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Abstract

We study the monetary instrument problem in a model of optimal discretionary fiscal and monetary policy. The policy problem is cast as a dynamic game between the central bank, the fiscal authority, and the private sector. We show that, as long as there is a conflict of interest between the two policy-makers, the central bank's monetary instrument choice critically affects the Markov-perfect Nash equilibrium of this game. Focussing on a scenario where the fiscal authority is impatient relative to the monetary authority, we show that the equilibrium allocation is typically characterized by a public spending bias if the central bank uses the nominal money supply as its instrument. If it uses instead the nominal interest rate, the central bank can prevent distortions due to fiscal impatience and implement the same equilibrium allocation that would obtain under cooperation of two benevolent policy authorities. Despite this property, the welfare-maximizing choice of instrument depends on the economic environment under consideration. In particular, the money growth instrument is to be preferred whenever fiscal impatience has positive welfare effects, which is easily possible under lack of commitment.

JEL classifications: E31, E32, E61, E63
Keywords: monetary instrument problem; central bank independence; dynamic game; optimal fiscal and monetary policy; lack of commitment

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1 Introduction

A prominent question in macroeconomics is whether a central bank should use the nominal money supply or the nominal interest rate as intermediate target for its policy decisions. This question, commonly referred to as the *monetary instrument problem*, was first raised forty years ago by Poole (1970). While Poole's original analysis was cast in a simple IS-LM framework, subsequent research has examined the implications of rational expectations (Sargent and Wallace, 1975; McCallum, 1981) and variations in the economic environment (Canzoneri, Henderson, and Rogoff, 1983; Carlstrom and Fuerst, 1995; Collard and Dellas, 2005). Moreover, some recent contributions also investigate how the (in)determinacy of rational expectations equilibria may depend on the central bank's instrument choice, and how these properties hinge on the interaction with fiscal policy.[1] All these studies point out that the desirability of money growth versus interest rate rules may depend on the source and relative importance of macroeconomic shocks, but they do not deliver an unambiguous conclusion about which instrument to prefer.

In this paper, we identify a novel dimension of the monetary instrument problem, which is completely independent from the existence of stochastic shocks. Instead, we argue that this problem arises typically in models of optimal discretionary fiscal and monetary policy implemented by separate - independent - authorities. Specifically, casting the optimal policy framework as a dynamic non-cooperative game between the fiscal authority, the central bank, and the private sector, we show that the allocation implemented in a Markov-perfect Nash equilibrium is critically affected by the monetary instrument choice. The pertinent welfare implications are non-trivial.

Our modelling approach differs from the one commonly adopted in the literature concerned with the characterization of optimal monetary and fiscal policies.[2] There, the policy problem is formalized as a constrained planning problem where a 'monolithic' policy maker chooses

[1]Benhabib, Schmitt-Grohe, and Uribe (2001) characterize conditions under which interest rate feedback rules ensure uniqueness of the rational expectations equilibrium, while Schabert (2006) provides results for money supply rules.

[2]Prominent examples of this literature include Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991), Schmitt-Grohe and Uribe (2004), Siu (2004), Khan, King, and Wolman (2003) and Klein, Krusell, and Rios-Rull (2008).

among all allocations that are consistent with a market equilibrium. As a consequence (i) strategic interactions between separate policy makers governing monetary and fiscal policy, respectively, are absent and (ii) the question of how desirable allocations can actually be implemented through instruments directly available to these policy makers is not addressed. We believe that this is an important shortcoming because, in most developed economies, monetary and fiscal policies are determined by independent policy authorities with their own respective mandates, but without direct control over allocations. It is therefore important to understand which allocations are implementable within a given institutional framework and to assess the corresponding welfare implications.

In our model, the instrument problem emerges since the two interacting policy makers' objectives are not perfectly aligned, which gives rise to a conflict of interest between them. Specifically, we assume that the monetary and fiscal policy makers agree on desirable allocations at a given point in time, but that there may be disagreement over the intertemporal trade-offs inherent in policy making: we focus on the case where the central bank benevolent and the fiscal authority is impatient in the sense of discounting future utility at a higher rate than society.[3] We show that the monetary instrument choice has a strong effect on the distortion introduced by fiscal impatience. Under a money growth policy, fiscal impatience leads to a government spending bias, i.e., the level of government expenditures is higher than in the equilibrium allocation that would obtain under a single, benevolent government authority (respectively, under cooperation of benevolent fiscal and monetary authorities). Conversely, under an interest rate policy fiscal impatience turns out to have no effect on the equilibrium allocation.

In the economy under consideration, the irrelevance of fiscal impatience under an interest rate policy obtains as the central bank, by choosing the return on bonds, fully determines the private sector's asset allocation decision, i.e., how much money relative to bonds to carry into the next period. The portfolio composition fully determines the future state of the economy such that the fiscal authority's optimal policy problem becomes static; the fiscal time preference rate is therefore irrelevant for fiscal policy decisions and the equilibrium allocation. Finally,

[3]We view fiscal impatience as a short-cut to introduce politico-economic frictions such as electoral uncertainty into the economy. Malley, Philippopoulos, and Woitek (2007) provide empirical evidence that such frictions actually induce policies which resemble the behavior of an impatient fiscal policy maker.

we show that, even though the interest rate instrument eliminates distortions due to fiscal impatience, it does not necessarily dominate the money supply instrument in terms of private sector welfare. Rather, the optimal choice of instrument depends on the specific economic environment under consideration; we identify the intertemporal elasticity of substitution as an important determinant of the relevant welfare ranking.

In terms of methodology, our paper contributes to a recent literature which studies optimal discretionary policies in dynamic macroeconomic models. This literature formulates the policy problem as a game between successive governments, one for each time period, and analyzes Markov-perfect equilibria of this game. Absent interaction with fiscal policy, King and Wolman (2004) have established that a Markov-perfect monetary policy may fail to implement a unique equilibrium through the control of nominal money balances.[4] Dotsey and Hornstein (2008) show that this non-uniqueness of Markov-perfect equilibria is sensitive to the instrument employed by the monetary authority: If the monetary authority chooses nominal interest rates rather than the nominal money supply, there is a unique Markov-perfect equilibrium. Adam and Billi (2007) examine optimal monetary policy when the zero floor on nominal interest rates is fully taken into account. Klein, Krusell, and Rios-Rull (2008) and Ortigueira (2006), in turn, study optimal fiscal policy, i.e., public expenditures and taxation, in non-monetary economies. Finally, Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008), Martin (2009), Niemann, Pichler, and Sorger (2009), Adam and Billi (2008), and Niemann (2009) examine monetary-fiscal interactions from an optimal taxation perspective. The latter two contributions examine optimal discretionary policy when fiscal and monetary policies are implemented by separate authorities engaged in a non-cooperative game. Their focus is on the role of inflation conservatism in settings without respectively with nominal government debt, yet they abstract from the possibility of a monetary instrument problem.

The remainder of this paper is organized as follows. In Section 2, we describe the economic environment under consideration and the first best allocation. In Section 3, we discuss equilibrium in different scenarios; we first examine the cooperative equilibrium and then study two non-cooperative equilibria which differ with respect to the monetary instrument employed. In

[4] Relatedly, Albanesi, Chari, and Christiano (2003) investigate 'expectation traps', i.e., the occurrence of multiple expectations-driven equilibria, in a model of discretionary monetary policy.

Section 4, we provide two numerical examples which illustrate the non-trivial welfare implications of the monetary instrument choice. We conclude in Section 5.

2 Model formulation

In this section we describe the framework of our analysis, which is a variant of the model studied by Nicolini (1998).[5] We start by describing the basic assumptions as well as the first-best solution that would be chosen by a benevolent social planner. We then turn to the setting of a decentralized market economy and discuss in detail how the private sector and the government behave.

2.1 The basic environment

We consider a discrete-time model of an economy which consists of a government and a continuum (of measure 1) of identical private agents. The private agents are producer-consumers who can transform labor into output at a unitary rate. The government purchases (part of the) output and transforms it into a public good at a one-to-one rate. In period t, the representative yeoman farmer supplies n_t units of labor and consumes c_t units of the private good, whereas the government provides g_t units of the public good. The aggregate resource constraint (output market clearing condition) of the economy is therefore given by

$$c_t + g_t = n_t. \tag{1}$$

The private agents derive utility $u(c_t, g_t) - \alpha n_t$ in period t, where α is a positive constant, and where u is a utility function depending on the consumption of private and public goods, respectively. We assume u to be continuous, increasing, and concave on its domain $\mathbb{R}^2_+$, and twice continuously differentiable, strictly increasing, and strictly concave on the interior of its domain. The private agents' time-preference factor is given by $\beta \in (0, 1)$ such that their lifetime

[5] Whereas Nicolini (1998) has studied his model under the assumption of commitment, we shall consider the case of discretion. See Ellison and Rankin (2007), Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008), and Martin (2009) for recent papers analyzing discretionary policy in very similar frameworks.

utility (welfare) is measured by

$$\sum_{t=0}^{+\infty} \beta^t [u(c_t, g_t) - \alpha n_t]. \tag{2}$$

The first-best (optimal) allocation is that allocation which maximizes the objective functional in (2) subject to the resource constraint (1). It is characterized by the necessary and sufficient first-order optimality conditions[6]

$$u_1(c_t, g_t) = u_2(c_t, g_t) = \alpha \tag{3}$$

and the constraint (1). Let (c^*, g^*, n^*) denote the unique solution of these three equations. If the government were benevolent and able to directly choose the allocation, then it would implement the first-best allocation.

In what follows, however, we postulate that the government cannot directly choose the allocation. Instead, we assume that the allocation must be decentralized via a restricted set of fiscal and monetary policy instruments which are controlled by two separate authorities. We shall refer to these authorities as the fiscal authority and the monetary authority, respectively.

Before discussing the policy instruments, we need to specify the set of assets that are available in the economy. There are two such assets: money (cash) and one-period nominal government bonds. A bond issued in period t promises to pay one unit of money in period $t+1$. We denote by q_t the price of a bond issued in period t (this is the reciprocal value of the gross nominal interest rate on bond holdings from t to $t+1$).

2.2 The private agents

We denote by M_t and B_t the amounts of money and bonds, respectively, owned by the representative producer-consumer at the start of period t. The private agents maximize the utility functional in (2) with respect to $(c_t, n_t, M_{t+1}, B_{t+1})_{t=0}^{+\infty}$ subject to a cash-in-advance constraint

$$M_t \geq P_t c_t, \tag{4}$$

[6]The notation u_i denotes the partial derivative of u with respect to the i-th argument. For the derivatives of other functions we shall use an analogous notation.

a flow budget constraint

$$P_t c_t + M_{t+1} + q_t B_{t+1} = P_t n_t + M_t + B_t, \tag{5}$$

and a solvency condition

$$\lim_{t \to +\infty} D_t B_t \geq 0, \tag{6}$$

where P_t is the price level in period t and D_t is the nominal discount factor defined by $D_0 = 1$ and $D_t = \prod_{s=0}^{t-1} q_s$ for $t \geq 1$.

The cash-in-advance constraint (4) says that consumption purchases must be made with cash carried over from the previous period. Alternative but equivalent interpretations of (4) are that, in any given period, the agents cannot trade bonds for money before making consumption purchases, or that the goods market opens before the asset market; see Svensson (1985). The flow budget constraint (5) shows how period-t labor income and financial wealth carried over from period $t-1$ (right-hand side) can be used for purchasing consumption goods and assets to be taken into the next period (left-hand side). Finally, the solvency condition (6) stipulates that the private agents must have non-negative wealth in the long-run (in present value terms).

Instead of the solvency condition (6) one could also impose the lifetime budget constraint

$$\sum_{t=0}^{+\infty} D_t P_t c_t + \sum_{t=0}^{+\infty} D_t (M_{t+1} - M_t) \leq B_0 + \sum_{t=0}^{+\infty} D_t P_t n_t. \tag{7}$$

The left-hand side of (7) is the present value of lifetime consumption plus the present value of all net purchases of money. The right-hand side is the initial bond holdings plus the present value of lifetime earnings. Under the assumption that (5) holds and that all infinite sums in (7) converge, conditions (6) and (7) are equivalent.

The Lagrangian function for the private agents' optimization problem is

$$L = \sum_{t=0}^{+\infty} \beta^t \left\{ u(c_t, g_t) - \alpha n_t + \lambda_t (P_t n_t + M_t + B_t - P_t c_t - M_{t+1} - q_t B_{t+1}) + \nu_t (M_t - P_t c_t) \right\},$$

where λ_t and ν_t are non-negative multipliers. The corresponding first-order conditions are

$$u_1(c_t, g_t) - (\lambda_t + \nu_t) P_t = 0, \tag{8}$$

$$-\alpha + \lambda_t P_t = 0, \tag{9}$$

$$-\lambda_t + \beta(\lambda_{t+1} + \nu_{t+1}) = 0, \tag{10}$$

$$-\lambda_t q_t + \beta \lambda_{t+1} = 0. \tag{11}$$

Using (9) to eliminate λ_t and λ_{t+1} from (11) it follows that the gross real interest rate from period t to $t+1$ is

$$r_t = P_t/(q_t P_{t+1}) = 1/\beta. \tag{12}$$

Equation (12) has several implications. First, it shows that the gross real interest rate must be constant and equal to $1/\beta$. Second, we have

$$1/q_t = r_t(P_{t+1}/P_t) = (1/\beta)(P_{t+1}/P_t), \tag{13}$$

which is the Fisher equation. Since the real interest rate is constant over time, the gross nominal interest rate $1/q_t$ and the gross rate of inflation P_{t+1}/P_t are proportional to each other. Finally, (12) implies that

$$D_t P_t/P_0 = \prod_{s=0}^{t-1} r_s^{-1} = \beta^t. \tag{14}$$

Combining (8)-(10) and (12) we obtain

$$u_1(c_{t+1}, g_{t+1}) = \alpha/q_t. \tag{15}$$

When compared to (3), equation (15) demonstrates that deviations of q_t below one are distortionary. A high nominal interest rate from period t to $t+1$ (low value of q_t) causes high opportunity costs of holding money, because the money has to be held across periods to satisfy the cash-in-advance constraint. As a consequence, private agents equalize the marginal utility of next period's consumption to the opportunity cost of holding money until next period. This discussion also makes clear that private agents do not hold more money than necessary whenever the nominal interest rate is positive. In other words, the cash-in-advance constraint (4) must be binding whenever $q_{t-1} < 1$. When $q_{t-1} = 1$, on the other hand, the opportunity cost of holding money vanishes and the agents are indifferent as to whether to hold financial wealth in the form of money or in the form of bonds. In order to have a well-defined money demand function also in this case, we simply assume that the agents hold the minimal amount of money that is consistent with optimal behavior even if $q_{t-1} = 1$. In other words, we assume that the cash-in-advance constraint holds with equality for all $t \geq 1$.

If the cross partial derivative $u_{12}(c, g)$ does not vanish, (15) implies that fiscal policy is distortionary, too. Any change of public expenditure g_{t+1} directly affects the marginal utility of private consumption in period $t+1$. On the other hand, if the utility function u is additively separable with respect to private and public goods consumption, then it follows that fiscal policy has no direct effect on the behavior of the private sector.

2.3 The government

Let us denote by $\bar{M}_t$ and $\bar{B}_t$ the cash in circulation and the amount of public debt outstanding at the start of period t. It is assumed that $\bar{M}_0$ is strictly positive. The consolidated flow budget constraint of the public sector is

$$P_t g_t + \bar{B}_t = \bar{M}_{t+1} - \bar{M}_t + q_t \bar{B}_{t+1}. \tag{16}$$

The left-hand side consists of public expenditures plus redemption of debt and the right-hand side is seignorage income plus newly issued debt. There are no taxes. As in the case of the private agents, we can augment this flow budget constraint either by the solvency condition

$$\lim_{T \to +\infty} D_T \bar{B}_T \leq 0 \tag{17}$$

or by the lifetime budget constraint

$$\sum_{t=0}^{+\infty} D_t P_t g_t + \bar{B}_0 \leq \sum_{t=0}^{+\infty} D_t (\bar{M}_{t+1} - \bar{M}_t). \tag{18}$$

Given that (16) holds, the two conditions (17) and (18) are equivalent.

Having described the budget constraint of the government, we now turn to its internal structure and its goals. First of all, we assume that the policy makers do not have any commitment power; that is, we consider discretionary policy. In this situation, the government in period t can choose period-t policy variables, but it cannot control policy variables for the future. The usual way to model this is to assume that there exists a separate government in each period and that each of these governments takes the policy rules of all future governments as given. Optimal policy in such an environment therefore corresponds to a Nash equilibrium between successive governments.

As for the behavior of the period-t government, we shall consider three scenarios. In the first one the government chooses the period-t instrument variables so as to maximize

$$\sum_{s=t}^{+\infty}(\beta^G)^{s-t}[u(c_s, g_s) - \alpha n_s], \tag{19}$$

where $\beta^G \in (0, 1)$ is the government's time-preference factor, which may or may not coincide with the private agents' time-preference factor β. If $\beta^G = \beta$ holds, the government is benevolent. The key assumption of this scenario is that monetary and fiscal policy decisions in each period are made by a single authority, which is why we call this scenario the cooperative solution.

In the second and third scenario, on the other hand, we shall assume that the government in each period consists of two separate authorities, one in charge of monetary policy and the other one in charge of fiscal policy. The difference between the second and the third scenario consists in the different policy instruments that are available to the monetary authority. In the second scenario we will assume that the central bank sets the gross money growth rate $\mu_t = \bar{M}_{t+1}/\bar{M}_t$; conversely, in the third scenario we will assume that it sets the bond price q_t (or, equivalently, the gross nominal interest rate $1/q_t$). In either case, the monetary authority in period t seeks to maximize

$$\sum_{s=t}^{+\infty}(\beta^M)^{s-t}[u(c_s, g_s) - \alpha n_s], \tag{20}$$

while the fiscal authority seeks to maximize

$$\sum_{s=t}^{+\infty}(\beta^F)^{s-t}[u(c_s, g_s) - \alpha n_s], \tag{21}$$

where $\beta^M \in (0, 1)$ and $\beta^F \in (0, 1)$ are the time-preference factors of the two authorities. We allow for the possibility that β^M differs from β^F and that one or both of these time-preference factors may be different from the private agents' discount factor β. We are especially interested in the case of fiscal impatience and a benevolent central bank, which is characterized by $\beta^F < \beta^M = \beta$.

3 Equilibrium conditions

In this section we define and analyze the equilibrium of the model. Since the government authorities take the behavior of the private agents as well as the market clearing mechanism as given, we start our discussion with a description of private-sector equilibrium for given policy variables. After that, we explain how optimal policy is determined by the interplay of the infinitely many incarnations of the fiscal and monetary authorities. Finally, having derived the equilibrium dynamics and the continuation value functions, we discuss the three scenarios (cooperative solution, money growth policy, and interest rate policy) in turn.

3.1 Private-sector equilibrium

A private-sector equilibrium is an allocation $(c_t, n_t)_{t=0}^{+\infty}$ and a price sequence $(P_t)_{t=0}^{+\infty}$ which satisfy the feasibility and optimality conditions for the private agents as well as all market clearing conditions for given policy instruments $(g_t, q_t, \bar{B}_{t+1}, \bar{M}_{t+1})_{t=0}^{+\infty}$ and given initial values $\bar{B}_0$ and $\bar{M}_0 > 0$. Furthermore, we require that the government's feasibility conditions (16)-(18) are satisfied.

The output market clearing condition is given by (1). The two asset market clearing conditions are

$$\bar{B}_t = B_t \qquad \text{and} \qquad \bar{M}_t = M_t. \tag{22}$$

Substituting these conditions into (7) we see that the private agents' lifetime budget constraint (7) and the consolidated government lifetime budget constraint (18) are just two sides of the same coin and that, in equilibrium, both of these equations have to hold with equality. The same argument is obviously also true for the solvency conditions (6) and (17).

For our analysis it will be convenient to reformulate the lifetime budget constraint in a different way. More specifically, condition (7) (holding with equality) can equivalently be written as[7]

$$\sum_{t=0}^{+\infty} D_t P_t c_t + \sum_{t=1}^{+\infty} D_t M_t (1/q_{t-1} - 1) = B_0 + M_0 + \sum_{t=0}^{+\infty} D_t P_t n_t.$$

If we divide this equation by P_0 and use (1) and (14) we get

$$\sum_{t=1}^{+\infty} \beta^t (M_t/P_t)(1/q_{t-1} - 1) = (M_0/P_0)(1 + b_0) + \sum_{t=0}^{+\infty} \beta^t g_t, \tag{23}$$

[7]The equivalence between the two formulations of the lifetime budget constraint holds under assumptions that ensure the absolute convergence of the infinite sums. We assume these conditions to be satisfied whenever we use the equivalence.

where $b_0 = B_0/M_0 = \bar{B}_0/\bar{M}_0$ is the public debt-to-money ratio in period 0.

Lemma 1 *The first-best allocation can be supported as a private-sector equilibrium if and only if*

$$-1 - g^*/[(1-\beta)c^*] \le b_0 < -1.$$

Proof: If the first-best allocation is a private-sector equilibrium allocation, then it follows that conditions (3) and (15) must hold simultaneously. Among other things this implies $c_t = c^*$, $g_t = g^*$, and $q_t = 1$ for all t. Substituting this into (23) we get

$$g^*/(1-\beta) = -(1+b_0)(M_0/P_0).$$

Since $c^* > 0$, $g^* > 0$, and $M_0/P_0 \ge c^*$ (the latter inequality follows from the cash-in-advance constraint), this can only hold if $-g^*/[(1-\beta)c^*] \le 1 + b_0 < 0$.

Conversely, let P_0 be a positive number to be specified below, and define $c_t = c^*$, $g_t = g^*$, $n_t = n^*$, $q_t = 1$, $M_t = \bar{M}_t = \bar{M}_0\beta^t$, $P_t = P_0\beta^t$, $\lambda_t = \alpha/P_t$, and $\nu_t = 0$ for all t. It is easy to see that (8)-(11) hold with these specifications. Furthermore, we have $D_t = 1$ for all t, and equation (16) yields

$$\bar{B}_T = \bar{B}_0 + [P_0 g^* + \bar{M}_0(1-\beta)](1-\beta^T)/(1-\beta).$$

Together with $D_t = 1$ for all t this demonstrates that the solvency condition (17) holds as an equality if and only if $P_0 = -(1-\beta)(\bar{B}_0 + \bar{M}_0)/g^* > 0$, whereby the strict positivity of P_0 follows from $b_0 < -1$. It remains to verify the cash-in-advance constraint (4). Substituting the above specifications, one obtains $1 \ge -(1-\beta)(1+b_0)c^*/g^*$, which holds by assumption. Since all equilibrium conditions are satisfied, it follows that the first-best allocation can be supported as a private-sector equilibrium. □

The lemma shows that the first-best allocation can be supported as a private-sector equilibrium if and only if the government has a strictly positive initial asset position $-(\bar{B}_0+\bar{M}_0)$ and private agents' initial financial debt $-(B_0 + M_0)$ is not too large.[8] To understand this finding, first observe that the first-best allocation can only be implemented under the Friedman rule, $q_t = 1$

[8] More precisely, private debt must not exceed $M_0 g^*/[(1-\beta)c^*]$.

for all t. This follows from (3) and (15). Moreover, because $\bar{M}_0 > 0$ is assumed, a non-positive asset position of the government would imply that it has strictly positive debt. Since the government has real expenditures $g^* > 0$ in each period, the present value of its debt cannot converge to zero as required by the solvency condition (17). On the other hand, if initial debt of the private agents is too large, they will not be able to satisfy the solvency condition (6) even if they sell g^* units of the final good to the government in each period. In what follows, we want to rule out that the first-best solution can be achieved, and we do so by assuming that $b_0 = \bar{b}_0 > -1$.

So far, we have described the behavior of the private sector in dependence of the bond price q_t, see equation (15). For later purposes, however, it will be convenient to rewrite the private agents' optimality condition in terms of the money growth rate μ_t. To this end, recall that we have assumed that the cash-in-advance constraint (4) holds as an equality in all periods, even if the nominal interest rate is equal to zero (i.e., if $q_t = 1$). Together with the money market clearing condition in (22) this implies that $P_t c_t = \bar{M}_t$ and $P_{t+1} c_{t+1} = \bar{M}_{t+1}$. Dividing the latter equation by the former and using (12) it follows that

$$\mu_t = \beta c_{t+1}/(q_t c_t). \tag{24}$$

Substituting this into (15) we obtain

$$\beta u_1(c_{t+1}, g_{t+1}) c_{t+1}/\alpha = \mu_t c_t. \tag{25}$$

This equation describes how private agents optimally react to the money growth rate μ_t.

3.2 Equilibrium dynamics and continuation value functions

Up to now we have assumed given settings of the policy instruments. We now turn to the characterization of optimal policy. As we have already mentioned in Subsection 2.3, the government's lack of commitment implies that optimality has to be understood in the sense of a Nash equilibrium in a game between successive governments. Since this is a dynamic game, strategies can in principle depend on the entire history of the game. It is common, however, to restrict attention to those strategies that depend only on a minimal payoff-relevant state.

From (4), only money can be used to make purchases in the goods market. Therefore, the state must summarize the composition of private agents' nominal asset portfolios. For this reason, the debt-to-money ratio is an appropriate state variable for the model, and we will express all period-t variables as functions of the period-t state $b_t = B_t/M_t$.[9] In particular, we will adopt a notation of the form $c_t = \mathcal{C}(b_t)$, $q_t = \mathcal{Q}(b_t)$, $g_t = \mathcal{G}(b_t)$, $\mu_t = \mathcal{M}(b_t)$, etc.

From the flow budget constraint (16), the asset market clearing conditions (22), the cash-in-advance constraint (4) holding with equality, and the definition $b_t = B_t/M_t$ we obtain

$$g_t + (1 + b_t)c_t = \mu_t c_t(1 + q_t b_{t+1}) \tag{26}$$

for all t. In equilibrium it holds that $g_t = \mathcal{G}(b_t)$, $q_t = \mathcal{Q}(b_t)$, $\mu_t = \mathcal{M}(b_t)$, and $c_t = \mathcal{C}(b_t)$. Hence, under equilibrium behavior, equation (26) can be solved uniquely for b_{t+1} as a function of b_t. We shall denote this solution by $b_{t+1} = \mathcal{B}(b_t)$ and will refer to $\mathcal{B}$ as the equilibrium state dynamics.

Since the strategies that form a policy equilibrium must induce a private-sector equilibrium, conditions (15) and (24) must hold identically for all possible states $b > -1$. By using these conditions in (26) we obtain

$$\mathcal{G}(b) + (1 + b)\mathcal{C}(b) = F(\mathcal{B}(b)),$$

where

$$F(b) = \beta\mathcal{C}(b)[u_1(\mathcal{C}(b), \mathcal{G}(b))/\alpha + b]. \tag{27}$$

Hence, it follows that

$$\mathcal{B}(b) = F^{-1}(\mathcal{G}(b) + (1 + b)\mathcal{C}(b)).$$

Differentiating this equation with respect to b and evaluating it at $b = b_t$ it follows that

$$\mathcal{B}_1(b_t) = [\mathcal{G}_1(b_t) + c_t + (1 + b_t)\mathcal{C}_1(b_t)]/F_1(b_{t+1}) \tag{28}$$

must hold for all t.

Even if the government (or the two authorities) in period t can only choose the instruments in that period, they care about welfare derived throughout the entire infinite planning horizon

[9] Notice also that the allocation implemented in the model under commitment depends on the value b_0 (see, e.g., Ellison and Rankin, 2007).

$\{t, t+1, \ldots\}$ as specified by their objective functionals in (19)-(21). Consider, for example, the government's objective functional in (19) and note that it can be rewritten as

$$u(c_t, g_t) - \alpha(c_t + g_t) + \beta^G V^G(b_{t+1}), \tag{29}$$

where the market clearing condition (1) has already been used and where $V^G(b_{t+1})$ is the continuation value for the period-t government, i.e., the part of the period-t government's objective functional that it can only affect indirectly via the state variable b_{t+1}. This continuation value function must satisfy the recursive equation

$$V^G(b) = u(\mathcal{C}(b), \mathcal{G}(b)) - \alpha[\mathcal{C}(b) + \mathcal{G}(b)] + \beta^G V^G(\mathcal{B}(b)).$$

Since the above equation must hold identically for all values of b, we may also differentiate it with respect to b. Evaluating the result at $b = b_t$ and introducing the notation $w_t^c = u_1(c_t, g_t) - \alpha$ and $w_t^g = u_2(c_t, g_t) - \alpha$ one gets

$$V_1^G(b_t) = w_t^c \mathcal{C}_1(b_t) + w_t^g \mathcal{G}_1(b_t) + \beta^G V_1^G(b_{t+1})\mathcal{B}_1(b_t) \tag{30}$$

for all t.

Note that w_t^c and w_t^g are the wedges between the marginal utility of consumption of private and public goods in period t and the marginal cost of producing these goods. These wedges must be zero along the first-best allocation, but they are typically not equal to zero in an equilibrium. Furthermore, it follows from (8)-(9) that $w_t^c \geq 0$ must hold for all t.

Equations (29)-(30) have been derived under the assumptions characterizing the first scenario, i.e., that there is a single government authority in each period which has a time-preference parameter β^G and evaluates allocations according to (19). In a completely analogous way we can derive corresponding equations for scenarios 2 and 3 in which there are two separate authorities with the objective functionals (20) and (21), respectively. These equations are given by

$$u(c_t, g_t) - \alpha(c_t + g_t) + \beta^M V^M(b_{t+1}) \tag{31}$$

and

$$V_1^M(b_t) = w_t^c \mathcal{C}_1(b_t) + w_t^g \mathcal{G}_1(b_t) + \beta^M V_1^M(b_{t+1})\mathcal{B}_1(b_t) \tag{32}$$

for the monetary authority, and by

$$u(c_t, g_t) - \alpha(c_t + g_t) + \beta^F V^F(b_{t+1}) \tag{33}$$

and

$$V_1^F(b_t) = w_t^c \mathcal{C}_1(b_t) + w_t^g \mathcal{G}_1(b_t) + \beta^F V_1^F(b_{t+1})\mathcal{B}_1(b_t) \tag{34}$$

for the fiscal authority, respectively. We shall now discuss the three scenarios in turn.

3.3 Scenario 1: cooperative solution

In this subsection we assume that both fiscal and monetary policies in period t are determined by a single authority with objective functional (19) or, equivalently, (29). The solution in this case, which we call the cooperative solution, is characterized by the following proposition.

Proposition 1 *If $(c_t, g_t, b_t)_{t=0}^{+\infty}$ is an equilibrium outcome in scenario 1, then it must satisfy the following system of difference equations:*

$$g_t + (1 + b_t)c_t = F(b_{t+1}), \tag{35}$$

$$w_t^c = (1 + b_t)w_t^g, \tag{36}$$

$$w_t^g F_1(b_{t+1}) = \beta^G c_{t+1} w_{t+1}^g. \tag{37}$$

Proof: The problem of the period-t government in the present scenario is to maximize the objective function in (29) subject to the flow budget constraint (26) as well as the implementability conditions (15) and (25). Following a primal approach, we eliminate the policy variables q_t and μ_t using (15) and (25). This allows us to write the government's optimization problem as the maximization with respect to (c_t, g_t, b_{t+1}) of the objective function in (29) subject to the single constraint (35). The first-order conditions for this optimization problem are (36) and

$$\beta^G V_1^G(b_{t+1}) = -w_t^g F_1(b_{t+1}). \tag{38}$$

It follows that the cooperative solution is characterized by equations (30), (35)-(36), and (38) holding for all t. Using (28), (36), and (38), we can rewrite (30) as (37). This completes the proof of the proposition. □

The system of three difference equations (35)-(37) in the variables (c_t, g_t, b_t) fully describes the cooperative solution. Note that these conditions are generalized Euler equations because the terms $F(b_{t+1})$ and $F_1(b_{t+1})$ in (35) and (37) contain the unknown policy functions $\mathcal{C}$ and $\mathcal{G}$ as well as their derivatives. In Section 4 below we shall use a numerical approach to compute these policy functions in examples with parametrically specified utility functions. But even without knowing the policy functions $\mathcal{C}$ and $\mathcal{G}$, it is possible to derive some analytical results from Proposition 1. The following corollary is one such example.

Corollary 1 *If $(\bar{c}, \bar{g}, \bar{b})$ is a steady state solution of equations (35)-(37), then it follows that*

$$\frac{\bar{b}\mathcal{C}_1(\bar{b})}{\bar{c}} = \frac{\beta^G - \beta}{\beta} + \frac{1}{\alpha}\left[\left(\frac{1}{\bar{\sigma}} - 1\right) u_{11}(\bar{c}, \bar{g})\mathcal{C}_1(\bar{b}) - u_{12}(\bar{c}, \bar{g})\mathcal{G}_1(\bar{b})\right], \tag{39}$$

where $\bar{\sigma} = -\bar{c}u_{11}(\bar{c}, \bar{g})/u_1(\bar{c}, \bar{g})$ is the elasticity of the marginal utility of private consumption with respect to c evaluated at the steady state.

Proof: Differentiating (27) with respect to b one gets

$$F_1(b) = \beta\mathcal{C}_1(b)\left[\frac{u_1(\mathcal{C}(b), \mathcal{G}(b))}{\alpha} + b\right] + \beta\mathcal{C}(b)\left[\frac{u_{11}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{C}_1(b) + u_{12}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{G}_1(b)}{\alpha} + 1\right].$$

This equation can equivalently be written as

$$F_1(b) = \beta\mathcal{C}(b)\left\{1 + \frac{\mathcal{C}_1(b)b}{\mathcal{C}(b)} + \frac{u_{11}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{C}_1(b)[1 - 1/\sigma(b)] + u_{12}(\mathcal{C}(b), \mathcal{G}(b))\mathcal{G}_1(b)}{\alpha}\right\},$$

where $\sigma(b) = -\mathcal{C}(b)u_{11}(\mathcal{C}(b), \mathcal{G}(b))/u_1(\mathcal{C}(b), \mathcal{G}(b))$. Using this expression for $F_1(b)$, one can rewrite (37) as

$$\frac{\beta^G w_{t+1}^g}{\beta w_t^g} = 1 + \frac{\mathcal{C}_1(b_{t+1})b_{t+1}}{c_{t+1}} + \frac{u_{11}(c_{t+1}, g_{t+1})\mathcal{C}_1(b_{t+1})[1 - 1/\sigma(b_{t+1})] + u_{12}(c_{t+1}, g_{t+1})\mathcal{G}_1(b_{t+1})}{\alpha}.$$

In a steady state this equation simplifies to (39). □

The left-hand side of (39) is the elasticity of private consumption with respect to the debt-to-money ratio. Provided the cross-derivative $u_{12}(\bar{c}, \bar{g})$ is non-negative, consumption of private goods can be shown to be a strictly decreasing function of the debt-to-money ratio such that

$\mathcal{C}_1(b) < 0$ in the neighborhood of a stable steady state.[10] Moreover, in all our numerical examples we found private consumption to be strictly decreasing in b even for $u_{12}(\bar{c}, \bar{g}) < 0$. The implication of $\mathcal{C}_1(\bar{b}) < 0$ is that the right-hand side of (39) and the steady state value $\bar{b}$ must have opposite signs, which allows us to draw some interesting conclusions. First, if the government is benevolent ($\beta^G = \beta$) and the utility function u is additively separable ($u_{12}(\bar{c}, \bar{g}) = 0$), then it follows that the sign of the steady-state debt is solely determined by the value of $\bar{\sigma}$. More specifically, $\bar{b}$ is positive, zero, or negative depending on whether $\bar{\sigma}$ is greater than, equal to, or smaller than one.[11]

Equation (39) furthermore shows that this clear-cut characterization of the sign of the long-run debt level breaks down if the government's time-preference factor differs from that of the private sector, or if the utility function is not additively separable. In particular, (39) suggests that, at least in the additively separable case, an impatient government ($\beta^G < \beta$) induces upward pressure on the long-run debt-to-money ratio.

3.4 Scenario 2: money growth policy

Now let us consider the case in which there are two separate authorities which seek to maximize their respective objective functionals in (31) and (33). The two authorities act non-cooperatively and under discretion, which means that both of them take their opponent's policy function as well as the policy functions of all future authorities as given. Furthermore, we assume that the central bank sets the money growth rate μ_t.

[10] To see this, observe that stability of a steady state at $\bar{b} > -1$ requires $\mathcal{B}_1(\bar{b}) < 1$, while (28) implies $\mathcal{B}_1(\bar{b}) = [\mathcal{G}_1(\bar{b}) + \bar{c} + (1+\bar{b})\mathcal{C}_1(\bar{b})]/F_1(\bar{b})$. Hence, since (37), evaluated at $\bar{b}$, implies $F_1(\bar{b}) = \beta^G \bar{c}$, stability requires $\mathcal{G}_1(\bar{b}) + (1+\bar{b})\mathcal{C}_1(\bar{b}) < 0$. Next, differentiate (36) to obtain

$$\left[u_{11}(\bar{c}, \bar{g}) - (1+\bar{b})u_{21}(\bar{c}, \bar{g})\right]\mathcal{C}_1(\bar{b}) + \left[u_{12}(\bar{c}, \bar{g}) - (1+\bar{b})u_{22}(\bar{c}, \bar{g})\right]\mathcal{G}_1(\bar{b}) = w^g > 0.$$

Here, note that $u_{11}, u_{22} < 0$ and $u_{12} = u_{21} \geq 0$ together imply that the first expression in squared brackets is negative, while the second one is positive. Now suppose $\mathcal{C}_1(\bar{b}) > 0$. To satisfy the above equation, we then must have $\mathcal{G}_1(\bar{b}) > 0$ such that $\mathcal{G}_1(\bar{b}) + (1+\bar{b})\mathcal{C}_1(\bar{b}) > 0$. Since this is a contradiction to the stability requirement, it follows that $\mathcal{C}_1(\bar{b}) < 0$.

[11] This is a result that has also been derived in a similar framework by Martin (2009, Proposition 5).

Since q_t is not an instrument variable, we eliminate it using (24). This turns the flow budget constraint (26) into

$$g_t + (1 + b_t)c_t = \mu_t c_t + \beta b_{t+1}\mathcal{C}(b_{t+1}). \tag{40}$$

The monetary authority maximizes the utility function (31) with respect to μ_t, c_t, and b_{t+1} and subject to the flow budget constraint (40) and the implementability constraint (25), whereby it takes b_t and g_t as given. The next lemma presents the first-order optimality conditions for this optimization problem.

Lemma 2 *Optimal behavior of the monetary authority in scenario 2 implies*

$$g_t + (1 + b_t)c_t = F(b_{t+1}), \tag{41}$$

$$\beta^M V_1^M(b_{t+1}) = -w_t^c F_1(b_{t+1})/(1 + b_t). \tag{42}$$

PROOF: Using (25) to eliminate the money growth rate μ_t from (40) we obtain (41). The monetary authority's optimization problem can therefore equivalently be formulated as the maximization of (31) with respect to (c_t, b_{t+1}) and subject to the single constraint (41). The first-order optimality condition for this problem is (42). □

Let us now turn to the fiscal authority's problem. It maximizes (33) with respect to (c_t, g_t, b_{t+1}) and subject to (40) and (25), whereby it takes b_t and μ_t as given. The next lemma derives the optimality conditions for this problem. To simplify its statement, let us define the function H by

$$H(b) = F(b) - \beta\mathcal{C}(b)b,$$

where F is defined in (27).

Lemma 3 *Optimal behavior of the fiscal authority in scenario 2 implies*

$$\beta^F V_1^F(b_{t+1}) = -[w_t^c - (1 + b_t)w_t^g]H_1(b_{t+1})/\mu_t - w_t^g F_1(b_{t+1}). \tag{43}$$

PROOF: First note that (27) implies that

$$H(b) = \beta u_1(\mathcal{C}(b), \mathcal{G}(b))\mathcal{C}(b)/\alpha.$$

Using this observation, it is straightforward to verify that the two constraints (40) and (25) together are equivalent to $c_t = H(b_{t+1})/\mu_t$ and $g_t = K(b_{t+1}; b_t, \mu_t)$, where

$$K(b'; b, \mu) = H(b')[1 - (1+b)/\mu] + \beta \mathcal{C}(b')b'.$$

Thus, one can write the fiscal authority's optimization problem as the unconstrained maximization with respect to b_{t+1} of

$$u(H(b_{t+1})/\mu_t, K(b_{t+1}; b_t, \mu_t)) - \alpha[H(b_{t+1})(1 - b_t/\mu_t) + \beta \mathcal{C}(b_{t+1})b_{t+1}] + \beta^F V^F(b_{t+1}).$$

The first-order optimality condition is

$$\begin{aligned} &u_1(c_t, g_t)H_1(b_{t+1})/\mu_t + u_2(c_t, g_t)K_1(b_{t+1}; b_t, \mu_t) \\ &-\alpha[H_1(b_{t+1})(1 - b_t/\mu_t) + \beta \mathcal{C}_1(b_{t+1})b_{t+1} + \beta c_{t+1}] + \beta^F V_1^F(b_{t+1}) = 0. \end{aligned}$$

Using the definitions of K and H, this condition can also be written in the form of (43). □

We are now ready to collect all equilibrium conditions for the money growth scenario. In order to compare them more easily to those for the cooperative solution, we again formulate them as a system of three difference equations in the variables (b_t, c_t, g_t).

Proposition 2 *If $(c_t, g_t, b_t)_{t=0}^{+\infty}$ is an equilibrium outcome in scenario 2, then it must satisfy the following system of difference equations:*

$$g_t + (1+b_t)c_t = F(b_{t+1}), \quad (44)$$

$$\frac{w_t^c F_1(b_{t+1})}{1+b_t} = \frac{\beta^M c_{t+1} w_{t+1}^c}{1+b_{t+1}} + \frac{\beta^M [w_{t+1}^c - (1+b_{t+1})w_{t+1}^g]\mathcal{G}_1(b_{t+1})}{1+b_{t+1}}, \quad (45)$$

$$w_t^g F_1(b_{t+1}) + \frac{[w_t^c - (1+b_t)w_t^g]H_1(b_{t+1})c_t}{H(b_{t+1})} = \beta^F c_{t+1} w_{t+1}^g + \beta^F [w_{t+1}^c - (1+b_{t+1})w_{t+1}^g]A_{t+1}, \quad (46)$$

where

$$A_{t+1} = \frac{H_1(b_{t+2})c_{t+1}}{H(b_{t+2})F_1(b_{t+2})}[\mathcal{G}_1(b_{t+1}) + c_{t+1} + (1+b_{t+1})\mathcal{C}_1(b_{t+1})] - \mathcal{C}_1(b_{t+1}).$$

Proof: From the above analysis we know that equilibrium in the second scenario is described by equations (25), (32), (34), (41), (42), and (43). Equation (44) is simply a restatement of

(41) and therefore has to hold. Using (28) and (42) to eliminate V_1^M from (32) yields (45). Analogously, we use (28) and (43) to eliminate V_1^F from (34) which yields

$$w_t^g F_1(b_{t+1}) + [w_t^c - (1+b_t)w_t^g]H_1(b_{t+1})/\mu_t = \beta^F c_{t+1}w_{t+1}^g + \beta^F[w_{t+1}^c - (1+b_{t+1})w_{t+1}^g]\tilde{A}_{t+1},$$

where

$$\tilde{A}_{t+1} = \frac{H_1(b_{t+2})}{\mu_{t+1}F_1(b_{t+2})}[\mathcal{G}_1(b_{t+1}) + c_{t+1} + (1+b_{t+1})\mathcal{C}_1(b_{t+1})] - \mathcal{C}_1(b_{t+1}).$$

Finally, we use (25) to eliminate μ_t and μ_{t+1} from the above equation to obtain (46). □

Due to the complexity of the equilibrium conditions for this scenario, we were unable to derive any substantial analytical results for the general case. The following corollary therefore considers the special case in which the utility function is additively separable as well as logarithmic in private consumption.[12]

Corollary 2 *Suppose that $u(c,g) = \gamma\ln(c) + v(g)$. If $(\bar{c},\bar{g},\bar{b})$ is a steady state solution of equations (44)-(46), then it follows that*

$$\frac{\bar{b}\mathcal{C}_1(\bar{b})}{\bar{c}} = \frac{\beta^M - \beta}{\beta} + \frac{\beta^F - \beta^M}{\beta}\left[1 + \frac{\beta^F\bar{w}^c\mathcal{C}_1(\bar{b})}{\beta^M\bar{w}^g\mathcal{G}_1(\bar{b})}\right]^{-1}. \tag{47}$$

PROOF: When the utility function has the form specified in the corollary, then it follows that $F_1(b) = \beta[\mathcal{C}(b) + b\mathcal{C}_1(b)]$, $H_1(b) = 0$, and $A_{t+1} = -\mathcal{C}_1(b_{t+1})$. Substituting these expressions into (45)-(46) and evaluating at the steady state one gets

$$\beta\bar{w}^c[\bar{c} + \bar{b}\mathcal{C}_1(\bar{b})] = \beta^M\left\{\bar{c}\bar{w}^c + [\bar{w}^c - (1+\bar{b})\bar{w}^g]\mathcal{G}_1(\bar{b})\right\},$$
$$\beta\bar{w}^g[\bar{c} + \bar{b}\mathcal{C}_1(\bar{b})] = \beta^F\left\{\bar{c}\bar{w}^g - [\bar{w}^c - (1+\bar{b})\bar{w}^g]\mathcal{C}_1(\bar{b})\right\}.$$

These two equations can equivalently be written as

$$\left[1 - (1+\bar{b})(\bar{w}^g/\bar{w}^c)\right]\mathcal{G}_1(\bar{b}) = (\beta/\beta^M)[\bar{c} + \bar{b}\mathcal{C}_1(\bar{b})] - \bar{c},$$
$$\left[1 - (1+\bar{b})(\bar{w}^g/\bar{w}^c)\right]\mathcal{G}_1(\bar{b})\left[\beta^M + \beta^F\frac{\bar{w}^c\mathcal{C}_1(\bar{b})}{\bar{w}^g\mathcal{G}_1(\bar{b})}\right] = \bar{c}(\beta^F - \beta^M).$$

Using the first of these equations to eliminate the term $\left[1 - (1+\bar{b})(\bar{w}^g/\bar{w}^c)\right]\mathcal{G}_1(\bar{b})$ from the second equation one obtains after simple rearrangements equation (47). □

[12] In Section 4 below we shall return to the general case and solve it using numerical methods.

It is instructive to compare the above corollary with the corresponding result for the cooperative solution. Under the assumptions of Corollary 2, equation (39) in Corollary 1 boils down to $\bar{b}\mathcal{C}_1(\bar{b})/\bar{c} = (\beta^G - \beta)/\beta$. Obviously, equation (47) coincides with this result in the case where both authorities have the common time-preference factor $\beta^F = \beta^M = \beta^G$. The second term on the right-hand side of (47), however, emerges only if the two authorities' objectives diverge, that is, if $\beta^M \neq \beta^F$. This term therefore captures the effect of the strategic interaction of the two authorities on the long-run debt-to-money ratio. Equation (47) furthermore demonstrates that the two authorities' time-preference rates play quite different roles in the determination of equilibrium, an observation that will recur in even more dramatic form in the next subsection.

3.5 Scenario 3: interest rate policy

Finally, we consider the case where the two authorities act non-cooperatively but where the central bank sets the bond price q_t (or, equivalently, the gross nominal interest rate $1/q_t$). Because μ_t is not an instrument variable, we eliminate it using (24). This turns the flow budget constraint (26) into

$$g_t + (1 + b_t)c_t = \beta\mathcal{C}(b_{t+1})(1/q_t + b_{t+1}). \tag{48}$$

Let us again start with the monetary authority's optimization problem. It consists of choosing q_t, c_t, and b_{t+1} so as to maximize the objective function (31) subject to the flow budget constraint (48) and the implementability constraint (15), whereby g_t and b_t are taken as given. The following lemma demonstrates that this problem is identical to the monetary authority's optimization problem from scenario 2 such that the conditions stated in Lemma 2 remain valid.

Lemma 4 *Optimal behavior of the monetary authority in scenario 3 implies that conditions (41)-(42) hold.*

Proof: Using (15) to eliminate the bond price q_t from (48), the monetary authority is seen to maximize (31) with respect to (c_t, b_{t+1}) and subject to the single constraint (41). This is the same problem as that from scenario 2. Hence, the first-order condition (42) must hold. □

Now let us turn to the fiscal authority's optimization problem. This authority maximizes the objective function (33) with respect to (c_t, g_t, b_{t+1}) and subject to (48) and (15), whereby it takes b_t and q_t as given. It is important to note that (15) determines b_{t+1} independently of the fiscal authority's actions. Indeed, using the fact that the monetary authority in period t and all future authorities as well as the private agents use their respective equilibrium strategies, equation (15) can be written as

$$u_1(\mathcal{C}(b_{t+1}), \mathcal{G}(b_{t+1})) = \alpha/\mathcal{Q}(b_t).$$

Given b_t, this is an equation for the single unknown variable b_{t+1}, and it does not involve the fiscal authority's other control variable g_t. Although there could, in principle, exist multiple solutions to this equation, the solutions are generically isolated. As a matter of fact, in all our numerical examples studied below, the above equation is satisfied by a unique value b_{t+1}. In other words, the fiscal authority in period t has to take b_{t+1} as given. This, in turn, implies that the term $\beta^F V^F(b_{t+1})$ in the fiscal authority's objective functional (33) is irrelevant for the maximization problem such that we can drop it along with the decision variable b_{t+1}. Finally, we observe that by eliminating q_t from the two constraints (48) and (15), one obtains (35). Hence, the fiscal authority chooses (c_t, g_t) so as to maximize $u(c_t, g_t) - \alpha(c_t + g_t)$ subject to (35). The first-order condition for this optimization problem is $w_t^c = (1 + b_t)w_t^g$, which is identical to condition (36) from the cooperative solution. The above observations are summarized in the following lemma.

Lemma 5 *Optimal behavior of the fiscal authority in scenario 3 implies that conditions (35)-(36) hold.*

Summarizing the results for the interest rate policy scenario we obtain the following proposition.

Proposition 3 *(a) If $(c_t, g_t, b_t)_{t=0}^{+\infty}$ is an equilibrium outcome in scenario 3, then it must satisfy the following system of difference equations:*

$$g_t + (1 + b_t)c_t = F(b_{t+1}), \tag{49}$$

$$w_t^c = (1 + b_t)w_t^g, \tag{50}$$

$$w_t^g F_1(b_{t+1}) = \beta^M c_{t+1} w_{t+1}^g. \tag{51}$$

(b) The equilibrium conditions in scenario 3 are identical to those of scenario 1 when β^G is replaced by β^M.

(c) Equilibrium values in scenario 3 are independent of the fiscal authority's time-preference factor β^F.

(d) If $\beta^F \neq \beta^M$, then there is an instrument problem for the monetary authority, that is, its choice of the policy instrument affects the equilibrium allocation of the economy.

PROOF: (a) The results derived so far show that equilibrium in scenario 3 is described by equations (32), (35), (36), (41), and (42). Both equations (35) and (41) are identical to (49), and equation (36) is identical to (50). Using (28), (36), and (42) we can write (32) as (51). This completes the proof of part (a).

(b) This statement follows immediately from a comparison of conditions (35)-(37) and (49)-(51), respectively.

(c) This statement is obvious from the equilibrium conditions stated in part (a).

(d) In order to prove this statement, we show that, whenever a sequence of variables (c_t, g_t, b_t) simultaneously satisfies conditions (44)-(46) and (49)-(51), then it follows that $\beta^F = \beta^M$. Since (50) holds for all t, equations (45) and (46) simplify to

$$w_t^c F_1(b_{t+1})/(1 + b_t) = \beta^M c_{t+1} w_{t+1}^c/(1 + b_{t+1})$$

and

$$w_t^g F_1(b_{t+1}) = \beta^F c_{t+1} w_{t+1}^g. \tag{52}$$

Using (50) again, it is easily seen that the former equation is equivalent to

$$w_t^g F_1(b_{t+1}) = \beta^M c_{t+1} w_{t+1}^g.$$

Comparing this equation to (52) it becomes apparent that $\beta^F = \beta^M$ must hold. □

Proposition 3 is the main analytical result of this paper. Parts (a)-(c) together reveal that the strategic interaction between the fiscal and the monetary authority does not create any additional distortion when the central bank uses the nominal interest rate as its instrument. Instead, the equilibrium in this scenario coincides with the one that emerges if a single government authority decides on both fiscal and monetary policies (provided that the single government's time-preference factor coincides with that of the monetary authority in scenario 3). Interestingly, the fiscal authority's time-preference factor is completely irrelevant for the equilibrium in scenario 3. The key to understanding these findings is the observation pointed out already earlier in this subsection: if the monetary authority sets the interest rate, the fiscal authority's optimization problem becomes a static one because the interest rate fully determines next period's debt-to-money ratio. As a consequence of Proposition 3(c), it does not matter at all for the equilibrium whether or not the fiscal authority displays stronger impatience than the monetary authority.

Proposition 2 and parts (b) and (d) of Proposition 3 show that, in the case where the monetary authority controls the money growth rate, the strategic interaction between the two authorities manifests itself in a distortion from the cooperative solution as long as the two authorities do not share the same objective functional (i.e., the same time-preference factor). If they do share the same time-preference rate, there is no strategic conflict between the two authorities, and the Nash equilibrium coincides with the cooperative solution.

Proposition 3(d) raises the question of whether one instrument is to be preferred over the other in terms of welfare. To examine this question first recall that the monetary authority can eliminate distortions through fiscal impatience by choosing the interest rate as its instrument; see parts (b)-(c) of Proposition 3. One might think that, due to this property, the interest rate policy is preferable in terms of private-sector welfare. However, this is not obvious under discretionary policy-making. Under lack of commitment, a non-benevolent policy-maker could potentially implement better policies than a benevolent one, as has been demonstrated by Rogoff (1985). The intuition behind this result is that the departure from benevolence may mitigate the policy-maker's time-inconsistency problem. Thus, in the present context, whether or not the interest rate instrument is preferred over the money growth instrument will crucially depend on the nature of the time-inconsistency problem faced by the policy makers which, in turn, depends on the specific economic environment under consideration. We shall investigate this property in the following section using numerical examples.

4 Numerical examples

In this section we present some numerical results for the case in which the utility function takes the form

$$u(c,g) = \frac{\left(c^{\gamma_1} g^{1-\gamma_1}\right)^{1-\gamma_2} - 1}{1-\gamma_2},$$

where $\gamma_1 \in (0,1)$ and $\gamma_2 > 0$ are exogenous parameters. The parameter γ_1 measures the relative weight of private versus public consumption in the Cobb-Douglas aggregate $c^{\gamma_1} g^{1-\gamma_1}$. The parameter γ_2 determines the (constant) elasticity of intertemporal substitution with respect to this aggregate. Moreover, γ_2 determines the sign of the cross partial derivative

$$u_{12}(c,g) = \frac{\gamma_1(1-\gamma_1)(1-\gamma_2)\left(c^{\gamma_1} g^{1-\gamma_1}\right)^{1-\gamma_2}}{cg}.$$

Thus, depending on the value taken by γ_2, c and g enter the utility function as substitutes ($\gamma_2 > 1$) or as complements ($0 < \gamma_2 < 1$).

We shall present results for two examples that differ from each other in the numerical values assumed for γ_2, and thus in the substitutability of c and g. In the first example we assume $\gamma_2 = 1$, which implies that the utility function takes the additively separable form $u(c,g) = \gamma_1 \log c + (1-\gamma_1) \log g$. In this case the cross partial derivative $u_{12}(c,g)$ vanishes. Note that this case allows for some analytical results which are not available for other values of γ_2; see Corollaries 1 and 2. In the second example we examine the case $\gamma_2 = 0.4$, where the elasticity of intertemporal substitution is relatively high and private and public consumption are complementary goods. This case is interesting because it leads to fundamentally different normative conclusions than those obtained for $\gamma_2 = 1$.[13]

4.1 Additively separable utility with unit-elastic preferences

As explained above, unit-elastic preferences obtain for $\gamma_2 = 1$. For both the money growth scenario 2 and the interest rate scenario 3 we compute numerical approximations to the equilibrium

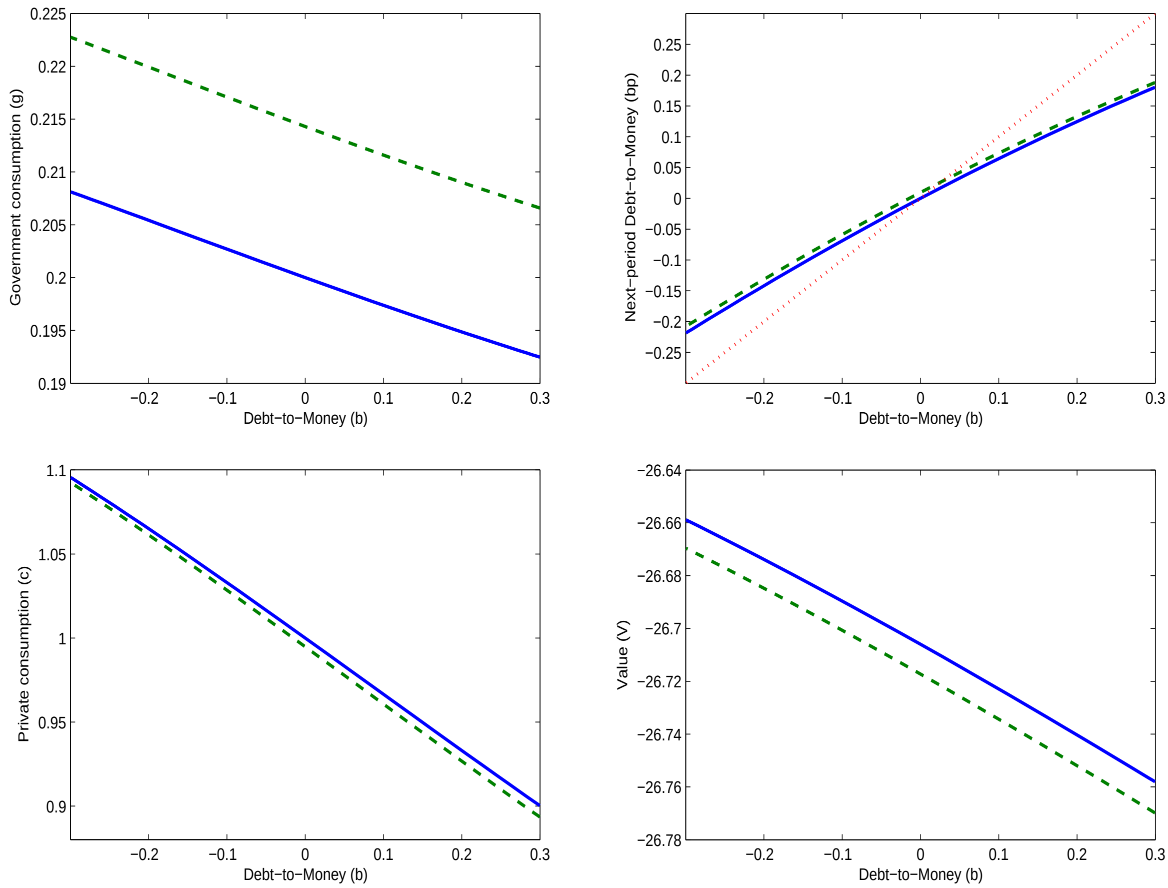
[13] We will not separately discuss results for the case $\gamma_2 > 1$. We have experimented with several values for γ_2 larger than one and found the results to be qualitatively similar to those for $\gamma_2 = 1$. For space considerations these results are omitted from the paper, but they are available upon request.

policy functions $\mathcal{C}$, $\mathcal{G}$, and $\mathcal{B}$ as well as the private sector value function V using collocation projection methods as described in Judd (1992, 1998). This requires, first, to postulate values for the remaining model parameters β, β^M, β^F, α, and γ_1. Since the nature of our numerical exercise is mainly illustrative, we choose these values in a simple fashion. We set $\beta = \beta^M = 0.96$, which corresponds to an annual real interest rate of close to 4%. Note that the monetary authority is assumed to be benevolent. As for the fiscal authority, we assume that it is more impatient, which is reflected by $\beta^F = 0.8$. Finally, we choose $\alpha = 2/3$ and $\gamma_1 = 5/6$, which implies that that steady-state consumption levels of private and public goods in the cooperative solution with a benevolent government are given by $\bar{c} = 1$ and $\bar{g} = 1/5$, respectively.[14]

Figure 1 displays approximations to $\mathcal{G}$, $\mathcal{B}$, $\mathcal{C}$ and V. To interpret these functions, it is useful to recall that the equilibrium obtained under the interest rate instrument coincides with the equilibrium that would obtain under a single, benevolent government authority deciding over both fiscal and monetary policies. This equilibrium is thus independent of β^F; in other words, fiscal impatience is irrelevant under an interest rate instrument choice. By contrast, under the money growth instrument, fiscal impatience does affect the equilibrium policy functions and allocation. Inspecting Figure 1, we observe that this property manifests itself in a *higher* level of public consumption (a public spending bias) and a *higher* level of debt issuance under the money growth instrument choice compared to the interest rate instrument choice. Moreover, it leads to a *lower* level of private consumption. Intuitively, this results from fiscal impatience distorting the private sector's optimal trade-off between current and future utilities, i.e., current utility is too high relative to future utility, and the household responds to this misallocation by reducing private consumption. Finally, the bottom-right panel of Figure 1 shows that, independent of the level of b, private-sector welfare is lower under the money growth instrument than under the interest rate instrument. Thus the interest rate is to be preferred over the money growth rate as the monetary instrument.

[14]To see this, observe that in the present case where $u(c, g) = \gamma_1 \log c + (1 - \gamma_1) \log g$, Corollary 1 implies that $\bar{b} = 0$ provided that $\beta^G = \beta$. Using this fact, it is easily seen that the steady-state versions of equations (35)-(36) can be written as $\bar{g} + \bar{c} = \beta\gamma_1/\alpha$ and $\gamma_1/\bar{c} = (1 - \gamma_1)/\bar{g}$, respectively, which yields the stated values for $\bar{c}$ and $\bar{g}$.

Figure 1: Equilibrium policy and value functions ($\gamma_2 = 1$)



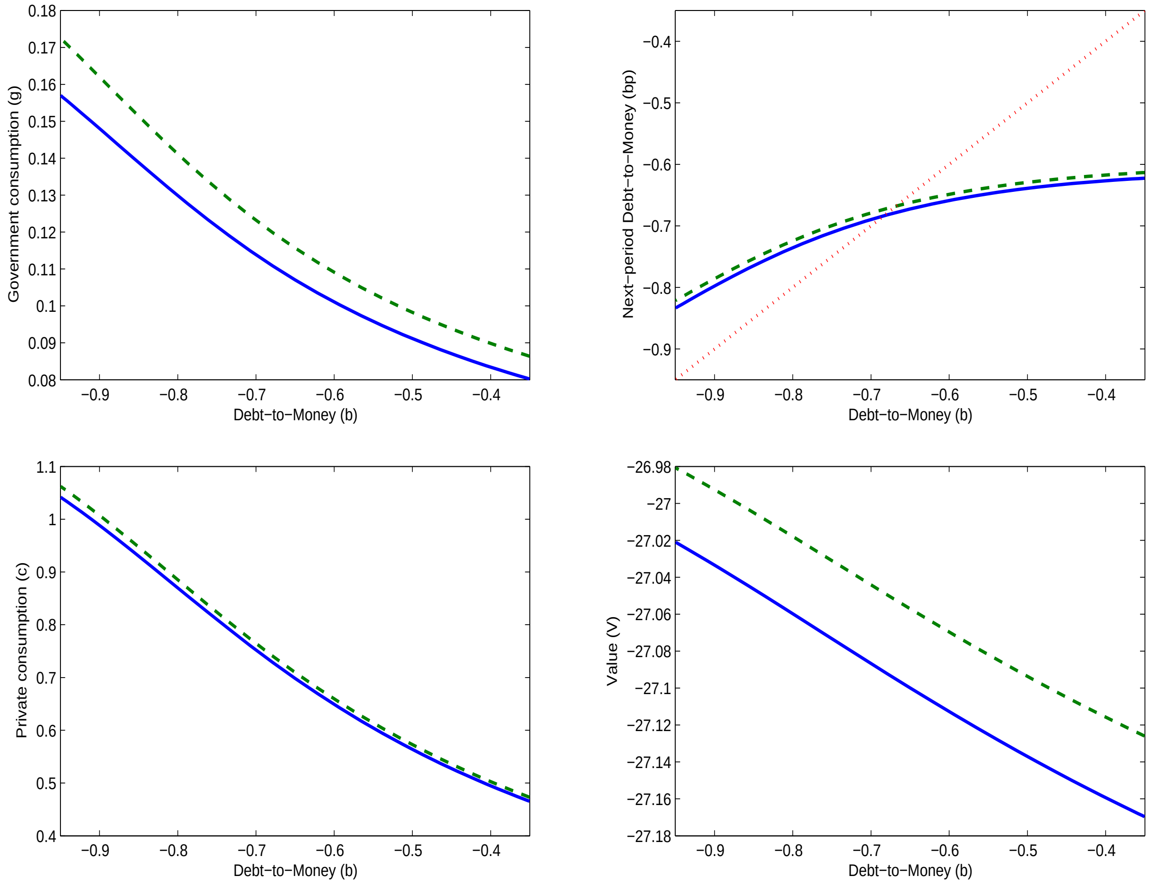
Notes: the figure displays numerical approximations to the public consumption policy $\mathcal{G}(b)$ (top-left panel), the debt policy $\mathcal{B}(b)$ (top-right panel), the private consumption policy $\mathcal{C}(b)$ (bottom-left panel), and the value function $V(b)$ (bottom-right panel) under the interest rate instrument policy (solid line) and the money growth instrument policy (dashed line). The underlying parameters are $\beta = \beta^M = 0.96$, $\beta^F = 0.8$, $\alpha = 2/3$, $\gamma_1 = 5/6$, $\gamma_2 = 1$.

4.2 Non-separable utility with elastic preferences

Let us next consider the case where $\gamma_2 = 0.4$.[15] Notice that $\gamma_2 < 1$ implies $u_{12}(c, g) > 0$, such that private and public consumption enter the utility function as complements. We keep the structural parameters of the model, except for γ_2, at the same values also used in our first example, and we compute again approximations to the equilibrium policy functions $\mathcal{G}$, $\mathcal{B}$, $\mathcal{C}$

[15]Given $\gamma_1 = 5/6$, this parameterization implies $\sigma = -cu_{11}(c, g)/u_1(c, g) = 1 - \gamma_1(1 - \gamma_2) = 0.5$.

Figure 2: Equilibrium policy and value functions ($\gamma_2 = 0.4$)



Notes: the figure displays numerical approximations to the public consumption policy $\mathcal{G}(b)$ (top-left panel), the debt policy $\mathcal{B}(b)$ (top-right panel), the private consumption policy $\mathcal{C}(b)$ (bottom-left panel), and the value function $V(b)$ (bottom-right panel) under the interest rate instrument policy (solid line) and the money growth instrument policy (dashed line). The underlying parameters are $\beta = \beta^M = 0.96$, $\beta^F = 0.8$, $\alpha = 2/3$, $\gamma_1 = 5/6$, $\gamma_2 = 0.4$.

and the value function V. Figure 2 displays these functions.

As in the case $\gamma_2 = 1$, we observe that the money growth instrument leads to a *higher* level of public consumption and debt compared to the interest rate instrument. In contrast to the case $\gamma_2 = 1$, however, the money growth instrument now leads to a *higher* level of private consumption. Intuitively, this results from private and public consumption being complements: the high level of fiscal spending increases the marginal utility of private consumption, making it attractive for the private sector to raise its consumption level. Moreover, as revealed by the bottom-right panel of Figure 2, the money growth instrument choice leads also to a *higher* level of private-sector welfare. Thus, unlike in the example with $\gamma_2 = 1$, the money growth rate turns out to be the optimal monetary instrument.

The intuition behind this finding can be understood as follows. Independent of the monetary instrument choice, the policy-makers in the economy face a time-inconsistency problem. With future consumption being more elastic than current consumption, this time-inconsistency problem leads to a sub-optimally low level of consumption under discretionary policy-making.[16] In the present example, fiscal impatience counteracts this time-inconsistency problem. It generates a public spending bias and, as private and public consumption are complements, leads to a higher level of private consumption. Fiscal impatience thus influences the equilibrium allocation in a way that moves this allocation closer to the second-best, which ultimately has a beneficial effect on private-sector welfare. Allowing for this positive effect on the equilibrium allocation, the money growth instrument turns out to be preferable to the interest rate for the specific economy under consideration (featuring $\gamma_2 = 0.4$).

Taken together, the two examples discussed above demonstrate that the welfare ranking across monetary instruments is not unambiguous. This is true even though we allow for fiscal impatience, whose effect on the equilibrium allocation can be eliminated if the central bank adopts the nominal interest rate as its instrument. Actually, the second example illustrates that a fiscal spending bias, which only unfolds in a money growth regime, can be welfare improving because it counteracts the monetary time-inconsistency problem.

[16]To see this, consider a government with commitment power. When current consumption is relatively inelastic, this government would choose a policy plan that features higher distortions in the initial period than in later periods, and thus lower consumption in the initial period than in later periods (i.e., it would choose an increasing consumption path). Absent commitment, the incentive to choose a low level of current consumption is present in every period, and thus consumption will be sub-optimally low in every period. A detailed discussion of this mechanism is provided, among others, in Nicolini (1998) and Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

5 Conclusion

This paper has studied the monetary instrument problem in the context of a dynamic game between the fiscal authority, the central bank, and the private sector. We have shown that, as long as there is a conflict of interest between the two policy makers, the choice of monetary instrument affects the equilibrium outcome. In particular, when the fiscal authority's preferences are characterized by relative impatience, the central bank can prevent distortions arising from the bias in fiscal preferences by resorting to the interest rate as its instrument. Nevertheless, the optimal choice of instrument critically depends on the economic environment under consideration: the money growth instrument is preferable whenever fiscal impatience has positive welfare effects, which is easily possible under lack of commitment.

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