TIMING SYNCHRONIZATION AT THE RELAY NODE IN PHYSICAL LAYER NETWORK CODING

A Thesis

by

ASHISH BASIREDDY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2012

Major Subject: Electrical Engineering

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ABSTRACT

Timing Synchronization at the Relay Node in Physical Layer Network Coding.
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Chair of Advisory Committee: Dr. Scott L.Miller

In recent times, there has been an increased focus on the problem of information exchange between two nodes using a relay node. The introduction of physical layer network coding has improved the throughput efficiency of such an exchange. In practice, the reliability of information exchange using this scheme is reduced due to synchronization issues at the relay node. In this thesis, we deal with timing synchronization of the signals received at the relay node. The timing offsets of the signals received at the relay node are computed based on the propagation delays in the transmitted signals. However, due to the random attenuation of signals in a fading channel, the near far problem is inherent in this situation. Hence, we aim to design near far resistant delay estimators for this system. We put forth four algorithms in this regard. In all the algorithms, propagation delay of each signal is estimated using a known preamble sent by the respective node at the beginning of the data packet. In the first algorithm, we carefully construct the preamble of each data packet and apply the MUSIC algorithm to overcome the near far problem. The eigenstructure of the correlation matrix is exploited to estimate propagation delay. Secondly, the idea of interference cancellation is implemented to remove the near far problem and delay is estimated using a correlator. Thirdly, a modified decorrelating technique is presented to negate the near far problem. Using this technique we aim to obtain an estimate of the weak user's delay that is more robust to errors in the strong user's delay estimate. In the last algorithm, pilot signals with desired autocorrelation and cross

correlation functions are designed and a sliding correlator is used to estimate delay. Even though this approach is not near far resistant, performance results demonstrate that for the length's of preamble considered, this algorithm performs similar to the other algorithms.

To my parents and my elder brother

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First and foremost, I would like to express my sincere gratitude to my advisor Dr. Scott Miller for introducing me to research and being so patient and supportive, especially in the tough times. The detailed feedback from him on my results and progress helped me significantly during this work. This thesis would not have been possible without his guidance and support. I would also like to thank the members of my advisory committee for the intriguing questions and suggestions about my work. Thanks to the staff in the Department of Electrical Engineering for making my academic life here so convenient and to faculty for providing me with the important knowledge needed for research. I am also thankful to my parents, without whom I would not have been able to accomplish this task. A special thanks to my friends Arvind Yedla, Ajay Kumar and my grand mother, Aruna Reddy for their support and encouragement which made this work possible.

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CHAPTER I

INTRODUCTION

During the past decade there has been a major shift in technology from wired networks to wireless. In the midst of various topologies of the wireless networks, there has been a growing interest in relay networks. In these networks the source and destination are connected by an intermediate node to aid in information exchange. The throughput efficiency of the information exchange in these networks is defined as the ratio of the amount of information exchanged to the number of time slots required to carry out this process. Inorder to improve the throughput efficiency of relay networks, a few techniques were introduced in [1], [2], [3], [4]. Digital network coding and physical layer network coding (PLNC) are two techniques that result in better throughput efficiency. In this chapter, we provide a brief overview of these schemes and also present the scenarios and assumptions considered in this thesis.

A. Information exchange mechanisms in a relay network

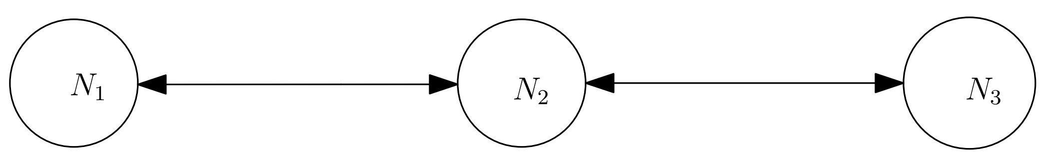

Fig. 1. Relay network

A simple relay network comprises of three nodes wherein two nodes exchange information with the help of an intermediate node. Figure 1 shows a relay network. The

This thesis follows the Journal style of *IEEE Transactions on Automatic Control.*

intermediate node is also known as a relay node. In practice, a relay network is used in scenarios where the wireless range is limited or the nodes have an obstruction or the channel does not allow reliable data transmission. A primitive function of the relay node is to amplify the received signal and forward it to the desired node. This is known as the standard transmission technique. Using this transmission technique, two time slots are required to transmit a frame or bits (S_1) from N_1 to N_2. We define a single time slot as the time taken to transmit a frame from one node to the closest adjacent node. Hence a total of four time slots are needed to exchange two frames between N_1 and N_2. A few other efficient schemes were introduced later. One of them known as digital network coding uses bit encoding and decoding techniques to improve the throughput efficiency. In this scheme, initially, N_1 and N_3 transmit their respective frames to N_2 consecutively in two time slots. Denote these frames as S_1 and S_3 respectively. After receiving S_1 and S_3, N_2 encodes frame S_2 as

$$S_2 = S_1 \oplus S_3,$$

where $\oplus$ denotes bitwise binary addition over the entire frames of S_1 and S_3. N_2 then broadcasts S_2 to both N_1 and N_3 during the third time slot. Once N_1 receives S_2, it decodes S_3 from S_2 using the local information S_1.

$$S_1 \oplus S_2 = S_1 \oplus (S_1 \oplus S_3) = S_3.$$

Similarly, N_3 can decode S_1. Figure 2 shows a schematic of this scheme. Note that a total of three time slots are required for this scheme as compared to four in the standard transmission technique. Hence there is a throughput improvement of 33% in comparison to standard transmission technique.

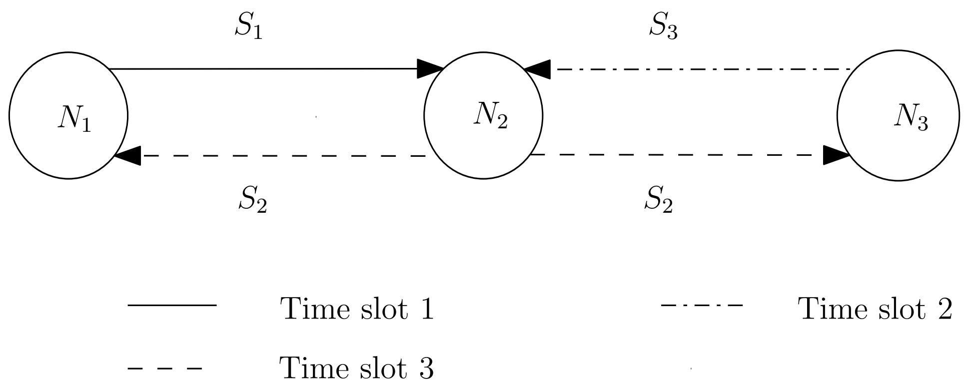

Fig. 2. Digital network coding scheme

Physical layer network coding (PLNC) is an another scheme designed for information exchange. This scheme is based on an idea similar to digital network coding but involves electromagnetic signals. The received signal can be perceived as the sum of two transmitted signals with different phases, amplitudes and time delays. The relay can then either broadcast the received signal after amplification (called the amplify-and-forward technique), or it can decode the modulo-2 sum of the bits from the received signal and then broadcast a signal constructed from the modulo-2 sum of the bits (called the decode-and-forward technique). Regardless of the technique

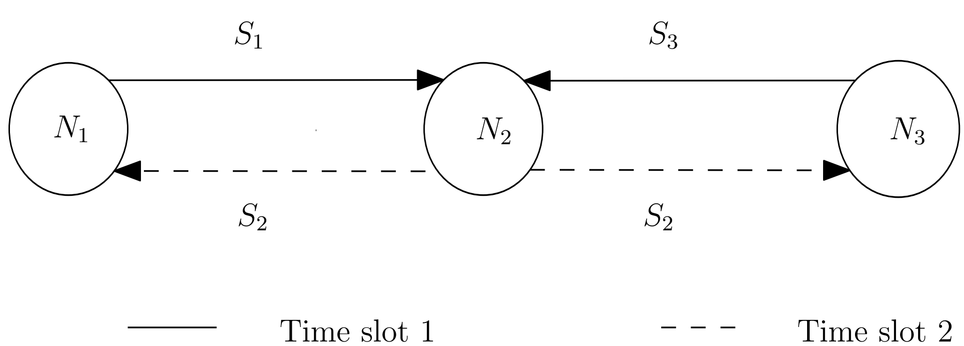

Fig. 3. PLNC scheme

used, ideally PLNC provides a throughput improvement of 50% improvement over the standard transmission scheme. This is depicted in Figure 3.

B. Scenarios and assumptions

In this work, we assume that the relay network divides time into fixed length slots and data is transmitted in packets that will fit into a time slot. Each data packet consists of a preamble of bits known to both the transmitter and receiver followed by the information carrying data bits which would obviously be known to the transmitter but not to the receiver. These bits are transmitted using binary phase shift keying (BPSK). Note that each node would need to synchronize its internal clock to some common reference so that their packet transmissions will align with the appropriate time slot and not overlap (interfere) with the transmissions of other nodes in adjacent time slots in this network. However, due to varying propagation delays between the transmitting nodes and the relay, it may be difficult for the transmitting nodes to synchronize their transmissions to the point where the two packets received at the relay node align in time to a precision that is significantly smaller than a data symbol interval. The main contribution of this thesis is to present algorithms to estimate relative timing offsets in such a situation. Further, the near far problem is inherent in these scenarios. Therefore, in this thesis we aim to use delay estimators that can negate the affect of the near far problem to obtain the timing offsets.

In order to motivate the set of assumptions used in this work, consider a scenario where two mobile nodes, N_1 and N_3, are communicating through a fixed relay node, N_2. Furthermore suppose that the distance between the mobile nodes and the relay can be as much as ten kilometers so that variations in the propagation delays between the mobile nodes and the relay node may be as much as about 50 microseconds. We could then transmit data at symbol rates up to something on the order of 100kbps without encountering propagation delays of more than five bit intervals. Further, if we assume that the time slots are on the order of a few tens of milliseconds, then each data packet may contain a few thousand data bits. Therefore, it is reasonable to consider preambles containing less than 100 bits. Note that we consider only the signal corresponding to the preamble to estimate delay. Hence fading can be assumed to be constant over this duration as the fading rates encountered by the system may be as much as perhaps a hundred Hz or possibly slightly higher depending on what frequency band is used.

C. Outline of thesis

This thesis contains three more chapters. In Chapter II, we briefly review the correlation properties of pilot signals used later in the thesis. We also illustrate the affect of near far problem on delay estimation using a correlation approach for a two user system. Chapter III comprises of the proposed algorithms used to estimate delay at the relay node. Simulation results corresponding to each algorithm are also demonstrated. Lastly in Chapter IV, the performance of all algorithms is compared for the scenario described in the previous section and conclusive remarks are presented.

CHAPTER II

BACKGROUND MATERIAL AND PROBLEM MOTIVATION

A. Correlation properties of pilot signals

In general, pilot signals are used to estimate propagation delay. For the delay estimation problem, it is desirable to have pilot signals that posses the following properties.

***Property-1*:** Each signal in the set is easy to distinguish from a time-shifted version of itself.

***Property-2*:** Each signal in the set is easy to distinguish from every other signal and their time shifted versions.

The complex envelope of pilot signals obtained by modulating bits using BPSK, have a structure similar to a square pulse train. Note that for such signals, the above mentioned properties can be quantified by their correlation properties. Hence, we study about the correlation properties of such signals in this section. A periodic square pulse train is of the form

$$c(t) = \sum_{k=-\infty}^{\infty} c_k p(t - kT_b),$$

where $p(t)$ is a square pulse and takes a non-zero constant value only over an interval of one bit T_b. The pulse $p(t) = 0$, if $t < 0$ or $t > T$, and $c_k \in \{\pm 1\}$ is a periodic sequence with period $N = \frac{T}{T_b}$ and T is the period of $c(t)$. The continuous time periodic autocorrelation function of c(t) is given by

$$R_{c,c}(\tau) = \frac{1}{T}\int_0^T c(t)c(t+\tau)dt.$$

Define a periodic discrete time autocorrelation function as

$$\theta_{c,c}(n) = \frac{1}{N}\sum_{k=0}^{N-1} c_k c_{k+n}.$$

Using the above definition, the continuous time autocorrelation can be written in terms of the discrete time autocorrelation function as

$$R_{c,c}(\tau) = \theta_{c,c}(n)\frac{1}{N}\int_0^{T_b-\delta} p(t)p(t+\delta)dt + \theta_{c,c}(n+1)\frac{1}{T_b}\int_{(T_b-\delta)}^{T_b} p(t)p(t+\delta-T_b), \tag{2.1}$$

where $\tau = nT_b + \delta$, $0 < \delta \leq T_b$ and $n \in \mathbb{N}$. Further, from [5], the above equation can be simplified as

$$R_{c,c}(T) = \theta_{c,c}(n)\left(1 - \frac{\delta}{T_b}\right) + \theta_{c,c}(n+1)\frac{\delta}{T_b}.$$

Therefore we can conclude that the periodic autocorrelation functions of such signals can be represented using corresponding discrete correlation functions. Also note that these functions are piecewise linear and they assume the same values as the respective discrete functions at integer multiples of a bit interval.

$$\begin{aligned} R_{c,c}(nT_b) =& \theta_{c,c}(n), \text{ and} \\ R_{c,c}((n+1)T_b) =& \theta_{c,c}(n+1). \end{aligned}$$

Such an autocorrelation function is shown in Figure 4 for an arbitrary sequence.

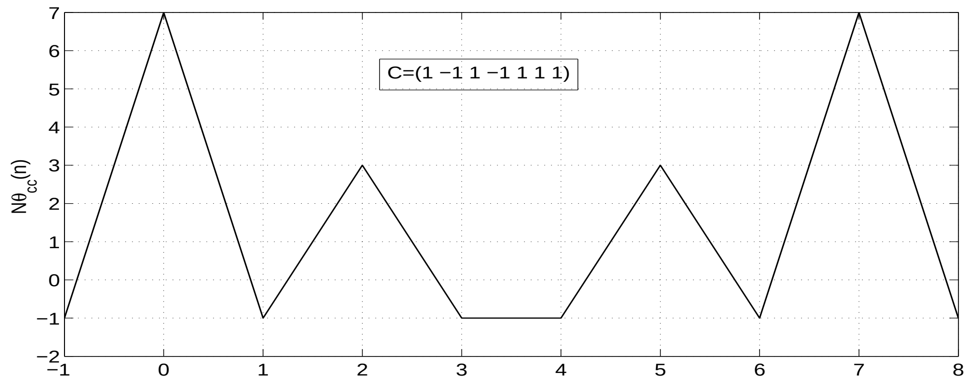

Fig. 4. Continuous time autocorrelation function

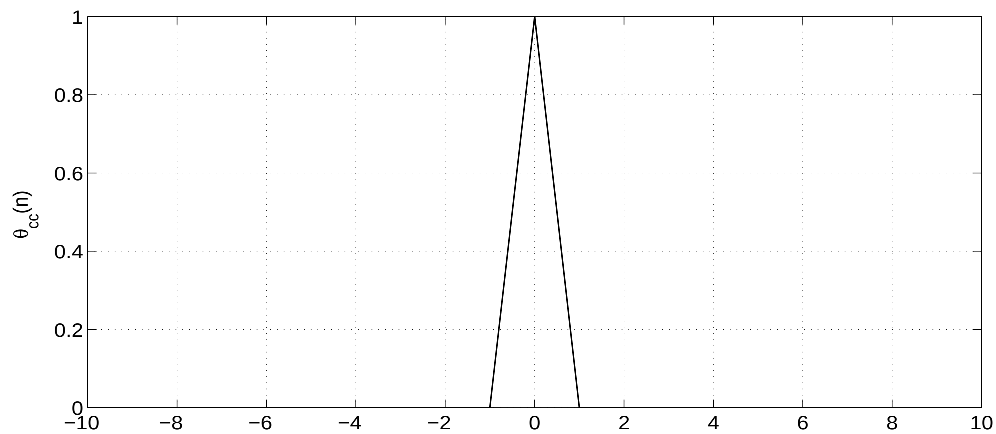

Fig. 5. Desired autocorrelation function

In-order to design signals with Property 1, we choose c(t) such that $\theta_{c,c}(n)$ is as small as possible for all $n \neq 0$. Ideally, such signals would have an autocorrelation function depicted in Figure 5 i.e., the autocorrelation function has only one large peak at the origin and almost zero throughout.

Large *m-sequences* posses similar property [5]. Following an identical methodology, we can also conclude that the periodic crosscorrelation function of two signals with similar structures is also piecewise linear and can be completely represented using a discrete crosscorrelation function (defined similarly). Note that signals with Property-2 are such that each of the signals is orthogonal to other signals and time shifted versions of them. However such signals are not possible in reality. Hence signals with low (close to zero) crosscorrelation are said to posses property-2. Therefore for the delay estimation problem, pilot signals with an autocorrelation function containing a peak at the origin and very low(close to zero) elsewhere and a crosscorrelation function that is close to zero everywhere are desirable.

B. Correlation approach to estimate delay

The idea of employing the sliding correlator to estimate propagation delay is presented in this section. Initially, this idea is analysed for a single user system in a slow fading channel and later extended to a multi-user system. Through this analysis, we aim to illustrate the affect of the near far problem in delay estimation.

For this analysis, consider pilot signals generated by modulating K bits using BPSK. Denote the pilot signal (complex envelope) of user-i by $s_i(t)$. The pilot signal is spread over a duration of KT seconds, where T is the bit interval. Note that the receiver has prior information of these signals. Let the maximum possible delay be T_d (known apriori). Further, we assume that the propagation delay of each user is uniformly distributed over $[0, T_d]$. Let the transmission delay of user-i's signal be τ_i and an estimate of this delay be denoted as $\hat{\tau}_i$. In Figure 6 we show a pictorial representation of the notation followed for a pilot signal considered.

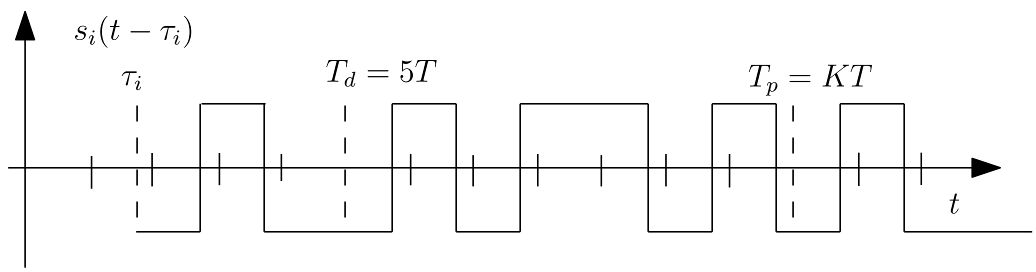

Fig. 6. Diagrammatic representation of the notation.

The simulation results demonstrated later are based on Monte Carlo simulation. Errors in delay estimation are highly varied due to noise and fading. Therefore outlier probability is used as a measure to analyze large errors. This measure will inform us about the frequency of occurrence of large errors. An outlier is said to occur if the error in delay estimation is greater than T_o, where T_o is known as the outlier duration. Mathematically, an outlier occurs if $|\tau_i - \hat{\tau}_i| > T_o$. To account for errors less than T_o, the root mean square error is computed, subject to a condition of non-occurrence of an outlier. Hence forth, a delay estimator is said to perform better if the outlier probability is low and the conditional RMSE is small. We now study about the estimation of propagation delay in the transmitted signals based on the signal received at the receiver.

1. Single user in a slow fading channel

In this system, a user transmits a pilot signal over a slow fading channel. To be consistent with the notation, denote this user as user-1. The complex envelope of the delayed signal received at the receiver is given by

$$r_1(t) = \rho e^{j\theta} e^{-j2\pi f_c \tau_1} s_1(t-\tau_1) + n(t),$$

where f_c is the carrier frequency and $n(t)$ is a complex additive white noise process. ρ is Rayleigh random variable and θ is a uniform random variable in $[0, 2\pi]$. Note that, ρ and θ are independent. As the propagation delay is uniformly distributed over $[0, T_d]$, the received signal over the duration of $[T_d, T_p]$ is assured to contain a portion of the pilot signal for any delay in the range assumed. Therefore the received signal over the duration of $[T_d, T_p]$ is used to estimate delay by the receiver. We follow this procedure in all the delay estimation algorithms presented later in the thesis. Denote this signal by $r(t)$. Mathematically,

$$r(t) = \begin{cases} r_1(t), & T_d \le t \le T_p, \\ 0, & \text{otherwise.} \end{cases}$$

By principles discussed in [6], an estimate of τ_1 is computed using the maximum likelihood estimation technique. The likelihood function used to estimate delay is given by,

$$\Lambda(\tau_1) = \eta \int_0^{\infty} \int_0^{2\pi} e^{-\left\{\frac{\int |r(t) - \rho e^{j\theta} S(t,\tau_1)|^2 dt}{\beta}\right\}} f_{\rho,\theta}(\rho, \theta) d\rho d\theta,$$

where η and β are positive constants and $S(t, \tau_1) = e^{-j2\pi f_c \tau_1} s_1(t - \tau_1)$. From [6], the estimate of the propagation delay is given by,

$$\begin{aligned} \hat{\tau_1} &= \arg \max_{0<\tau_1<T_d} \Lambda(\tau_1) \\ &= \arg \max_{0<\tau_1<T_d} \left| \int_{T_d}^{KT} r(t) S(t, \tau_1) dt \right| \\ &= \arg \max_{0<\tau_1<T_d} \left| \int_{T_d}^{KT} r(t) s_1(t - \tau_1) dt \right|. \end{aligned}$$

Such an estimator is known as a sliding correlator [6]. Figure 7 shows the simulation results of delay estimation using this method.

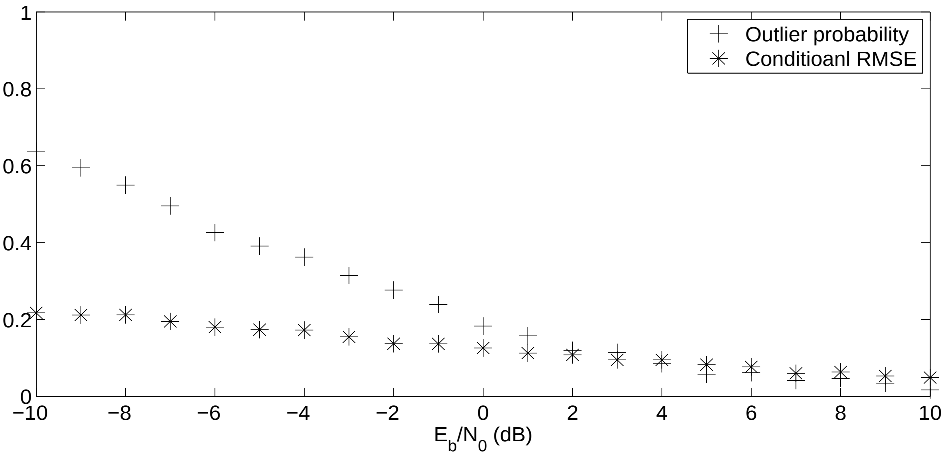

Fig. 7. Simulation results for delay estimation in a single user system. $K = 31, T_d = 5T, T_o = \frac{T}{2}$.

2. Two-user system in a slow fading channel

We now consider the problem of propagation delay estimation in a two user system. In this system, both the users simultaneously transmit their respective pilot signals through independent slow fading channels. The complex envelope of the received signal $r(t)$ is given by,

$$\begin{aligned} r(t) =& \rho_1 e^{j\theta_1} e^{-j2\pi f_c \tau_1} s_1(t-\tau_1) + \rho_2 e^{j\theta_2} e^{-2j\pi f_c \tau_2} s_2(t-\tau_2) + n(t) \\ =& A_1 s_1(t-\tau_1) + A_2 s_2(t-\tau_2) + n(t), \end{aligned}$$

where ρ_i {i=1,2} are IID rayleigh variables and θ_i {i=1,2} are IID uniform random variable on $[0, 2\pi]$, $A_i = \rho_i e^{j\theta_i} e^{-j2\pi f_c \tau_i}$.

For this case, following a similar procedure described in the single user case, we compute a joint estimate of both the user delays. The likelihood function $\Lambda(\tau_1, \tau_2)$ is given by,

$$\Lambda(\tau_1,\tau_2) = \eta \int_0^{\infty}\int_0^{\infty}\int_0^{2\pi}\int_0^{2\pi} e^{-\frac{1}{\beta}\left\{\int |r(t)-S(t,\tau_1,\tau_2)|^2 dt\right\}} f_{\rho_1,\theta_1,\rho_2,\theta_2}(\rho_1,\theta_1,\rho_2,\theta_2) d\theta_1 d\theta_2 d\rho_1 d\rho_2, \tag{2.2}$$

where $S(t,\tau_1,\tau_2) = \rho_1 e^{j\theta_1} e^{-j2\pi f_c \tau_1} s_1(t-\tau_1) + \rho_2 e^{j\theta_2} e^{-j2\pi f_c \tau_2} s_2(t-\tau_2)$. A joint estimate of the propagation delays in both the transmitted signals is given by,

$$\{\hat{\tau}_1, \hat{\tau}_2\} = \arg \max_{0<\tau_1,\tau_2<T_d} \Lambda(\tau_1,\tau_2).$$

Equation (2.2) is not easy to compute and the joint estimate requires a two dimensional search over all possible delays. Hence, we resort to less complex sub-optimal estimation methods. One such technique applies the single-user correlation approach to estimate delay. In this method, we treat the system as a single user system and use a sliding correlator to estimate delay. Mathematically,

$$\hat{\tau}_i = \arg \max_{0\le\tau_i\le T_d} \left| \int_{T_d}^{KT} r(t) s_i(t-\tau_i) dt \right|. \tag{2.3}$$

The performance of this method can be improved by choosing signals that posses the desired autocorrelation function and crosscorrelation functions as pilot signals. However, the performance of this method is limited due to the attenuation of the transmitted signals in a slow fading channel. Different attenuations result in dissimilar powers of the transmitted signals being received at the receiver. To understand this better we consider the performance of this approach for both strong and weak users. We define a strong user as follows. Consider a function $\Lambda_i(\tau)$ given by,

$$\Lambda_i(\tau) = \left| \int_{T_d}^{KT} r_l(t) s_i(t-\tau) dt \right|.$$

User-i is said to be stronger than user-j if

$$\max_{0 \le \tau \le T_d} \Lambda_i(\tau) > \max_{0 \le \tau \le T_d} \Lambda_j(\tau).$$

The performance using this approach is depicted in Figure 8. In these simulation results, bits are chosen such that the pilot signals have desirable autocorrelation properties. Observe that there is a considerable difference in the performance of two users. The stronger user's signal and noise degrade the performance of the delay estimation of weak user. This is due to the near-far problem. Therefore we aim to design delay estimators that can overcome this problem. The following chapter elaborates on such methods.

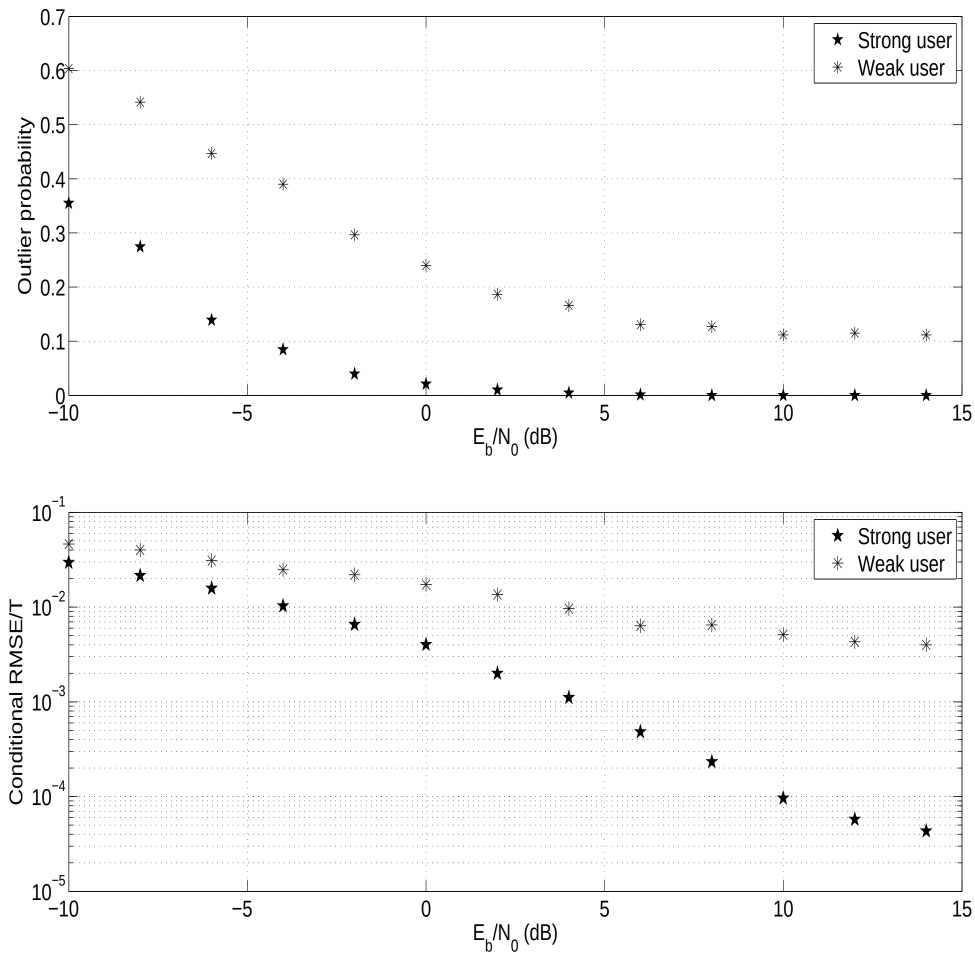

Fig. 8. Simulation results of delay estimation of both users using correlation approach in a slow fading channel. $K = 31$, $T_d = 5T$, $T_o = \frac{T}{2}$.

CHAPTER III

DELAY ESTIMATION ALGORITHMS

We put forth four algorithms to estimate propagation delay in this chapter. Section-A of this chapter, comprises of a description of the system model. A comprehensive study of the algorithms is carried out in Section-B. Simulation results for each algorithm are also presented in this section.

A. System model

A relay network wherein two nodes exchange information using a relay node is our system model. The primary focus of the algorithms presented in this chapter is to estimate propagation delays in the signals received at the relay node. In this system each node (user) transmits a data packet of length N consisting of a preamble ($K < N$ bits) and data bits. The transmitted signal is generated by modulating the bits of the data packet using BPSK. The portion of the entire signal corresponding to the preamble of the data packet is considered as the pilot signal. Let the BPSK modulated variable of the k^{th} bit of node-i be denoted as $c_{k,i}$. We define the pilot sequence of each node as a finite length sequence of these variables corresponding to the preamble (bits) of respective nodes. The pilot signal of user-i (complex envelope) is given by

$$s_i(t) = \sum_{k=0}^{K} c_{k,i} p(t - kT),$$

where T is the bit interval and $p(t)$ denotes a square pulse. The transmitted signal is obtained by multiplying $s_i(t)$ with a carrier $\sqrt{2}\cos(2\pi f_c t)$. The signal received at the receiver considered for delay estimation is

$$r(t) = \Re\left\{\sum_{i=1}^{2} s_i(t-\tau_i)\rho_i \exp\left(j(2\pi f_c(t-\tau_i)+\theta_i)\right)\right\} + n(t), \qquad T_d \leq t \leq KT. \tag{3.1}$$

Note that all the quantities in the above equation are identical to those described in Chapter II for a two user system. The noise waveform $n(t)$, is a white noise waveform with two-sided power spectral density $\frac{N_0}{2}$. Denote the lowpass equivalent signal of $r(t)$ as $r_l(t)$. For delay estimation, we assume $r(t)$ to be zero outside the specified interval.

B. Algorithms

1. MUSIC algorithm

In this algorithm, an idea based on the multiple signal classification(MUSIC) algorithm [7] is presented to estimate the propagation delay of each user. The preamble of the data packet for each user is constructed by concatenating blocks of bits. With such preambles, the digitized received signal is divided into a set of vectors, where each vector contains samples corresponding to a block of the received signal. These vectors are modeled as a linear combination of the signal vectors plus noise. Using the MUSIC algorithm, the eigenspace of the correlation matrix of the received vectors is partitioned into a signal subspace (the subspace spanned by the signal components of all users in the received vector) and it's orthogonal complement (known as the noise subspace). The orthogonality property of the signal and noise subspaces is exploited to estimate propagation delay. Simulation results portray that such a delay estimator is near far resistant. A similar idea was presented in [8], [9], [10], [11] for a multi-user CDMA systems in different applications. A detailed description of this algorithm, for the system model described in the previous section is presented below.

Initially we present this algorithm for a single user system. Denote this user as user-1. Generate a preamble of length $K = ML, \text{where} M, L \in \mathbb{N}$, for this user by appending a block of M bits, L times periodically. Mathematically, the complex envelope of the pilot signal of user-1 generated by using BPSK is given by,

$$s_1(t) = \sum_{l=0}^{L} q_1(t - lMT), \tag{3.2}$$

where

$$q_1(t) = \sum_{m=0}^{M-1} c_{m,1} p(t - mT).$$

The signal received at the receiver that is considered for delay estimation in this case is given by,

$$r(t) = \Re\left\{s_1(t-\tau_1)\rho_1 \exp\left(j(2\pi f_c(t-\tau_1)+\theta_1)\right)\right\} + n(t), \qquad T_d \leq t \leq KT. \tag{3.3}$$

This signal is down converted to baseband and digitized by using a standard IQ-mixing stage followed by an integrate and dump section as shown in Figure 9. These samples can be expressed as,

$$R(k) = A_1 S_1^{\tau_1}(k) + N(k), \tag{3.4}$$

Note that $S_1^{\tau_1}(k) = \frac{1}{T}\int_{kT}^{(k+1)T} s_1(t-\tau_1)dt$, $A_1 = \rho_1 e^{j\theta_1} e^{-j2\pi f_c \tau_1}$ and $N(k)$ is zero mean white complex Gaussian variable with variance $\sigma^2 = \frac{N_0}{T}$.

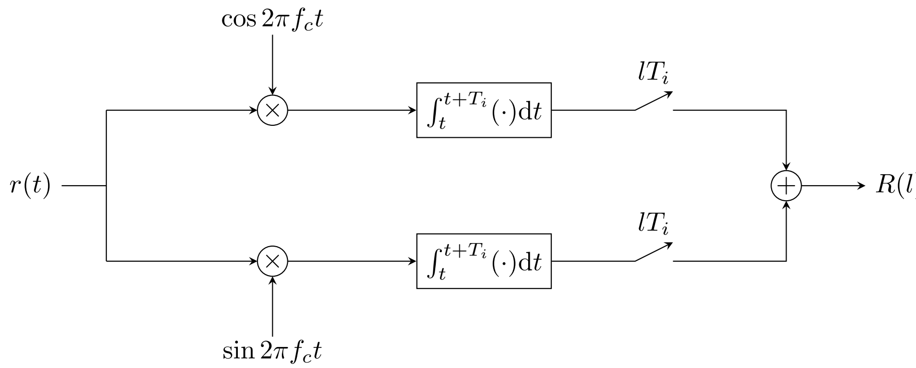

Fig. 9. Receiver front end.

Consider a vector $\mathbf{r}_j$ $(jMT > T_d)$ containing the samples of $r(t)$,

$$\mathbf{r}_i = \begin{bmatrix} R(jM) & R(jM+1) & R(jM+2) & \ldots & R(jM+M-1) \end{bmatrix}^T \in \mathbb{C}^M.$$

The vector $\mathbf{r}_j$ can also be expressed as

$$\mathbf{r}_j = A_1 \mathbf{s}_1^{\tau_1} + \mathbf{n}_j,$$

where $\mathbf{s}_1^{\tau}$ is the vector obtained by digitizing the delayed pilot signal $s_1(t-\tau)$ over the duration $[jMT, (j+1)MT]$. Note that A_1 is a random variable but constant for all vectors. The correlation matrix of the received vector is given by,

$$\begin{aligned} \mathbf{R} =& E\left[\mathbf{r}\mathbf{r}^H\right] \\ =& \mathbf{A}\mathbf{A}^H + \sigma^2 \mathbf{I}, \end{aligned}$$

where $\mathbf{A} = A_1 \mathbf{s}_1^{\tau_1}$. Using the MUSIC algorithm [7], the eigenspace of $\mathbf{R}$ can be partitioned into signal subspace $(V_s \in \mathbf{R}^{M\times 1})$ and noise subspace $(V_n \in \mathbf{R}^{M\times M-1})$. Further, these two subspaces are orthogonal. This property can be used to estimate delay. Consider the cost function $\gamma_1(\tau)$ given by,

$$\gamma_1(\tau) = \left|\left|V_n^T \mathbf{s}_1^{\tau}\right|\right|^2. \tag{3.5}$$

Note that the vector $\mathbf{s}_1^{\tau_1}$ belongs to the signal subspace. Therefore, we can find $\tau_1 \in [0, T_d]$ as the solution to $\left|\left|V_n^T \mathbf{s}_1^{\hat{\tau_1}}\right|\right|^2 = 0$. It is also important to note that $\mathbf{S}_1^{\tau_1+MT}$ also belongs to the signal subspace due to the structure of the preamble used in this procedure. Therefore, the solution is not unique if $T_d > MT$. Hence the block length must be adjusted based on T_d to overcome this issue.

Now we extend this procedure to the system model considered. Using similar preambles as in the single user case for each user (different for each user), the received vector $\mathbf{r}_j$ in this case is,

$$\mathbf{r}_j = A_1 \mathbf{s}_1^{\tau_1} + A_2 \mathbf{s}_2^{\tau_2} + \mathbf{n}_j,$$

and the correlation matrix is given by,

$$\begin{aligned} \mathbf{R} =& E\left[\mathbf{r}\mathbf{r}^H\right] \\ =& \mathbf{B}\mathbf{B}^H + \sigma^2 \mathbf{I}. \end{aligned}$$

where $\mathbf{B} = A_1 \mathbf{s}_1^{\tau_1} + A_2 \mathbf{s}_2^{\tau_2}$. Even in this case, using the MUSIC algorithm the eigenspace of $\mathbf{R}$ can be partitioned into a subspace ($W \in \mathbf{R}^{M \times 1}$) containing $\mathbf{B}$ and another subspace ($V \in \mathbf{R}^{M \times M-1}$) orthogonal to it. However, note that $\mathbf{s}_1^{\tau_1}$ and $\mathbf{s}_2^{\tau_2}$ are not contained in either W or V individually. Therefore the orthogonality property cannot be used to estimate delay in this case. Hence, we modify the structure of the preambles such that the eigenspace of the correlation matrix can be partitioned into a subspace containing vectors $\mathbf{s}_1^{\tau_1}$, $\mathbf{s}_2^{\tau_2}$ and it's orthogonal complement. Without any loss of generality, construct the preamble of user-2 by appending a bit reversed block (obtained by flipping each bit in the original block) alternately and that of user-1 by repeating the same block. Mathematically, the complex envelope of pilot signal corresponding to user-i obtained using such preambles can be expressed as,

$$s_i(t) = \sum_{l=0}^{L} d_{l,i} q_i(t - lMT), \tag{3.6}$$

where $q_i(t)$ and $d_{l,i}$ are given by

$$q_i(t) = \sum_{m=0}^{M-1} c_{m,i} p(t - mT). \tag{3.7}$$

$$d_{l,i} = \begin{cases} 1, & \text{for i=1,} \\ (-1)^l, & \text{for i=2.} \end{cases}$$

The received vector $\mathbf{r}_i$ in this case can be modeled as,

$$\mathbf{r}_i = A_1 \mathbf{s}_1^{\tau_1} + bA_1 \mathbf{s}_2^{\tau_2} + \mathbf{n}_i$$

where $\mathbf{s}_i^{\tau_i}$ is the signal component of user-i and $b \in \{-1, 1\}$ is a random variable such that $Pr(b = 1) = Pr(b = -1) = \frac{1}{2}$. Hence the correlation matrix is given by

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^H] \tag{3.8}$$

$$= ||A_1||^2 \mathbf{s}_1^{\tau_1} \left[\mathbf{s}_1^{\tau_1}\right]^H + E\left[b^2\right] ||A_2||^2 \mathbf{s}_2^{\tau_2} \left[\mathbf{s}_2^{\tau_2}\right]^H + E\left[\mathbf{n}\mathbf{n}^H\right] \tag{3.9}$$

$$= BPB^H + \sigma^2 I, \tag{3.10}$$

where the matrix $B = [\mathbf{s}_1^{\tau_1} \;\; \mathbf{s}_2^{\tau_2}]$ and P is a diagonal matrix. In the above equation the matrix BPB^H is real symmetric and has a rank 2 if the vectors $\mathbf{s}_1^{\tau_1}, \mathbf{s}_2^{\tau_2}$ are linearly independent (preamble can be appropriately designed to achieve this). Hence there is an eigenvalue decomposition of BPB^H such that,

$$BPB^H = \begin{bmatrix} E_s & E_n \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_s & E_n \end{bmatrix}^H$$

where $E_s \in \mathbf{R}^{M\times 2}$ and $E_n \in \mathbf{R}^{M\times M-2}$ are such that $[E_s \;\; E_n]$ is orthogonal. Define the signal subspace to be the subspace spanned by the columns of B. The noise subspace is defined as the orthogonal complement of the signal subspace. It can be deduced that *range*(B)=*range*(BPB^H)=*range*(E_s) and the noise subspace is the range of E_n. From (3.10) it is easy to conclude that the eigenvector of BPB^H is also an eigenvector of R. Therefore, the eigenvalue decomposition of R is given by,

$$R = \begin{bmatrix} E_s & E_n \end{bmatrix} \begin{bmatrix} \Lambda_s & 0 \\ 0 & \Lambda_n \end{bmatrix} \begin{bmatrix} E_s & E_n \end{bmatrix}^H \tag{3.11}$$

where $\Lambda_s = P + \sigma^2 I_2$ and $\Lambda_n = I_{M-2}$. In this case the eigenspace of the correlation matrix can be divided into a set of eigenvectors ($\mathbf{E}_s$) that span the signal subspace and another set of eigenvectors ($\mathbf{E}_n$) spanning the noise subspace based on the eigenvalues. Therefore, we can estimate the delay of each user following a similar procedure as described in the single user case i.e. $\tau_i \in [0, T_d]$ is the solution to $\left|\left|E_n^T \mathbf{S}_{i,j}^{\hat{\tau}_i}\right|\right|^2 = 0$. This solution is unique if the matrix B has full rank for all possible delays. As in the single user case the solution is not unique if $T_d > MT$. In practice, $\mathbf{R}$ is unknown and therefore estimated based on $J < L$ observations as,

$$\hat{\mathbf{R}} = \frac{1}{J}\sum_{l=1}^{l=J} \mathbf{r}_l \mathbf{r}_l^H.$$

A consistent estimate of E_n is obtained by eigendecomposition of $\hat{\mathbf{R}}$.

$$\hat{\mathbf{R}} = \hat{E}_s \hat{\Lambda}_s \hat{E}_s^H + \hat{E}_n \hat{\Lambda}_n \hat{E}_n^H,$$

where columns of $\hat{E}_n$ are eigenvectors corresponding to $M-2$ smallest eigenvalues of $\hat{\mathbf{R}}$. Note that the columns of B will now be approximately orthogonal to columns of $\hat{E}_n$. Hence, we compute the estimate $\hat{\tau}_i$ by minimizing the cost function in (3.5). Mathematically,

$$\begin{aligned} \hat{\tau}_i &= arg \min_{0<\tau_1<T_d} \hat{\gamma}_i(\tau) \\ &= arg \min_{0<\tau_1<T_d} \left|\left| \hat{E}_n^T \mathbf{S}_{i,j}^{\tau} \right|\right|^2 . \end{aligned}$$

Further, an appropriate choice of the blocks (of bits) used to generate the preamble can also improve performance of this algorithm in comparison to a random choice of blocks. From Chapter II we can infer that the preambles of each user that result in signal components(vectors) with desired correlation properties are desirable. However, as the signal components in the received vector are obtained from a portion of the pilot signals over a duration of MT seconds, it is sufficient to analyse the correlation properties of the pilot signal over this duration. Define the autocorrelation and crosscorrelation functions as

$$R_{i,i}(\tau) = \frac{1}{T} \int_{jMT}^{jMT+MT} s_i(t) s_i(t-\tau) dt,$$

$$R_{i,j}(\tau) = \frac{1}{T} \int_{jMT}^{jMT+MT} s_i(t) s_j(t-\tau) dt,$$

where $j \in \mathbb{N}$. For a given block length M, as the number of different blocks of bits are finite, a computer program can be used to search for the suitable blocks. The blocks for which these functions closely resemble the desired functions are preferred. The simulation results corresponding to this algorithm are presented in Figure 10 for different preamble. For M=7, the correlation properties of the blocks used are shown in Figure 11. In Figure 12 we compare the performance of this algorithm for different block lengths. Note that all the simulation results are presented for user-1 as both users have similar performance. These results also demonstrate the fact that the delay estimator is near far resistant. Observe that the performance improves with increase in preamble length in Figure 10 due to a better estimate of the correlation matrix for $K = 126$. Further, from Figure 12 it is clear that this algorithm performs better with an increase in block length. This can be attributed to an increase in the dimensionality of noise subspace. However, there is a trade off between block length and the estimate of the correlation matrix i.e. increase in block length will degrade the estimate of the correlation matrix for a fixed length preamble. Therefore block length must be appropriately chosen in this algorithm.

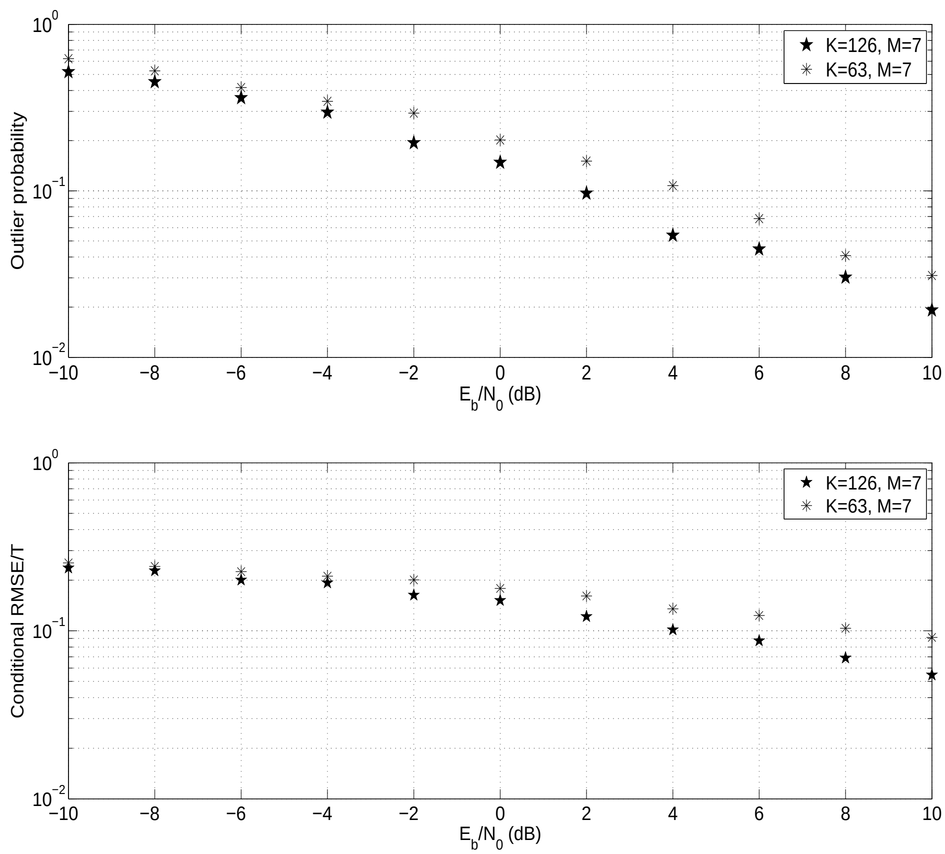

Fig. 10. Performance comparison over different lengths of preamble. $T_o = \frac{T}{2}$. For $M = 7$ blocks of both users are given by binary representation of 3 and 86.

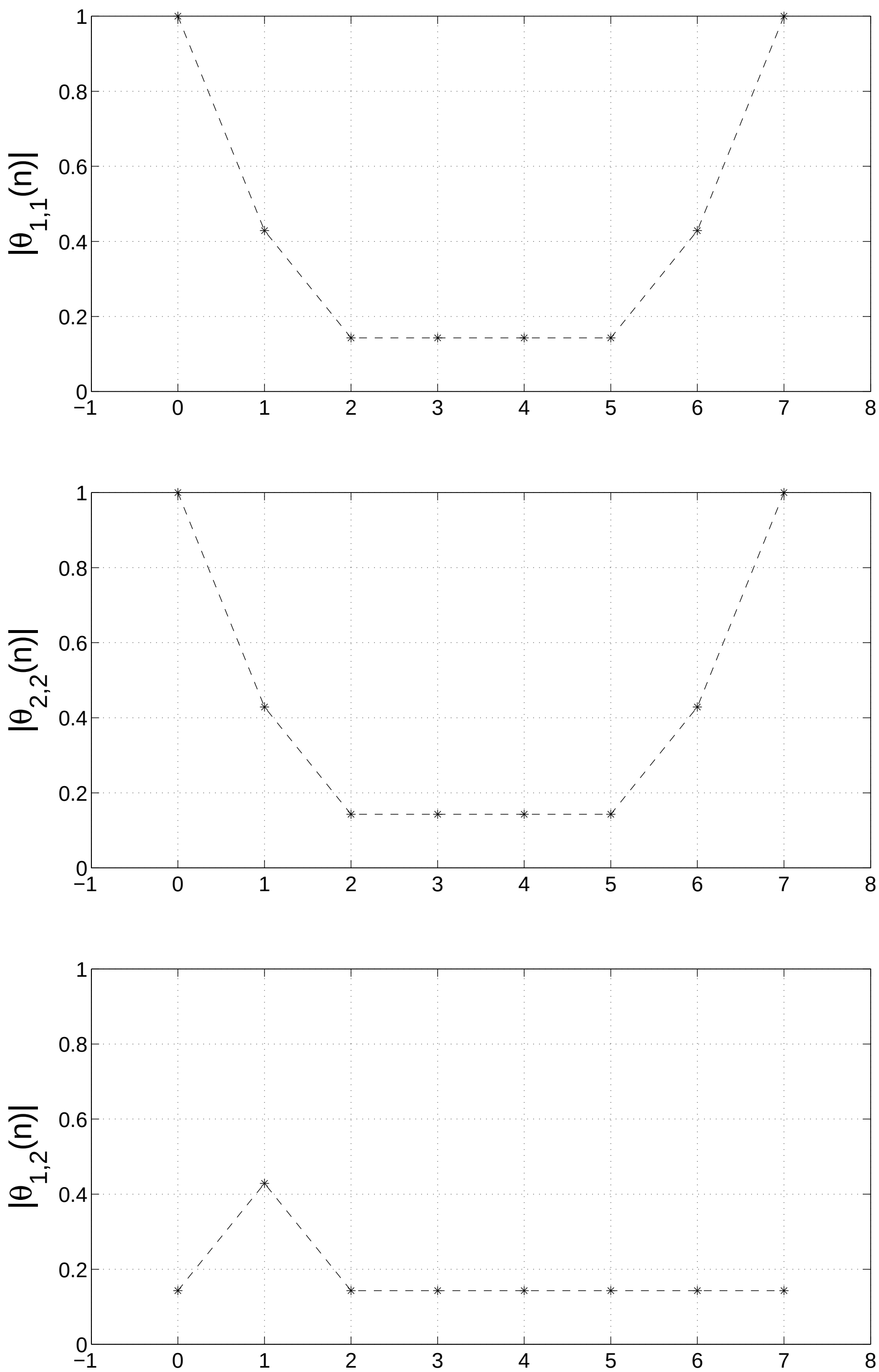

Fig. 11. Correlation functions of the pilot signals used in above simulation for $M = 7$.

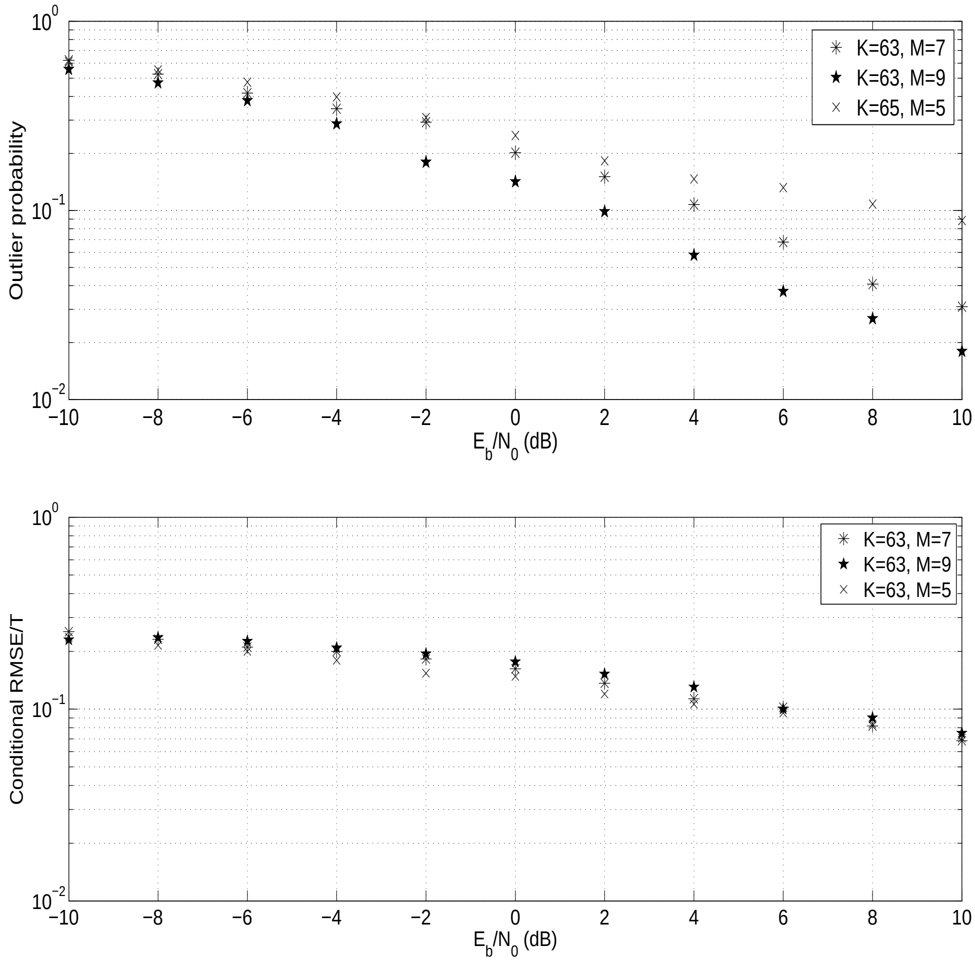

Fig. 12. Performance comparison over different block lengths. $T_o = \frac{T}{2}$. For $K = 63, M = 5$, blocks with binary representation of 3 and 13 are used. $K = 63, M = 7$, blocks with binary representation of 3 and 86 are used, $K = 63, M = 9$, blocks with binary representation of 11 and 198 are used.

2. Interference cancellation

Throughout this algorithm we follow a terminology introduced in Chapter II. In this algorithm the strong user's delay is estimated following a correlation approach. The weak user's delay is estimated with the help of the interference cancellation technique. Using this technique we aim to remove the near-far problem. This algorithm is demonstrated in more detail in the rest of this section.

For the two user system, without any loss of generality, assume user-1 is stronger than user-2. In this algorithm, the estimate of propagation delay($\hat{\tau}_1$) of user-1 is computed using (2.3),

$$\hat{\tau}_1 = arg \max_{0 \leq \tau_1 \leq T_d} \left| \int r_l(t) s_i(t - \tau_1) dt \right|. \tag{3.12}$$

The propagation delay of the weak user is also estimated using a sliding correlator, however, a modified received signal obtained by removing the estimated interference due to user-1 from the received signal is used in this case. The interference due to the stronger user signal is computed by estimating the respective delay and fading parameter as the receiver has prior information of the preamble of this user. Considering the delay estimated in (3.12), the corresponding fading parameter is estimated by treating the system as a single user system and using the maximum likelihood estimation technique. Following this procedure, the parameter $\hat{A}_1 = \hat{\rho}_1 e^{j\hat{\theta}_1} e^{-j2\pi f_c \hat{\tau}_1}$ is calculated as,

$$\hat{A}_1 = \frac{\int r_l(t) s_1^*(t - \tau_1) dt}{\int s_1(t - \tau_1) s_1^*(t - \tau_1) dt}.$$

Using this information, the estimate of the interference due to user-1, $x(t)$, is given by, $x(t) = \hat{A}_1 s_1(t - \hat{\tau}_1)$. Denote the modified received signal, obtained after subtracting the estimated interference due to the strong user, by $y(t)$. Mathematically,

$$y(t) = r_l(t) - x(t). \tag{3.13}$$

The propagation delay estimate of the weak user is given by,

$$\hat{\tau}_2 = \arg \max_{0 \leq \tau_2 \leq T_d} \left| \int y(t) s_2(t - \tau_2) dt \right|.$$

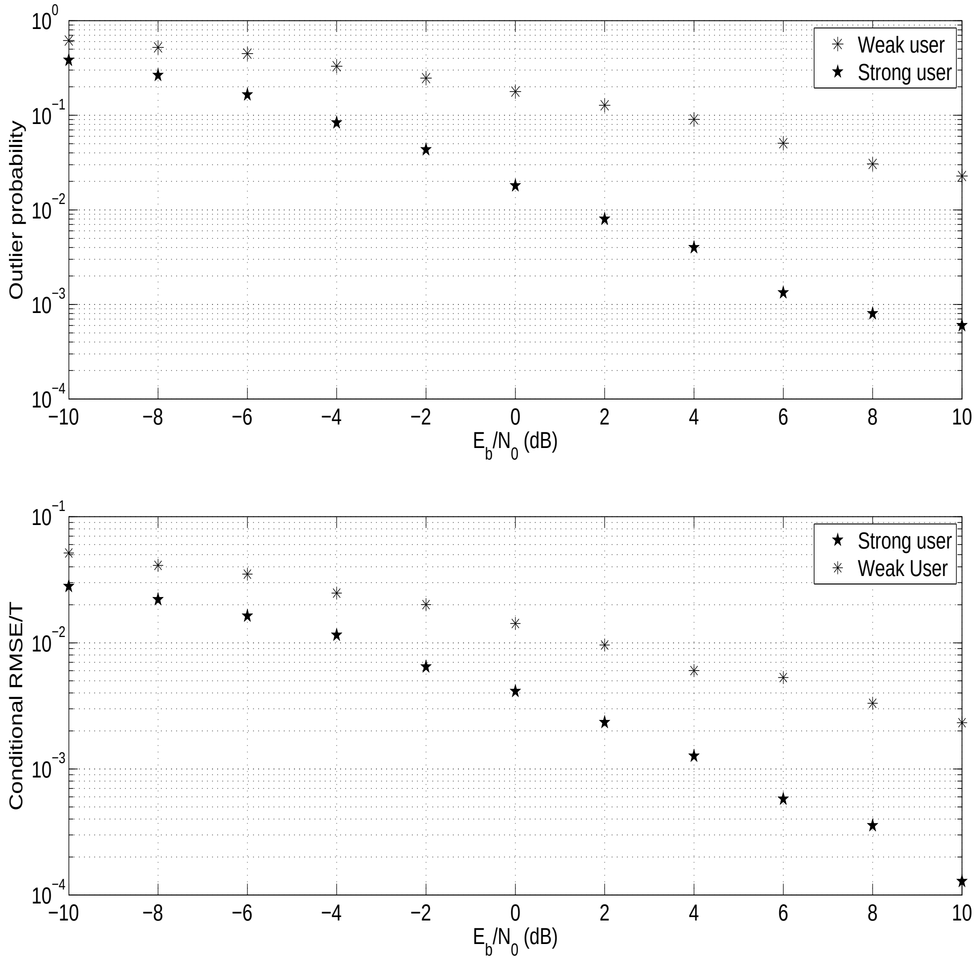

Fig. 13. Simulation results of delay estimation of both the users using Interference cancellation algorithm. $K = 31, T_d = 5T, T_o = \frac{T}{2}$.

It is also important to note that the performance of the weak user's delay estimation in this algorithm is dependent on the accuracy of the estimates of delay and the fading parameter of the strong user's signal. It is hoped that the interference due to the strong user is estimated correctly and hence the amount of interference removed is greater than the amount we create. Figure 13 displays the simulation results of delay estimation of both the users using this algorithm. In all the simulation results presented, we use preambles derived from m-sequences as pilot sequences. The

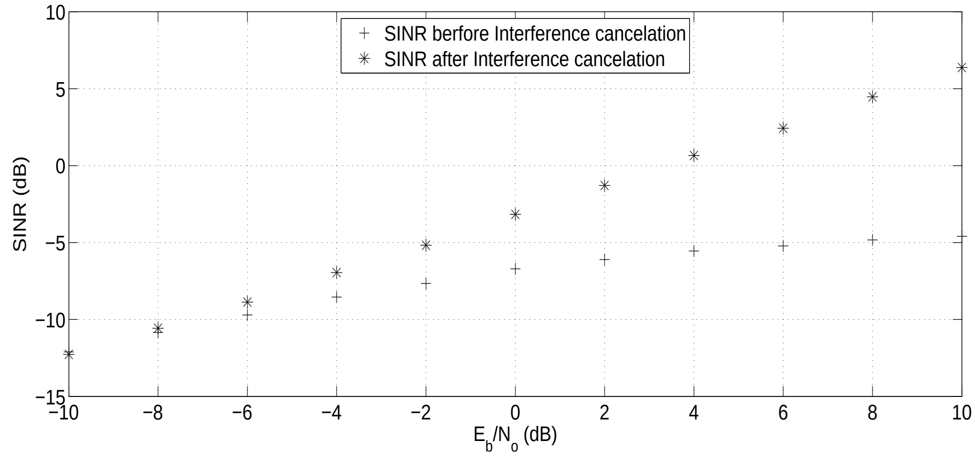

Fig. 14. Signal to Interference ratio of weak user before and after interference cancellation for the above case.

resulting pilot signals posses better correlation properties in comparison to random choice of bits as preamble. Observe that the delay estimator is near-far resistant. Further, to understand the effect of the interference cancellation technique better, we demonstrate the SINR of the weak user before and after removing the estimated strong user's signal in Figure 14.

This algorithm has a disadvantage. The performance of this algorithm will degrade if the users have similar received powers. In this case the cross correlation properties will have a considerable impact on the estimate of fading parameter. Thereby, affecting the weak user's delay estimation. To overcome this issue we follow a decorrelating procedure to estimate weak user's delay. This is discussed in the next algorithm. In Figure 15 and Figure 16 we compare the performance of this algorithm for different length preambles.

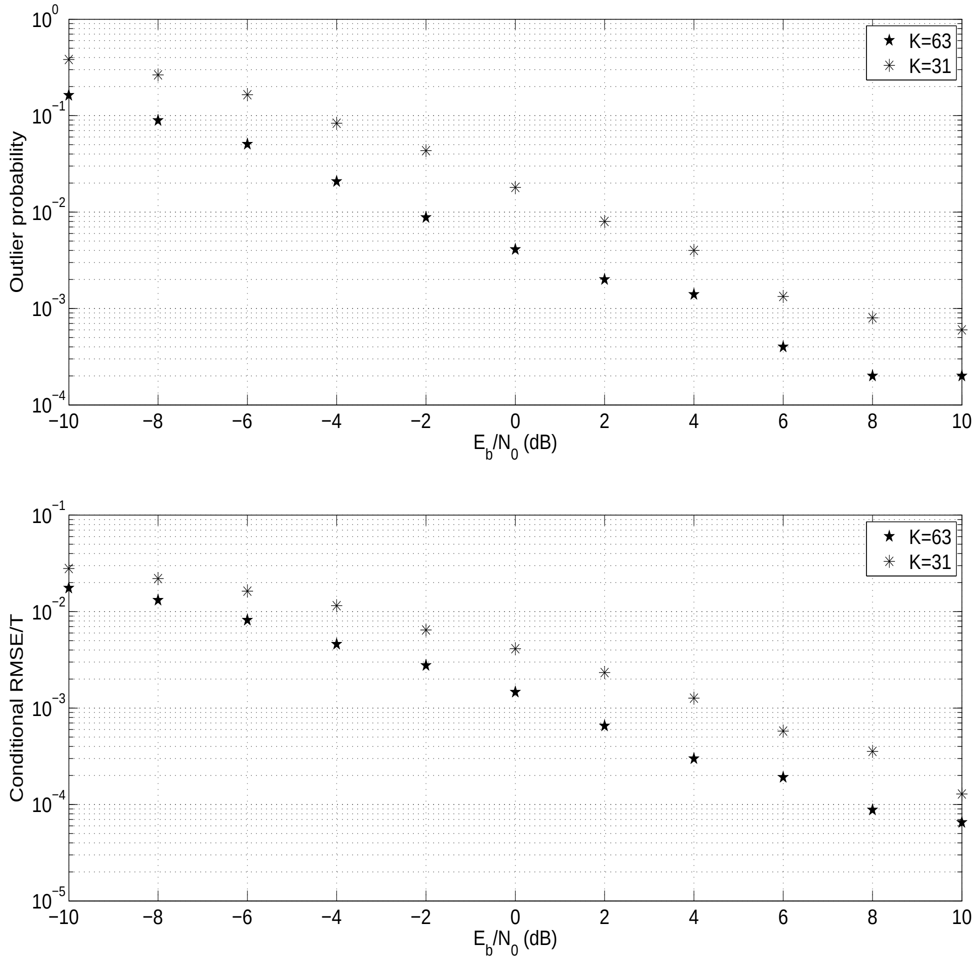

Fig. 15. Performance comparison of delay estimation of *strong user* using the interference cancellation algorithm for different preambles. $T_d = 5T, T_o = \frac{T}{2}$.

3. Decorrelating method

We present a decorrelating approach to overcome the near far problem and estimate the weak user's delay in this section [12]. The strong user's delay is estimated following a similar procedure outlined in the interference cancellation algorithm. Principles identical to those in a decorrelating receiver are used to estimate the weak user's delay. In this procedure, the weak user's delay estimate is dependent on the strong user's delay estimate. Hence, in this algorithm we design a new decorrelating technique

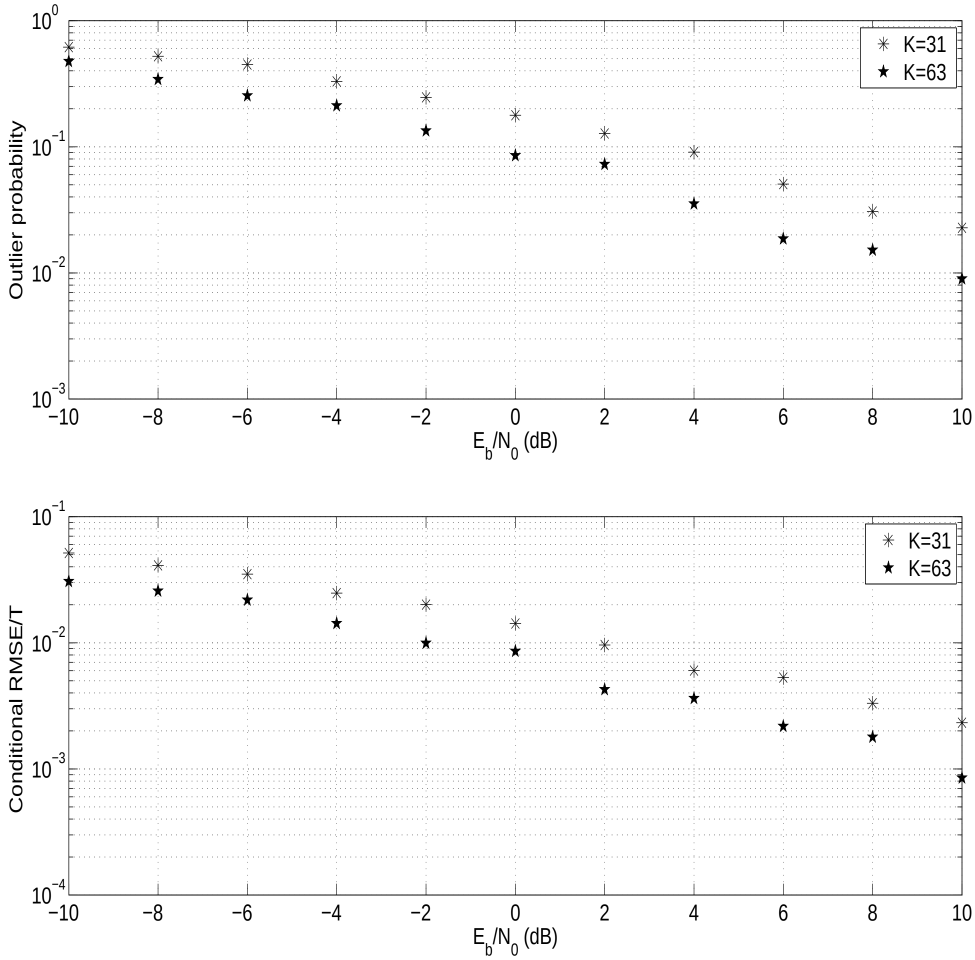

Fig. 16. Performance comparison of delay estimation of *weak user* using the interference cancellation algorithm for different preambles. $T_d = 5T, T_o = \frac{T}{2}$.

that can accommodate errors in strong user's delay estimation. The pulse shape of the pilot signals is exploited to design a more robust decorrelating technique. A near far resistant delay estimator based on this algorithm is explained in detail below.

The received signal at the relay node is given by (3.3). To simplify the analysis of this algorithm, assume $T_d = qT, q \in \mathbb{N}$. Without any loss of generality, assume user-1 is stronger than user-2. The delay estimate of user-1, $\hat{\tau}_1$, is

$$\hat{\tau}_1 = varg \max_{0 \leq \tau_1 \leq T_d} \left| \int r_l(t) s_i(t - \tau_1) dt \right|.$$

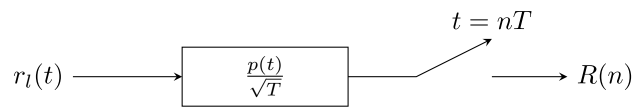

Fig. 17. Matched filter.

Consider the digitized received signal obtained at the output of the matched filter in Figure 17. Let $R(k)$ be a sample of the received signal. $R(k)$ is given by,

$$R(k) = A_1 S_1^{\tau_1}(k) + A_2 S_2^{\tau_2}(k) + N(k), \tag{3.14}$$

where $S_i^{\tau}(k) = \frac{1}{T} \int_{(q+k)T}^{(q+k+1)T} s_i(t - \tau) dt$ and $N(k)$ is a zero mean complex Gaussian random with variance $\frac{N_0}{T}$. Let $\mathbf{r}$ denote a vector containing all the samples of the received signal,

$$\mathbf{r} = \begin{bmatrix} R(0) & R(1) & R(2) & R(3) \ldots & R(K-q-2) & R(K-q-1) \end{bmatrix}^T$$

and $\mathbf{S}_i^{\tau}$ be a vector given by,

$$\mathbf{S}_i^{\tau} = \begin{bmatrix} S_i^{\tau}(0) & S_i^{\tau}(1) & S_i^{\tau}(2) & S_i^{\tau}(3) \ldots & S_i^{\tau}(K-q-2) & S_i^{\tau}(K-q-1) \end{bmatrix}^T.$$

Using the estimate of the strong user's delay a modified weak user signal ($\hat{\mathbf{S}}_2^{\tau}$) is obtained by decorrelating the corresponding weak user signal ($\mathbf{S}_2^{\tau}$) and the estimated delayed strong user signal ($\mathbf{S}_1^{\hat{\tau}_1}$) for each delay in the range $[0, T_d]$. Mathematically,

$\hat{\mathbf{S}}_2^{\tau}$ is computed as,

$$\hat{\mathbf{S}}_2^{\tau} = \mathbf{S}_2^{\tau} - \frac{< \mathbf{S}_1^{\hat{\tau}_1} \mathbf{S}_2^{\tau_2} >}{< \mathbf{S}_1^{\hat{\tau}_1} \mathbf{S}_1^{\hat{\tau}_1} >} \mathbf{S}_1^{\hat{\tau}_1}. \tag{3.15}$$

$< .,. >$ denotes the standard inner product. For column vectors $\mathbf{x}$ and $\mathbf{y} \in \mathcal{C}^{1 \times K-q}$, it is defined as

$$< \mathbf{x}, \mathbf{y} >= \mathbf{x} * \mathbf{y}^H$$

With the help of this signal, the delay estimate of the weak user can be computed using a sliding correlator. Consider the cost function $Z(\tau)$, given by

$$Z(\tau) =< \mathbf{r}, \hat{\mathbf{S}}_2^{\tau} >, \forall \tau \in [0, T_d].$$

The propagation delay estimate of the weak user is given by,

$$\hat{\tau_2} = \arg \max_{0 \le \tau \le T_d} |Z(\tau)| \,. \tag{3.16}$$

Note that $Z(\tau)$ will not contain any component of $\mathbf{S}_1^{\hat{\tau}_1}$. However, it is desirable to have $\hat{\mathbf{S}}_2^{\tau}$ orthogonal to $\mathbf{S}_1^{\tau_1}$ (actual strong user signal component) to obtain a more reliable estimate of the weak user's delay. Therefore we put-forth a new idea to obtain $\hat{\mathbf{S}}_2^{\tau}$ that can accommodate for errors in estimation of strong user's delay.

As the square pulse is used to generate the pilot signal, note that $S_1^{\hat{\tau}_1}(n)$ can also be expressed as,

$$S_1^{\hat{\tau}_1}(n) = \left(1 - \frac{\delta}{T}\right) C_{1,1}^{\hat{\tau}_1}(n) + \frac{\delta}{T} C_{1,2}^{\hat{\tau}_1}(n), \tag{3.17}$$

where $C_{1,1}^{\hat{\tau}_1}(n), C_{1,2}^{\hat{\tau}_1}(n) \in [-1, 1]$, corresponds to the bit information in the interval $[(n+q)T, (n+q+1)T]$ of the delayed user-1 signal for $\tau_1 = pT + \delta$, $p \in \mathbb{N}$. From

(3.17), $\mathbf{S}_1^{\hat{\tau}_1}$ can be represented as,

$$\mathbf{S}_1^{\hat{\tau}_1} = \mathbf{C}_{1,1}^{\hat{\tau}_1}\left(1 - \frac{\delta}{T}\right) + \mathbf{C}_{1,2}^{\hat{\tau}_1}\frac{\delta}{T}.$$

$\mathbf{C}_{1,1}^{\hat{\tau}_1}, \mathbf{C}_{1,2}^{\hat{\tau}_1}$ are column vectors$(1 \times (K-q))$ containing the bit information. Generate $\hat{\mathbf{S}}_2^{\tau}$ by projecting the vector $\mathbf{S}_2^{\tau}$ onto the subspace orthogonal to both $\mathbf{C}_{1,1}^{\hat{\tau}_1}$ and $\mathbf{C}_{1,2}^{\hat{\tau}_1}$. From the above relation it is clear that this vector is orthogonal to the estimated strong user signal component, $\mathbf{S}_1^{\hat{\tau}_1}$. However, it also true that the vector $\hat{\mathbf{S}}_2^{\tau}$ is orthogonal to all vectors $\mathbf{S}_1^{t}$, $\forall\, t \in [pT, (p+1)T]$. Hence, if the actual delay and the estimated delay of the strong user belong to the same bit interval, we can obtain the desired modified weak user signal. Therefore, this method is resilient to errors in delay estimation of strong user's signal. The following procedure is used to compute $\hat{\mathbf{S}}_2^{\tau}$ in this algorithm. Let A be a matrix with $\mathbf{C}_{1,1}^{\hat{\tau}_1}$ and $\mathbf{C}_{1,2}^{\hat{\tau}_1}$ as column vectors.

$$A = \begin{bmatrix} \mathbf{C}_{1,1}^{\hat{\tau}_1} & \mathbf{C}_{1,2}^{\hat{\tau}_1} \end{bmatrix}$$

Denote the projection matrix of A by P. P is given by,

$$P = A\left(A^T A\right)^{-1} A^T.$$

From linear algebra, the projection matrix corresponding to the subspace orthogonal to the column space of A, is given by I-P. Therefore the projection of $\mathbf{S}_2^{\tau}$ onto this subspace is,

$$\hat{\mathbf{S}}_2^{\tau} = (I - P)\mathbf{S}_2^{\tau}.$$

Following a similar procedure, the propagation delay estimate of the weak user is given by,

$$\hat{\tau}_2 = arg \max_{0 \le \tau \le T_d} |Z(\tau)| \tag{3.18}$$

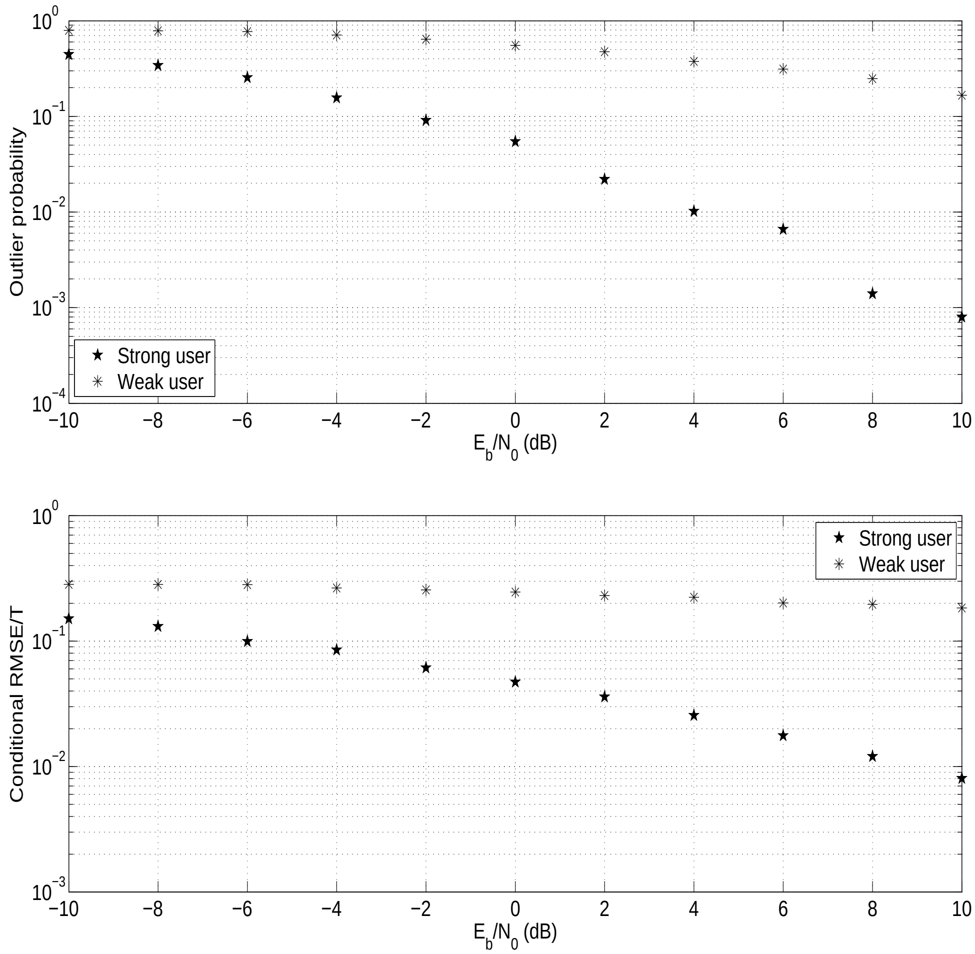

Fig. 18. Simulation results of delay estimation of both the users using decorrelation technique. $K = 31, T_d = 5T, T_o = \frac{T}{2}$.

However, the implementation of (3.18) can be simplified with the help of an approximation technique using the samples of the function $Z(\tau)$. Consider the samples of $Z(\tau)$ at integer multiples of T. Denote these as $z(n)$.

$$z(n) = Z(nT)$$

It is reasonable to assume that the propagation delay of the weak user is closest to the largest sample due to the structure of the pilot signals. Therefore, choose two samples such that one of them is largest of all samples and the other is largest among the two adjacent samples of the largest sample. Using these two samples fit a triangle function passing through them. The timing instant of the peak of this triangle is considered as the estimate of the delay. There is a small amount of inherent error in this estimate. The following simulation results depict this issue. The performance of this technique can be improved by considering more samples of $Z(\tau)$. However complexity also increases. In the simulation results presented, we considered samples separated by T seconds to estimate delay. Further, pilot signals similar to those used in the interference cancellation algorithm are used in simulations. The simulation results for the delay estimation of both the users using this algorithm are presented in Figure 18. In Figure 19, the performance of the weak user's delay estimation using the traditional decorrelation approach and the proposed new technique is presented. Their performance is almost identical. Therefore the proposed approach did not improve the performance of the weak user's delay estimate. Lastly, simulation results for different preamble lengths are shown in Figure 20.

The performance of the strong user's delay estimate is identical to interference cancellation. Observe that this estimator near-far resistant. Further, in the case of weak user's delay estimation, this algorithm does not require the estimate of the fading parameter of the strong user signal component. Therefore, the performance of the weak user's delay estimator is dependent only on the delay estimate of the strong user. Hence in scenarios where both the users have similar received powers, the performance of this algorithm is better in comparison to interference cancellation.

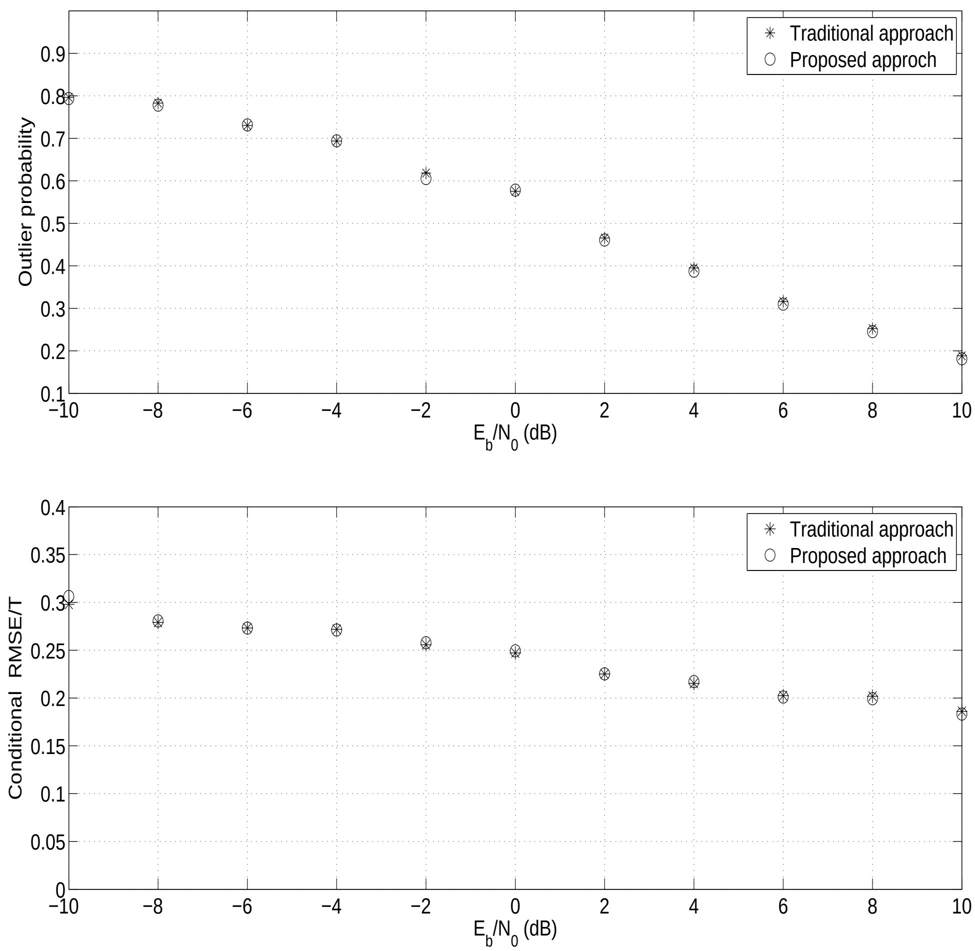

Fig. 19. Performance comparison of delay estimation of *weak user* using traditional and proposed technique. $K = 31, T_d = 5T, T_o = \frac{T}{2}$.

4. Algorithm based on periodic extension of Preamble

In the previous algorithms, pilot signals with property-1 listed in Chapter II were considered to improve the performance. In this algorithm, we design the preamble of each user such that the generated pilot signals over the duration $[T_d, KT]$ posses both the properties (Property-1 and Property-2) for the entire range of timing uncertainty. Using such signals, a sliding correlator is employed to estimate propagation delay of both the users. This algorithm is illustrated below in more detail.

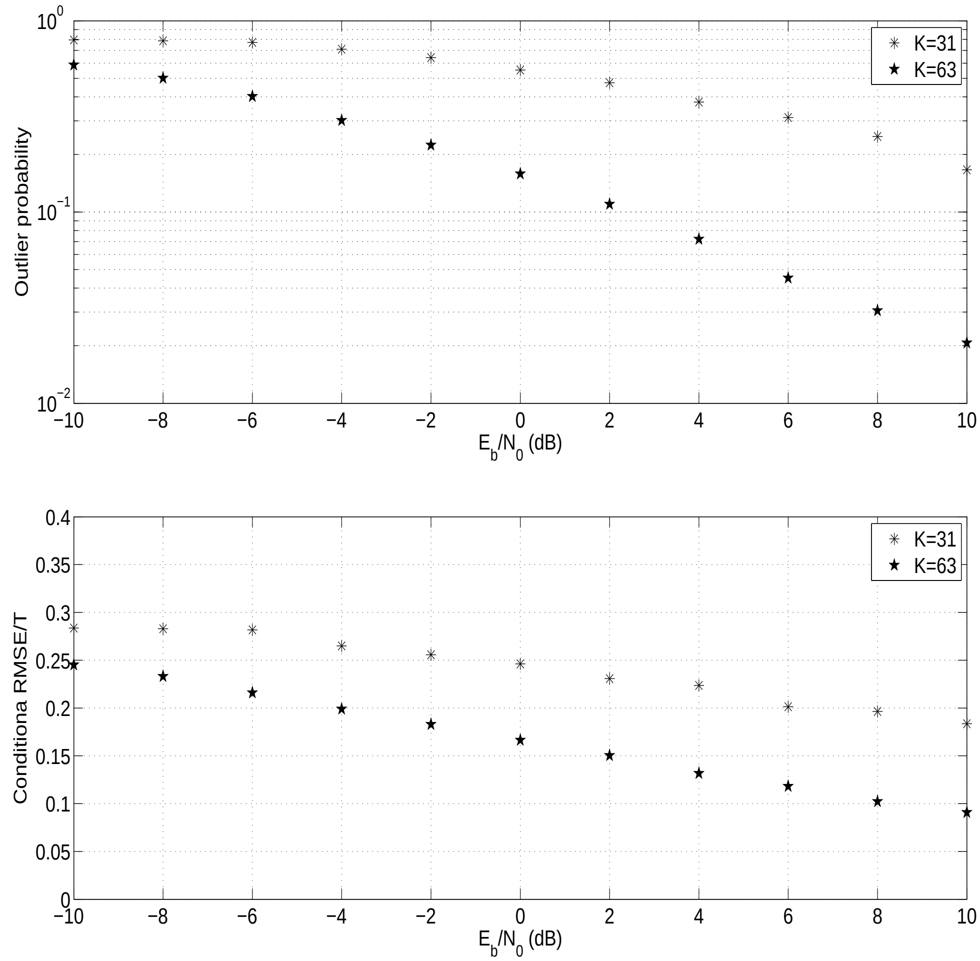

Fig. 20. Performance comparison of *weak user's* delay estimation using the decorrelation technique for different preamble lengths. $T_d = 5T$, $T_o = \frac{T}{2}$.

From Chapter II it is clear that pilot signals with m-sequences as pilot sequences posses the desired autocorrelation function for delay estimation. Further, by a property of m-sequences, any cyclic shift of it is also an m-sequence. Using this fact, preambles of both the users are chosen such that they result in an m-sequence and a shifted version of the same m-sequence as their respective pilot sequences. For such a choice, the crosscorrelation function is a time shifted version of the autocorrelation function. Therefore, an appropriate cyclic shift of the preamble will achieve low cross correlation and desired autocorrelation property of the pilot signals over the range of timing uncertainty.

Consider a block of bits that generate an m-sequence over a period of length M as a pilot sequence. Let p be the smallest integer such that $p \geq \frac{T_d}{T}$. In this algorithm, for user-1, a preamble is generated by concatenating the above mentioned block of bits and the first p bits of the same block. In the case of user-2, the block of bits obtained by performing $n = \frac{M-1}{2}$ cyclic shifts of the original block, in used to generate the preamble in a similar way. Note that the length of the preamble of each user is given by $K = M + p$. As the received signal over a duration of $[T_d, KT]$ is considered for delay estimation, it is sufficient to analyse the correlation properties of the pilot signals in this duration. Define the autocorrelation and crosscorrelation functions of the pilot signals over this duration as follows.

$$R_{i,i}(\tau) = \frac{1}{MT} \int_{T_1}^{T_1+MT} s_i(t) s_i(t-\tau) dt,$$

$$R_{i,j}(\tau) = \frac{1}{MT} \int_{T_1}^{T_1+MT} s_i(t) s_j(t-\tau) dt.$$

Observe that these functions are similar to the periodic correlation functions discussed in Chapter II. The correlation functions defined above are shown in Figure 21 and Figure 22 for $M = 15, n = 7, p = 5$. From these figures it is clear that the pilot signals posses the desired correlation properties in the range $[-5T, 5T]$.

For the system model considered, with such preambles, a sliding correlator is used to estimate the propagation delay of each user in this algorithm. The received signal used to estimate delay is given by (3.3). The delay estimate of user-i is given by,

$$\hat{\tau}_i = arg \max_{0 \leq \tau_i \leq T_d} \left| \int_{T_1}^{T_1+MT} r_1(t) s_i(t-\tau_i) dt \right|.$$

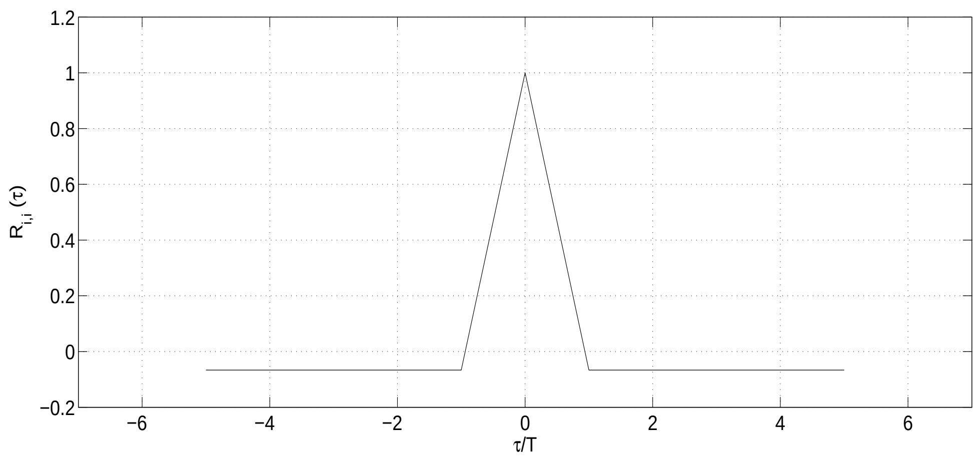

Fig. 21. Autocorrelation function

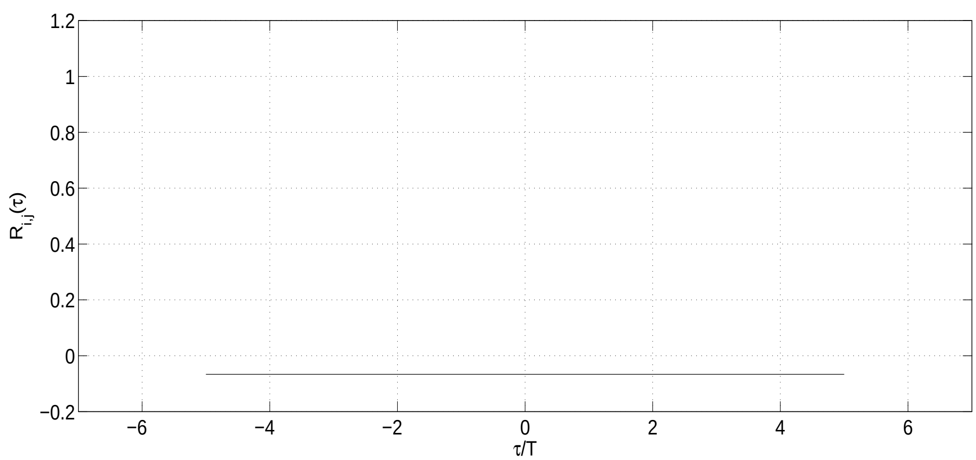

Fig. 22. Crosscorrelation function

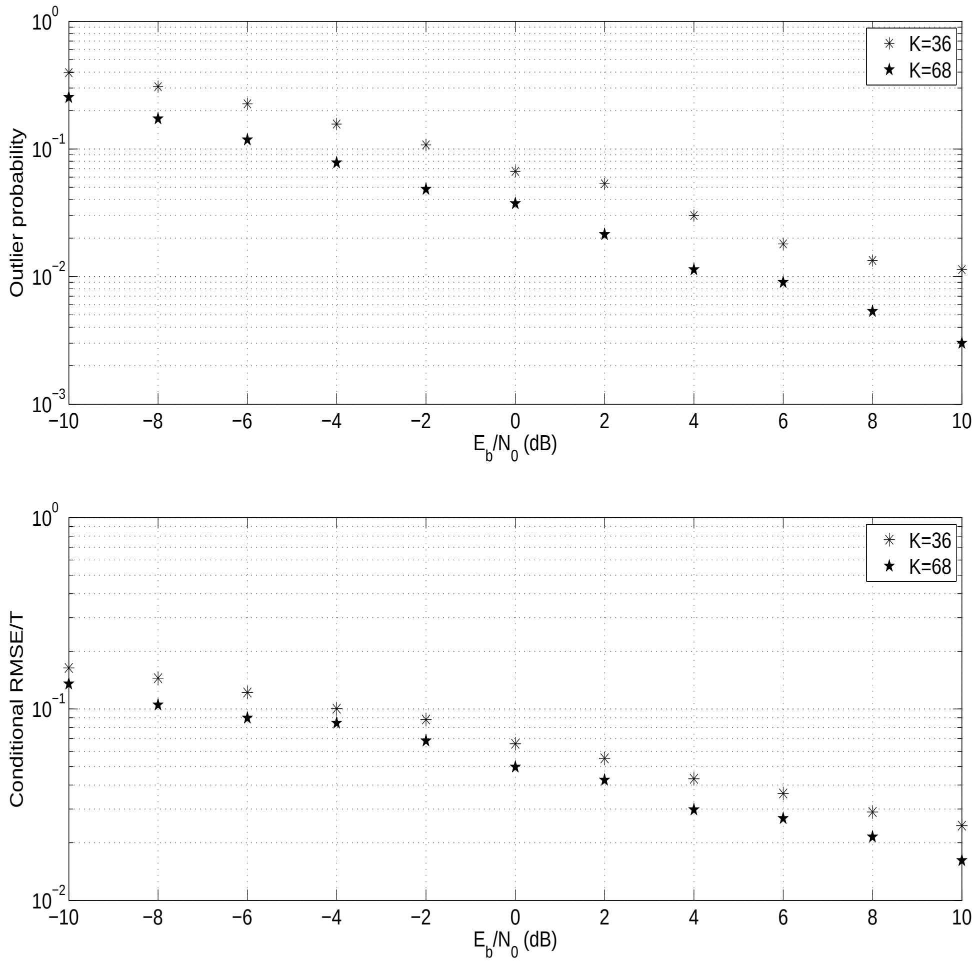

Fig. 23. Performance comparison of delay estimation using the algorithm based on periodic extension of preamble. $T_d = 5T$, $T_o = \frac{T}{2}$.

Simulation results for this algorithm are demonstrated in Figure 23. However, there are some important issues to be noted in this algorithm. First, the length of the preamble is dependent on the maximum timing uncertainty. In this algorithm, the preamble of each user is designed such that the resulting pilot signals posses the desired correlation properties over the range $[-T_d, T_d]$. Therefore, MT should be greater than $2T_d$, as the peaks in the correlation functions must be separated by atleast $2T_d$. Further, the value of n is chosen such that it can accommodate the largest possible range of timing uncertainty for a given value of M. Secondly, due to the structure of the preambles used in this algorithm, preambles of certain lengths can be used for a given value of maximum timing uncertainty. Lastly, this algorithm is not near-far resistant i.e. it might fail if the ratio of the received power of the weak user and the strong user is less than $\frac{1}{M}$. The probability of occurrence of such an event can be reduced by increasing the length of the block used to generate the preamble. Therefore, there exists a trade off between the length of the preamble and performance of this algorithm.

CHAPTER IV

COMPARISONS AND CONCLUSIONS

A. Comparison

In this chapter, the performance of all the proposed algorithms is compared for a scenario described in Chapter I. It is reasonable to assume $T_d = 5T$ in these scenarios. Figure 24 demonstrates the performance of all four algorithms. For fair comparison, all simulations use preambles of roughly 65 bits. The parameters employed for each algorithm are as follows. In the case of the MUSIC algorithm, $K = 63, M = 9$. For the algorithms based on interference cancellation and decorrelation, $K = 63$. For this choice, preambles generating different m-sequences as pilot sequences are used as they perform better in comparison to random block of bits. In the algorithm based on the periodic extension of the preamble, $K = 68, M = 63$.

From the results shown in Figure 24 observe that the interference cancellation algorithm performs better than the rest. However, the decorrelation technique and MUSIC algorithm have lower complexity as they deal with sampled signals. The weak user's delay estimate in the case of interference cancellation and decorrelation is dependent on the strong user's delay estimate. In the MUSIC algorithm and the algorithm involving periodic extension of the preamble, both the users delay can be estimated simultaneously. From the simulation results presented in Chapter III, it is evident that increasing the length of the preamble will improve the performance of each algorithm. However the assumption of constant fading over the duration of the preamble limits the length of the preamble.

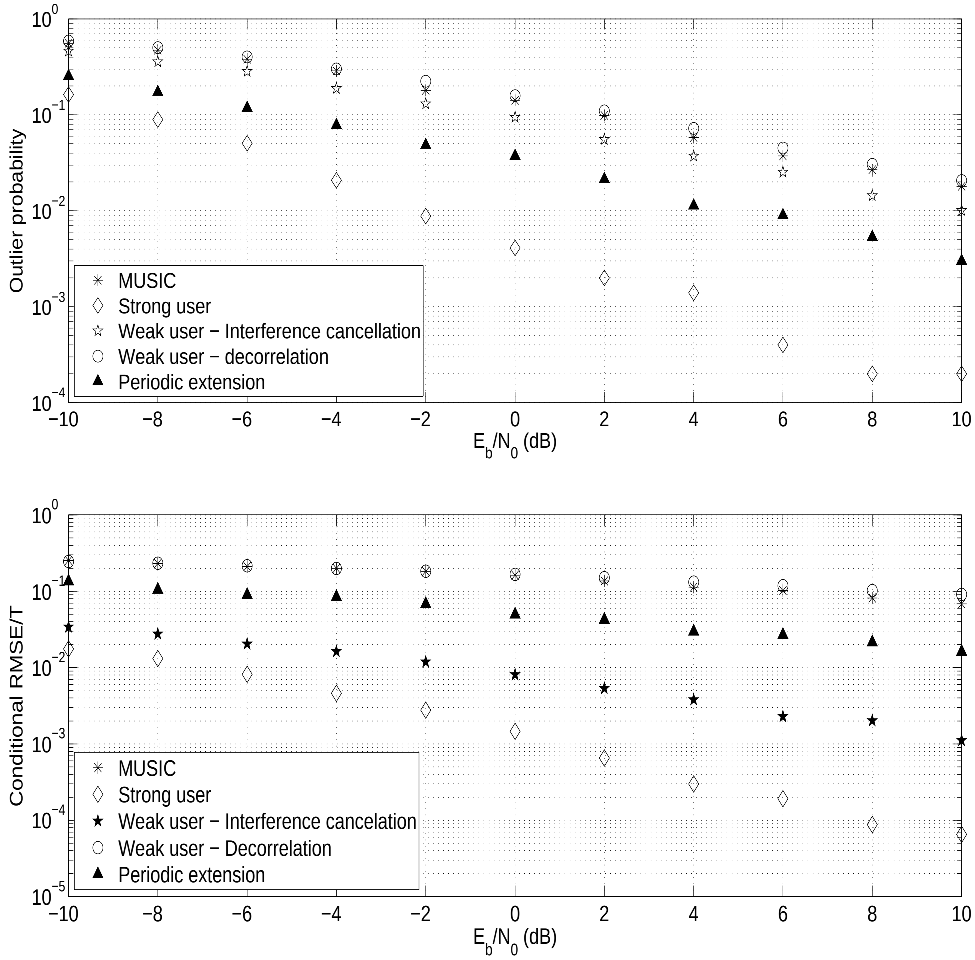

Fig. 24. Performance comparison of all the algorithms. $T_d = 5T, T_0 = \frac{T}{2}$.

B. Future application

In physical layer network coding, the synchronisation of signals at the relay node is a vital part of practical implementation. Synchronisation includes both estimation of timing offsets and tracking fading parameters. Recently, some research has been done to tackle this issue. One such algorithm was presented in [13]. In this algorithm, it was assumed that the symbols are off by at-most a symbol duration and both timing offset and fading were estimated. The estimation technique in [13] was based on an initial guess of both users timing offset. Using these guesses fading parameters are estimated through MLE. Later, the estimated parameters are used to estimate timing offsets. This process is iteratively performed to obtain the true estimates. Note that the convergence of the iterative procedure is dependent on the initial guesses of the timing offsets. In such scenarios the designed delay estimation algorithms can be used to obtain a reasonable estimate of the timing offsets. Therefore both the algorithms can be used in conjunction to overcome the synchronisation issues at the relay node.

REFERENCES

[1] S. Zhang, S. Liew, and P. Lam, "Physical layer network coding," *Proc. 12th Annual International Conference on Mobile Computing and Networking (ACM MobiCom 2006)*, Sep. 2006.

[2] Y. Wu, P. Chou, and S. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," *Conference on Information Sciences and Systems*, Mar. 2005.

[3] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the air: Practical wireless network coding," *IEEE/ACM Transactions on Networking*, vol. 16, no. 3, pp. 497–510, Jun. 2008.

[4] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," *Proceedings of the 2007 CONFERENCE on Applications, Technologies, Architectures, and Protocols for Computer Communications (SIGCOMM-07)*, pp. 397–408, Aug. 2007.

[5] D. V. Sarwate and M. B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences," *Proceedings of the IEEE*, vol. 68, no. 5, pp. 593–619, May 1980.

[6] J. Proakis, *Digital Communications, 5th ed.* New York: Mc-Graw Hill, Nov. 2007.

[7] J. Proakis, and D. Manolakis, *Digital Signal Processing 4th ed.* New York: Mc-Graw Hill, Apr. 2006.

[8] E. G. Strom, S. Parkvall, S. L. Miller, and B. E. Ottersten, "Propagation delay estimation in asynchronous direct sequence code division multiple access systems," *IEEE Trans. Communications*, vol. 44, pp. 84–93, Jan. 1996.

[9] S. E. Bensely and B. Aazhang, “Subspaced-based channel estimation for code division multiple access communication systems,” *IEEE Trans. Communications*, vol. 44, pp. 1009–1020, Aug. 1996.

[10] E. G. Strom, S. Parkvall, S. L. Miller,and B. E. Ottersten, “DS-CDMA synchronisation in a time varying fading channel,” *IEEE Journal on Selected Areas in Communication,* vol. 14, no. 8, pp. 1636–1642, Oct. 1996.

[11] D. Zheng, J. Li, and S. Miller, “An efficient propagation delay estimator for DS-CDMA signals,”*IEEE Transaction on Signal Processing,* vol. 40, no. 1, pp. 82–89, Jan. 1997.

[12] S. Verdu, *Multiuser Detection 3rd ed.* New York: Cambridge University Press, Aug. 1998.

[13] M. Jain, “Parameter estimation and tracking in physical layer network coding,” Master’s thesis, Texas A&M University, College Station, TX, May 2011.

VITA

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The typist for this thesis was Ashish Basireddy.