jr} - \sum_j e^{v^{V'}_{js}} R_{jr} X^V_{jr} + 2\sum_j e^{v^{V'}_{js} + t'_{jrs}} R_{jr} X^V_{jr} + \sum_q e^{w'_{qs}} D_{qrs} + v_{0s} \end{aligned} \tag{3.19.2}$$

$$\begin{aligned} &\sum_q e^{u'_{qs}} (y^E_{qrs} + \varepsilon) = \sum_q e^{u'_{qs}} \left[y_{qrs}(1 - z1_{qrs}) + \left[D_{qrs} - y^c_{qrs} \right] z1_{qrs} + \varepsilon \right] \\ &= \sum_q e^{u'_{qs}} \left[y_{qrs}(1 - z1_{qrs}) + \left[D_{qrs} - [(y_{qrs} - D_{qrs}) z3_{qrs} + D_{qrs}(1 - z3_{qrs})] \right] z1_{qrs} + \varepsilon \right] \\ &= \sum_q e^{u'_{qs}} \left[(Y_{qr} + \sum_j \beta^V_{qjr} d_{jrs})(1 - z1_{qrs}) + \left[D_{qrs} - [((Y_{qr} + \sum_j \beta^V_{qjr} d_{jrs}) - D_{qrs}) z3_{qrs} + D_{qrs}(1 - z3_{qrs})] \right] z1_{qrs} + \varepsilon \right] \\ &= \sum_q e^{u'_{qs}} Y_{qr} + \sum_q e^{u'_{qs}} \sum_j \beta^V_{qjr} d_{jrs} - \sum_q e^{u'_{qs}} Y_{qr} z1_{qrs} - \sum_q e^{u'_{qs}} \sum_j \beta^V_{qjr} d_{jrs} z1_{qrs} \\ &+ \sum_q e^{u'_{qs}} D_{qrs} z1_{qrs} - \sum_q e^{u'_{qs}} Y_{qr} z3_{qrs} z1_{qrs} - \sum_q e^{u'_{qs}} \sum_j \beta^V_{qjr} d_{jrs} z3_{qrs} z1_{qrs} + \sum_q e^{u'_{qs}} D_{qrs} z3_{qrs} z1_{qrs} \\ &- \sum_q e^{u'_{qs}} D_{qrs} z1_{qrs} + \sum_q e^{u'_{qs}} D_{qrs} z3_{qrs} z1_{qrs} + \sum_q e^{u'_{qs}} \varepsilon \\ &= \sum_q e^{u'_{qs}} Y_{qr} + \sum_q e^{u'_{qs}} \sum_j (\beta^{V+}_{qjr} z2_{jrs} + \beta^{V-}_{qjr}(1 - z2_{jrs}))(-R_{jr} X^V_{jr} + e^{t'_{jrs}} 2R_{jr} X^V_{jr}) \\ &- \sum_q e^{u'_{qs}} Y_{qr} z1_{qrs} - \sum_q e^{u'_{qs}} \sum_j (\beta^{V+}_{qjr} z2_{jrs} + \beta^{V-}_{qjr}(1 - z2_{jrs}))(-R_{jr} X^V_{jr} + e^{t'_{jrs}} 2R_{jr} X^V_{jr}) z1_{qrs} \\ &- \sum_q e^{u'_{qs}} Y_{qr} z3_{qrs} z1_{qrs} - \sum_q e^{u'_{qs}} \sum_j (\beta^{V+}_{qjr} z2_{jrs} + \beta^{V-}_{qjr}(1 - z2_{jrs}))(-R_{jr} X^V_{jr} + e^{t'_{jrs}} 2R_{jr} X^V_{jr}) z3_{qrs} z1_{qrs} + 2\sum_q e^{u'_{qs}} D_{qrs} z3_{qrs} z1_{qrs} + \sum_q e^{u'_{qs}} \varepsilon \\ &= \sum_q e^{u'_{qs}} Y_{qr} - \sum_q \sum_j e^{u'_{qs}} \beta^{V+}_{qjr} z2_{jrs} R_{jr} X^V_{jr} + \sum_q \sum_j e^{u'_{qs}} \beta^{V+}_{qjr} z2_{jrs} e^{t'_{jrs}} 2R_{jr} X^V_{jr} - \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} R_{jr} X^V_{jr} \\ &+ \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} z2_{jrs} R_{jr} X^V_{jr} \\ &+ \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} e^{t'_{jrs}} 2R_{jr} X^V_{jr} - \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} z2_{jrs} e^{t'_{jrs}} 2R_{jr} X^V_{jr} - \sum_q e^{u'_{qs}} Y_{qr} z1_{qrs} \\ &+ \sum_q \sum_j e^{u'_{qs}} \beta^{V+}_{qjr} z2_{jrs} R_{jr} X^V_{jr} z1_{qrs} - \sum_q \sum_j e^{u'_{qs}} \beta^{V+}_{qjr} z2_{jrs} e^{t'_{jrs}} 2R_{jr} X^V_{jr} z1_{qrs} + \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} R_{jr} X^V_{jr} z1_{qrs} \\ &- \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} z2_{jrs} R_{jr} X^V_{jr} z1_{qrs} \\ &- \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} e^{t'_{jrs}} 2R_{jr} X^V_{jr} z1_{qrs} + \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} z2_{jrs} e^{t'_{jrs}} 2R_{jr} X^V_{jr} z1_{qrs} \\ &- \sum_q e^{u'_{qs}} Y_{qr} z3_{qrs} z1_{qrs} + \sum_q \sum_j e^{u'_{qs}} \beta^{V+}_{qjr} z2_{jrs} R_{jr} X^V_{jr} z3_{qrs} z1_{qrs} \\ &- \sum_q \sum_j e^{u'_{qs}} \beta^{V+}_{qjr} z2_{jrs} e^{t'_{jrs}} 2R_{jr} X^V_{jr} z3_{qrs} z1_{qrs} + \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} R_{jr} X^V_{jr} z3_{qrs} z1_{qrs} \\ &- \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} z2_{jrs} R_{jr} X^V_{jr} z3_{qrs} z1_{qrs} \\ &- \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} e^{t'_{jrs}} 2R_{jr} X^V_{jr} z3_{qrs} z1_{qrs} + \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} z2_{jrs} e^{t'_{jrs}} 2R_{jr} X^V_{jr} z3_{qrs} z1_{qrs} + 2\sum_q e^{u'_{qs}} D_{qrs} z3_{qrs} z1_{qrs} + \sum_q e^{u'_{qs}} \varepsilon \end{aligned}$$

$$
\begin{aligned}
&= \sum_q e^{u'_{qs}} Y_{qr} - \sum_q \sum_j b2_{qjrs} \beta^{V+}_{qjr} R_{jr} X^V_{jr} + \sum_q \sum_j g2_{qjrs} \beta^{V+}_{qjr} 2R_{jr} X^V_{jr} - \sum_q \sum_j e^{u'_{qs}} \beta^{V-}_{qjr} R_{jr} X^V_{jr} \\
&+ \sum_q \sum_j b2_{qjrs} \beta^{V-}_{qjr} R_{jr} X^V_{jr} + \sum_q \sum_j e^{u'_{qs}+t'_{jrs}} \beta^{V-}_{qjr} 2R_{jr} X^V_{jr} - \sum_q \sum_j g2_{qjrs} \beta^{V-}_{qjr} 2R_{jr} X^V_{jr} - \sum_q b1_{qrs} Y_{qr} \\
&+ \sum_q \sum_j b12_{qjrs} \beta^{V+}_{qjr} R_{jr} X^V_{jr} - \sum_q \sum_j g12_{qjrs} \beta^{V+}_{qjr} 2R_{jr} X^V_{jr} + \sum_q \sum_j b1_{qrs} \beta^{V-}_{qjr} R_{jr} X^V_{jr} \\
&- \sum_q \sum_j b12_{qjrs} \beta^{V-}_{qjr} R_{jr} X^V_{jr} - \sum_q \sum_j g1_{qjrs} \beta^{V-}_{qjr} 2R_{jr} X^V_{jr} + \sum_q \sum_j g12_{qjrs} \beta^{V-}_{qjr} 2R_{jr} X^V_{jr} \\
&- \sum_q b13_{qrs} Y_{qr} + \sum_q \sum_j b123_{qjrs} \beta^{V+}_{qjr} R_{jr} X^V_{jr} - \sum_q \sum_j g123_{qjrs} \beta^{V+}_{qjr} 2R_{jr} X^V_{jr} + \sum_q \sum_j b13_{qrs} \beta^{V-}_{qjr} R_{jr} X^V_{jr} \\
&- \sum_q \sum_j b123_{qjrs} \beta^{V-}_{qjr} R_{jr} X^V_{jr} - \sum_q \sum_j g13_{qjrs} \beta^{V-}_{qjr} 2R_{jr} X^V_{jr} + \sum_q \sum_j g123_{qjrs} \beta^{V-}_{qjr} 2R_{jr} X^V_{jr} + 2\sum_q b13_{qrs} D_{qrs} + \sum_q e^{u'_{qs}} \varepsilon
\end{aligned}
\tag{3.19.3}
$$

$$
\sum_i v^F_{is} X^F_{ik} + \sum_j e^{v^{V'}_{js}} X^V_{jk} - \sum_q e^{u'_{qs}} Y_{qks} + v_{0s} \ge 0,\ \forall k \setminus r \tag{3.19.4}
$$

$$
\begin{aligned}
&\sum_i v^F_{is} X^F_{ir} + \sum_j e^{v^{V'}_{js}} (X^V_{jr} - R_{jr} X^V_{jr} + e^{t'_{jrs}} 2R_{jr} X^V_{jr}) - \sum_q e^{u'_{qs}} (y^E_{qrs} + \varepsilon) + v_{0s} \\
&= \sum_i v^F_{is} X^F_{ir} + \sum_j e^{v^{V'}_{js}} X^V_{jr} - \sum_j e^{v^{V'}_{js}} R_{jr} X^V_{jr} + \sum_j e^{v^{V'}_{js}+t'_{jrs}} 2R_{jr} X^V_{jr} - \sum_q e^{u'_{qs}} (y^E_{qrs} + \varepsilon) + v_{0s} \ge 0
\end{aligned}
\tag{3.19.5}
$$

$$ e^{u'_{qs}} - M(1 - z1_{qrs}) \le b1_{qrs} \le e^{u'_{qs}} + M(1 - z1_{qrs}),\ \forall q \tag{3.19.6} $$

$$ -Mz1_{qrs} \le b1_{qrs} \le Mz1_{qrs},\ \forall q \tag{3.19.7} $$

$$ e^{u'_{qs}} - M(1 - z2_{jrs}) \le b2_{qjrs} \le e^{u'_{qs}} + M(1 - z2_{jrs}),\ \forall j, \forall q \tag{3.19.8} $$

$$ -Mz2_{jrs} \le b2_{qjrs} \le Mz2_{jrs},\ \forall j, \forall q \tag{3.19.9} $$

$$ e^{u'_{qs}} - M(1 - a12_{qjrs}) \le b12_{qjrs} \le e^{u'_{qs}} + M(1 - a12_{qjrs}),\ \forall j, \forall q \tag{3.19.10} $$

$$ -Ma12_{qjrs} \le b12_{qjrs} \le Ma12_{qjrs},\ \forall j, \forall q \tag{3.19.11} $$

$$ z1_{qrs} - M(1 - z2_{jrs}) \le a12_{qjrs} \le z1_{qrs} + M(1 - z2_{jrs}),\ \forall j, \forall q \tag{3.19.12} $$

$$ -Mz2_{jrs} \le a12_{qjrs} \le Mz2_{jrs},\ \forall j, \forall q \tag{3.19.13} $$

$$ e^{u'_{qs}} - M(1 - a13_{qrs}) \le b13_{qrs} \le e^{u'_{qs}} + M(1 - a13_{qrs}),\ \forall q \tag{3.19.14} $$

$$ -Ma13_{qrs} \le b13_{qrs} \le Ma13_{qrs},\ \forall q \tag{3.19.15} $$

$$ z1_{qrs} - M(1 - z3_{qrs}) \le a13_{qrs} \le z1_{qrs} + M(1 - z3_{qrs}),\ \forall q \tag{3.19.16} $$

$$ -Mz3_{qrs} \le a13_{qrs} \le Mz3_{qrs},\ \forall q \tag{3.19.17} $$

$$ e^{u'_{qs}} - M(1 - a123_{qjrs}) \le b123_{qjrs} \le e^{u'_{qs}} + M(1 - a123_{qjrs}),\ \forall j, \forall q \tag{3.19.18} $$

$$ -Ma123_{qjrs} \le b123_{qjrs} \le Ma123_{qjrs},\ \forall j, \forall q \tag{3.19.19} $$

$$ a12_{qjrs} - M(1 - z3_{qrs}) \le a123_{qjrs} \le a12_{qjrs} + M(1 - z3_{qrs}),\ \forall j, \forall q \tag{3.19.20} $$

$$ -Mz3_{qrs} \le a123_{qjrs} \le Mz3_{qrs},\ \forall j, \forall q \tag{3.19.21} $$

$$ e^{u'_{qs}+t'_{jrs}} - M(1 - z1_{qrs}) \le g1_{qjrs} \le e^{u'_{qs}+t'_{jrs}} + M(1 - z1_{qrs}),\ \forall j, \forall q \tag{3.19.22} $$

$$-Mz1_{qrs} \le g1_{qjrs} \le Mz1_{qrs},\ \forall j, \forall q \tag{3.19.23}$$

$$e^{u'_{qs}+t'_{jrs}} - M(1-z2_{jrs}) \le g2_{qjrs} \le e^{u'_{qs}+t'_{jrs}} + M(1-z2_{jrs}),\ \forall j, \forall q \tag{3.19.24}$$

$$-Mz2_{jrs} \le g2_{qjrs} \le Mz2_{jrs},\ \forall j, \forall q \tag{3.19.25}$$

$$e^{u'_{qs}+t'_{jrs}} - M(1-a12_{qjrs}) \le g12_{qjrs} \le e^{u'_{qs}+t'_{jrs}} + M(1-a12_{qjrs}),\ \forall j, \forall q \tag{3.19.26}$$

$$-Ma12_{qjrs} \le g12_{qjrs} \le Ma12_{qjrs},\ \forall j, \forall q \tag{3.19.27}$$

$$e^{u'_{qs}+t'_{jrs}} - M(1-a13_{qrs}) \le g13_{qjrs} \le e^{u'_{qs}+t'_{jrs}} + M(1-a13_{qrs}),\ \forall j, \forall q \tag{3.19.28}$$

$$-Ma13_{qrs} \le g13_{qjrs} \le Ma13_{qrs},\ \forall j, \forall q \tag{3.19.29}$$

$$e^{u'_{qs}+t'_{jrs}} - M(1-a123_{qjrs}) \le g123_{qjrs} \le e^{u'_{qs}+t'_{jrs}} + M(1-a123_{qjrs}),\ \forall j, \forall q \tag{3.19.30}$$

$$-Ma123_{qjrs} \le g123_{qjrs} \le Ma123_{qjrs},\ \forall j, \forall q \tag{3.19.31}$$

Similar to the term $\sum_q e^{u'_{qs}}(y^E_{qrs}+\varepsilon)$ and equations (3.19.3)-(3.19.31), we can give exponential-based formulation of $\sum_q e^{w'_{qs}}(y^E_{qrs}+\varepsilon)$ and add new variables and constraints to handle constraint $\sum_q e^{u'_{qs}}(y^E_{qrs}+\varepsilon)+\sum_q e^{w'_{qs}}(y^E_{qrs}+\varepsilon)=1$.

$$D_{qrs} - (y_{qrs} - D_{qrs}) < Mz3_{qrs},\ \forall q \tag{3.19.32}$$

$$D_{qrs} - (y_{qrs} - D_{qrs}) \ge -M(1-z3_{qrs}),\ \forall q \tag{3.19.33}$$

$$z3_{qrs} \in \{0,1\},\ \forall q \tag{3.19.34}$$

$$Y_r + \sum_j \beta^V_{jr} d_{jrs} - D_{qrs} < Mz1_{qrs},\ \forall q \tag{3.19.35}$$

$$Y_r + \sum_j \beta^V_{jr} d_{jrs} - D_{qrs} \ge -M(1-z1_{qrs}),\ \forall q \tag{3.19.36}$$

$$d_{jrs} < Mz2_{jrs},\ \forall j \tag{3.19.37}$$

$$d_{jrs} \ge -M(1-z2_{jrs}),\ \forall j \tag{3.19.38}$$

$$d_{jrs} = d^+_{jrs} - d^-_{jrs},\ \forall j \tag{3.19.39}$$

$$d_{jrs} = -R_{jr}X^V_{jr} + e^{t'_{jrs}} 2R_{jr}X^V_{jr},\ \forall j \tag{3.19.40}$$

$$\ln\varepsilon \le u'_{qs},\ \ln\varepsilon \le w'_{qs},\ \forall q, \text{ and } \ln\varepsilon \le v^{V'}_{js},\ \ln\varepsilon \le t'_{jrs} \le 0,\ \forall j \tag{3.19.41}$$

$$z1_{qrs}, z2_{jrs}, z3_{qrs} \in \{0,1\},\ \forall j,\ \forall q \tag{3.19.42}$$

$$d^+_{jrs}, d^-_{jrs}, v^F_{is} \ge 0,\ \forall i, \forall j \tag{3.19.43}$$

Miscellaneous

The minimum function $y_{qrs}^{c} = \min(y_{qrs} - D_{qrs}, D_{qrs})$ in constraint (3.9.6) can be transformed into (3.17.3)-(3.17.8).

$$\begin{aligned} y_{qrs}^{c} &= (y_{qrs} - D_{qrs})z3_{qrs} + D_{qrs}(1 - z3_{qrs}) \\ &= w1_{qrs} - D_{qrs}z3_{qrs} + D_{qrs}(1 - z3_{qrs}),\ \forall q \end{aligned} \tag{3.17.3}$$

$$D_{qrs} - (y_{qrs} - D_{qrs}) < Mz3_{qrs},\ \forall q \tag{3.17.4}$$

$$D_{qrs} - (y_{qrs} - D_{qrs}) \geq -M(1 - z3_{qrs}),\ \forall q \tag{3.17.5}$$

$$z3_{qrs} \in \{0,1\},\ \forall q \tag{3.17.6}$$

$$y_{qrs} - M(1 - z3_{qrs}) \leq w1_{qrs} \leq y_{qrs} + M(1 - z3_{qrs}),\ \forall q \tag{3.17.7}$$

$$-Mz3_{qrs} \leq w1_{qrs} \leq Mz3_{qrs},\ \forall q \tag{3.17.8}$$

Then, constraint (3.9.6) can be linearized further as follows.

$$\begin{aligned} y_{qrs}^{E} &= y_{qrs}(1 - z1_{qrs}) + \left[D_{qrs} - y_{qrs}^{c}\right]z1_{qrs} \\ &= y_{qrs} - w2_{qrs} + D_{qrs}z1_{qrs} - w3_{qrs},\ \forall q \end{aligned} \tag{3.17.9}$$

$$y_{qrs} - M(1 - z1_{qrs}) \leq w2_{qrs} \leq y_{qrs} + M(1 - z1_{qrs}),\ \forall q \tag{3.17.10}$$

$$-Mz1_{qrs} \leq w2_{qrs} \leq Mz1_{qrs},\ \forall q \tag{3.17.11}$$

$$y_{qrs}^{c} - M(1 - z1_{qrs}) \leq w3_{qrs} \leq y_{qrs}^{c} + M(1 - z1_{qrs}),\ \forall q \tag{3.17.12}$$

$$-Mz1_{qrs} \leq w3_{qrs} \leq Mz1_{qrs},\ \forall q \tag{3.17.13}$$

APPENDIX C

SUPPORTING DISCUSSIONS FOR CHAPTER IV

Instrumental Variables

The inverse demand function in an oligopolistic market depends on the output levels of all firms. However, in some cases other factors influence the price. Three cases are discussed below. In the body of the paper we assume the simplest case, (1) estimating the inverse demand function with no omitted variables, $P_i = P^0 - \alpha \mathcal{Y}_i + \varepsilon_i$, and $E(\mathcal{Y}_i \varepsilon_i) = 0$, $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ i.i.d. and a regression model using Ordinary Least Squares (OLS) and $E(P_i | \mathcal{Y}_i = Y_i) = P^0 - \alpha Y_i$. However, if there exist omitted variables W_i which affect price P_i , such that $P_i = P^0 - \alpha \mathcal{Y}_i + \beta W_i + \eta_i$ where $\eta_i \sim N\left(0, \sigma_\eta^2\right)$, alternative cases need to be considered.[22] In case (2), OLS can still provide consistent estimates when $E(\mathcal{Y}_i \eta_i) = 0$, $E(W_i \eta_i) = 0$ and the quantity variable $\mathcal{Y}_i$ is uncorrelated with the omitted variable, i.e. $E(\mathcal{Y}_i W_i) = 0$. Thus, the regression generates $E(P_i | \mathcal{Y}_i = Y_i) = P_W^0 - \alpha Y_i$, where $P_W^0 = P^0 + \beta E(W_i)$. In case (3), if $\mathcal{Y}_i$ is correlated with the omitted variable $E(\mathcal{Y}_i W_i) \neq 0$, let $\varepsilon_i = \beta W_i + \eta_i$, which results in $E(\mathcal{Y}_i \varepsilon_i) \neq 0$.

Case 3 is termed endogeneity in econometrics; OLS provides inconsistent, biased, and inefficient estimates for the α parameters of interest (Greene, 2011). To address this issue, we use an instrumental variable Z_i that is highly correlated with $\mathcal{Y}_i$ but independent of W_i and η_i, specifically $E(Z_i W_i) = 0$, $E(Z_i \eta_i) = 0$. The regression

[22] The omitted variable could be the price or quantity of substitute products or other contextual factors that could affect the price of output *q*.

model can be rewritten as $P_i = P^0 - \alpha Z_i + \beta W_i + \eta_i$, and OLS can provide consistent estimates and $E(P_i|Z_i = z_i) = P_W^0 - \alpha z_i$. As described in section 3.1, our paper focuses on an inverse demand function expressed and estimated by a linear function $P(Y) = P^0 - \alpha Y$, where P^0 is the intercept corresponding to case 1. However, if endogeneity exists in the inverse demand model, we can identify instrumental variables using the methods described in Goldberger (1972), Morgan (1990), and Angrist and Krueger (2001). Note that when output quantity changes, we assume a change in supply curve rather than a change in quantity supplied.

Weak, Moderate, and Strong Dominance Properties

Lemma 4.3: In the two-output product case, if matrix $\boldsymbol{\alpha}$ satisfies MDD and symmetric properties, then matrix $\boldsymbol{\alpha}$ satisfies SDD.

Proof: If matrix $\boldsymbol{\alpha}$ satisfies MDD and symmetric properties, it will spontaneously lead to the transitivity property, which implies that the main effect of each product dominates the minor effect of the other products, i.e. the SDD property.

Lemma 4.4: If price sensitivity matrix $\boldsymbol{\alpha}$ satisfies the SDD property, then solving MCP (4.5) generates a solution such that $y_{rq} \geq 0$ and $P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) \geq 0$ where $\forall (X_{ri}, y_{rq}) \in \tilde{T}$.

Proof: $P_q(Y_q, \boldsymbol{Y}_{(-q)}) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq} = 0$, that is $P_q(Y_q, \boldsymbol{Y}_{(-q)}) \geq \alpha_{qq} y_{rq} + \sum_{h \neq q} \alpha_{hq} y_{rh}$. If $y_{rq}, y_{rh} \geq 0, q \neq h$, then $P_q(Y_q, \boldsymbol{Y}_{(-q)}) \geq 0$ and the revenue function is nonnegative. The case $y_{rq} \geq 0, y_{rh} < 0, q \neq h$ will not happen because, given $P_q(Y_q, \boldsymbol{Y}_{(-q)}) = P_q^0 - \alpha_{qq} Y_q - \sum_{h \neq q} \alpha_{qh} Y_h$ the increase of revenue through the increase of price $P_q(Y_q, \boldsymbol{Y}_{(-q)}) \geq 0$ cannot exceed the decrease of revenue through the increase of price $P_h(Y_h, \boldsymbol{Y}_{(-h)}) \geq 0$ when y_{rh} becomes smaller and smaller and $y_{rh} < 0$ since price $P_q(Y_q, \boldsymbol{Y}_{(-q)})$ is not sensitive with respect to y_{rh} by symmetric $\boldsymbol{\alpha}$ and $\alpha_{qq} \gg \alpha_{qh}$ for all q. Thus, the preferred solution for revenue maximization problem is $y_{rh} = 0$. Similar to the impossible case $y_{rq} < 0, y_{rh} \geq 0, q \neq h$, by the SDD property we have $P_q(Y_q, \boldsymbol{Y}_{(-q)}) \geq 0$ which will cause $y_{rq} = 0$ to maximize revenue. If $y_{rq} < 0, y_{rh} < 0, q \neq h$, then we have $P_q(Y_q, \boldsymbol{Y}_{(-q)}) > 0$. Similar to lemma 4.2, we have solution $y_{rq} = y_{rh} = 0$ to maximize the revenue function $\sum_q P_q(Y_q, \boldsymbol{Y}_{(-q)}) y_{rq} > 0$. Therefore, the case $y_{rq} < 0, y_{rh} < 0$ will not happen. Moreover, if price sensitivity matrix $\boldsymbol{\alpha}$ is symmetric and satisfies the WDD property, and $\alpha_{qq} \gg \alpha_{qh}$ for all q, then, as lemma 4.3, solving MCP (4.5) will automatically generate $y_{rq} \geq 0$ and $P_q(Y, \boldsymbol{Y}_{(-q)}) \geq 0$ where $\forall (X_{ri}, y_{rq}) \in \tilde{T}$.

Lemma 4.4 shows a case of SDD of sensitivity matrix $\boldsymbol{\alpha}$. The WDD or MDD properties are not enough to ensure $y_{rq} \geq 0$ in MCP (4.5). That is, if $\boldsymbol{\alpha}$ is not symmetric or violates MDD, then for some y_{rq} a Nash equilibrium solution may set $y_{rq} < 0$. We illustrate this in two cases below.

Case 1: If price sensitivity matrix $\boldsymbol{\alpha}$ satisfies MDD but not symmetry and we know that if $y_{rq} \neq 0$, then $P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq} = 0$. We have $y_{rq} = \frac{P_q(Y_q, \boldsymbol{Y}_{(-q)}) - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}}{\alpha_{qq}} = \frac{P_q^0 - \alpha_{qq} Y_q - \sum_{h \neq q} \alpha_{qh} Y_h - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}}{\alpha_{qq}}$. Finally, $y_{rq} = \frac{P_q^0}{2\alpha_{qq}} - \frac{\sum_{k \neq r} y_{kq}}{2} - \sum_{h \neq q} \frac{\alpha_{qh}}{2\alpha_{qq}} Y_h - \sum_{h \neq q} \frac{\alpha_{hq}}{2\alpha_{qq}} y_{rh} - \frac{\mu 1_{rq}}{2\alpha_{qq}}$. Thus, y_{rq} might be less than zero and $P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) < 0$ for the revenue maximization problem as $\alpha_{hq} \gg \alpha_{qq}$ and $y_{rh} > 0$.

Case 2: If price sensitivity matrix $\boldsymbol{\alpha}$ satisfies WDD and symmetric properties, in an extreme case, if for one product q, we have $\frac{\alpha_{qh}}{\alpha_{qq}} \to 1^-$, this notation means the ratio approaches 1 from the left-hand side. We know $y_{rq} = \frac{P_q(Y_q, \boldsymbol{Y}_{(-q)}) - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}}{\alpha_{qq}} = \frac{P_q^0 - \alpha_{qq} Y_q - \sum_{h \neq q} \alpha_{qh} Y_h - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}}{\alpha_{qq}}$. Then, $y_{rq} \approx \frac{P_q^0}{2\alpha_{qq}} - \frac{\sum_{k \neq r} y_{kq}}{2} - \frac{\sum_{h \neq q} Y_h}{2} - \frac{\sum_{h \neq q} y_{rh}}{2} - \frac{\mu 1_{rq}}{2\alpha_{qq}}$. Thus, y_{rq} might be less than zero and $P_q\left(Y_q, \boldsymbol{Y}_{(-q)}\right) > 0$ for the revenue maximization problem since α_{qq} and α_{hh} are large, and $\alpha_{qq} > \alpha_{hh}$ and $y_{rh} > 0$.

Therefore, to ensure $y_{rq} \geq 0$ from formulation (4.5), the SDD property provides a sufficient condition based on lemma 4. If matrix $\boldsymbol{\alpha}$ satisfies SDD and given an extreme case, for all q, $\frac{sum(\boldsymbol{\alpha}) - tr(\boldsymbol{\alpha})}{\alpha_{qq}} \to 0^+$ this notation means the ratio approaches 0 from the right-hand side. If we know $y_{rq} = \frac{P_q(Y_q, \boldsymbol{Y}_{(-q)}) - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq}}{\alpha_{qq}}$, then we can obtain an estimate of $y_{rq} = \frac{P_q^0 - \alpha_{qq}\sum_{k\neq r} y_{kq} - \sum_{h\neq q}\alpha_{qh}Y_h - \sum_{h\neq q}\alpha_{hq}y_{rh} - \mu 1_{rq}}{2\alpha_{qq}} \approx \frac{P_q^0 - \alpha_{qq}\sum_{k\neq r} y_{kq} - \mu 1_{rq}}{2\alpha_{qq}} > 0$ for the revenue maximization problem.

Cost minimization case

In the case of a single fixed input and a single variable input and a given output level, figure C1 illustrates the Nash equilibrium solution obtained by minimizing costs. Each firm attempts to adjust its variable input to reach the isoquant, holding a fixed input constant in the short run.

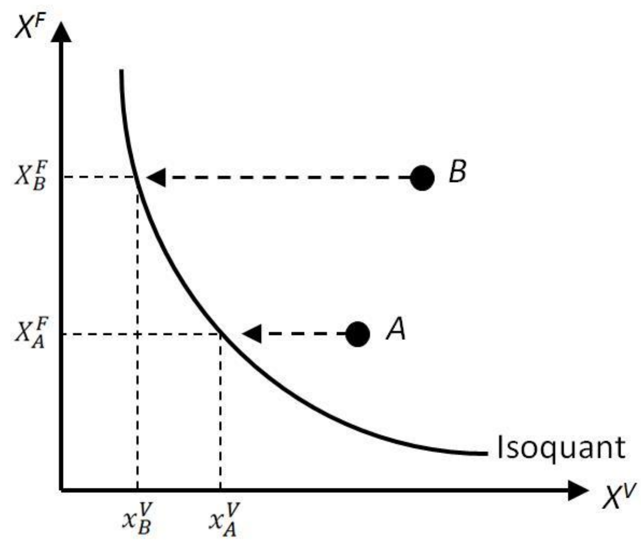

Figure C1 Adjusted variable input in Nash equilibrium

We construct a multi-input cost model to identify a Nash equilibrium solution using MCP. The result shows that the Nash equilibrium solution is on the production frontier regardless of the $\boldsymbol{\beta}$ matrix selected. In particular, to formulate the MCP with multiple variable inputs, first we define the Lagrangian function as:

$L_r\left(x_{rj}^V, \lambda_{rk}, \mu 1_r, \mu 2_{ri}, \mu 3_{rj}, \mu 4_r\right) = -\sum_r \sum_j P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) x_{rj}^V - \sum_r \sum_q \mu 1_{rq}\left(Y_{rq} - \sum_k \lambda_{rk} Y_{kq}\right) - \sum_r \sum_i \mu 2_{ri} \left(\sum_k \lambda_{rk} X_{ki}^F - X_{ri}^F\right) - \sum_r \sum_j \mu 3_{rj} \left(\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V\right) - \sum_r \mu 4_r \left(\sum_k \lambda_{rk} - 1\right).$

Then the resulting MCP problem is:

$$0 \geq \frac{\partial L_r}{\partial x_{rj}^V} = \left(-P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) - \beta_{jj} x_{rj}^V - \sum_{l \neq j} \beta_{lj} x_{rl}^V + \mu 3_{rj}\right) \perp x_{rj}^V \geq 0 \quad \forall r, j$$

$$0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = \left(\sum_q \mu 1_{rq} Y_{kq} - \sum_i \mu 2_{ri} X_{ki}^F - \sum_j \mu 3_{rj} X_{kj}^V - \mu 4_r\right) \perp \lambda_{rk} \geq 0 \quad \forall r, k$$

$$0 \geq \left(Y_{rq} - \sum_k \lambda_{rk} Y_{kq}\right) \perp \mu 1_{rq} \geq 0 \quad \forall r, q \qquad (4.15)$$

$$0 \geq \left(\sum_k \lambda_{rk} X_{ki}^F - X_{ri}^F\right) \perp \mu 2_{ri} \geq 0 \quad \forall r, i$$

$$0 \geq \left(\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V\right) \perp \mu 3_{rj} \geq 0 \quad \forall r, j$$

$$0 = \left(\sum_k \lambda_{rk} - 1\right) \quad \forall r$$

Theorem 4.9: In the cost minimization case a Nash equilibrium generated from MCP (4.15) exists on the production frontier, given an arbitrary $\boldsymbol{\beta}$ matrix with all nonnegative components satisfying WDD.

Proof: Proving the existence of a Nash equilibrium is similar to theorem 4.7. The Nash equilibrium generated from MCP will stay on the production frontier, given an arbitrary $\boldsymbol{\beta}$ matrix satisfying WDD. If an equilibrium output vector exists and $x_{rj}^V > 0$, then it must satisfy the first order condition of MCP. From the complementary condition, we have the following first order condition:

$$-P_j^{X^V}\left(X_j^V,\boldsymbol{X}_{(-j)}^V\right)-\beta_{jj}x_{rj}^V-\sum_{l\neq j}\beta_{lj}x_{rl}^V+\mu 3_{rj}=0\quad\forall r,j$$

which can be expressed in matrix notation as:

$$-\boldsymbol{P}^{X_0^V}-\boldsymbol{\beta}\boldsymbol{x}^V\boldsymbol{e}-\boldsymbol{\beta}^T\boldsymbol{x}_r^V+\boldsymbol{\mu 3}_r=\boldsymbol{0}\quad\forall r$$

where$\boldsymbol{x}^V$ is a matrix with $(\boldsymbol{x}_1^V,\dots,\boldsymbol{x}_K^V)$ and each vector $\boldsymbol{x}_k^V=(x_{k1}^V,\dots,x_{kJ}^V)^T$. $\boldsymbol{e}$ is a vector $(1,\dots,1)^T$ with K elements. $\boldsymbol{P}^{X_0^V}$ is a price vector with elements $P_j^{X_0^V}$. $\boldsymbol{\mu 3}_r$ is a vector of the Lagrangian multiplier with elements $\mu 3_{rj}$. If $\boldsymbol{x}^{V*}$ is the solution obtained from the first order condition, we need to show that $P_j^{X^V}\left(X_j^{V*},\boldsymbol{X}_{(-j)}^{V*}\right)=P_j^{X_0^V}+\beta_{jj}X_j^{V*}+\sum_{l\neq j}\beta_{jl}X_l^{V*}\geq 0$ for all j. We express this equation in the matrix notation $\boldsymbol{P}^{X_0^V}+\boldsymbol{\beta}\boldsymbol{x}^{V*}\boldsymbol{e}$. Obviously, the first order condition gives $\boldsymbol{P}^{X_0^V}+\boldsymbol{\beta}\boldsymbol{x}^{V*}\boldsymbol{e}+\boldsymbol{\beta}^T\boldsymbol{x}_r^{V*}=\boldsymbol{\mu 3}_r>\boldsymbol{0}$ if $\boldsymbol{P}^{X_0^V}>0$ and $\boldsymbol{\beta}$ have nonnegative elements. This implies that it is necessary to set $\left(\sum_k\lambda_{rk}X_{kj}^V-x_{rj}^V\right)=0$ in terms of MCP; the upper bound of input level is characterized by the least value at the free disposability hull of inputs and the lower bound is the input level described by the free disposability hull of outputs shown in theorem 6. Because $\left(\sum_k\lambda_{rk}X_{kj}^V-x_{rj}^V\right)=0$, that is, for cost minimization, the whole quantity of supply market would be minimized to reach a lower price at the inverse supply function, a firm's best strategy is to reduce its input level and to produce on the production frontier.

Generalized profit model as revenue maximization case

In a special case of the revenue model, we assume that the output level directly follows the variable input, namely, the level of variable input determines and controls the level of output. For example, in the semiconductor manufacturing industry raw silicon wafers are released into the production line to generate the actual die output. If the yield is 100%, the output level is a linear function of the variable input level. Assuming a constant unit cost of the variable input, we formulate the profit maximization model as:

$$PF^* = \max_{y_{rq}} \left\{ \begin{matrix} \sum_r \sum_q P_q^Y\big(Y_q, \boldsymbol{Y}_{(-q)}\big) y_{rq} \\ - \sum_r \sum_q P_q^{X^V}\big(Y_q, \boldsymbol{Y}_{(-q)}\big) y_{rq} \end{matrix} \middle| \begin{matrix} \sum_k \lambda_{rk} Y_{kq} \geq y_{rq} \quad \forall q, r; \\ \sum_k \lambda_{rk} X_{ki}^F \leq X_{ri}^F \quad \forall i, r; \\ \sum_k \lambda_{rk} = 1 \quad \forall r; \\ \lambda_{rk} \geq 0 \quad \forall k, r; \end{matrix} \right\} \tag{4.16}$$

where $P_q^{X^V}\big(Y_q, \boldsymbol{Y}_{(-q)}\big)$ becomes a constant and presents a unit cost of variable input $x_{rj}^V = \rho y_{rq}$, and ρ is a coefficient to change the units to a linear function. Intuitively, model (4.16) is quite similar to formulation (4.4), the revenue maximization model. The profit function $\sum_r \sum_q \big[P_q^Y\big(Y_q, \boldsymbol{Y}_{(-q)}\big) - P_q^{X^V}\big(Y_q, \boldsymbol{Y}_{(-q)}\big)\big] y_{rq}$ is a concave function because $P_q^{X^V}\big(Y_q, \boldsymbol{Y}_{(-q)}\big)$ is a constant and $P_q^{X^V}\big(Y_q, \boldsymbol{Y}_{(-q)}\big) y_{rq}$ is a linear function. Thus, a Nash equilibrium exists and is unique. See sections 4.3 and 4.4 in the body of the paper.

APPENDIX D

PROOFS FOR CHAPTER IV

Lemma 4.1: Let output levels be decision variables denoted by y_{rq} as output q of firm r and $y_{rq} \geq 0$; further, let input levels be decision variables denoted by x_{ri} as input i of firm r, $x_{ri} \geq 0$, and $(x_{ri}, y_{rq}) \in \tilde{T}$. Define $P_q^Y(y_{rq})y_{rq}$ as a concave function of y_{rq} and assume that either the inverse demand function $P_q^Y(y_{rq})$ is a non-increasing or a convex function of y_{rq}. Thus, for each $Y_{(-r)q} > 0$, where $Y_{(-r)q} = \sum_{k \neq r} y_{kq}$, $P_q^Y(y_{rq} + Y_{(-r)q})y_{rq}$ is a concave function of y_{rq} for $y_{rq} \geq 0$. Similarly, let $P_i^X(x_{ri} + X_{(-r)i})x_{ri}$ be a convex function of x_{ri} for $x_{ri} \geq 0$, where $X_{(-r)i} = \sum_{k \neq r} x_{ki}$ and $P_i^X(x_{ri})$ is an inverse supply function. Further, if either $P_q^Y(y_{rq})$ is strictly decreasing or is strictly convex, then $P_q^Y(y_{rq} + Y_{(-r)q})y_{rq}$ is a strictly concave function on the nonnegative $y_{rq} \geq 0$ and $\sum_q P_q^Y(y_{rq} + Y_{(-r)q})y_{rq} - \sum_i P_i^X(x_{ri} + X_{(-r)i})x_{ri}$ is a concave function on $(x_{ri}, y_{rq}) \in \tilde{T}$.

Proof: Murphy *et al*. (1982) prove the single output product case that when $y_r \geq 0$ and $Y_{(-r)} > 0$, the revenue function $R_r = P^Y(y_r + Y_{(-r)})y_r$ is a concave function of y_r for $y_r \geq 0$ on the nonnegative real line since $\frac{\partial^2 R_r}{\partial y_r^2} < 0$. In our special case of Murphy *et al*. proven in their lemma 1, the production possibility set $(\boldsymbol{x}, \boldsymbol{y})$ is a convex set and the boundary is a piece-wise linear concave function which characterizes a production function with diminishing returns. Thus, $P_q^Y(y_{rq} + Y_{(-r)q})y_{rq}$ is a concave function of

y_{rq} for $y_{rq} \geq 0$ and $(x_{ri}, y_{rq}) \in \tilde{T}$ since, given fixed input levels, firm r can expand output only by increasing y_{rq}. Similarly, we can prove a convex cost function of i^{th} input resource $P_i^X(x_{ri} + X_{(-r)i})x_{ri}$ and concave profit function $\sum_q P_q^Y\left(y_{rq} + Y_{(-r)q}\right)y_{rq} - \sum_i P_i^X(x_{ri} + X_{(-r)i})x_{ri}$.

Theorem 4.1: If the profit function of firm r, $\theta_r(x_{ri}, y_{rq}) = \sum_q P_q^Y\left(Y_q\right)y_{rq} - \sum_i P_i^X(X_i)x_{ri}$ is concave with respect to (x_{ri}, y_{rq}) and continuously differentiable, where $Y_q = \sum_k y_{kq}$ and $X_i = \sum_k x_{ki}$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*) \in \tilde{T}$ is a Nash-Cournot oligopolistic market equilibrium if and only if it satisfies the set of VI $\langle F\left((\boldsymbol{x}^*, \boldsymbol{y}^*)\right), (\boldsymbol{x}, \boldsymbol{y}) - (\boldsymbol{x}^*, \boldsymbol{y}^*)\rangle \geq 0$, $\forall(\boldsymbol{x}, \boldsymbol{y}) \in \tilde{T}$. That is,

$$\sum_k F_k\left((\boldsymbol{x}^*, \boldsymbol{y}^*)\right)((\boldsymbol{x}_k, \boldsymbol{y}_k) - (\boldsymbol{x}_k^*, \boldsymbol{y}_k^*)) \geq 0 \quad \forall(\boldsymbol{x}_k, \boldsymbol{y}_k) \in \tilde{T},$$

where

$F_k\left((\boldsymbol{x}, \boldsymbol{y})\right) = (-\nabla_{x_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}), -\nabla_{y_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}))$, $\nabla_{x_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}) = \left(\frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial x_{k1}}, \dots, \frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial x_{kI}}\right)$ and $\nabla_{y_k}\theta_k(\boldsymbol{x}, \boldsymbol{y}) = \left(\frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial y_{k1}}, \dots, \frac{\partial\theta_k(\boldsymbol{x},\boldsymbol{y})}{\partial y_{kQ}}\right)$.

Proof: To simplify the proof, we first focus on revenue function with a single output. If the revenue function $P(Y)y_r$ is concave with respect to y_r and continuously differentiable, then $(X_i, y^*) \in \tilde{T}$ is a Nash-Cournot oligopolistic market equilibrium if and only if it satisfies the set of VI $\langle F(\boldsymbol{y}^*), y - y^*\rangle \geq 0$, $\forall(X_{ki}, y_k) \in \tilde{T}$. That is, $\sum_k F_k(\boldsymbol{y}^*)(y_k - y_k^*) \geq 0 \quad \forall(X_{ki}, y_k) \in \tilde{T}$; $F_k(\boldsymbol{y}^*) = -P(Y^*) - y_k^*P'(Y^*)$. Since the revenue function $P(Y)y_r$ is a continuously differentiable function and concave with respect to y_r , for a fixed r , the Nash equilibrium condition $P\left(y_r^*, y_{(-r)}^*\right)y_r^* - P\left(y_r, y_{(-r)}^*\right)y_r \geq 0$, $\forall(X_{ri}, y_r) \in \tilde{T}$ is equivalent to the variational inequality problem $F_r(\boldsymbol{y}^*)(y_r - y_r^*) \geq 0$, that is, $\langle F_r(\boldsymbol{y}^*), y_r - y_r^*\rangle \geq 0, \forall(X_{ri}, y_r) \in \tilde{T}$. Then, summing over all firms k generates $\langle F(\boldsymbol{y}^*), y - y^*\rangle \geq 0,\ \forall(X_{ki}, y_k) \in \tilde{T} \geq 0$. This result can be extended to prove the VI of the profit function.

Theorem 4.2: Consider an oligopoly with K firms, with an inverse demand function $P^Y(\cdot)$ that is strictly decreasing and continuously differentiable in y, and an inverse supply function $P^X(\cdot)$ that is strictly increasing and continuously differentiable in x. Since lemma 1 shows that the profit function $\theta_k(x_k, y_k)$ is concave and $x_k, y_k \geq 0$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*) = ((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*), \dots, (\boldsymbol{x}_K^*, \boldsymbol{y}_K^*))$ is a Nash equilibrium solution if and only if

$$\nabla_{x_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq 0 \text{ and } \nabla_{y_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq 0 \quad \forall k;$$

$$\boldsymbol{x}_k^*\left[\nabla_{x_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*)\right] = 0 \text{ and } \boldsymbol{y}_k^*\left[\nabla_{y_k}\theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*)\right] = 0 \quad \forall k$$

where $(\boldsymbol{x}_k^*, \boldsymbol{y}_k^*) \in \tilde{T}$.

Proof: We derive the formulas above based on the KKT conditions. Note that the KKT conditions are both necessary and sufficient conditions for a unique global optimum since the model maximizes a strictly concave profit function over a convex polyhedral set (the production possibility set). The detail of existence and uniqueness of a Nash equilibrium is addressed in section 4.4 of the paper.

Lemma 4.2 : A Nash solution to MCP problem (4.3) will satisfy $y_r \geq 0$ and $P(Y) \geq 0$.

Proof: $P(Y) - \alpha y_r - \mu 1_r = 0$, that is $P(Y) \geq \alpha y_r$ since $\mu 1_r \geq 0$. If $y_r \geq 0$, then $P(Y) \geq 0$ and the revenue function is nonnegative. If $y_r \leq 0$ and $P(Y) \geq 0$, a firm's best strategy to maximize the revenue function is to make $y_r = 0$. The case $y_r \leq 0$ and $P(Y) < 0$ will not happen because if $(Y) < 0$, then there exists at least one firm generating $y_k > 0, k \neq r$ such that $P(Y) = P^0 - \alpha Y < 0$. However, to maximize its revenue, firm k prefers to produce $y_k = 0$. In other words, $P(Y) = P^0 - \alpha Y \geq 0$ if $y_r \leq 0$. In addition, if α is a large positive number, y_r can be very small but positive to ensure a positive revenue function. Thus, any solution to this MCP (3) model enforces that y_r and $P(Y)$ are nonnegative.

Theorem 4.3: If $P(Y) = P^0 - \alpha Y \geq 0$ and α is a small enough positive parameter, the Nash equilibrium solution is for all firms to produce on the production frontier.

Proof: In MCP, $\frac{\partial L_r}{\partial y_r} = (P(Y) - \alpha y_r - \mu 1_r) = 0, \forall r$; where α is small enough, then $P(Y) - \alpha y_r = \mu 1_r \geq 0$. In the extreme case, $\alpha = 0$, then $P(Y) = P^0 = \mu 1_r > 0$. By MCP, $0 \geq (y_r - \sum_k \lambda_{rk} Y_k) \perp \mu 1_r > 0, \forall r$, which gives $y_r - \sum_k \lambda_{rk} Y_k = 0$. Once again, a firm's best strategy is to produce on the production frontier.

Theorem 4.4: If $P(Y) = P^0 - \alpha Y \geq 0$ and α is a large enough positive parameter, the MCP will lead to a benchmark output level with $y_r = \overline{y_r}$ close to zero, where $\overline{y_r}$ defines a truncated output level.

Proof: Since $P^0 - \alpha Y \geq 0$ from lemma 4.2 and P^0 is a constant, then $\leq \frac{P^0}{\alpha}$, meaning that a larger α will result in a smaller Y. In the MCP, $0 \geq (y_r - \sum_k \lambda_{rk} Y_k) \perp \mu 1_r \geq 0, \forall r$. If Y is small, then $(y_r - \sum_k \lambda_{rk} Y_k) < 0$, i.e. $\mu 1_r = 0$. In other words, we can increase α until no firm would choose to produce on the production frontier in a Nash equilibrium solution, and then all $\mu 1_r = 0, \forall r$. Proving this results in a truncated benchmark output level and requires us to show that if α increases, then y_r decreases and approaches zero. Since $\mu 1_r = 0$ and we know $y_r \geq 0$ by lemma 4.2, $P(Y) - \alpha y_r = 0$ in the MCP and $y_r = \frac{P(Y)}{\alpha}$. In addition, $y_r = \frac{P(Y)}{\alpha} = \frac{P^0 - \alpha \sum_{k \neq r} y_k}{2\alpha}$. If there are only two firms in the market, $y_1 = \frac{(P^0/\alpha) - y_2}{2}$ and $y_2 = \frac{(P^0/\alpha) - y_1}{2}$, then $y_1 = y_2 = \frac{P^0}{3\alpha}$. This constant $\frac{P^0}{3\alpha}$ identifies the truncation output level for production. If there are K firms in the market, $y_r = \frac{P^0 - \alpha \sum_{k \neq r} y_k}{2\alpha}$ and $y_r = \frac{\left(\frac{P^0}{\alpha}\right) - (K-1)\left(\frac{P^0}{2\alpha}\right) + \left(\frac{K-1}{2}\right) y_r + \left(\frac{K-2}{2}\right) \sum_{k \neq r} y_k}{2}$, then $\sum_{k \neq r} y_k = \frac{2}{K-2}\left(2y_r - \left(\frac{P^0}{\alpha}\right) + (K-1)\left(\frac{P^0}{2\alpha}\right) - \left(\frac{K-1}{2}\right) y_r\right) = \frac{2}{K-2}\left(\left(\frac{5-K}{2}\right) y_r + \left(\frac{P^0}{2\alpha}\right)(K-3)\right)$. We replace $\sum_{k \neq r} y_k$ in equation y_r, thus $y_r = \frac{P^0 - \alpha \sum_{k \neq r} y_k}{2\alpha} = \frac{\frac{P^0}{\alpha} - \frac{(K-3)P^0}{(K-2)\alpha}}{2 + \frac{5-K}{K-2}} = \frac{P^0}{(K+1)\alpha}$. Therefore, for K firms $y_r = \frac{P^0}{(K+1)\alpha} = \overline{y_r}$ and this constant $\overline{y_r}$ identifies the benchmark output level. As α goes to infinity, $y_r = \frac{P^0}{(K+1)\alpha} \rightarrow 0$.

Theorem 4.5: If the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies WDD but is not necessarily symmetric, then the MCP (4.6) generates $(X_{ri}, y_{rq}) \in \tilde{T}$ where y_{rq} will approach the efficient frontier for small enough values of α_{qq}; $y_{rq} = \bar{y}_{rq}$ is the truncated benchmark output level that approaches zero as α_{qq} approaches infinity.

Proof: This is similar to theorems 4.3 and 4.4. We know $\frac{\partial L_r}{\partial y_r} = P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} \leq \mu 1_{rq}, \forall r$. If the α_{qq} value is small enough and we consider a special case $\alpha_{qq} = 0$, and the $\boldsymbol{\alpha}$ matrix is diagonally dominant, then $0 < P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) = P_q^0 \leq \mu 1_r$. Referring to the MCP, $0 \geq \big(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\big) \perp \mu 1_{rq} > 0 \quad \forall r, q$, meaning $y_{rq} - \sum_k \lambda_{rk} Y_{kq} = 0$, or a firm's best strategy is to produce on the production frontier except for the portion associated with positive slacks and dual variables equal to zero on the output constraints since increasing output does not affect the price reduction. On the other hand, if the α_{qq} value is large enough, $P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) = P_q^0 - \alpha_{qq} Y_q - \sum_{h \neq q} \alpha_{qh} Y_h \geq 0$ and P_q^0 is a constant, then $Y_q \leq \frac{P_q^0 - \sum_{h \neq q} \alpha_{qh} Y_h}{\alpha_{qq}}$. As α_{qq} becomes larger, Y_q approaches zero. Referring to the MCP, $0 \geq \big(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\big) \perp \mu 1_{rq} \geq 0 \quad \forall r, q$. If Y_q is small, then $\big(y_{rq} - \sum_k \lambda_{rk} Y_{kq}\big) < 0$ and $\mu 1_{rq} = 0$. In other words, we can increase α_{qq} until no firm would choose to produce on the production frontier in a Nash equilibrium solution, and then all $\mu 1_{rq} = 0, \forall r, q$. To show this result, as α_{qq}increases, then y_{rq}, the truncated output level becomes smaller and approaches zero. Since $P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq} \leq 0$, and we know $\mu 1_{rq} = 0$ and $y_{rq} \geq 0$, thus $P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} = 0$ in MCP (4.6) and $0 \leq y_{rq} = \frac{P_q^0 - \alpha_{qq} \sum_{k \neq r} y_{kq} - \sum_{h \neq q} \alpha_{qh} Y_h - \sum_{h \neq q} \alpha_{hq} y_{rh}}{2\alpha_{qq}}$. For the two-output products example, $y_{r1} = \frac{P_1^0}{2\alpha_{11}} - \frac{\sum_{k \neq r} y_{k1}}{2} - \frac{\alpha_{12}}{2\alpha_{11}} Y_2 - \frac{\alpha_{21}}{2\alpha_{11}} y_{r2}$ and $y_{r2} = \frac{P_2^0}{2\alpha_{22}} - \frac{\sum_{k \neq r} y_{k2}}{2} - \frac{\alpha_{21}}{2\alpha_{22}} Y_1 - \frac{\alpha_{12}}{2\alpha_{22}} y_{r1}$; replacing y_{r2} in equation y_{r1} gives $y_{r1} = \left(1 - \frac{\alpha_{12}\alpha_{21} + \alpha_{21}^2}{4\alpha_{11}\alpha_{22}}\right)^{-1} \left[\left(\frac{P_1^0}{2\alpha_{11}} - \frac{\alpha_{21}}{2\alpha_{11}} \frac{P_2^0}{2\alpha_{22}}\right) - \left(\frac{1}{2} - \frac{\alpha_{21}^2}{4\alpha_{11}\alpha_{22}}\right) \sum_{k \neq r} y_{k1} - \frac{\alpha_{12}}{2\alpha_{11}} Y_2 + \frac{\alpha_{21}}{4\alpha_{11}} \sum_{k \neq r} y_{k2}\right]$. Also, $Y_2 = y_{r2} + \sum_{k \neq r} y_{k2}$ finally gives $y_{r1} = \left(1 - \frac{2\alpha_{12}\alpha_{21} + \alpha_{12}^2 + \alpha_{21}^2}{4\alpha_{11}\alpha_{22}}\right)^{-1} \left[\left(\frac{P_1^0}{2\alpha_{11}} - \frac{(\alpha_{12} + \alpha_{21}) P_2^0}{4\alpha_{11}\alpha_{22}}\right) - \left(\frac{1}{2} - \frac{\alpha_{21}^2}{4\alpha_{11}\alpha_{22}} - \frac{\alpha_{12}\alpha_{21}}{4\alpha_{11}\alpha_{22}}\right) \sum_{k \neq r} y_{k1} + \left(\frac{\alpha_{12}}{4\alpha_{11}} + \frac{\alpha_{21}}{4\alpha_{11}} - \frac{\alpha_{12}}{2\alpha_{11}}\right) \sum_{k \neq r} y_{k2}\right]$. Based on WDD, $y_{r1} \approx \left(\frac{P_1^0}{2\alpha_{11}}\right) - \left(\frac{1}{2}\right) \sum_{k \neq r} y_{k1} + \left(\frac{\alpha_{21}}{4\alpha_{11}}\right) \sum_{k \neq r} y_{k2}$. This result shows that y_{r1} is a function of y_{k1} and y_{k2} , not a variable of index r. Thus, y_{r1} is limited by a truncated level $\bar{y}_{r1}$ for all firms, since for all firms r the same equation applies as does y_{r1} for revenue maximization. Similar equation can be derived for y_{r2}. In addition, as α_{11} approaches infinity $y_{r1} \approx \frac{-\sum_{k \neq r} y_{k1}}{2}$. That is, $y_{r1} = \bar{y}_{r1}$ should be equal to zero. We can extend this result to outputs of more than two. Therefore, the truncation point approaches zero as α_{qq} becomes large.

Corollary 4.1: If the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies the MDD property and $\alpha_{qq} \gg \alpha_{hh}, q \neq h$, then the solution to the MCP (4.6) will satisfy $y_{rq} < y_{rh}\ \forall r, q$.

Proof: Theorem 4.5 proves Corollary 4.1.

Theorem 4.6: Given arbitrary price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ that satisfy WDD, MCP (4.9) generates all allocatively efficient Nash solutions $(X_{ri}^F, x_{rj}^{V*}, y_{rq}^*) \in \tilde{T}$. These solutions are on the frontier including the weakly efficient frontier, but excluding the portion of the frontier associated with positive slacks and dual variables equal to zero on the input constraints.

Proof: Based on theorem 4.5, if $y_{rq} > 0$, then $x_{rj}^V > 0$ because there is no free lunch axiom in production theory (Färe *et al.*, 1985). According to formulation (4.11) $-P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) - \beta_{jj}x_{rj}^V - \sum_{l\neq j}\beta_{lj}x_{rl}^V + \mu 3_{rj} = 0$, that is, $P_j^{X^V}\left(X_j^V, \boldsymbol{X}_{(-j)}^V\right) + \beta_{jj}x_{rj}^V + \sum_{l\neq j}\beta_{lj}x_{rl}^V = \mu 3_{rj}$. Consider that $\beta_{jl} \geq 0$, $P_j^{X_0^V} > 0$ and the $\boldsymbol{\beta}$ matrix is diagonally dominant; then $\mu 3_{rj} > 0$. Referring to MCP (4.9), $0 \geq \left(\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V\right) \perp \mu 3_{rj} > 0 \quad \forall r, j$, which gives $\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V = 0$. Based on theorem 5 we know that $\mu 1_{rq}$ might/might not be equal to zero for all r, q according to the price sensitivity matrix $\boldsymbol{\alpha}$, i.e. equation $y_{rq} - \sum_k \lambda_{rk} Y_{kq} \leq 0$. Thus, a firm's best strategy is to adjust its variable input and output levels approaching the production frontier. The solution becomes allocatively efficient. Further, $\sum_k \lambda_{rk} X_{kj}^V - x_{rj}^V = 0$ implies that the slacks of the input constraints are equal to zero and the feasible region of the Nash solution is the production possibility set $\tilde{T}$ excluding the region for which the input level is larger than the least value at the free disposability hull of the inputs. This exception of the free disposability hull of inputs implies an upper bound of adjustable input level. Note that the points on the free disposability hull of inputs, except the anchor points, have positive slacks and dual variables equal to zero on the inputs' constraints. Therefore, all Nash equilibrium solutions $(X_{ri}^{F}, x_{rj}^{V*}, y_{rq}^{*})$ belong to $\tilde{T}$ excluding the input level larger than the anchor point at the free disposability hull of inputs.

Theorem 4.7: MCP (4.9) generates a Nash equilibrium solution $(\boldsymbol{X}^{F}, \boldsymbol{x}^{V*}, \boldsymbol{y}^{*}) \in \tilde{T}$.

Proof: If an equilibrium output vector exists and $x_{rj} > 0, y_{rq} > 0$, it must satisfy the first order condition of MCP (4.9). The complementary condition gives the following first order condition on the output side:

$$P_q\big(Y_q, \boldsymbol{Y}_{(-q)}\big) - \alpha_{qq} y_{rq} - \sum_{h \neq q} \alpha_{hq} y_{rh} - \mu 1_{rq} = 0 \quad \forall r, q$$

This condition can be expressed in matrix notation as:

$$\boldsymbol{P}^{Y_0} - \boldsymbol{\alpha y e} - \boldsymbol{\alpha}^{T} \boldsymbol{y}_r - \boldsymbol{\mu 1}_r = \boldsymbol{0} \quad \forall r$$

where $\boldsymbol{y}$ is a matrix with $(\boldsymbol{y}_1, \dots, \boldsymbol{y}_K)$ and each vector $\boldsymbol{y}_k = (y_{k1}, \dots, y_{kQ})^T$. $\boldsymbol{e}$ is a vector $(1, \dots, 1)^T$ with K elements. $\boldsymbol{P}^{Y_0}$ is a price vector with elements $P_q^{Y_0}$. $\boldsymbol{\mu 1}_r$ is a vector of the Lagrangian multiplier with elements $\mu 1_{rq}$. If $\boldsymbol{y}^{*}$ is the solution obtained from the first order condition, we need to show that $P_q^{Y}\big(Y_q^{*}, \boldsymbol{Y}_{(-q)}^{*}\big) = P_q^{Y_0} - \alpha_{qq} Y_q^{*} -$

$\sum_{h\neq q}\alpha_{qh}Y_h^* \geq 0$ for all q. This equation can be expressed in matrix notation as $\boldsymbol{P}^0 - \boldsymbol{\alpha y}^*\boldsymbol{e}$. Obviously, the first order condition gives $\boldsymbol{P}^0 - \boldsymbol{\alpha y}^*\boldsymbol{e} = \boldsymbol{\alpha}^T\boldsymbol{y}_r^* + \boldsymbol{\mu 1}_r \geq \boldsymbol{0}$ if $\boldsymbol{y}_r^* \geq \boldsymbol{0}$ for all r by lemma 4.2.

Similar to the first order condition on the variable input side

$$-\boldsymbol{P}^{X_0^V} - \boldsymbol{\beta x}^V\boldsymbol{e} - \boldsymbol{\beta}^T\boldsymbol{x}_r^V + \boldsymbol{\mu 3}_r = \boldsymbol{0} \quad \forall r$$

where $\boldsymbol{x}^V$ is a matrix with $(\boldsymbol{x}_1^V, \dots, \boldsymbol{x}_K^V)$ and each vector $\boldsymbol{x}_k^V = (x_{k1}^V, \dots, x_{kJ}^V)^T$. $\boldsymbol{P}^{X_0^V}$ is a price vector with elements $P_j^{X_0^V}$. $\boldsymbol{\mu 3}_r$ is a vector of the Lagrangian multiplier with elements $\mu 3_{rj}$. If $\boldsymbol{x}^{V*}$ is the solution obtained from the first order condition, we need to show that $P_j^{X^V}\left(X_j^{V*}, \boldsymbol{X}_{(-j)}^{V*}\right) = P_j^{X_0^V} + \beta_{jj}X_j^{V*} + \sum_{l\neq j}\beta_{jl}X_l^{V*} \geq 0$ for all j. This equation can be expressed in matrix notation as $\boldsymbol{P}^{X_0^V} + \boldsymbol{\beta x}^V\boldsymbol{e}$. Obviously, the first order condition gives $-\boldsymbol{\beta}^T\boldsymbol{x}_r^V + \boldsymbol{\mu 3}_r = \boldsymbol{P}^{X_0^V} + \boldsymbol{\beta x}^V\boldsymbol{e} \geq \boldsymbol{0}$ if $\boldsymbol{x}_r^{V*} \geq \boldsymbol{0}$ for all r by the estimated production possibility set $\tilde{T}$ describing a positive lower bound of input level. Therefore, if an equilibrium vector exists, it must equal $(\boldsymbol{x}^{V*}, \boldsymbol{y}^*)$.

To show that $(\boldsymbol{x}^{V*}, \boldsymbol{y}^*)$ is indeed an equilibrium vector, for any nonnegative vector $(\boldsymbol{X}^F, \hat{\boldsymbol{x}}^V, \hat{\boldsymbol{y}}) \in \tilde{T}$ where $(\boldsymbol{X}^F, \hat{\boldsymbol{x}}^V, \hat{\boldsymbol{y}}) \neq (\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*)$, we consider $(\boldsymbol{X}^F, \hat{\boldsymbol{x}}^V, \hat{\boldsymbol{y}})$ in which all the elements are equal to $(\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*)$ except for some $\boldsymbol{x}_r^V, \boldsymbol{y}_r$ columns. We need to show that

$$\sum_r\sum_q P_q^Y\left(\hat{Y}_q, \hat{\boldsymbol{Y}}_{(-q)}\right)\hat{y}_{rq} - \sum_r\sum_j P_j^{X^V}\left(\hat{X}_j^V, \hat{\boldsymbol{X}}_{(-j)}^V\right)\hat{x}_{rj}^V$$
$$\leq \sum_r\sum_q P_q^Y\left(Y_q^*, \boldsymbol{Y}_{(-q)}^*\right)y_{rq}^* - \sum_r\sum_j P_j^{X^V}\left(X_j^{V*}, \boldsymbol{X}_{(-j)}^{V*}\right)x_{rj}^{V*}$$

for all r. Since PF is a strictly concave function under concavity and differentiability assumptions for the maximization problem, and $(\boldsymbol{x}^{V*}, \boldsymbol{y}^*)$ satisfies the first order condition and the KKT condition, then $(\boldsymbol{x}^{V*}, \boldsymbol{y}^*)$ must be a global optimum, i.e. the complementary condition provides a Nash equilibrium solution: $\sum_r \sum_q P_q^Y(\hat{Y}_q, \widehat{\boldsymbol{Y}}_{(-q)})\hat{y}_{rq} - \sum_r \sum_j P_j^{X^V}(\hat{X}_j^V, \widehat{\boldsymbol{X}}_{(-j)}^V)\hat{x}_{rj}^V \leq \sum_r \sum_q P_q^Y(Y_q^*, \boldsymbol{Y}_{(-q)}^*)y_{rq}^* - \sum_r \sum_j P_j^{X^V}(X_j^{V*}, \boldsymbol{X}_{(-j)}^{V*})x_{rj}^{V*}$ for all r and $(\boldsymbol{X}^F, \widehat{\boldsymbol{x}}^V, \widehat{\boldsymbol{y}}) \in \tilde{T}$.

Theorem 4.8: If the profit function is a strictly concave function on $(\boldsymbol{X}^F, \boldsymbol{x}^V, \boldsymbol{y}) \in \tilde{T}$ that is continuous and differentiable and the price sensitivity matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ satisfy the WDD property, then the Nash equilibrium solution found using MCP (4.9) is unique if a solution exists for the maximization problem.

Proof: To prove the uniqueness, let two vectors $(\boldsymbol{X}^F, \widehat{\boldsymbol{x}}^V, \widehat{\boldsymbol{y}})$ and $(\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*) \in \tilde{T}$ be solutions and $(\boldsymbol{X}^F, \widehat{\boldsymbol{x}}^V, \widehat{\boldsymbol{y}}) \neq (\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*)$ satisfy the variational inequality:

$\sum_k F_k((\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*))^T \cdot ((\boldsymbol{X}_k^F, \boldsymbol{x}_k^{V\prime}, \boldsymbol{y}_k') - (\boldsymbol{X}_k^F, \boldsymbol{x}_k^{V*}, \boldsymbol{y}_k^*)) \geq 0 \quad \forall (\boldsymbol{X}_k^F, \boldsymbol{x}_k^{V\prime}, \boldsymbol{y}_k') \in \tilde{T}$; (4.17)

$\sum_k F_k((\boldsymbol{X}^F, \widehat{\boldsymbol{x}}^V, \widehat{\boldsymbol{y}}))^T \cdot ((\boldsymbol{X}_k^F, \boldsymbol{x}_k^{V\prime}, \boldsymbol{y}_k') - (\boldsymbol{X}_k^F, \widehat{\boldsymbol{x}}_k^V, \widehat{\boldsymbol{y}}_k)) \geq 0 \quad \forall (\boldsymbol{X}_k^F, \boldsymbol{x}_k^{V\prime}, \boldsymbol{y}_k') \in \tilde{T}$; (4.18)

Substituting $\widehat{\boldsymbol{x}}_k^V, \widehat{\boldsymbol{y}}_k$ for $\boldsymbol{x}_k^{V\prime}, \boldsymbol{y}_k'$ in (4.17) and $\boldsymbol{x}_k^{V*}, \boldsymbol{y}_k^*$ for $\boldsymbol{x}_k^{V\prime}, \boldsymbol{y}_k'$ in (4.18) and adding the resulting inequalities gives

$\sum_k (F_k((\boldsymbol{X}^F, \boldsymbol{x}^{V*}, \boldsymbol{y}^*)) - F_k((\boldsymbol{X}^F, \widehat{\boldsymbol{x}}^V, \widehat{\boldsymbol{y}})))^T \cdot ((\boldsymbol{X}_k^F, \widehat{\boldsymbol{x}}_k^V, \widehat{\boldsymbol{y}}_k) - (\boldsymbol{X}_k^F, \boldsymbol{x}_k^{V*}, \boldsymbol{y}_k^*) \geq 0$

However, this inequality does not satisfy the definition of strict monotonicity.

Thus, $\widehat{\boldsymbol{x}}^V = \boldsymbol{x}^{V*}$, $\widehat{\boldsymbol{y}} = \boldsymbol{y}^*$ and the solution is unique.

Corollary 4.2: Assume all input and output variables are normalized to eliminate unit dependence, and the price of outputs dominates the price of inputs to ensure a positive marginal profit. Given a production frontier including three portions: IRS, CRS, and DRS, the MCP (4.9) generates a Nash equilibrium solution that is characterized by DRS when the inverse demand and supply functions are less sensitive, or the Nash equilibrium is characterized by IRS when the inverse demand and supply functions are more sensitive.

Proof: Intuitively, for one-variable-input one-output production process if the inverse demand and supply functions are less sensitive, this is illustrated in a special case where both price sensitivity matrix $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are equal to zero; the profit function PF can be written as maximizing $PF = P^{Y_0}y - P^{X_0^V}x^V$. Let PF^* denote the optimal value of profit function. Thus, we can express the function $y = \frac{PF^* + P^{X_0^V}x^V}{P^{Y_0}}$, where $\frac{P^{X_0^V}}{P^{Y_0}}$ indicates the slope of profit function. Given the price of outputs dominating the price of inputs, in a special case the slope $\frac{P^{X_0^V}}{P^{Y_0}} \to 0^+$, the optimal solution of profit maximization problem will show the flat profit line tangent to the production possibility set. Since $\frac{P^{X_0^V}}{P^{Y_0}} \to 0^+$, a firm would like to generate a Nash solution on the DRS frontier for profit maximization based on theorems 4.5 and 4.6, i.e., in extreme case, the input level of the Nash solution has to be on the upper bound defined by the least value of the free disposability hull of inputs (see firm A in figure 4.4). DRS is associated with the insensitive inverse demand and supply function. Therefore, the Nash equilibrium solution generated from MCP (4.9) presents the DRS with respect to MPSS and $\sum_{k^{MPSS}} \lambda_{rk^{MPSS}}^{CRS*} = \sum_k \lambda_{rk}^{CRS*} > \sum_k \lambda_{rk}^{*} = 1$. Similarly, we can show that the Nash solutions present the IRS when more sensitive inverse demand and supply functions occur, i.e., a profit function with larger slope. The result can be extended to the multiple-input multiple-output case.

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