View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by University of Essex Research Repository

ISSN 1755-5361

University of Essex

Department of Economics

Discussion Paper Series

No. 667 April 2009

Dynamic Monetary-Fiscal Interactions and the Role of Monetary Conservatism

Stefan Niemann

Note : The Discussion Papers in this series are prepared by members of the Department of Economics, University of Essex, for private circulation to interested readers. They often represent preliminary reports on work in progress and should therefore be neither quoted nor referred to in published work without the written consent of the author.

Dynamic Monetary-Fiscal Interactions and the Role of Monetary Conservatism

Stefan Niemann*
University of Essex

April 28, 2009

Abstract

The present paper reassesses the role of monetary conservatism in a setting with nominal government debt and endogenous fiscal policy. We assume that macroeconomic policies are chosen by monetary and fiscal policy makers who interact repeatedly but cannot commit to future actions. The real level of public liabilities is an endogenous state variable, and policies are chosen in a non-cooperative fashion. We focus on Markov-perfect equilibria and investigate the role of fiscal impatience and monetary conservatism as determinants of the economy's steady state and the associated welfare implications. Fiscal impatience creates a tendency of accumulating debt, and monetary conservatism actually exacerbates such excessive debt accumulation. Increased conservatism implies that any given level of real liabilities can be sustained at a lower rate of inflation. However, since this is internalized by the fiscal authority, the Markov-perfect equilibrium generates a steady state with higher indebtedness. As a result, increased monetary conservatism has adverse welfare implications.

JEL classification: E61, E63, H21, H63
Keywords: government debt, inflation, time-consistency, Markov-perfect equilibrium, monetary conservatism

*Department of Economics, University of Essex, Wivenhoe Park, Colchester CO4 3SQ, Essex, United Kingdom. Email: sniem@essex.ac.uk. Phone: +44-1206-87-3351. Fax: +44-1206-87-2724.

1 Introduction

In the context of monetary time-consistency problems, the implications of central bank inflation aversion ('conservatism') have been extensively studied. The predominant view is that delegation to a weight-conservative central banker has positive welfare effects (Rogoff, 1985; Adam and Billi 2008a,b). However, it has not yet been investigated whether central bank conservatism remains desirable in an environment with *endogenous fiscal policy and an explicit role for the dynamics of government debt*.

The present paper examines this question within the deterministic framework of a flexible price economy with government debt but without capital. We assume that macroeconomic policies are implemented sequentially and that there is no intertemporal commitment binding the hands of future policy makers. In contrast to most of the existing literature, however, we presume that monetary and fiscal instruments are directed by separate policy authorities whose objectives may not be perfectly aligned. This is in line with the institutional setup in most developed economies, where monetary and fiscal policies are determined by independent policy authorities with their own respective mandates.

Our starting point is to consider two deviations from the purely benevolent benchmark, where both policy authorities would share the representative household's objective function. In particular, we allow for 'fiscal impatience', modelled via a reduced discount factor for the fiscal authority, and 'monetary conservatism', modelled in terms of an explicitly inflation-averse central bank à la Rogoff (1985). Generally, the rationale for devising central banks as independent and conservative institutions is to mitigate the adverse welfare effects of monetary discretion and to insulate them from pressures due to fiscal profligacy. While we take fiscal impatience as a given primitive of the model, it is important to recognize that such impatience can arise endogenously in a politico-economic context (electoral concerns among politicians, fiscal institutions which disperse decision power over public expenditures, etc.). In the present paper, we focus on the implications of such fiscal impatience for the interaction with monetary policy and the desirability of monetary conservatism.

Adopting a public finance perspective, our model confronts the policy authorities with the task of financing an exogenously given stream of public expenditures via a set of distortionary monetary and fiscal policy instruments. In the face of the two authorities' diverging objectives, monetary and fiscal policies are chosen in a non-cooperative fashion. Since we allow for the issuance of government debt, the ensuing monetary-fiscal interaction is dynamic, with the level of real debt taking the role of an endogenous state variable. Under our maintained assumption of no commitment, we restrict attention to Markov-perfect equilibria of the dynamic game. Hence, at any point of time, the set of current monetary and fiscal policy choices may depend on the past only via the inherited endogenous state variable. By definition of a Markov-perfect equilibrium, the corresponding policy rules are time-consistent.

The mechanics of the intertemporal government budget constraint require that the stock of government debt must be covered by future revenues. In our economy, such revenues can only be generated by distortionary activity. Therefore, outstanding liabilities have adverse welfare effects.[1] Taking the stock of inherited debt as given, the current authorities choose their respective policies such as to balance current and future distortions (as perceived by

[1]Since the economy under consideration is deterministic, there is no role for stabilization policies in general, or for debt as a shock absorber in particular.

them). This, in turn, introduces a motive for manipulating the future state of the economy and thus the policies implemented by future authorities.

Our analysis investigates the role of fiscal impatience and monetary conservatism as determinants of the economy's steady state and the associated welfare implications. The central findings are as follows. The policies implemented in the Markov-perfect equilibrium by the monetary and fiscal authority, respectively, can be characterized by two distinct *generalized Euler equations*. Accordingly, the steady state level of real debt must balance both authorities' accumulation and decumulation incentives. In the special case where both authorities pursue the same purely benevolent objective, the Markov-perfect equilibrium allocation coincides with the one implemented by a single benevolent authority. Introducing the perturbed and non-aligned objective functions leads to higher steady state debt. Importantly, monetary conservatism exacerbates the impatient fiscal authority's tendency of excessive debt accumulation.

The key mechanism behind these results is the interaction of the monetary time-consistency problem with the consolidated government budget constraint, which implies that the equilibrium rate of monetary expansion is an increasing function of the level of public debt. In our model, the monetary time-consistency problem is caused by fiscal deficits. Thus, at the steady state, the elasticity of money growth depends on the fiscal discount factor, but not on the degree of monetary conservatism. On the other hand, the steady state level of debt itself is determined by the degree of monetary conservatism, but immune to changes in the fiscal discount factor. Accordingly, increased monetary conservatism does not affect the steady state elasticity of money growth, but induces an increase in debt and inflation. The intuition behind this finding is that government debt is a state variable influenced by endogenous fiscal policy: The *direct effect* of increased conservatism is that any given level of real liabilities can be sustained at a lower rate of inflation. However, since this is internalized by the fiscal authority, the *indirect effect* is that the Markov-perfect equilibrium generates a steady state with higher indebtedness. Consequently, the welfare gains during the transition to the steady state must be weighted against the costs of inducing a steady state with higher accumulation of real liabilities. For a calibrated economy, we find that the indirect effect dominates such that increased monetary conservatism has adverse welfare implications. This is in contrast to findings in the previous literature, which neglects the endogeneity of government debt.

The dynamic inconsistency of optimal plans is a pervasive phenomenon in models of macroeconomic policy making. The literature has investigated a number of potential solutions to address the distortions caused by the lack of monetary commitment. Delegation to weight-conservative central bankers (Rogoff, 1985), inflation targets (Svensson, 1997) and incentive contracts (Walsh, 1995) have received particular attention in this context. However, all these approaches abstract from fiscal policy or take it as exogenously given. On the other hand, there is a growing literature analyzing time-consistent fiscal policies in dynamic general equilibrium models (Chari and Kehoe, 1990; Klein, Krusell and Ríos-Rull, 2008; Ortigueira and Pereira, 2008). Similar to the present paper, this literature characterizes Markov-perfect equilibria of the dynamic game between successive governments in terms of *generalized Euler equations* for the government, but it typically studies models without money.

This dichotomy in the analysis of the monetary and fiscal aspects of macroeconomic time-consistency problems is somewhat surprising, given that the intertemporal government budget constraint is an important source of monetary-fiscal interactions (Sargent and Wallace, 1981; Lucas and Stokey, 1983). Only recently, a number of papers has investigated optimal time-consistent policies in settings with a meaningful role for both monetary and fiscal instruments (Díaz-Giménez et al., 2008; Martin, 2009; Niemann et al., 2008). However, these papers proceed under the assumption of a single policy authority deciding about the complete set of monetary and fiscal policy instruments. Hence, they provide only little information on the strategic interaction between monetary and fiscal policy makers in general and the role of monetary conservatism in particular.

The work by Adam and Billi (2008a,b), who consider separate policy authorities, constitutes a notable exception. Specifically, they find that monetary conservatism can help to eliminate the inflation and spending biases arising from monetary and fiscal commitment problems. However, their analysis is cast in terms of a stabilization problem in the spirit of Barro and Gordon (1983) and invokes either lump-sum taxes or alternatively a periodically balanced budget. Consequently, their environment effectively boils down to a setting without government debt such that monetary-fiscal interactions do not operate via the intertemporal government budget constraint. By contrast, in allowing for endogenous fiscal policy with non-balanced budgets, the present paper advises caution with respect to the welfare implications of monetary conservatism derived by Adam and Billi (2008a,b) as well as the earlier literature which abstracts from fiscal policy altogether.

The rest of the paper is organized as follows. The next section sets up the model economy, defines a competitive equilibrium and introduces the objectives pursued by fiscal and monetary policy makers. Section 3 lays out the structure of the policy game and formally defines an equilibrium in time-consistent policies. Section 4 describes the equilibrium outcomes and presents a set of quantitative experiments for the calibrated economy. Finally, Section 5 draws some institutional implications and concludes. Technical details are delegated to the Appendix.

2 The model

We consider a dynamic monetary general equilibrium economy whose basic structure is identical to the one in Díaz-Giménez et al. (2008). The economy is made up of a private sector and a government. There is no capital, and in each period labor n_t can be transformed into private consumption c_t or public consumption g_t at a constant rate, which we assume to be unitary. Consequently, in each period $t \geq 0$ aggregate feasibility is reflected by the following linear resource constraint:

$$c_t + g_t \leq n_t. \tag{1}$$

On the private side, the economy is inhabited by a continuum of measure one of identical infinitely-lived households whose preferences over sequences of consumption c_t and labor n_t can be represented by the following additively separable expression:

$$\sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) - v(n_t) \right\}, \tag{2}$$

where the discount factor β satisfies $0 < \beta < 1$. Government expenditure g_t yields no utility. In each period t, each household faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - P_t(1 + \tau_t^c)c_t + B_t(1 + R_t) + W_t n_t, \tag{3}$$

where P_t and W_t are the price level and the nominal wage prevailing at time t; B_{t+1} and M_{t+1} are nominal government debt and nominal money balances carried from period t to period $t+1$; R_t is the nominal interest rate on government debt held from period $t-1$ to period t. A demand for money arises due to the assumption that the gross-of-tax consumption expenditure in period t must be financed using currency carried over from period $t-1$. This gives rise to the following cash-in-advance (CIA) constraint:

$$M_t \geq P_t(1+\tau_t^c)c_t. \tag{4}$$

The timing protocol underlying this CIA constraint follows Svensson (1985) and requires that the goods market operates and closes before the asset market opens. Consequently, household purchases of goods have to be undertaken before nominal balances can be reshuffled optimally. However, the information about a money injection leads to an immediate price reaction. Hence, the effects of inflationary money expansions are twofold: First, expected inflation leads to a distortion via its effect on the nominal interest rate. Second, surprise inflation is distortionary since the households are constrained in their consumption decisions by the value of the money balances taken over from the previous period. Finally, we assume that each consumer faces a no-Ponzi condition that prevents him from running explosive consumption/debt schemes:

$$lim_{T\to\infty}\beta^T \frac{B_{T+1}}{P_T} \geq 0. \tag{5}$$

The government sector consists of a monetary authority and a fiscal authority who take their decisions independently. The policy instrument controlled by the monetary authority is the supply of money M_{t+1}^a. (Throughout, the superscript a is used to distinguish an aggregate variable from an individual variable.) The fiscal authority collects consumption taxes τ_t^c in order to finance an exogenously given stream of public expenditures g_t. For simplicity, we let public spending be deterministic and constant over time such that $g_t = g$ for all $t \geq 0$. The two authorities interact via the consolidated budget constraint of the government sector. Seignorage revenues from money creation by the monetary authority accrue to the consolidated government budget. Thus, we restrict attention to the public finance role of monetary policy in order to focus on the implications of decentralized decision power among the two independent authorities. Finally, we assume that the fiscal authority, besides its tax policy, issues nominal one-period bonds B_{t+1}^a, whereby the quantity of bonds issued must satisfy the following sequence of budget constraints for the government sector for all $t \geq 0$:

$$M_{t+1}^a + B_{t+1}^a + P_t\tau_t^c c_t \geq M_t^a + B_t^a(1+R_t) + P_t g. \tag{6}$$

Moreover, the consolidated government sector faces the following no-Ponzi condition:

$$lim_{T\to\infty}\beta^T \frac{B_{T+1}^a}{P_T} \leq 0. \tag{7}$$

A government policy is represented by a sequence $\{\tau_t^c, M_{t+1}^a, B_{t+1}^a\}_{t=0}^{\infty}$, whereby the initial stock of money M_0^a and the initial debt liabilities $B_0^a(1+R_0)$ are given.[2] We are now ready to define a competitive equilibrium for the economy.

[2] However, we impose the additional consistency condition that, in equilibrium, there is no surprise inflation in the initial ($t=0$) period; thus, by linking the nominal interest rate R_0 to the equilibrium rate of inflation in the first period, we prevent the authorities from taking advantage of the inelasticity of the amount of oustanding nominal balances M_0 and B_0 in the first period.

Definition 1 *A competitive equilibrium for this economy is composed of the government sector's policies* $\{\tau_t^c, M_{t+1}^a, B_{t+1}^a\}_{t=0}^{\infty}$*, an allocation* $\{c_t, n_t, B_{t+1}, M_{t+1}\}_{t=0}^{\infty}$*, and prices* $\{R_{t+1}, P_t\}_{t=0}^{\infty}$ *such that:*

(i) given M_0^a*,* $B_0^a(1 + R_0)$ *and* g*, the policies and the prices satisfy the sequence of budget constraints of the government sector described in expression (6) as well as the no-Ponzi condition (7);*

(ii) when households take M_0*,* $B_0(1+R_0)$ *and prices as given, the allocation solves the household problem of maximizing (2) subject to the private budget constraint (3), the CIA constraint (4) and the no-Ponzi condition (5);*

(iii) markets clear, i.e., $M_t^a = M_t$*,* $B_t^a = B_t$*, and* g *and* $\{c_t, n_t\}_{t=0}^{\infty}$ *satisfy the economy's resource constraint (1) for all* $t \geq 0$*.*

The linearity of the production technology implies that the equilibrium real wage is $w_t \equiv \frac{W_t}{P_t} = 1$ for all $t \geq 0$. In the competitive equilibrium allocation the household budget constraint (3) and the aggregate resource constraint (1) both hold at equality. Moreover, the first order conditions characterizing the solution to the household problem are both necessary and sufficient. Finally, whenever $R_t > 0$, the CIA constraint (4) is binding. The competitive equilibrium allocation can then be determined from the government budget constraint (6), the aggregate resource constraint (1) and the following conditions that must hold for all $t \geq 0$:

$$M_t = P_t(1 + \tau_t^c)c_t, \tag{8}$$

$$\frac{u'(c_t)}{v'(n_t)} = (1 + R_t)(1 + \tau_t^c), \tag{9}$$

$$(1 + R_{t+1}) = \frac{v'(n_t)}{\beta v'(n_{t+1})} \frac{P_{t+1}}{P_t}. \tag{10}$$

Furthermore, the competitive equilibrium must obey the following transversality condition:

$$lim_{T\to\infty}\beta^T \left(\frac{M_{T+1} + B_{T+1}}{P_T}\right) = 0. \tag{11}$$

Above definition of a competitive equilibrium makes clear that macroeconomic policies are crucial in determining the equilibrium allocation and hence the welfare enjoyed by private households. We now turn to the description of the objectives pursued by the fiscal and monetary policy authorities when implementing their respective policies.

2.1 Fiscal authority

The fiscal authority is impatient insofar as it tries to maximize the discounted sum of the household's period utilities $\{u(c_t) - v(n_t)\}$, whereby its discount factor $\delta < \beta$ is distorted downwards as compared to the one employed by the representative household. The fiscal objective function is:

$$\sum_{t=0}^{\infty} \delta^t \{u(c_t) - v(n_t)\}. \tag{12}$$

We see this payoff function as a shortcut for introducing politico-economic frictions into the model. Examples include electoral concerns, dynamic common pool problems or fiscal institutions which disperse the decision power over debt and deficits. We point out that a rationale for fiscal impatience can be explicitly derived in a politico-economic context.[3] For the purposes of this paper, though, we simply take fiscal impatience as a primitive of the model. A divergence in the discount factors of the form $\delta < \beta$ then reflects the systematic tendency towards policy choices that shift distortions into the future.

2.2 Monetary authority

As regards the monetary authority, our starting point are the statutes of many independent central banks which ascribe importance to the task of stabilizing the inflation rate $\pi_t = \frac{P_t}{P_{t-1}} - 1$ at a low level, but at the same time also refer to further indicators for general economic performance. We parameterize this by defining the monetary authority's objective function as follows:

$$\sum_{t=0}^{\infty} \beta^t \left\{ -\gamma \left(\frac{1+\pi_t}{1+\tilde{\pi}_t} \right)^2 + (1-\gamma)[u(c_t) - v(n_t)] \right\}. \tag{13}$$

Here, $\gamma \in (0,1)$ is a weight which balances the relative impacts on the monetary authority's payoff of general welfare (as measured by the representative household's lifetime utility) and a loss term resulting from deviations of the realized inflation rate π_t from the inflation rate $\tilde{\pi}_t$ that was expected by the public. This specification is a particular interpretation of a weight-conservative central banker (Rogoff, 1985), according to which the monetary aversion against *surprise* inflation implies a reluctance to use the inflation tax as a lump-sum instrument.

In the present environment featuring endogenous fiscal policy and a monetary time-consistency problem, this form of monetary conservatism against surprise inflation is arguably a natural specification: The inflationary loss term does not punish absolute inflation π_t *per se*; rather, for given private-sector expectations $\tilde{\pi}_t$, it punishes deviations of the form $\pi_t > \tilde{\pi}_t$. This is consistent with the nature of distortions at work in the economy. Specifically, the private sector optimality condition (9) reveals that the interest rate and tax wedges are equivalent with respect to the decision margin they distort. However, once the nominal interest rate is determined, this equivalence breaks down because of the ex-post incentives to inflate the economy faced by the monetary authority. Therefore, by punishing deviations of actual inflation from expected inflation, monetary conservatism provides incentives to reduce the costs which arise as a consequence of the Svensson-type CIA constraint. In other words, γ parameterizes the degree of commitment in the face of the monetary time-consistency problem. But despite its specific mission the monetary authority is assumed to be also responsive to general economic conditions, i.e., $\gamma \in (0,1)$. So, the commitment generated by monetary conservatism remains incomplete.

[3] For example, Malley et al. (2007) argue that fiscal incumbents with uncertain prospects of reelection find it optimal to follow shortsighted fiscal policies. Using US data from 1947 to 2004, they find a statistically and economically significant link between electoral uncertainty and macroeconomic policy outcomes, whereby the specific mechanism at work is that increased electoral uncertainty maps into decreased discount factors. See Persson and Tabellini (2000) for a discussion of other politico-economic mechanisms shaping the conduct of fiscal policy.

3 Equilibrium

We presume that policy choices are the outcome of a dynamic game between the monetary and the fiscal authority and assume that the authorities do not have access to an intertemporal commitment technology. Hence, implemeted policies must be sequentially optimal. Our interest here is in time-consistent policy rules, and we limit the analysis to (differentiable) Markov-stationary policy rules.

Markov-stationary policy rules are characterized by time-invariant mappings from the current aggregate state into policy choices.[4] The aggregate state must be informative along two dimensions: First, with respect to the composition of nominal claims with which households enter period t, and secondly, with respect to the real value of the government's debt burden inherited from the past. These considerations imply that $z_t^a \equiv \frac{B_t^a(1+R_t)}{M_t^a}$ serves as aggregate state variable in period t.[5] Note that z_t^a is part of the agents' information set at the end of period $t-1$; hence, expectations as of time $t-1$ are measurable with respect to z_t^a.

We now switch to recursive notation, whereby current variables are denoted without subscript, while variables pertaining to the next period are denoted with a prime. In each period fiscal policy is denoted by a function $\varphi_f(z^a)$ that delivers the current tax rate $\tau^c > -1$ as a function of the aggregate state z^a.[6] Similarly, monetary policy is denoted by a function $\varphi_m(z^a)$ that delivers the current rate of money growth $\mu \equiv \frac{M^{a\prime}}{M^a} - 1 > -1$ as a function of the aggregate state z^a. To summarize:

$$z^a = \frac{B^a(1+R)}{M^a}, \qquad \tau^c = \varphi_f(z^a), \qquad \mu = \varphi_m(z^a).$$

We solve for the model's Markov-perfect equilibrium (MPE) by following the approach outlined in Klein et al (2005). We thus proceed in three steps. First, we compute the private sector equilibrium for given arbitrary policy rules. In the second step, we determine the optimal equilibrium policy instruments τ^c and μ for the current period when future policy is determined by arbitrary rules. Hence, we solve for the optimal current rules given future rules. Finally, we solve for the time-consistent equilibrium by the requirement that the current optimal rules must coincide with the future rules (policy fixed point). In the following, we describe each of these steps in detail.

3.1 Equilibrium for arbitrary policy rules

Assume that fiscal taxes and money growth are determined by arbitrary functions $\varphi_f(z^a)$ and $\varphi_m(z^a)$, respectively. To ease notation it will be useful to collect fiscal and monetary policy choices into a combined policy function $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$.

[4] The implication is that history does not matter except via its influence on the current aggregate state. It is precisely this restriction that rules out reputational mechanisms. Note also that the household is representative; hence, the household's individual state and the aggregate state coincide in equilibrium.

[5] To understand why z_t^a serves as the aggregate state variable, notice that neither nominal variables such as money and bonds nor their real values are sufficient statistics. The reason is that the contemporaneous price level, being endogenous, cannot be used to normalize nominal variables. Moreover, due to the CIA constraint only money is available for current consumption expenditure such that information on the composition of the nominal asset portfolio held by private agents is needed. Finally, at the aggregate level, the CIA constraint is binding in every competitive equilibrium; therefore, real money does not enter as a separate state variable.

[6] The quantity of bonds traded is determined by private demand.

3.1.1 The household problem

In order to define the household problem recursively, we first introduce the household's individual state variable $z = \frac{B(1+R)}{M}$. Different from the aggregate level, however, there is *a priori* no mechanism that guarantees that an individual household's CIA constraint will be binding at equality. So it must be imposed as a separate constraint, and there is an additional state variable $m = \frac{M}{P_{-1}}$ in the household problem. Using these definitions, the household budget constraint (3) can be rewritten as follows:

$$\frac{m}{(1+\pi)}[1+z] - (1-\tau^c)c + wn - m'[1+z'(1+R')^{-1}] \geq 0, \tag{14}$$

where $w = \frac{W}{P}$ is the real wage and $\pi = \frac{P}{P_{-1}} - 1$ is the rate of inflation from the previous to the current period. Given the policies φ, the household takes the real wage, the rate of inflation and the nominal interest rate R' as given functions of the aggregate state; formally: $(w, \pi, R')' = \mathcal{G}(z^a; \varphi)$. Moreover, the household presumes a perceived law of motion for the endogenous aggregate state variable: $z^{a\prime} = \mathcal{H}(z^a; \varphi)$.

The household problem then reads as follows:

$$V(z, m, z^a; \varphi) = \max_{c,n,z',m'} \{[u(c) - v(n)] + \beta V(z', m', z^{a\prime}; \varphi)\} \tag{15}$$

subject to:

$$\begin{aligned}
&\frac{m}{(1+\pi)}[1+z] - (1-\tau^c)c + wn - m'[1+z'(1+R')^{-1}] \geq 0, \\
&\frac{m}{(1+\pi)} - (1-\tau^c)c \geq 0, \\
&w = \mathcal{G}_w(z^a; \varphi), \qquad \pi = \mathcal{G}_\pi(z^a; \varphi), \qquad R' = \mathcal{G}_{R'}(z^a; \varphi), \\
&z^{a\prime} = \mathcal{H}(z^a; \varphi), \qquad \tau^c = \varphi_f(z^a), \qquad \mu = \varphi_m(z^a).
\end{aligned}$$

The solution to the household problem is given by decision rules which, given the authorities' policy rules φ, map the states $(z, m, z^a)'$ into choices for c, n, z', m'. Letting λ and ν denote the associated multipliers on the household's budget and CIA constraints, respectively, the household decision rules can then be collected in a vector-valued function f with the following components:

$$\begin{aligned}
c &= f_c(z, m, z^a; \varphi), \qquad n = f_n(z, m, z^a; \varphi), \qquad z' = f_{z'}(z, m, z^a; \varphi), \\
m' &= f_{m'}(z, m, z^a; \varphi), \qquad \lambda = f_\lambda(z, m, z^a; \varphi), \qquad \nu = f_\nu(z, m, z^a; \varphi).
\end{aligned}$$

Specifically, once the functions φ, $\mathcal{G}$ and $\mathcal{H}$ are specified, the household decision rules are characterized by:

$$\frac{u'(c)}{v'(n)} = (1+R)(1+\tau^c), \tag{16}$$

$$(1+R') = \frac{v'(n)}{\beta v'(n')}(1+\pi'), \tag{17}$$

$$\nu\left[\frac{m}{(1+\pi)} - (1-\tau^c)c\right] = 0. \tag{18}$$

3.1.2 Private sector equilibrium for given policies

We restrict attention to symmetric equilibria where all households have idential initial conditions and make identical choices. Hence, since there is a continuum of measure one of households, individual variables correspond to economy-wide aggregates, i.e., households are representative. For the purposes of defining an equilibrium, we introduce the vector-valued function F describing economy-wide aggregates if households are identical; specifically:

$$
\begin{aligned}
F_c(z^a;\varphi) &= f_c(z,m,z^a;\varphi), \qquad F_n(z^a;\varphi) = f_n(z,m,z^a;\varphi), \qquad F_{z'}(z^a;\varphi) = f_{z'}(z,m,z^a;\varphi),\\
F_{m'}(z^a;\varphi) &= f_{m'}(z,m,z^a;\varphi), \qquad F_\lambda(z^a;\varphi) = f_\lambda(z,m,z^a;\varphi), \qquad F_\nu(z^a;\varphi) = f_\nu(z,m,z^a;\varphi).
\end{aligned}
$$

Similarly, the function G characterizes the realized price vector consistent with optimal household behavior: $(w,\pi,R')' = G(z^a;\varphi)$.

We are now in a position to define a private sector equilibrium for given policy rules φ.

Definition 2 *A recursive competitive equilibrium for given policy rules $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$ consists of a household value function $V(z,m,z^a;\varphi)$, household decision rules $f(z,m,z^a;\varphi)$, and perceived functions $\mathcal{G}(z^a;\varphi)$ and $\mathcal{H}(z^a;\varphi)$ such that:*

(i) households optimize, i.e., given states (z,m,z^a), policies $\varphi(z^a)$ and the perceived functions $\mathcal{G}(z^a;\varphi)$ and $\mathcal{H}(z^a;\varphi)$, the value function $V(z,m,z^a;\varphi)$ and decision rules $f(z,m,z^a;\varphi)$ solve the household problem;

(ii) households are representative, i.e., $z = z^a$;

(iii) households are rational, i.e., the perceived equations for the real wage, the inflation rate and the interest rate as well as the perceived law of motion for the aggregate state coincide with their respective actual equations:

$$
\begin{aligned}
\mathcal{G}_w(z^a;\varphi) &= G_w(z^a;\varphi),\\
\mathcal{G}_\pi(z^a;\varphi) &= G_\pi(z^a;\varphi),\\
\mathcal{G}_{R'}(z^a;\varphi) &= G_{R'}(z^a;\varphi),\\
\mathcal{H}(z^a;\varphi) &= F_{z'}(z^a;\varphi);
\end{aligned}
$$

(iv) markets clear:

$$
\begin{aligned}
F_{z'}(z^a;\varphi) &= z^{a\prime},\\
F_{m'}(z^a;\varphi) &= (1+\varphi_m(z^a))(1+\varphi_f(z^a))F_c(z^a;\varphi),\\
F_n(z^a;\varphi) &= F_c(z^a;\varphi) + g;
\end{aligned}
\tag{19}
$$

(v) the policies $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$ are feasible, i.e., they satisfy the government budget constraint:

$$
\begin{aligned}
(1+\varphi_m(z^a))(1+\varphi_f(z^a))F_c(z^a;\varphi)[1+F_{z'}(z^a;\varphi)(1+G_{R'}(z^a;\varphi))^{-1}]&\\
\geq F_c(z^a;\varphi) + z^a(1+\varphi_f(z^a))F_c(z^a;\varphi) + g.&
\end{aligned}
$$

3.1.3 The authorities' value functions

Having defined a private sector equilibrium for given policy rules φ, we can proceed to characterize the authorities' value functions, again for given policy rules φ. The fiscal authority's value function must satisfy:

$$V^f(z^a;\varphi) = u(F_c(z^a;\varphi)) - v(F_n(z^a;\varphi)) + \delta V^f(F_{z'}(z^a;\varphi);\varphi). \tag{20}$$

Similarly, the monetary authority's value function must satisfy:

$$V^m(z^a;\varphi) = -\gamma\left(\frac{1+G_\pi(z^a;\varphi)}{1+\mathcal{G}_\pi(z^a;\varphi)}\right)^2 + (1-\gamma)[u(F_c(z^a;\varphi)) - v(F_n(z^a;\varphi))] + \beta V^m(F_{z'}(z^a;\varphi);\varphi). \tag{21}$$

3.2 Optimal current policy rules for given future policy rules

When implementing their current policies, the fiscal and monetary authority take the policy rules φ governing the behavior of future authorities as given; this is the implication of the policy makers' lack of commitment. In addition, the current fiscal authority takes the current rate of money growth μ as given, while the current monetary authority takes the current tax rate τ^c as given; this is the essence of decentralized authority between fiscal and monetary decision makers. In this section, we characterize the current authorities' behavior in the face of these restrictions. Assuming that the policy rules are given by $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$ from the next period onwards, we start by defining an equilibrium for arbitrary feasible policies today. Let $\psi(z^a) = \{\psi_f(z^a), \psi_m(z^a)\}$ denote these current policy rules.

3.2.1 The household problem

We first consider the problem solved by individual households when the two authorities choose arbirtrary policies ψ in the current period followed by arbitrary policies φ from the next period onwards. In this problem, the next period value function is therefore given by the value function obtained as the solution to the household problem (15) derived in the previous section. Conversely, the current period value function is indexed by the current policy rule ψ. Following Klein et al. (2005), we denote functions, which are affected by the current policies ψ, with hats. We then have:

$$\hat{V}(z,m,z^a;\psi,\varphi) = \max_{c,n,z',m'} \{[u(c) - v(n)] + \beta V(z',m',z^{a\prime};\varphi)\} \tag{22}$$

subject to:

$$\begin{aligned}
&\frac{m}{(1+\pi)}[1+z] - (1-\tau^c)c + wn - m'[1+z'(1+R')^{-1}] \geq 0,\\
&\frac{m}{(1+\pi)} - (1-\tau^c)c \geq 0,\\
&w = \hat{\mathcal{G}}_w(z^a;\psi,\varphi), \qquad \pi = \hat{\mathcal{G}}_\pi(z^a;\psi,\varphi), \qquad R' = \hat{\mathcal{G}}_{R'}(z^a;\psi,\varphi),\\
&z^{a\prime} = \hat{\mathcal{H}}(z^a;\psi,\varphi), \qquad \tau^c = \psi_f(z^a), \qquad \mu = \psi_m(z^a).
\end{aligned}$$

The solution to problem (22) is given by two sets of decision rules, one for the current period, $\hat{f}(z,m,z^a;\psi,\varphi)$, and one for the future, $f(z,m,z^a;\varphi)$. As stated above, in conjunction with the future value function the future decision rules constitute the solution to problem (15). On the other hand, by their forward-looking nature, the current decision rules $\hat{f}(z, m, z^a; \psi, \varphi)$ as well as the aggregate functions $\hat{\mathcal{G}}(z^a; \psi, \varphi)$ and $\hat{\mathcal{H}}(z^a; \psi, \varphi)$ depend on both current and future policies.

With appropriate notational changes, the private sector equilibrium for arbitrary policies ψ followed by policies φ is defined as in the previous section. In particular, in the symmetric equilibrium households optimize and are rational and representative. Thus, private sector behavior can be characterized via aggregate decision rules $\hat{F}(z, m, z^a; \psi, \varphi)$ for the current period and $F(z, m, z^a; \varphi)$ for the future.

We can now proceed to analyze the two authorities' respective problems of choosing optimal current policies ψ when facing arbitrary feasible continuation policy rules φ. Since fiscal and monetary policies are implemented in a non-cooperative fashion, each of the two current policy makers needs to take as given not only the continuation play φ, but also the policy instrument employed by the other current authority.

3.2.2 The fiscal authority's problem

The current fiscal authority faces the following problem:

$$\hat{V^f}(z^a; \psi, \varphi) = \max_{\tau^c} \left\{ u(\hat{F}_c(z^a; \psi, \varphi)) - v(\hat{F}_n(z^a; \psi, \varphi)) + \delta V^f(\hat{F}_{z'}(z^a; \psi, \varphi); \varphi) \right\} \tag{23a}$$

subject to:

$$\begin{aligned}(1+\mu)(1+\tau^c)\hat{F}_c(z^a; \psi, \varphi)[1 + \hat{F}_{z'}(z^a; \psi, \varphi)(1 + \hat{G_{R'}}(z^a; \psi, \varphi))^{-1}]& \\ \geq \hat{F}_c(z^a; \psi, \varphi) + z^a(1+\tau^c)\hat{F}_c(z^a; \psi, \varphi) + g&,\end{aligned} \tag{23b}$$

where $\mu = \psi_m(z^a)$ and the function V^f satisfies (20).

3.2.3 The monetary authority's problem

The current monetary authority faces the following problem:

$$\begin{aligned}\hat{V^m}(z^a; \psi, \varphi) = \max_{\mu} \Bigg\{ & - \quad \gamma \left(\frac{1 + \hat{G_\pi}(z^a; \psi, \varphi)}{1 + \hat{\mathcal{G}}_\pi(z^a; \psi, \varphi)} \right)^2 + (1-\gamma)[u(\hat{F}_c(z^a; \psi, \varphi)) - v(\hat{F}_n(z^a; \psi, \varphi))] \\ & + \quad \beta V^m(\hat{F}_{z'}(z^a; \psi, \varphi); \varphi) \Bigg\}\end{aligned} \tag{24a}$$

subject to:

$$\begin{aligned}(1+\mu)(1+\tau^c)\hat{F}_c(z^a; \psi, \varphi)[1 + \hat{F}_{z'}(z^a; \psi, \varphi)(1 + \hat{G_{R'}}(z^a; \psi, \varphi))^{-1}]& \\ \geq \hat{F}_c(z^a; \psi, \varphi) + z^a(1+\tau^c)\hat{F}_c(z^a; \psi, \varphi) + g&,\end{aligned} \tag{24b}$$

where $\tau^c = \psi_f(z^a)$ and the function V^m satisfies (21).

3.2.4 Nash equilibrium

In each period, the current fiscal authority faces a problem of type (23), whereas the current monetary authority faces a problem of type (24). Due to their dependence on the policy instrument employed by the respective other authority, these decision problems are strategically interdependent. They therefore constitute a *stage game*. In every such stage game, the two authorities simultaneously choose their current policy instruments to maximize their value functions. This leads to the following definition:

Definition 3 *Given the functions $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$, a Nash equilibrium of the stage game is a pair of functions $\psi^*(z^a) = \{\psi_f^*(z^a), \psi_m^*(z^a)\}$ such that (i) $\psi_f^*(z^a)$ maximizes $\hat{V}^f(z^a; \psi, \varphi)$, given $\psi_m^*(z^a)$, and (ii) $\psi_m^*(z^a)$ maximizes $\hat{V}^m(z^a; \psi, \varphi)$, given $\psi_f^*(z^a)$.*

Note that, by construction, the Nash equilibrium involves feasible policy choices.

3.3 Policy fixed point

We have now all the elements required to define the equilibrium time-consistent policy rules.

Definition 4 *The policy functions $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$ define time-consistent policies if they are the Nash solution of the stage game when the two authorities expect $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$ to determine future policies. Formally:*

$$\varphi_f(z^a) = \psi_f^*(z^a), \qquad \varphi_m(z^a) = \psi_m^*(z^a). \tag{25}$$

A MPE of the policy game is a profile of time-consistent Markov strategies for the two authorities that yields a Nash equilibrium in every stage game. It is these time-consistent policies $\varphi(z^a) = \{\varphi_f(z^a), \varphi_m(z^a)\}$ and the associated equilibrium outcomes that we are interested in.

4 Equilibrium outcomes

In what follows, we assume $u(c) = \log(c)$ and $v(n) = \alpha n$. The assumption of linear disutility of labor is made to sharpen the discussion, but implies also that the authorities cannot affect the real interest rate, which is given by $\frac{v'(n)}{\beta v'(n')} = \frac{1}{\beta}$. The assumption of logarithmic utility from consumption allows to focus on the role of nominal debt as a source of time-inconsistency and to abstract from the effects due to private holdings of nominal money balances. That is, we abstract from seignorage on base money and focus on the implications of changing the real value of nominal debt. This focus is consistent with the situation in most developed economies where government debt is arguably more important than money holdings as a source of dynamically inconsistent incentives.[7]

[7] See Nicolini (1998) for an instructive exposition of the nature of the time-inconsistency of monetary policy.

4.1 Primal approach

Starting from above assumptions on preferences, the competitive equilibrium conditions (18), (16) and (17) can be combined to the following condition governing private sector behavior:

$$(1+\mu)(1+\tau^c)c = \frac{\beta}{\alpha}. \tag{26}$$

Specifically, condition (26) describes money market equilibrium (compare equation (19)). Conditional on the policy μ employed by the current monetary authority, it defines a mapping from current fiscal policy τ^c into the household's consumption allocation. Equivalently, for a given fiscal policy τ^c, there results a mapping from current monetary policy μ into the allocation. This insight enables us to reformulate the authorities' policy problems and adopt a *primal approach*.[8] Accordingly, taking future policies φ and the policy implemented by the other authority as given, each authority maximizes directly over the allocation rather than over its actual policy instrument.

Condition (26) must be satisfied in any competitive equilibrium; therefore, it can be used to substitute in the constraints (23b) and (24b). We then obtain two distinct *implementability constraints* faced by the fiscal and monetary authority, respectively. For the fiscal authority, we get

$$\frac{\beta}{\alpha} + \beta\frac{\beta}{\alpha}\frac{z^{a\prime}}{(1+\mu(z^{a\prime}))} - c - \frac{\beta}{\alpha}\frac{z^a}{(1+\mu)} - g = 0, \tag{27}$$

where $c = \hat{F}_c(z^a;\psi,\varphi)$, $z^{a\prime} = \hat{F}_{z'}(z^a;\psi,\varphi)$ and where we used (26) along with (16) to substitute for the fiscal policy instrument and the nominal interest rate in (23b). For the monetary authority, we get

$$\frac{\beta}{\alpha} + \beta z^{a\prime}(1+\tau^c(z^{a\prime}))c(z^{a\prime}) - c - z^a(1+\tau^c)c - g = 0, \tag{28}$$

where we used (26) along with (16) to substitute for the monetary policy instrument and the nominal interest rate in (24b).

Note that the implementability constraints inherit the key property of condition (26): Since each implementability constraint is contingent on the policy instrument chosen by the other authority, it establishes a mapping between the own policy instrument and the implemented allocation $\{c, z^{a\prime}\}$. Put differently, for a given value of the policy instrument chosen by the other authority, each implementability constraint imposes a restriction on compatible choices of c and $z^{a\prime}$.

Finally, the economy's aggregate resource constraint (1) can be used to substitute for the representative household's labor supply, $n = c + g$. This *feasibility constraint* is exploited to rewrite the current fiscal authority's problem (23) as follows:

$$\begin{aligned} &\hat{V}^f(z^a;\varphi) = \max_{c,z^{a\prime}} \left\{u(c) - v(c+g) + \delta V^f(z^{a\prime};\varphi)\right\} \\ &\text{subject to}\,(27). \end{aligned} \tag{29}$$

[8]See Chari and Kehoe (1999) for a general exposition of the methodology.

In the same fashion, the current monetary authority's problem (24) is:

$$\hat{V^m}(z^a;\varphi) = \max_{c,z^{a\prime}} \Big\{ -\gamma \left(\frac{(1+\tilde{\tau}^c(z^a))\tilde{c}(z^a)}{(1+\tau^c)c} \right)^2 + (1-\gamma)[u(c) - v(c+g)] + \beta V^m(z^{a\prime};\varphi) \Big\}$$
$$\text{subject to (28),} \tag{30}$$

where the expression $(1+\tilde{\tau}^c(z^a))\tilde{c}(z^a)$ in the nominator of the inflationary loss term denotes the household's planned gross-of-tax expenditure for the current period.[9] Importantly, as an implication of the Svensson-type CIA constraint, this expenditure plan must be made in the preceeding period and is therefore a function of the information at the end of that period as summarized by the state variable z^a.

4.2 Necessary conditions

Sequential implemetation of policies under decentralized authority implies that each authority $i = f, m$ takes the continuation play φ and the current behavior of the respective other authority ψ_{-i} as given. Equilibrium in the dynamic game between the two authorities then requires that, in each stage game, the allocations $\{c, z^{a\prime}\}$ preferred by the current policy makers coincide. This is guaranteed via condition (26), which ensures that the two authorities' dynamic programs and their respective solutions are mutually consistent and decentralizable as a competitive equilibrium. Indeed, condition (26) implies that, in equilibrium, the distinct implementability constraints (27) and (28) coincide.

Concentrating on differentiable Markov-stationary policy rules, it is useful to consider the two authorities' first order conditions with respect to their primal choice variables c and $z^{a\prime}$ for a given continuation policy φ (step 2 above - see Section 3.2). These first order conditions, together with the respective implementability constraints, are necessary conditions characterizing any differentiable MPE.[10] A similar approach has been adopted in the literature on time-consistent fiscal policy (Klein et al. (2008)), where MPE outcomes are characterized in terms of generalized Euler equations (GEEs).

The fiscal authority's optimality conditions with respect to c and $z^{a\prime}$ can be combined to yield the following expression:

$$\frac{1}{c} - \alpha = -\frac{\frac{\delta}{\beta} V_z^f(z^{a\prime};\varphi)}{(1+\tau^c(z^{a\prime}))c(z^{a\prime})} \left[1 - \varepsilon_\mu(z^{a\prime})\right]^{-1}, \tag{31}$$

where $\varepsilon_\mu(z^{a\prime}) \equiv \frac{\partial(1+\mu(z^{a\prime}))/\partial z^{a\prime}}{(1+\mu(z^{a\prime}))/z^{a\prime}}$ is the elasticity of future monetary growth in response to a change in the aggregate state $z^{a\prime}$. According to this condition, the marginal gain from increased current consumption is equated to the marginal cost of entering the next period with a higher stock of real debt $z^{a\prime}$, which is (i) scaled down by the factor $\frac{\delta}{\beta} < 1$ due to the fiscal authority's relative

[9] Formally, the substitutions in the inflationary loss term are as follows. In the nominator, exploiting (18), we have: $1 + \hat{G_\pi}(z^a;\psi,\varphi) = 1 + \pi = \frac{P}{P_{-1}} = \frac{M/(1+\tau^c)c}{P_{-1}} = \frac{m}{(1+\tau^c)c}$. Analogously, in the denominator, we have: $1 + \hat{\mathcal{G}_\pi}(z^a;\psi,\varphi) = 1 + \tilde{\pi} = \frac{\tilde{P}}{P_{-1}} = \frac{M/(1+\tilde{\tau}^c)\tilde{c}}{P_{-1}} = \frac{m}{(1+\tilde{\tau}^c)\tilde{c}}$. Division yields the inflationary loss term in (30).

[10] The first order conditions are derived in Appendix A.1.

impatience and (ii) scaled up by the factor $[1-\varepsilon_\mu(z^{a\prime})]^{-1} \geq 1$.[11] This latter amplification results from the adverse expectational effects of increased outstanding liabilities: A higher debt burden increases the future monetary authority's incentives to resort to the inflation tax, which is anticipated by the public and leads to an upward distortion in nominal interest rates; this, in turn, constitutes an opportunity cost of consumption due to the CIA constraint.

The fiscal GEE is obtained by employing an envelope condition in order to substitute for the value function term in (31):

$$\frac{1}{c} - \alpha = \frac{\delta}{\beta}\left(\frac{1}{c(z^{a\prime})} - \alpha\right)[1-\varepsilon_\mu(z^{a\prime})]^{-1}. \tag{32}$$

Equation (32) reveals that, even though the current fiscal authority itself is not subject to a time-consistency problem, it will not act like a policy maker with intertemporal commitment. Rather, in trying to smooth distortions over time, it will also take into account the incentive problems of future (monetary) policy makers. However, since the fiscal authority attaches a lower relative weight to the time when the commitment problem is relevant, this strategic rationale is discounted.

As for the fiscal authority, the monetary authority's first order conditions for c and $z^{a\prime}$ can be combined to a single expression:

$$\frac{2\gamma\frac{1}{c} + (1-\gamma)\left(\frac{1}{c} - \alpha\right)}{[1 + z^a(1+\tau^c)]} = -\frac{V_z^m(z^{a\prime};\varphi)}{(1+\tau^c(z^{a\prime}))c(z^{a\prime})}[1-\varepsilon_\mu(z^{a\prime})]^{-1}. \tag{33}$$

Again, this condition has the interpretation that, in each period, the monetary authority tries to equate the marginal gain from higher current consumption to the marginal cost associated with higher debt in the next period. The marginal benefit from current consumption as perceived by the monetary authority (LHS) consists of three components: First, there is the direct effect via current household utility as reflected by the expression $(1-\gamma)\left(\frac{1}{c} - \alpha\right)$. Secondly, the term $2\gamma\frac{1}{c}$ captures the fact that, for given private sector expectations, higher consumption implies lower surprise inflation and hence affects the inflationary loss term. Finally, the expression in the denominator (which exceeds one for $z^a > 0$) reflects a discounting of the benefits from increased consumption due to the following mechanism: By virtue of the CIA constraint, higher consumption requires that the current monetary policy maker has to reduce the inflation tax, which would operate in a lump-sum fashion on the outstanding liabilities; instead, its successors are left with the task to satisfy the intertemporal budget constraint by means of future distortionary activity. The evaluation of the marginal cost of higher debt in the next period (RHS) again takes into account the commitment problems of future policy makers and thus comprises the amplification term $[1-\varepsilon_\mu(z^{a\prime})]^{-1} \geq 1$.

For $z^{a\prime} \neq 0$, substitution of the value function term in (33) via an envelope condition leads

[11] The equilibrium property $[1-\varepsilon_\mu(z^{a\prime})]^{-1} \geq 1$ follows from the fact that $1 > \varepsilon_\mu(z^{a\prime}) \geq 0$. Since any competitive equilibrium must be decentralized via distortionary policies, we have $[u'(c(z^a)) - v'(c(z^a)+g)] = [1/c(z^a) - \alpha] > 0$ for all $z^a > -1 - \frac{\alpha}{1-\beta}g$; then, as seen from (32), compatibility with fiscal optimality requires $1 > \varepsilon_\mu(z^{a\prime})$. Finally, $\varepsilon_\mu(z^{a\prime}) \geq 0$ is a consequence of the monetary authority's incentives to monetize outstanding government liabilities via the inflation tax being a non-decreasing function of the inherited debt burden z^a. Proposition 2 below will formally verify that $\varepsilon_\mu(z^a) > 0$ in a neighborhood of the steady state.

to the following monetary GEE:

$$\frac{2\gamma\frac{1}{c}+(1-\gamma)\left(\frac{1}{c}-\alpha\right)}{[1+z^a(1+\tau^c)]}=\left[\frac{2\gamma\frac{1}{c(z^{a\prime})}+(1-\gamma)\left(\frac{1}{c(z^{a\prime})}-\alpha\right)}{[1+z^{a\prime}(1+\tau^c(z^{a\prime}))]}-\frac{2\gamma\frac{\varepsilon_\mu(z^{a\prime})}{c(z^{a\prime})}}{z^{a\prime}(1+\tau^c(z^{a\prime}))}\right][1-\varepsilon_\mu(z^{a\prime})]^{-1}, \quad (34)$$

which dictates distortion smoothing subject to the twofold incentive constraint stemming from debt being nominal and policy implementation being sequential. The former distortion reflects the discretionary incentive to reduce the real value of government debt and enters (34) via the term $[1+z^a(1+\tau^c)]$, as explained above. The latter distortion arises as a consequence of future policies being responsive to the inherited stock of liabilities $z^{a\prime}$, which induces the current policy maker to take into account the associated consumption distortion due to increased nominal interest rates.

4.3 Steady state

It is useful to begin our discussion of steady states by drawing on a benchmark result established in Díaz-Giménez et al. (2008) and Martin (2009) for the case of a monolithic policy maker.[12] The following proposition extends these papers' results to the case of interacting policy makers.

Proposition 1 *For $\delta=\beta$ and $\gamma=0$, there are two steady states in differentiable policy rules. Real debt and consumption at these steady states can be characterized in closed form:*

$$\hat{z}_1^{a*}=-1-\frac{\alpha}{1-\beta}g, \qquad c_1(\hat{z}_1^{a*})=\frac{1}{\alpha}; \qquad \textit{and} \qquad \hat{z}_2^{a*}=0, \qquad c_2(\hat{z}_2^{a*})=\left(\frac{\beta}{\alpha}-g\right).$$

At z_1^{a*}, the government sector's net assets are sufficient to finance expenditures, and the allocation is undistorted.

Proof: See Appendix A.2. □

These steady state outcomes for the *unperturbed game* ($\delta=\beta$, $\gamma=0$), where both government authorities share the same benevolent objective function, coincide with the steady state outcomes when there is a monolithic authority deciding about both fiscal and monetary policies. Indeed, it is immediate that the MPE for the single-authority economy also constitutes a MPE with interacting authorities. This is because an optimal policy outcome under single-authority must have been decentralized by strategies for the fiscal and monetary policy instruments which are mutual best responses. If this was not the case, there would be an incentive to revise the policies.

In the *perturbed game* ($\delta<\beta$, $\gamma\in(0,1)$), however, neither of the two steady states above survives. Specifically, the undistorted steady state at $\hat{z}_1^{a*}$ breaks down because the monetary authority's inflationary loss term generates an excessive preference for consumption relative to leisure.[13] Thus, the monetary GEE (34) cannot be satisfied at $(\hat{z}_1^{a*}, c_1(\hat{z}_1^{a*}))$. Similarly, the

[12] These papers also characterize equilibria and steady states for general CRRA utility in consumption.

[13] This is apparent from the marginal utility term in (34): $2\gamma\frac{1}{c}+(1-\gamma)\left(\frac{1}{c}-\alpha\right)=(1+\gamma)\frac{1}{c}-(1-\gamma)\alpha$, which implies that the monetary authority favors a steady state allocation with $\tilde{c}=\frac{(1+\gamma)}{(1-\gamma)}\frac{1}{\alpha}>\frac{1}{\alpha}=c_1(\hat{z}_1^{a*})$.

distorted steady state z_2^{a*} breaks down because of the fiscal authority's relative impatience. As $z^a \to \hat{z}_2^{a*}$, the monetary GEE (34) demands $\varepsilon_\mu(z^a) \to 0$; however, since $\delta < \beta$, this is not compatible with the fiscal GEE (32).

In the economy under consideration, government expenditures and debt service need to be financed via distortionary taxation or seignorage. Hence, unless the government has a large net asset position vis-à-vis the private sector, which allows to finance expenditures via interest earnings, the allocation will be distorted. This implies that, generically, the marginal utility from consumption is higher than the marginal disutility from labor effort: $u'(c(z^a)) - v'(c(z^a) + g) = [1/c(z^a) - \alpha] > 0$. Moreover, government debt crowds out private consumption because it has to be serviced via distortionary activity. Thus, the function $c(z^a)$ is decreasing, and outstanding government liabilities have adverse welfare implications.[14] Indeed, the only role of government debt is as an instrument to defer taxation. Seen from this perspective, it is clear that the fiscal authority's relative impatience plays an important role for the dynamics of government debt: If the fiscal authority discounts the future at a higher rate than the private households and the monetary authority do ($\delta < \beta$), then its preferred policy systematically shifts policy distortions into the future at the cost of accumulating public debt.

Formally, given that $[1/c(z^a) - \alpha] > 0$ and $c(z^a)$ decreasing, the fiscal GEE (32) reveals that the fiscal authority prefers to accumulate debt as long as $\frac{\delta}{\beta}\left[1 - \varepsilon_\mu(z^{a\prime})\right]^{-1} < 1$. On the other hand, the same argument implies that there is a limit to the economy's tendency to accumulate debt. Indeed, for $\frac{\delta}{\beta}\left[1 - \varepsilon_\mu(z^{a\prime})\right]^{-1} > 1$ the fiscal GEE (32) dictates decumulation of debt. The model's long-run prediction therefore is that the economy converges to a stationary level of debt z^{a*} implicitly characterized by $\frac{\delta}{\beta}\left[1 - \varepsilon_\mu(z^{a*})\right]^{-1} = 1$. Solving for the steady state elasticity of money growth yields $\varepsilon_\mu(z^{a*}) = \frac{\beta-\delta}{\beta} > 0$.

Proposition 2 *For $\delta < \beta$ and $\gamma \in (0,1)$, the steady state level of debt z^{a*} is implicitly characterized by $\frac{\delta}{\beta}\left[1 - \varepsilon_\mu(z^{a*})\right]^{-1} = 1$. The steady state elasticity of money growth is given by $\varepsilon_\mu(z^{a*}) = \left.\frac{\partial(1+\mu(z^a))/\partial z^a}{(1+\mu(z^a))/z^a}\right|_{z^a = z^{a*}} = \frac{\beta-\delta}{\beta} > 0$. In the neighborhood of a stable steady state, the elasticity of money growth is strictly positive and increasing in z^a.*

Proof: See Appendix A.2. □

Proposition 2 highlights that the numerical value of the steady state elasticity of money growth is determined by the fiscal discount factor δ, whereas monetary institutions do not matter. This result suggests that inflation is ultimately a fiscal phenomenon: Although fiscal policy itself has no direct inflationary effects, by running deficits and accumulating public debt it can strategically manipulate the monetary authority's willingness to inflate the economy.

The private sector optimality condition (16) indicates that there is an equivalence between fiscal and monetary policies in the sense that the wedges introduced by these policies distort the same margins. Indeed, from a welfare perspective, it does not matter whether public expenditures are financed by fiscal or monetary policies. What creates additional welfare losses, though, is the fact that money growth systematically varies with the stock of real liabilities.

[14] This follows from condition (26), $(1+\mu(z^a))(1+\tau^c(z^a))c(z^a) = \frac{\beta}{\alpha}$: An increase in z^a requires higher taxes and/or seignorage, $(1+\mu(z^a))(1+\tau^c(z^a))$; (26) then implies that $c(z^a)$ must be lower. Since $u'(c(z^a)) - v'(c(z^a)+g) = [1/c(z^a) - \alpha] > 0$, the increase in z^a has adverse welfare consequences.

This is because the commitment problem faced by future monetary policy makers is anticipated by the public, which leads to interest rate distortions without generating any revenue for the government sector. For this reason, we call any policy rule characterized by $\varepsilon_\mu(z^a) > 0$ a rule that is (locally) subject to an *inflation bias*.

Note that this definition as well the content of Proposition 2 are not in terms of the level of money growth, but in terms of its elasticity. The preceeding discussion of the equivalence between fiscal and monetary policies hints at the general difficulty to explicitly characterize the rates of taxation and money growth, respectively. However, for the economy under consideration, the inflationary loss term in (13) breaks the equivalence from the monetary authority's perspective. Therefore, we are able to solve for the set of fiscal and monetary policy instruments that decentralize the MPE allocation. Proposition 3 provides further details.

Proposition 3 *For $\delta < \beta$ and $\gamma \in (0,1)$, taxation, money growth, nominal interest rates and inflation at the steady state are given by:*

$$(1+\tau^c(z^{a*})) = \left[\frac{1}{2\gamma} z^{a*}(1-\gamma)\left(1-\alpha c(z^{a*})\right)\right]^{-1}, \tag{35}$$

$$(1+\mu(z^{a*})) = \frac{\beta}{2\gamma\alpha} z^{a*}(1-\gamma)\left(\frac{1}{c(z^{a*})}-\alpha\right), \tag{36}$$

$$(1+R(z^{a*})) = \frac{1}{2\gamma\alpha} z^{a*}(1-\gamma)\left(\frac{1}{c(z^{a*})}-\alpha\right), \tag{37}$$

$$(1+\pi(z^{a*})) = \frac{\beta}{2\gamma\alpha} z^{a*}(1-\gamma)\left(\frac{1}{c(z^{a*})}-\alpha\right), \tag{38}$$

where $c(z^{a*})$, the steady state level of consumption, is implicitly defined as the positive solution to

$$\alpha c(z^{a*})^2 - \left[1+\frac{(1-\beta)2\gamma}{(1-\gamma)}+\alpha\left(\frac{\beta}{\alpha}-g\right)\right]c(z^{a*}) + \left(\frac{\beta}{\alpha}-g\right) = 0 \tag{39}$$

and hence independent of the degree of fiscal impatience δ.

Proof: See Appendix A.2. □

It is interesting to observe that the steady state level of consumption $c(z^{a*})$ does not depend on the degree of fiscal impatience δ. Accordingly, fiscal institutions, insofar as they are represented by δ, do not play a role in the determination of long-run consumption. Section 4.6 will come back to this point.

So, what can be said about the steady state level of real debt itself? It turns out that the steady state stock of real government liabilities z^{a*} cannot be determined in closed form. Nevertheless, a number of results characterizing the nature of the steady state are contained in the following Proposition.

Proposition 4 *Let $\mathcal{C}(z^a)$ and $\mathcal{Z}^{a\prime}(z^a)$ be the primal policy functions which jointly solve problems (29) and (30). The steady state level of real debt z^{a*} is a fixed point in the mapping $\mathcal{Z}^{a\prime}(z^a)$. For $\delta < \beta$ and $\gamma \in (0,1)$, and assuming differentiable policy rules,* *(i) if there is no positive solution to (39), a steady state does not exist;*

(ii) if $c^* = \mathcal{C}(z^{a*})$ is a positive solution to (39), the associated steady state level of real debt is implicitly characterized as $z^{a*} = \mathcal{C}^{-1}(c^*)$;

(iii) $z^{a*} \neq 0$;

(iv) if there is a unique positive solution to (39), the steady state is stable (unstable) if $z^{a*} > 0$ ($z^{a*} < 0$);

(v) if there are two positive solutions to (39), there are two steady states, z_1^{a*}, z_2^{a*}, where $z_1^{a*} < z_2^{a*}$ and

(a) if $0 < z_1^{a*} < z_2^{a*}$, then z_1^{a*} is stable, z_2^{a*} is unstable;

(b) if $z_1^{a*} < 0 < z_2^{a*}$, then z_1^{a*} is unstable, z_2^{a*} is stable;

(c) if $z_1^{a*} < z_2^{a*} < 0$, then z_1^{a*} is stable, z_2^{a*} is unstable.

PROOF: See Appendix A.2. □

Proposition 4 indicates that the existence and the properties of a steady state depend on the set of parameters $(\alpha, \beta, \gamma, g)$ in (39). In what follows, we assume these parameters are such that a steady state exists. Moreover, our subsequent numerical analysis will concentrate on the stable steady state. The justification for this is to view the deterministic economy under consideration as the limiting case of of a more general stochastic economy. In the long-run, such an economy will almost surely settle in the neighborhood of the stable steady state.

4.4 Calibration

In order to illustrate the dynamic evolution of the economy in the presence of nominal government debt, we calibrate the model and then invoke a simple numerical example. Our calibration strategy is to target empirical moments for the U.S. during 1962-2006 for the following variables: nominal interest rates, inflation, government expenditure over GDP and taxes over GDP. The value for debt over GDP is not calibrated, but inferred from the steady state value z^{a*} that satisfies the government budget constraint. Data are taken from the Economic Report of the President (2008) and the Federal Reserve (FRED).

The period length is set to be a year. The average annualized nominal interest rate for three-year constant maturity Treasury Bills during 1962-2006 was 6.7%. Annual inflation (based on the consumption deflator) over the same period averaged at 3.9%. By the Fisher equation (10), these two statistics imply an average real interest rate of 2.7%, consistent with a household discount factor of $\beta = 0.97$. The fraction of time devoted to labor is set to 0.3. Accordingly, from (1), GDP is $c + g = n = 0.3$. Average government outlays over 1962-2006 amounted to 20.4% of GDP. This implies a value of $g = 0.0612$. Federal revenue (which does not include loans or seignorage) over the sample period amouted to a fraction of 18.2% of GDP on average; this is consistent with a tax rate of $\tau^c = 22.9\%$.[15] From (9), we can then infer $\alpha = 3.19$. Finally, z^{a*} is obtained as the steady state level of real debt that satisfies the government budget constraint.

[15] While not used for the calibration, the average annual budget deficit over 1962-2006 was 2.2% of GDP.

In particular, given (α, β, g) and the steady state value of consumption $c(z^{a*}) = 0.2388$ and money growth $\mu(z^{a*}) = 0.039$, implementability constraint (28) delivers $z^{a*} = 0.4629$. In terms of end-of-period real debt, i.e., $b^{a\prime} \equiv \frac{B^{a\prime}}{P}$, this means that the steady state features real debt of $b^{a*} = z^{a*}\frac{\beta}{\alpha}(1+R)^{-1} = 0.1319$ or a debt-to-GDP ratio of 44.0%. This compares to an empirical average ratio of gross federal debt to GDP of 49.1%, whereby the statistic for debt held by the public (i.e., excluding holdings of federal agencies) is at 35.8%.

It remains to pin down the institutional parameters δ and γ. In want of a better theory, we take fiscal impatience as a given fact and fix the fiscal discount factor at $\delta = 0.96$. This reflects a fiscal authority which discounts the future at an annual rate of 4.2% compared to the real interest rate of 2.7%. Next, from Proposition 2, we infer that the steady state elasticity of money growth is given by $\varepsilon_\mu(z^{a*}) = \frac{\beta-\delta}{\beta} = 0.0103$. Given the other parameters and the induced steady state values, the value of γ is then implicitly defined via the monetary optimality condition (34) evaluated at the steady state. This procedure yields $\gamma = 0.0635$. Table 1 summarizes the calibrated parameters and the associated steady state values of endogenous variables.[16]

Table 1: Benchmark calibration and steady state statistics

α	β	γ	δ	g	c	z^a	$\frac{g}{c+g}$	b^a	$\frac{b^a}{c+g}$
3.19	0.97	0.0635	0.96	0.0612	0.2388	0.4629	0.204	0.1319	0.4397

4.5 Dynamics

Based on above calibration, we simulate the model in order to illustrate its dynamics. The key results of this exercise are displayed in the two plots of Figure 1. The left plot shows the dynamic evolution of the end-of-period stock of real government debt z^a. Starting from $z^a < z^{a*}$, the stock of real debt grows at a decreasing rate and converges to a debt ceiling corresponding to a debt-to-GDP ratio of 44.0%. The increasing distortions associated with the accumulation of debt - direct ones due to the need to service the debt via distortionary tax instruments and indirect ones stemming from the additional interest rate distortions - affect the pattern of consumption displayed in the right plot.

According to the monetary GEE (34), it must be the case that, at z^{a*}, the marginal losses incurred due to inflation and the marginal benefits from stabilizing the level of debt by monetizing fiscal deficits via the inflation tax are equal from the monetary authority's perspective. At the same time, the responsiveness of monetary policy to the level of debt makes the accumulation of debt increasingly unattractive for the fiscal authority, since it suffers from the distortionary effects caused by the public's increasing inflation expectations, too. Thus, despite its impatience, the fiscal authority has an incentive not to accumulate debt without bounds. In other words, the steady-state level z^{a*} of public liabilities must be such that the motives for debt accumulation and decumulation exactly balance each other from both authorities' perspectives.

[16]The calibrated parameters are consistent with existence of a steady state.

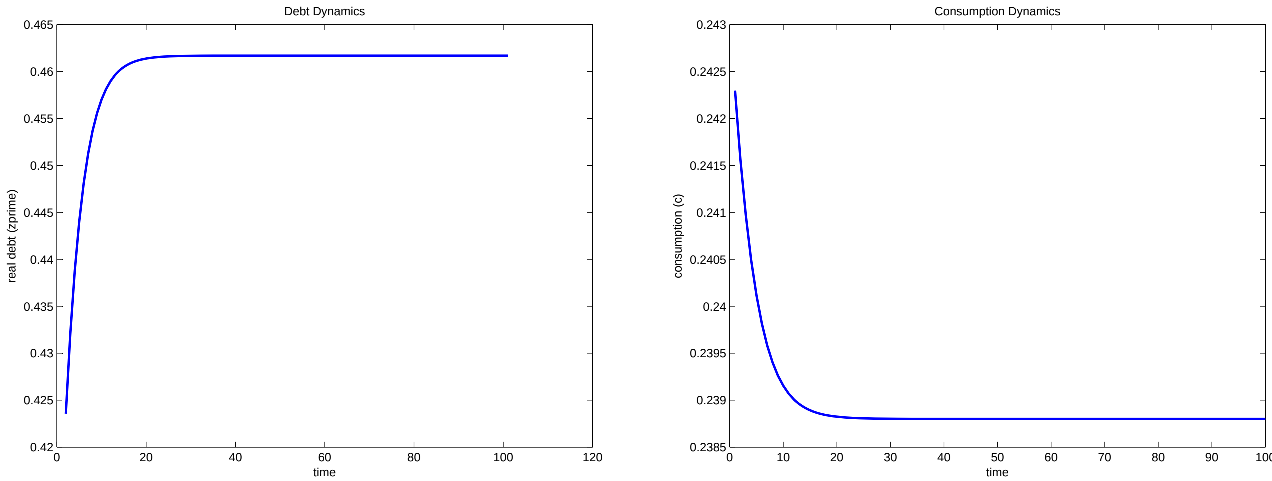

Figure 1: Debt and consumption dynamics

4.6 Comparative statics

Against the background of these results, it is interesting to investigate how changes in the two authorities' preference parameters impinge on the properties of the equilibrium outcomes. First, consider the effect of a lower fiscal discount factor δ, holding β fixed. From Proposition 3, steady state consumption $c(z^{a*})$ is unaffected by δ. In conjunction with condition (26), this means that government revenue at the steady state, as measured by $(1+\mu(z^{a*}))(1+\tau^c(z^{a*}))c(z^{a*})$, remains unchanged. However, this implies that the steady state amount of real debt z^{a*} itself must be unaffected by the change in δ. Finally, equations (35) to (38) reveal that this is also true for taxes, money growth, interest rates and inflation. Although the steady state values of the policy instruments remain unchanged, the drop in the ratio $\frac{\delta}{\beta}$ implies that $\varepsilon_\mu(z^{a*})$, the steady state elasticity of money growth, must increase, as can be inferred from the fiscal GEE (32). This means that a more impatient fiscal authority triggers a monetary policy which must be (locally) more responsive to variations in the stock of debt. Consequently, the indirect liability costs of government debt are accentuated. Since this is internalized by the fiscal authority, there are two forces at work: On the one hand, increased fiscal impatience gives rise to a more pronounced tendency to accumulate debt; but on the other hand, the extra liability costs of outstanding debt increase. It turns out that the two mechanisms just offset each other in terms of their effects on the steady state level of the endogenous state variable z^a. We can summarize our findings in the following Proposition.

Proposition 5 *Given β and γ, the steady state allocation $(z^{a*}, c(z^{a*}))$ is immune to changes in the fiscal discount factor δ. Steady state taxes and money growth are unaffected, but $\varepsilon_\mu(z^{a*})$, the steady state elasticity of money growth, increases as δ decreases.*

Next, consider what happens if γ, the monetary authority's aversion against surprise inflation, is increased. From Proposition 2, the degree of monetary responsiveness at the steady state is pinned down at $\varepsilon_\mu(z^{a*}) = \frac{\beta-\delta}{\beta}$. Since $\frac{\beta-\delta}{\beta}$ remains unchanged, the value for $\varepsilon_\mu(z^{a*})$ must be influenced by two countervailing effects which neutralize each other: The *direct effect* of a higher $\gamma \in (0,1)$ is to equip the monetary authority with an improved commitment capacity against inflationary monetary expansions. The reason for this is that a higher weight on the inflationary loss term amplifies the monetary authority's excess preference for consumption relative to leisure. But for a given fiscal policy, condition (26) implies that a higher consumption allocation is only possible with a reduced rate of money growth. Consequently, for any given level of real debt z^a, the value for $\varepsilon_\mu(z^a)$ and thus the indirect liability costs of sustaining any given level of real government debt decrease. Importantly, however, an additional *indirect effect* kicks in. Specifically, the strategic response of the fiscal authority to the monetary authority's increased (but incomplete) commitment capacity is to accumulate more debt. Hence, while the adverse welfare consequences of any given amount of real debt diminish, z^{a*}, the steady state level of real government debt, increases.

Proposition 6 *Given β and δ, the degree of monetary inflation aversion $\gamma \in (0,1)$ does not affect $\varepsilon_\mu(z^{a*})$, the steady state elasticity of money growth. An increase in γ implies that $\varepsilon_\mu(z^a)$ is lower for any given z^a. In the neighborhood of a stable steady state, since $\varepsilon_\mu(z^a)$ is increasing in z^a, an increase in γ induces the accumulation of a higher steady state level of debt z^{a*}.*

This proposition has the remarkable implication that a more conservative central bank, i.e., a monetary authority which is more averse against the surprise use of its inflation tax instrument, will generally not be more successful in containing the accumulation of public debt. This theoretical finding is confirmed by the first part of Table 2, which displays the change in steady state indebtedness and inflation induced by a variation in γ. The following pattern emerges: Monetary conservatism is a successful commitment device to constrain the monetary accommodation of fiscal profligacy. Hence, for any given level of debt z^a, the higher γ, the lower the recourse to the inflation tax. But since this advantageous commitment effect is internalized by the impatient fiscal authority, the latter has an incentive to accumulate more debt. This is because monetary conservatism helps to reduce the interest rate distortions and thus the crowding out of consumption associated with outstanding public debt. In the MPE, therefore, both the steady state stock of debt and the rate of inflation are increasing in γ.

For economic environments where a monetary time-consistency problem is a concern, the conventional presumption is that monetary conservatism has positive welfare implications. In contrast, the present analysis establishes that the welfare gains during the transition to the steady state (the direct effect) must be weighted against the costs of inducing a steady state with higher accumulation of real liabilities (the indirect effect). The second part of Table 2 presents results from this exercise. Row A compares steady state allocations only, while row B incorporates the effects of the transition. We find that the transitory gains from monetary conservatism are overcompensated by the long-run costs.

The endogenous determination of macroeconomic policies implies that the economies with fiscal impatience and monetary conservatism differ from the benchmark economy ($\gamma = 0$, $\delta = \beta$) not merely in terms of feedback rules in the neighborhood of a given reference point, but in their selection of distinct steady states. Thus, the welfare losses associated with the design of fiscal and monetary policy authorities are considerable. For example, doubling the inflation aversion parameter γ from 0.0635 to 0.127 increases the welfare loss from -0.0646 to -0.1413 percent of steady state consumption. Importantly, these welfare losses from monetary conservatism arise in a deterministic environment; in a stochastic economy, there result additional costs from suboptimal stabilization policies (Rogoff, 1985). In this sense, the welfare losses presented here should be interpreted as a lower bound.

Table 2: Steady state implications of monetary conservatism

γ	0.05	0.0635	0.08	0.1	0.127	0.015	0.2
z^a	0.3637	0.4629	0.5857	0.7359	0.9416	1.1218	1.5265
$\frac{b^a}{c+g}$	0.3454	0.4393	0.5553	0.6969	0.8903	1.0593	1.4372
π	0.0322	0.0361	0.0409	0.0462	0.0542	0.0610	0.0762
welfare loss							
A	-0.3026	-0.3914	-0.5017	-0.6243	-0.8128	-0.9740	-1.3422
B	-0.0472	-0.0646	-0.0863	-0.0995	-0.1413	-0.1764	-0.2721

All parameters except γ are at their benchmark values indicated in Table 1. Welfare losses are converted in consumption equivalents and calculated in percentage terms relative to consumption in the distorted steady state $\{\hat{z}_2^{a*} = 0,\, c_2(\hat{z}_2^{a*}) = (\beta/\alpha - g)\}$ of an economy with purely benevolent monetary and fiscal policy makers ($\gamma = 0$, $\delta = \beta$). Row A compares steady state allocations only. Row B takes into account the transition, starting from the steady state induced by the respective γ.

5 Conclusion

This paper examines dynamic monetary-fiscal interactions in a flexible price economy with nominal government debt. The starting point is the assumption that monetary and fiscal policies are directed by separate authorities who cannot commit over time and who pursue different objectives: The fiscal authority is relatively impatient, and the monetary authority is weight-conservative, i.e., inflation-averse. The government's stock of real liabilities is an endogenous state variable which shapes the intensity of the monetary time-consistency problem.

The paper's contribution is a positive one: Assuming fiscal impatience and monetary conservatism, it analyzes time-consistent policies in order to understand how monetary and fiscal institutions affect equilibrium outcomes. Our results challenge a number of the basic tenets of the existing literature on monetary-fiscal interactions. In part, this is reflects the fact that we do not consider stabilization policies in the neighborhood of a given steady state (e.g. Leeper, 1991), but rather focus on the determination of the steady state itself.

We find that the degree of fiscal impatience does not affect the steady state level of debt. However, in the MPE, it does determine the monetary responsiveness to variations in the stock of debt around its steady state level. In this sense, inflation is to be seen as a fiscal phenomenon. While the degree of monetary conservatism does not influence the steady state rate of monetary responsiveness, it is one key determinant of the economy's steady state level of debt. Specifically, increased conservatism implies that any given level of real liabilities can be sustained at a lower rate of inflation. However, since the impatient fiscal authority internalizes the economy's improved debt tolerance, the MPE outcome features a steady state with higher indebtedness and inflation. When assessing the desirability of monetary conservatism for a calibrated economy, we find that the latter effect dominates such that increased monetary conservatism has adverse welfare implications. This result has an obvious second-best nature and is similar in spirit to Lippi (2003). At the same time, it is in contrast to established findings in much of the previous literature.

Our analysis has taken the degree of fiscal impatience as a given primitive of the model. Such lack of fiscal discipline is generally seen as a rationale for the imposition of fiscal constraints. Within the framework considered, such constraints should be designed to provide a ceiling to the maximum admissible amount of real debt. Establishing limits on fiscal deficits can help as an auxiliary device, because, under a binding constraint on primary deficits, the transition to the steady state proceeds along a path featuring lower rates of inflation. However, we conjecture that deficit rules alone will be counterproductive: They require an increased responsiveness of taxes to variations in the stock of debt. Insofar as this induces an upper bound for the degree of monetary responsiveness at any given level of real debt, the present paper has established that the MPE outcome will involve the accumulation of higher amounts of debt, resulting in increased steady state inflation. Hence, similar to monetary conservatism also deficit rules could be harmful from a welfare perspective. We plan to formally address this question in future research.

Bibliography

Adam, K. and Billi, R., 2008a. Distortionary Fiscal Policy and Monetary Policy Goals. Mimeo, University of Mannheim.

Adam, K. and Billi, R., 2008b. Monetary Conservatism and Fiscal Policy. *Journal of Monetary Economics*, forthcoming.

Barro, R. and Gordon, D., 1983. A Positive Theory of Monetary Policy in a Natural Rate Model. *Journal of Political Economy* 91, 589-610.

Chari, V. and Kehoe, P., 1990. Sustainable Plans. *Journal of Political Economy* 98, 783-802.

Chari, V. and Kehoe, P., 1999. Optimal Fiscal and Monetary Policy. In: J. Taylor and Woodford, M. (Eds.), Handbook of Macroeconomics, Vol. 1, Elsevier, Amsterdam, pp. 1671-1745.

Council of Economic Advisers, 2008. Economic Report of the President. U.S. Government Printing Office, Wahsington.

Díaz-Giménez, J., Giovanetti, G., Marimon, R. and Teles, P., 2008. Nominal Debt as a Burden on Monetary Policy. *Review of Economic Dynamics* 11, 493-514.

Federal Reserve Bank of St. Louis, 2008. Federal Reserve Economic Data. At http://research.stlouisfed.org/fred2/.

Klein, P., Krusell, P. and Ríos-Rull, J.-V., 2008. Time-Consistent Public Policy. *Review of Economic Studies* 75, 789-808.

Klein, P., Quadrini, V. and Ríos-Rull, J.-V., 2005. Optimal Time-Consistent Taxation with International Mobility of Capital. *B.E. Journals - Advances in Macroeconomics* 5, No. 1, Article 2.

Kydland, F. and Prescott, E., 1977. Rules Rather Than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* 85, 473-492.

Leeper, E., 1991. Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies. *Journal of Monetary Economics* 27, 129-147.

Lippi, F., 2003. Strategic Monetary Policy with Non-Atomistic Wage Setters. *Review of Economic Studies* 70, 909-919.

Lucas, R. and Stokey, N., 1983. Optimal Fiscal and Monetary Policy in an Economy without Capital. *Journal of Monetary Economics* 12, 55-93.

Malley, J., Philippopoulos, A. and Woitek, U., 2007. Electoral Uncertainty, Fiscal Policy and Macroeconomic Fluctuations. *Journal of Economic Dynamics and Control* 31, 1051-1080.

Martin, F., 2009. A Positive Theory of Government Debt. *Review of Economic Dynamics*, forthcoming.

Nicolini, J.-P., 1998. More on the Time Consistency of Monetary Policy. *Journal of Monetary Economics* 41, 333-350.

Niemann, S., Pichler, P. and Sorger, G., 2008. Optimal Fiscal and Monetary Policy without Commitment. University of Essex Discussion Paper No. 654.

Ortigueira, S. and Pereira, J., 2007. Markov-Perfect Optimal Fiscal Policy: The Case of Unbalanced Budgets. Mimeo, European University Institute.

Persson, T. and Tabellini, G., 2000. *Political Economics: Explaining Economic Policy*, MIT Press, Cambridge.

Rogoff, K., 1985. The Optimal Degree of Commitment to an Intermediate Monetary Target. *Quarterly Journal of Economics* 100, 1169-1190.

Sargent, T. and Wallace, N., 1981. Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review* 5, 1- 17.

Svensson, L., 1985. Money and Asset Prices in a Cash-in-Advance Economy. *Journal of Political Economy* 93, 919-944.

Svensson, L., 1997. Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts. *American Economic Review* 87, 98-114.

Walsh, C., 1995. Optimal Contracts for Central Bankers. *American Economic Review* 85, 150-167.

A Appendix

A.1 The authorities' primal problems

A.1.1 The fiscal problem

The fiscal authority's problem is:

$$\hat{V}^f(z^a;\varphi) = \max_{c,z^{a\prime}} \left\{ u(c) - v(c+g) + \delta V^f(z^{a\prime};\varphi) \right\} \tag{40a}$$

subject to:

$$\frac{\beta}{\alpha} + \beta\frac{\beta}{\alpha}\frac{z^{a\prime}}{(1+\mu(z^{a\prime}))} - c - \frac{\beta}{\alpha}\frac{z^a}{(1+\mu)} - g = 0. \tag{40b}$$

Let λ^f denote the multiplier on the fiscal implementability constraint (40b). For $u(c) = \log(c)$ and $v(n) = \alpha n$, the first order conditions with respect to c and $z^{a\prime}$ then are:

$$\frac{1}{c} - \alpha - \lambda^f = 0, \tag{41}$$

$$\delta V_z^f(z^{a\prime};\varphi) + \lambda^f \beta\frac{\beta}{\alpha}(1+\mu(z^{a\prime}))^{-1}\left[1-\varepsilon_\mu(z^{a\prime})\right] = 0, \tag{42}$$

where $\varepsilon_\mu(z^{a\prime}) \equiv \frac{\partial(1+\mu(z^{a\prime}))/\partial z^{a\prime}}{(1+\mu(z^{a\prime}))/z^{a\prime}}$ is the elasticity of future money growth with respect to the aggregate state $z^{a\prime}$. The (forwarded) envelope condition is:

$$V_z^f(z^{a\prime};\varphi) = -\lambda^{f\prime}\frac{\beta}{\alpha}(1+\mu(z^{a\prime}))^{-1}. \tag{43}$$

Conditions (41) and (42) can be combined to eliminate λ^f, which yields:

$$\frac{1}{c} - \alpha = -\frac{\frac{\delta}{\beta}V_z^f(z^{a\prime};\varphi)}{\frac{\beta}{\alpha}(1+\mu(z^{a\prime}))^{-1}}\left[1-\varepsilon_\mu(z^{a\prime})\right]^{-1} = -\frac{\frac{\delta}{\beta}V_z^f(z^{a\prime};\varphi)}{(1+\tau^c(z^{a\prime}))c(z^{a\prime})}\left[1-\varepsilon_\mu(z^{a\prime})\right]^{-1}, \tag{44}$$

where the last transformation exploits (26). Finally, the envelope condition (43) and (41) can be used to substitute for the derivative of the value function to obtain the fiscal GEE:

$$\frac{1}{c} - \alpha = \frac{\delta}{\beta}\left(\frac{1}{c(z^{a\prime})} - \alpha\right)\left[1-\varepsilon_\mu(z^{a\prime})\right]^{-1}. \tag{45}$$

A.1.2 The monetary problem

The monetary authority's problem is:

$$\hat{V^m}(z^a;\varphi) = \max_{c,z^{a\prime}} \left\{ -\gamma\left(\frac{(1+\tilde{\tau}^c(z^a))\tilde{c}(z^a)}{(1+\tau^c)c}\right)^2 + (1-\gamma)[u(c)-v(c+g)] + \beta V^m(z^{a\prime};\varphi)\right\} \tag{46a}$$

subject to:

$$\frac{\beta}{\alpha} + \beta z^{a\prime}(1+\tau^c(z^{a\prime}))c(z^{a\prime}) - c - z^a(1+\tau^c)c - g = 0. \tag{46b}$$

Let λ^m denote the multiplier on the monetary implementability constraint (46b). For $u(c) = \log(c)$ and $v(n) = \alpha n$, the first order conditions with respect to c and $z^{a\prime}$ then are:

$$2\gamma\frac{1}{c} + (1-\gamma)\left(\frac{1}{c} - \alpha\right) - \lambda^m[1 + z(1+\tau^c)] \;=\; 0, \tag{47}$$

$$\beta V_z^m(z^{a\prime};\varphi) + \lambda^m\beta(1+\tau^c(z^{a\prime}))c(z^{a\prime})\left[1 - \varepsilon_\mu(z^{a\prime})\right] \;=\; 0, \tag{48}$$

where the derivation of (48) exploits that, as a consequence of (26), $-\varepsilon_\mu(z^{a\prime}) = \varepsilon_{(1+\tau^c)c}(z^{a\prime}) \equiv \frac{\partial(1+\tau^c(z^{a\prime}))c(z^{a\prime})/\partial z^{a\prime}}{(1+\tau^c(z^{a\prime}))c(z^{a\prime}))/z^{a\prime}}$. Again making use of this relationship, the (forwarded) envelope condition (for $z^{a\prime} \neq 0$) is:

$$V_z^m(z^{a\prime};\varphi) = 2\gamma\frac{\varepsilon_\mu(z^{a\prime})}{z^{a\prime}} - \lambda^{m\prime}(1+\tau^c(z^{a\prime}))c(z^{a\prime}). \tag{49}$$

Conditions (47) and (48) can be combined to eliminate λ^m, which yields:

$$\frac{2\gamma\frac{1}{c} + (1-\gamma)\left(\frac{1}{c} - \alpha\right)}{[1 + z(1+\tau^c)]} = -\frac{V_z^m(z^{a\prime};\varphi)}{(1+\tau^c(z^{a\prime}))c(z^{a\prime})}\left[1 - \varepsilon_\mu(z^{a\prime})\right]^{-1}. \tag{50}$$

Finally, the envelope condition (49) and (47) can be used to substitute for the derivative of the value function to obtain the monetary GEE (for $z^{a\prime} \neq 0$):

$$\frac{2\gamma\frac{1}{c} + (1-\gamma)\left(\frac{1}{c} - \alpha\right)}{[1 + z^a(1+\tau^c)]} = \left[\frac{2\gamma\frac{1}{c(z^{a\prime})} + (1-\gamma)\left(\frac{1}{c(z^{a\prime})} - \alpha\right)}{[1 + z^{a\prime}(1+\tau^c(z^{a\prime}))]} - \frac{2\gamma\frac{\varepsilon_\mu(z^{a\prime})}{c(z^{a\prime})}}{z^{a\prime}(1+\tau^c(z^{a\prime}))}\right]\left[1 - \varepsilon_\mu(z^{a\prime})\right]^{-1}. \tag{51}$$

A.2 Analytical results

A.2.1 Proof of Proposition 1

In a steady state, $z^a = z^{a\prime} = z^{a*}$ and $c(z^a) = c(z^{a\prime}) = c(z^{a*})$ such that the two GEEs (32) and (34) are satisfied. For $\delta = \beta$ and $\gamma = 0$, (32) and (34) coincide. Accordingly, both optimality conditions are satisfied in the following two cases:

(i) $c(z^{a*}) = \frac{1}{\alpha}$: Since this is an undistorted allocation, (16) and (17) together imply $(1 + \tau^c(z^{a*})) = 1$ and $(1 + \mu(z^{a*})) = \beta$ (the Friedman rule). Using this in either implementability constraint (27) or (28), we get $z^{a*} = -1 - \frac{\alpha}{1-\beta}g$.

(ii) $c(z^{a*}) < \frac{1}{\alpha}$ and $\varepsilon_\mu(z^{a*}) = \frac{\partial(1+\mu(z^a))/\partial z^a}{(1+\mu(z^a))/z^a}\big|_{z^a=z^{a*}} = 0$: Since this is a distorted allocation, we must have $\frac{\partial(1+\mu(z^a))}{\partial z^a}\Big|_{z^a=z^{a*}} = -\frac{\partial(1+\tau^c(z^a))c(z^a)}{\partial z^a}\big|_{z^a=z^{a*}} > 0$, for otherwise debt would be locally explosive. [This is formally seen from either implementability constraint (27) or (28).] Hence, since $\varepsilon_\mu(z^{a*}) = 0$, we get $z^{a*} = 0$. Either implementability constraint (27) or (28) then implies $c(z^{a*}) = \frac{\beta}{\alpha} - g$.

A.2.2 Proof of Proposition 2

The fiscal GEE (32) immediately implies that $\varepsilon_\mu(z^{a*}) = \frac{\beta-\delta}{\beta} > 0$. In the neighborhood of $z^a = z^{a*}$, if $\varepsilon_\mu(z^a) < \frac{\beta-\delta}{\beta}$, then it follows from (32) that $(1/c(z^a) - \alpha) < (1/c(z^{a\prime}) - \alpha)$. Since $c(z^a)$ is decreasing, it follows that $z^a < z^{a\prime}$. Hence, real debt is accumulated. If z^{a*} is a stable steady state, this means that $z^a < z^{a\prime} < z^{a*}$. Conversely, if $\varepsilon_\mu(z^a) > \frac{\beta-\delta}{\beta}$, then it follows from (32) that $(1/c(z^a) - \alpha) > (1/c(z^{a\prime}) - \alpha)$. Since $c(z^a)$ is decreasing, it follows that $z^a > z^{a\prime}$. Hence, real debt is decumulated. If z^{a*} is a stable steady state, this means that $z^a > z^{a\prime} > z^{a*}$. By continuity, these observations together imply that $\varepsilon_\mu(z^a)$ is an increasing function in the neighborhood of a stable steady state.

A.2.3 Proof of Proposition 3

From Proposition 2, the elasticity of money growth at the steady state is given by $\varepsilon_\mu(z^{a*}) = \frac{\beta-\delta}{\beta} > 0$. The monetary GEE (34), evaluated at the steady state, then implies:

$$(1 + \mu(z^{a*})) = \frac{\beta}{2\gamma\alpha} z^{a*}(1-\gamma)\left(\frac{1}{c(z^{a*})} - \alpha\right). \tag{52}$$

Making use of condition (26) in (52), we get:

$$(1 + \tau^c(z^{a*})) = \left[\frac{1}{2\gamma} z^{a*}(1-\gamma)\left(1 - \alpha c(z^{a*})\right)\right]^{-1}. \tag{53}$$

Next, combining (53) with the household decision rule (16), we get:

$$(1 + R(z^{a*})) = \frac{1}{2\gamma\alpha} z^{a*}(1-\gamma)\left(\frac{1}{c(z^{a*})} - \alpha\right). \tag{54}$$

Finally, (54) and the household decision rule (17) at the steady state together yield:

$$(1 + \pi(z^{a*})) \quad = \quad \frac{\beta}{2\gamma\alpha} z^{a*}(1-\gamma)\left(\frac{1}{c(z^{a*})} - \alpha\right). \tag{55}$$

In order to solve for steady state consumption $c(z^{a*})$, we use (52) / (53) to substitute in the implementability constraint (27) / (28). This yields a quadratic equation in $c(z^{a*})$:

$$\alpha c(z^{a*})^2 - \left[1 + \frac{(1-\beta)2\gamma}{(1-\gamma)} + \alpha\left(\frac{\beta}{\alpha} - g\right)\right] c(z^{a*}) + \left(\frac{\beta}{\alpha} - g\right) = 0. \tag{56}$$

Equation (56) implicitly characterizes steady state consumption $c(z^{a*})$, which does not depend on δ.

A.2.4 Proof of Proposition 4

Let $\mathcal{C}(z^a)$ and $\mathcal{Z}^{a\prime}(z^a)$ be the primal policy functions which jointly solve problems (29) and (30). Formally, the steady state level of real debt z^{a*} is a fixed point in the mapping $\mathcal{Z}^{a\prime}(z^a)$.

Equivalently, the steady state level of real debt is given by $z^{a*} = \mathcal{C}^{-1}(c(z^{a*}))$, where $c(z^{a*})$ solves (39).

Due to the non-negativity of consumption, negative roots of equation (39) are ruled out as admissible solutions. Hence, if there is no positive root, then there exists no steady state in differentiable policy rules. If there is only one positive root $c(z^{a*})$, then $z^{a*} = \mathcal{C}^{-1}(c(z^{a*}))$ is the unique steady state generated by differentiable policy rules. In the case of two positive roots, denote them by $c_1(z_1^{a*})$ and $c_2(z_2^{a*})$, where $c_2(z_2^{a*}) < c_1(z_1^{a*})$. Since $\mathcal{C}(z^a)$ is a decreasing function, it follows that $z_1^{a*} < z_2^{a*}$.

The monetary GEE (34) implies that, as $z^a \to 0$, $\varepsilon_\mu(z^a) \to 0$. From the fiscal GEE (32), $\varepsilon_\mu(z^a) = 0$ together with the fact that $\mathcal{C}(z^a)$ is decreasing implies that, at $z^a = 0$, $z^{a\prime} > 0$. Hence, $z^a = 0$ cannot be a steady state, and at $z^a = 0$, the mapping $\mathcal{Z}^{a\prime}(z^a)$ is above the 45^o line in the $(z^a, z^{a\prime})$ space. Focusing on differentiable policy rules, the mapping $\mathcal{Z}^{a\prime}(z^a)$ is continuous. The stability of steady states therefore hinges on whether the mapping $\mathcal{Z}^{a\prime}(z^a)$ intersects the 45^o line from above (stable) or below (unstable). If the steady state is unique, we have:

(i) If $0 < z^{a*}$, then z^{a*} is stable;

(ii) if $z^{a*} < 0$, then z^{a*} is unstable.

If there are two steady states, three cases can arise:

(i) If $0 < z_1^{a*} < z_2^{a*}$, then z_1^{a*} is stable, z_2^{a*} is unstable;

(ii) if $z_1^{a*} < 0 < z_2^{a*}$, then z_1^{a*} is unstable, z_2^{a*} is stable;

(iii) if $z_1^{a*} < z_2^{a*} < 0$, then z_1^{a*} is stable, z_2^{a*} is unstable.

This completes the description of steady states in differentiable policy rules.