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WAGE-EXPERIENCE CONTRACTS AND EMPLOYMENT STATUS

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Abstract

The objective of this paper is to study equilibrium in a labour market in which identical firms post wage-contracts and ex-ante identical workers search on the job. The main novelty of this paper is to generate dispersion in contract offers by allowing firms to condition their offers on workers' initial experience and employment status although these characteristics do not affect productivity. In this context I show that changes in firms' information set at the moment of recruiting can have strong effects on wage dispersion and turnover without changing the agents' payoffs. I construct an equilibrium in which firms compete in promotion contracts. Employed and more experience workers are offered better contracts with shorter time-to-promotion periods. This implies contract offers are disperse within and between experience levels. The earnings distribution within the firm is then such that workers who have acquired more "outside" firm experience and more tenure are higher in the earnings scale. This generates workers cohort effects within a firm that depend on the level of experience at which they where hired.

Keywords: Search, wage dispersion, recruitment, experience, employment status, promotions.
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1 Introduction

This paper contributes to the emerging equilibrium search literature that considers non-stationary firm wage policies to analyse wage distributions and labour market transitions. Using a model somewhat similar to Stevens (2004), I show that there exists an equilibrium in which firms offer contracts conditional upon workers' initial experience and employment status for reasons other than productivity. In this equilibrium firms compete for workers using promotion contracts. The distribution of outside offers for each experience level is described by two mass points, one for each employment status. Firms offer "bad" jobs with longer time-to-promotion periods to unemployed workers and "good" jobs with shorter ones to employed workers. Turnover occurs in the direction of bad towards good jobs. Furthermore, outside offers are disperse across experience levels and become more generous with experience for employed workers.

The main novelty of the paper is to provide a new foundation for equilibrium wage dispersion among ex-ante similar agents. Dispersion in contract offers is generated by considering experience and employment status to be focal points. As discussed by Schelling (1960) and Myerson (1997), focal points are exogenous characteristics that agents use to coordinate their actions and achieve a particular equilibrium. They can be determined by societal norms, the status quo, preplay communication, payoff differentials or even seemingly trivial aspects. For example, firms might expect other firms to contract upon experience and employment status because of well established recruitment policies. In turn, these beliefs may imply all firms contract upon them even though they do not affect workers' productivity.

In particular, the empirical evidence presented by Medoff and Abraham (1980) and Manning (2000) seems to suggest that firms do not necessarily contract upon experience and employment status because of human capital reasons. Using personnel data of two US manufacturing companies, the former study shows that wages among jobs of the same difficulty are positively correlated with workers' initial experience even after controlling for productivity. On the other hand, Manning shows that a simple model of on-the-job search and job displacement can explain around 50% of the observed experience-earnings profile without relying on human capital explanations.[1]

In recent years equilibrium search models á la Burdett and Mortensen (1998) have become increasingly popular due to their ability to provide a rich theory of wage dispersion and turnover.[2] In this framework, workers' on-the-job search is a necessary condition to obtain a continuous wage distribution. The implied dispersion in reservation wages among workers enables identical firms to differentiate their wage policies while obtaining the same total profits by trading-off profits per worker with the size of their labour force.[3]

However, this theory has been recently criticised in two important ways. First, bounding firms to offer a single wage is not profit maximising. It is precisely the possibility of continuous job search that gives firms the incentive to deviate from offering a constant wage. Given no financial markets, the optimal contract is described by an upward sloping wage-tenure profile. Stevens (2004) shows that in the case of risk neutral workers the optimal contract can be characterised by a step-contract. She shows that when agents are homogeneous the distribution of outside offers degenerates to a single mass point at the reservation value of unemployed workers, eliminating the effects of workers' on-the-job search.

[1] Moreover, Manning (2003) uses the Displaced Worker Survey for the US and shows that the earnings losses of these workers are positive correlated with experience, a prediction that contradicts the standard human capital model. Similar evidence was also found by Jacobson, LaLonde and Sullivan (1993) using administrative data for the state of Pennsylvania and by Burda and Mertens (2001) using the German Socioeconomic Panel.

[2] From a theoretical point of view this theory provides an important contribution that eludes the so called Diamond's paradox. Diamond (1971) showed that in a wage posting game with identical agents where workers search while unemployed there is no wage dispersion in equilibrium. The unique offer distribution is described by a single mass point at the common reservation wage. If search is costly, no matter how small the cost, workers will not participate in the market.

[3] An important literature has also been developed in testing these models empirically. See Van den Berg (1999), Mortensen (2003) and Manning (2003) for recent surveys and interesting applications.

A second critique comes from the assumptions made on the information available to the firm when recruiting workers and when confronted with outside competition for its employees. The Burdett and Mortensen framework assumes firms have no information about workers other than their productivity and opportunity cost of employment. As a result firms always offer the same wage to any worker, missing out potential profits. Postel-Vinay and Robin (2002) analyse the case of complete information and offer-matching, in which firms perfectly discriminate workers by their reservation wages. When agents are homogenous, workers are effectively offered a step-contract with a random promotion date. The wage offer distribution is described by a mixture of two mass points: one at the common reservation wage and the other at the worker's marginal productivity. As in Stevens, no worker ever quits his employer and firms are able to extract the entire match rents.

In the present paper firms pre-commit not to counter-offer any outside offers but observe workers' experience and employment status when posting their contract offers.[4] As mentioned earlier, in this case wage dispersion occurs within and between experience levels. Moreover, firms obtain the same profits as in Stevens and Postel-Vinay and Robin. Hence, this paper's second main result is to shows that when firms use optimal contracts, changes in their information set at the moment of recruiting can have strong effects on wage dispersion and turnover without changing the agents' payoffs.

In a related paper, Burdett and Coles (2003) extend Stevens' model and show that if workers are risk averse and liquidity constraint, firms' incentive to backload wages is tempered by workers' desire to smooth consumption over time. This trade-off implies an

[4] Atkinson, Giles and Meager (1996) present evidence for the UK showing that firms have information on these characteristics. Manning (2003) discusses why offer-matching seems to be a relatively rare practice in many labour markets, in particular in those of unskilled labour. Postel-Vinay and Robin (2004) developed a model in which a firm's choice to counteroffer is endogenously determined. They showed that under certain conditions firms that offer low-productivity jobs will not choose to counteroffer, while firms that offer high productivity jobs will.

upward sloping and concave wage-tenure profile. A continuous wage distribution survives in equilibrium for the same reasons as in Burdett and Mortensen.

The present paper differs from theirs in several aspects. As in Burdett and Mortensen, their model assumes firms have no information about workers. Second one of their aims is to show that risk neutrality is determinant for Stevens' degeneracy results. Adding risk aversion to the present framework is not a trivial task and beyond the scope of this paper. To the best of my knowledge, this paper is the first attempt to analyse recruitment and retention strategies in a non-stationary equilibrium search environment in which firms have a coarse information set about workers' characteristics. Furthermore, given it delivers reasonable empirical implications, I argue that this framework can be useful to study the effects of recruitment and retention policies on the distribution of wages observed within a firm and analyse how search frictions can influence a firms' internal wage structure.

The rest of the paper is divided as follows. The next sections describe the general framework and the workers' and firms' decision problems. Section 5 shows that the optimal wage-tenure contract is a step-contract for each level of initial experience and employment status. Sections 6 and 7, define and construct the market equilibrium. Section 8 shows existence of equilibrium. The last section further discusses the results and concludes.

2 Basic Framework

Consider a labour market in steady state in which time is continuous and there is a fixed number of workers and firms each of measure one. Workers and firms are homogeneous in that any firm generates revenue p for each worker it employs per unit of time.[5] Workers can either be employed (e) or unemployed (u) with experience, x, defined as total time spent in

[5] In an earlier version, see Carrillo-Tudela (2004a), I consider the case in which productivity does increase with experience and show that firms will still have incentives to condition their offers on workers' initial experience to exploit their monopsony power.

all previous employments. Firms post job offers at a zero cost on a take it or leave it basis. Both unemployed and employed workers of any experience search. Let $0 < \lambda < \infty$ denote the common Poisson arrival rate of these offers. Assume there is no recall should a worker quit or reject a job offer.

A job offer is described by a wage contract. Upon a meeting firms are able to observe the worker's experience and labour market status and condition their offers upon these characteristics. An important assumption is that firms pre-commit to pay the worker the wages specified in the contract and not to counter-offer any outside offer the worker might receive in the future. Contracts are then contingent on the worker's tenure, t, defined as time spent working on the firm. A job offer is fully described by a wage-tenure contract conditional on the worker's initial experience and employment status.

An important simplification is that workers are liquidity constrained and cannot borrow against future earnings. As in Stevens (2004) the lack of capital markets constrain the set of feasible contracts available to the firm. In particular, they rule out contracts that require entry fees or quitting payments from the worker. Formally, a wage contract is described by a right-continuous function w_i^x defined for all tenures t given employment status $i = u, e$ and starting experience x such that it is bounded from below by $\underline{w}$. It is useful to remember that in this environment a firm is constrained to offer the same w_i^x contract to any worker of type i, x it meets.

Both agents have a zero rate of time preference. Firms are risk neutral and infinitely lived. The objective of each firm is to maximize total steady state flow profits. Workers, on the other hand, are also risk neutral but their lives are of uncertain duration. Any worker's life is described by an exponential random variable with parameter $0 < \delta < \infty$. The inflow rate of new unemployed workers of zero experience into the market is δ. The objective of any worker is to maximize total expected lifetime utility. Finally, let b denote the opportunity cost of employment per unit of time and assume $p > b > \underline{w} \geq 0$.

3 Worker's Payoffs and Job Search Strategies

Given contact with a firm, a worker of employment status i and experience x observes the posted contract w_i^x. Let V_i^x denote his expected lifetime utility conditional on accepting it and using an optimal quit strategy in the future. Further, let $F_i(V_i^x \mid x)$ denote the distribution of starting payoffs offered by firms to workers with employment status i and experience x. Random matching implies, given contact with a firm, $F_i(V_i^x \mid x)$ describes the probability that the outside offer has a value no greater than V_i^x. Although $F_i(. \mid x)$ will be endogenously determine in equilibrium, at this stage assume it is continuous in x and has a bounded support. Let $\underline{V}_i^x$ and $\overline{V}_i^x$ denote the infimum and supremum of the support for each i, x.

First consider the case of an unemployed worker. Let $U(x)$ denote the expected lifetime payoff of this worker when he has experience x and follows an optimal search strategy. Conditional on receiving a job offer, the definition of $U(x)$ and the no recall assumption imply that his optimal policy is described by: accept a job offer if and only if $V_u^x \geq U(x)$ and reject a job offer otherwise. Since workers do not accumulate experience while unemployed, $U(x)$ then solves the following stationary Bellman equation

$$\delta U(x) = b + \lambda \int_{U(x)}^{\overline{V}_u^x} [V_u^x - U(x)] dF_u(V_u^x \mid x). \tag{1}$$

Now consider an employed worker who has been hired from state i with starting experience x on a wage contract w_x^i. Define $V_i^x(t; w_i^x)$ as this worker's expected lifetime payoff at tenure t when using an optimal quit strategy. Given any contract w_i^x and experience $x + t$ where $V_i^x(t; w_i^x) > U(x + t)$, the definition of $V_i^x(.; w_i^x)$ and the no recall assumption implies the worker's optimal strategy is to quit if and only if he receives a job offer which has starting value $V_e^{x+t} > V_i^x(t; w_i^x)$ and continue employment at the firm if and only if $V_e^{x+t} \leq V_i^x(t; w_i^x)$. Since the worker gains experience while employed, $V_i^x(.; w_i^x)$ then satisfies the following non-stationary Bellman equation

$$\delta V_i^x(t; w_i^x) = w_i^x(t) + \frac{dV_i^x(t; w_i^x)}{dt} + \lambda \int_{V_i^x(t; w_i^x)}^{\overline{V}_e^{x+t}} [V_e^{x+t} - V_i^x(t; w_i^x)] dF_e(V_e^{x+t} \mid x+t), \qquad (2)$$

for $i = u, e$, and note that $V_i^x(.; w_i^x)$ is right-differentiable with respect to t at the point in which w_i^x is discontinuous.[6]

However, if $V_i^x(t; w_i^x) < U(x+t)$ for some accumulated experience $x+t$, the worker's optimal strategy is to quit into unemployment. To allow for this possibility define the set Υ_i^x by

$$\Upsilon_i^x = \{t \in \Re_+ : V_i^x(t; w_i^x) < U(x+t)\} \quad \text{for } i = u, e$$

and let $\underline{t}_i^x = \inf \Upsilon_i^x$. Hence, $\underline{t}_i^x$ denotes the tenure at which an employed worker hired from state i with initial experience x optimally quits into unemployment. If $V_i^x(t; w_i^x) \geq U(x+t)$ for all t, then define $\underline{t}_i^x = \infty$ and the worker never quits into unemployment.

Note that given a wage contract w_i^x and tenure $t < \underline{t}_i^x$, a worker's hazard rate is $\delta + \lambda[1 - F_e(V_i^x(t; w_i^x) \mid x+t)]$ for $i = u, e$. Hence, for tenures $t < \underline{t}_i^x$, the survival probability

$$\psi_i^x(t; w_i^x) = e^{-\int_0^t [\delta + \lambda(1 - F_e(V_i^x(s; w_i^x) | x+s)] ds} \quad \text{for } i = u, e$$

describes the probability a newly employed worker hired from state i with starting experience x does not leave the firm before tenure t. If $\underline{t}_i^x < \infty$, then $\psi_i^x = 0$ for all $t \geq \underline{t}_i^x$ and $i = u, e$.

[6] See Van den Berg (1990)- Theorem 1 for a formal derivation of this equation and its properties.

4 Firm Payoffs and Optimal Strategies

As firms can perfectly discriminate by employment status and previous experience, to simplify the exposition consider for each employment status $i = u, e$ the market of experience x.[7] First lets analyse the market of unemployed workers with experience x. Define $\mu(x)$ as the steady state number of unemployed workers with experience x. Consider a firm which posts a contract w_u^x such that V_u^x denotes a worker's expected lifetime payoff by accepting it. If $V_u^x < U(x)$ the firm does not hire the worker and makes zero profit. However, if $V_u^x \geq U(x)$ the firm's steady state flow profit in this market is then given by

$$\Omega_u^x(V_u^x, w_u^x) = \lambda\mu(x)\left[\int_0^\infty \psi_u^x(t; w_u^x)[p - w_u^x(t)]dt\right],$$

where the first term describes the firm's hiring rate and the second term the firm's expected profit per new hire. The firm's total steady state flow profit in the market of unemployed workers is then obtained by integrating Ω_u^x across all experience markets

$$\Omega_u(W_u) = \int_0^\infty \lambda\mu(x)\left[\int_0^\infty \psi_u^x(t; w_u^x)[p - w_u^x(t)]dt\right]dx,$$

where W_u denotes the set of tenure contracts, w_u^x, the firm offers to unemployed workers for each experience $x \geq 0$ such that $V_u^x \geq U(x)$.

Next, consider the market of employed workers with experience x. Let $N(x)$ denote the steady state number of employed workers that have experience no greater than x and $1 - G(V \mid x)$ denote the steady state proportion of employed workers that currently have experience x and a lifetime expected payoff of at least V.[8] These two steady state measures include workers that where hired from unemployment and from a competing firm. Consider

[7] Note that at $x = 0$ the only offer distribution that is defined is $F_u(. \mid 0)$. As employed workers gain $x = 0^+$ experience firms have the possibility of hiring them from a competing firm and hence $F_e(. \mid x)$ is defined for all $x > 0$.

[8] Given that workers' lives follow a exponetial distribution, the $N(x)$ is also exponentially distributed.

a firm which posts a contract w_e^x such that V_e^x denotes a worker's expected lifetime payoff by accepting it. Note $G(V_e^x \mid x)dN(x)$ describes the steady state number of employed workers of experience x that will accept the firms offer. If $V_e^x < U(x)$ the firm is certain not to hire any worker and makes zero profit. If $V_e^x \geq U(x)$ the firm's steady state flow profit in the market of employed workers of experience x is then given by

$$\Omega_e^x(V_e^x, w_e^x) = \lambda G(V_e^x \mid x)dN(x) \left[\int_0^\infty \psi_e^x(t; w_e^x)[p - w_e^x(t)]dt \right].$$

The firm's total steady state flow profit in the market of employed workers is obtained by integrating Ω_e^x across experience markets

$$\Omega_e(W_e) = \int_0^\infty \lambda G(V_e^x \mid x)dN(x) \left[\int_0^\infty \psi_e^x(t; w_e^x)[p - w_e^x(t)]dt \right] dx,$$

where W_e denotes the set of tenure contracts, w_e^x, the firm offers to employed workers for each experience $x > 0$ such that $V_e^x \geq U(x)$.

Hence a firm's total steady state profit flow is given by

$$\Omega(W_u, W_e) = \Omega_u(W_u) + \Omega_e(W_e).$$

The objective of each firm is to choose two sets of wage contracts $\{W_u, W_e\}$, one for each employment status, to maximise $\Omega(W_u, W_e)$ given $F_u(. \mid x)$, $F_e(. \mid x)$, $U(x)$ for each market x and the turnover strategies of workers described in the previous section.[9] However, the no recall assumption implies a firm can maximise total steady state profits by choosing W_i independently to maximise $\Omega_i(W_i)$ for each $i = u, e$. Furthermore, no recall also implies that for a given i the firm can choose w_i^x to maximise Ω_i^x at each market x. This structure much simplifies the analysis as it allows us to focus on the firm's optimisation problem for each pair i, x.[10]

[9] Note that $\mu(x)$ and $G(. \mid x)$ are functions of $F_u(. \mid x)$ and $F_e(. \mid x)$.

[10] Without no recall, an employed worker hired with initial experience x' might want to quit and be rehired by the same firm in a future date because of a more attractive contract. Although a possibility in the decision theory of a firm, I will show that in equilibrium this never happens.

First, conditional on offering a new hire a starting payoff $V_i^x \geq U(x)$ an optimal contract in market x solves the programming problem

$$\max_{w_i^x(.)\geq \underline{w}} \int_0^{\infty} \psi_i^x(t; w_i^x)[p - w_i^x(t)]dt$$

subject to

$$V_i^x(0; w_i^x) = V_i^x.$$

Let $w_i^{x*}(.; V_i^x)$ denote this optimal contract and define $\Pi_i^{x*}(0; V_i^x)$ as the expected profit per new hire associated with it. The optimized steady state flow profit in the market of unemployed workers of experience x is then given by

$$\Omega_u^{x*}(V_u^x, w_u^{x*}) = \lambda\mu(x)\Pi_u^{x*}(0; V_u^x)$$

and the corresponding steady state flow profit in the market of employed workers of experience x is given by

$$\Omega_e^{x*}(V_e^x, w_e^{x*}) = \lambda G(V_e^x \mid x)dN(x)\Pi_e^{x*}(0; V_e^x).$$

The firm then chooses V_i^x to maximise Ω_i^{x*}. Let $\overline{\Omega}_i^x$ denote the maximised value of Ω_i^{x*}. It follows that by integrating $\overline{\Omega}_i^x$ across experience markets for each employment status we obtain $\overline{\Omega}_i(W_i)$ for each $i = u, e$ such that $\overline{\Omega}_u(W_u) + \overline{\Omega}_e(W_e) = \overline{\Omega}(W_u, W_e)$.

5 Optimal Wage Contracts

5.1 The Contracting Problem

Given a match is formed at any market, search frictions provide the firm with a dynamic monopsony power that enables it to extract quasi rents from the worker. The latter is able to recover those rents (or part of them) over time through job shopping. Hence, in this context quits are jointly inefficient. As firms cannot eliminate potential quits and workers are liquidity constraint, the optimal contract must then minimises the worker's quit rate by offering him an increasing share of the match rents.

In particular, when designing an optimal contract for a worker of employment status i and initial experience x, each firm takes as given the distribution of outside offers for each pair i, x, the expected lifetime utility of unemployed workers and the optimal quit strategy of employed workers. Formally, the firm's optimal contracting problem is defined as

$$\max_{w_i^x(.)} \int_0^\infty \psi_i^x(t)[p - w_i^x(t)]dt$$

subject to

$$\frac{dV_i^x(t)}{dt} = \delta V_i^x(t) - w_i^x(t) - \lambda \int_{V_i^x(t)}^{\overline{V}_e^{x+t}} \left[V_e^{x+t} - V_i^x(t)\right] dF_e(V_e^{x+t} \mid x+t)$$

$$\frac{d\psi_i^x(t)}{dt} = -\left[\delta + \lambda(1 - F_e(V_i^x(t) \mid x+t))\right] \psi_i^x(t)$$

and the initial conditions

$$V_i^x(0) = V_i^x \text{ and } \psi_i^x(0) = 1$$

and

$$w_i^x(.) \geq \underline{w}.$$

Let $J^x(t)$ denote the maximum expected value of a match between a firm and an employed worker of initial experience x at tenure t. Note $J^x(t)$ also describes the expected lifetime utility of a worker of tenure t and starting experience x that is paid $w_i^x(t) = p$ for all t and follows an optimal quit strategy. Such a contract is an optimal contract, it solves firm's maximisation problem conditional on $V_i^x = J^x(0)$, and is the only one that guarantees that the worker's privately optimal quit strategy is also jointly efficient. Using the same arguments as in (2) we obtain that $J^x(t)$ solves the following Bellman equation

$$\delta J^x(t) = p + \frac{dJ^x(t)}{dt} + \lambda \int_{J^x(t)}^{\overline{V}_e^{x+t}} [V_e^{x+t} - J^x(t)] dF_e(V_e^{x+t} \mid x+t). \tag{3}$$

However, since optimality implies no firm offer a starting payoff $V_i^x > J^x(0)$ for any $x \geq 0$, as doing so yields negative profits, (3) can be reduced to $J^x(t) = p/\delta$ for all $x, t \geq 0$.

Also note that, conditional on workers earning expected lifetime payoffs belonging to the set $[U(x), J^x(0))$, a firm can make strictly positive steady state profits in market i, x by offering a contract with a starting payoff $V_i^x \in [U(x), J^x(0))$. Moreover, such a starting payoff is feasible for all $x, t \geq 0$ since $p > b$ and (1) imply $J^x(t) > U(x+t)$ for all $x, t \geq 0$. Hence without loss of generality lets assume that in the above optimisation problem a firm conditions its contract on a starting payoff $V_i^x \in [U(x), J^x(0))$.

The next claim follows Burdett and Coles (2003) and establishes boundary conditions for the expected value of employment, $V_i^x(.; w_i^x)$ under an optimal contract.

CLAIM 1: *Given an employment status $i = u, e$, any initial experience $x \geq 0$, any profile $F_e(. \mid \varkappa)$ and $F_u(. \mid \varkappa)$ for $\varkappa \geq x$ and conditional on a $V_i^x \in [U(x), J^x(0))$, an optimal contract implies $U(x+t) \leq V_i^x(t; w_i^{x*}) \leq J^x(t)$ for all $t > 0$.*

Proof: See Appendix.

An important corollary of Claim 1 is that no worker employed in an optimal contract will quit into unemployment. Hence, $\underline{t}_i^x = \infty$ for all x, t and $i = u, e$ in an optimal contract.

5.2 Step-Contracts

Since $F_i(. \mid x)$ might have mass points at any x, standard dynamic optimisation techniques cannot be applied to obtain necessary or sufficient conditions that could help characterise the optimal contract. However, we can use similar arguments as in Stevens (2004). Given the worker is risk neutral and hence there is no gain in smoothing income, Proposition 1 shows that for an employment status i and experience x a *step-contract* is an optimal contract. In particular, a step-contract is fully characterised by a promotion tenure z and wages paid satisfy:

$$
\begin{aligned}
w_i^x(t) &= \underline{w} && \text{for } t < z, \\
w_i^x(t) &= p && \text{for } t \geq z;
\end{aligned}
$$

and the promotion tenure z is chosen so that the value of accepting the contract is V_i^x.

PROPOSITION 1: *Given an employment status $i = u, e$, any initial experience $x \geq 0$, any profile $F_e(. \mid \varkappa)$ and $F_u(. \mid \varkappa)$ for $\varkappa \geq x$ and conditional on a $V_i^x \in [U(x), J^x(0))$, a step-contract is an optimal contract.*

Proof: See Appendix.

Note that as long as $V_i^x(t; w_i^x) < J^x(t)$ for some t, any contract w_i^x generates inefficient quit behaviour, where a quit is jointly inefficient if the outside offer has value $V_e^{x+t} < J^x(t)$. The proof of Proposition 1 relies on showing that a step-contract maximises the expected profit per new hire by simply maximising the growth rate of $V_i^x(t; w_i^x)$ and hence minimising the deadweight loss caused by inefficient quit behaviour.

The step-contract property is useful as a firm's optimal contract for each i, x is now fully described by a singleton, z_i^x. The worker quits if an outside offer that promises an earlier promotion date is received. To simplify the analysis, consider the following re-normalisation. Let an employed worker be hired from state i with initial experience x. Define $T_i^x = x + z_i^x$ as the accumulated experience when promotion arrives. Note that the step-contract offer z_i^x is equivalent to promotion when the worker's accumulated experience $x + t$ reaches T_i^x. This re-normalisation is convenient since outside offers are conditioned on experience. A worker then quits if and only if he receives an outside offer at experience $x' < T_i^x$, where the corresponding promotion offer $T_e^{x'} = x' + z_e^{x'}$ satisfies $T_e^{x'} < T_i^x$.

As discussed by Stevens (2004), equilibrium can imply there is a continuum of equilibrium contracts which are payoff equivalent to a step-contract.[11] However, given a step-contract is always optimal and is strictly optimal if some firms offer a starting payoff $V_i^x > J^x(0)$ (as would be the case with heterogenous firms), there is no loss in generality to only consider

[11] With risk neutrality there is a continuum of payoff equivalent contracts for $t \geq z$, each of which implies the worker never quits and that $V_i^x(t, .) = p/\delta$. For example the firm could pay $w_i^x(t) = 0$ for $t \in [z, z']$ and $w_i^x(t) = w_H > p$ for $t > z'$ where z' and w_H are chosen so that $V_i^x(t, .) = p/\delta$ for all $t \geq z$.

equilibrium in step-contracts.

6 Market Equilibrium

Given step-contracts as described in the previous section, lets define the following notation. Let $V_i(x, T_i^x)$ denoted the expected value of an employed worker hired from state $i = u, e$ with experience x on a step-contract T_i^x; i.e. the worker will be promoted after $z_i^x = T_i^x - x$ further units of time (if the worker does not quit). Let $\Pi_i(x, T_i^x)$ denote the corresponding firm's expected profit. For workers of employment status i and experience x, the distribution of offers is described by $F_i(T_i^x \mid x)$, where $F_i(. \mid x)$ describes the probability that an outside offer implies promotion at accumulated experience no greater than T_i^x. Let $\underline{T}_i^x$ and $\overline{T}_i^x$ be the infimum and supremum of the support for each i, x. Note that offers always satisfy $T_i^x \geq x$. Also, conditional on experience $x \geq 0$, let $1 - G(T_i^x \mid x)$ denote the proportion of employed workers on a step-contract of at least T_i^x.

In the market for unemployed workers of experience x, a firm then offers T_u^x to maximise expected steady state flow profit

$$\Omega_u^x(T_u^x) = \lambda \mu(x) \Pi_u(x, T_u^x). \tag{4}$$

Similarly, in the market of employed workers with experience x, a firm offers T_e^x to maximise expected steady state flow profit

$$\Omega_e^x(T_e^x) = \lambda[1 - G(T_e^x \mid x)] dN(x) \Pi_e(x, T_e^x). \tag{5}$$

DEFINITION: *A Market Equilibrium in step-contracts requires:*

(a) an employed worker (x, T_e^x) quits if an outside offer $T_e^{x'} < T_e^x$ is received;

(b) optimal job search by unemployed workers of experience x, where

$$\delta U(x) = b + \lambda \int_{\underline{T}_u^x}^{\overline{T}_u^x} \max[V_u(x, T_u^x) - U(x), 0] dF_u(T_u^x \mid x),$$

and an unemployed worker with experience x accepts offer T_u^x if and only if $V_u(x, T_u^x) \geq U(x)$.

(c) $\mu(x)$ and $G(. \mid x)$ are consistent with the distribution of contract offers $F_i(. \mid x)$ and the optimal quit turnover strategies for each i, x;

(d) steady state flow profits satisfy

$$\begin{aligned} \Omega_i^x(T_i^x) &= \overline{\Omega}_i^x \text{ for all } T_i^x \text{ in the support of } F_i(. \mid x), \\ &\leq \overline{\Omega}_i^x \text{ otherwise, for } i = u, e. \end{aligned}$$

Consider an equilibrium where outside offers are deterministic for each pair i, x.[12] Note that Claim 1 implies once an unemployed worker with no previous experience is hired, he will not quit into unemployment at any positive experience. Hence $\mu(x) = 0$ for all $x > 0$ and $\mu(0) = \delta/(\delta + \lambda)$ describes the equilibrium unemployment rate. Let T_u^{0*} denote the equilibrium contract offered to unemployed workers and $T_e^*(x)$ denote the equilibrium contract offered to an employed worker with experience $x > 0$.[13] Optimality implies $T_e^*(x) < T_u^{0*}$ for experiences $x < T_u^{0*}$. Otherwise the offer is rejected by workers hired from unemployment and the firm makes zero profit.

Let T_e^* have the following properties:

$A1$: **For $x \in (0, T_u^{0*})$, T_e^* is continuously differentiable with**

(a) $T_e^*(0) = \underline{T} < T_u^{0*}$,

(b) it is strictly increasing with $T_e^*(x) > x$, and

(c) $\lim_{x \to T_u^{0*}} T_e^*(x) = T_u^{0*}$.

[12] Although an equilibrium with non-degenerate and continuous outside offers may exist, it is not trivial to construct one. It can be shown that if $F_u(. \mid 0)$ is assumed continuous with connected support $\left[\underline{T}_u^0, \overline{T}_u^0\right]$, the profile $F_e(. \mid x)$ around $x = 0$ must be degenerate. By constructing a candidate equilibrium in which all $F_e(. \mid x)$ defined for $x < \overline{T}_u^0$ are degenerate it can be shown using a contradiction argument that the only $F_u(. \mid 0)$ consistent with the profile $F_e(. \mid x)$ is degenerate at $\overline{T}_u^0$.

[13] Note that $T_e^*(x) \equiv T_e^{x*}$.

Hence, in equilibrium firms offer two type of jobs. A "bad" job -T_u^{0*} contracts- to unemployed workers of zero experience and "good" jobs -T_e^* contracts- to employed workers. Only those workers hired under T_u^{0*} contracts quit to firms that offer T_e^* contracts. Given $T_e^*(x)$ is increasing in x, once a worker is hired under a T_e^* contract he stays in that firm until retirement.

However, to fully characterise equilibrium we need to obtain the optimal promotions offered to potential unemployed workers of positive experience and determine $U(x)$ for $x \geq 0$. Let $T_u^*(x)$ denote the equilibrium contract firms offer to any potential unemployed worker with experience $x \geq 0$ and note that $T_u^*(0) = T_u^{0*}$. After solving for a T_e^* and T_u^{0*} such that $A1$ is satisfied, the solutions of $T_u^*(x)$ and $U(x)$ for all $x \geq 0$ are described. It is then easy to show that Claim 1 is indeed satisfied. As mentioned in the introduction, note that for each experience $x \in \left(0, T_u^{0*}\right)$ the distribution of contract offers is then define by a mixture of two mass points, $T_u^*(x)$ and $T_e^*(x)$. Finally I show existence of equilibrium and analyse some comparative statics by numerically solving the model.

7 Identifying a Market Equilibrium

7.1 Outside Offers for Employed Workers

Given workers' optimal search strategies, the properties of T_e^* as described by $A1$ and that at market $x = 0$ it is optimal to offer T_u^{0*} to unemployed workers, lets now characterise outside offers $T_e^*(x)$ for all $x \in (0, T_u^{0*})$. This is done by showing $T_e^*(x)$ maximises steady state flow profits for each $x \in (0, T_u^{0*})$. For simplicity assume $\underline{w} = 0$.

Step 1: Consider any market $x \in (0, T_u^{0*})$. We start by characterising G and Π_e. The next claim derives $1 - G(T_i^x \mid x)$, the equilibrium distribution of employed worker's reservation values.

CLAIM 2: *At any experience* $x \in (0, T_u^{0*})$:

$$1 - G(T_u^{0*} \mid x) = e^{-\lambda x}$$

and for $T_e^x \in (\underline{T}, T_u^{0*})$:

$$\begin{aligned} &\partial G(T_e^x \mid x)/\partial T_e^x = 0, && [1 - G(T_e^x \mid x)] = e^{-\lambda x} \quad \text{for } x < \kappa, \\ &\partial G(T_e^x \mid x)/\partial T_e^x = \lambda e^{-\lambda\kappa}/[dT_e^*(\kappa)/dx], && [1 - G(T_e^x \mid x)] = e^{-\lambda\kappa} \quad \text{for } x > \kappa, \end{aligned}$$

,where κ *is defined by* $T_e^*(\kappa) = T_e^x$.

Proof: See Appendix.

Next consider any market $x \in (0, T_u^{0*})$ in which a firm offers $T_e^x \in (\underline{T}, T_u^{0*})$. Similarly, define κ where $T_e^x = T_e^*(\kappa)$ and suppose $x < \kappa$. Conditional on hiring a worker with such a contract, $A1$ implies the firm's expected profit is:

$$\begin{aligned} \Pi_e(x, T_e^x) &= \int_x^\kappa e^{-(\lambda+\delta)(s-x)} p ds + e^{-(\lambda+\delta)(\kappa-x)} \int_\kappa^{T_e^x} e^{-\delta(s-\kappa)} p ds \\ &= \frac{p}{\lambda+\delta}\left[1 - e^{-(\lambda+\delta)(\kappa-x)}\right] + e^{-(\lambda+\delta)(\kappa-x)} \frac{p}{\delta}\left[1 - e^{-\delta(T_e^x-\kappa)}\right] \quad \text{if } x < \kappa, \end{aligned}$$

as the worker quits for experiences $x + s < \kappa$, does not quit thereafter, and the firm makes zero profit for experiences $x + s \geq T_e^x$. Now suppose $x > \kappa$. The firm's expected profit (conditional on a hire and $A1$) is

$$\Pi_e(x, T_e^x) = \int_x^{T_e^x} e^{-\delta(s-x)} p ds = \frac{p}{\delta}\left[1 - e^{-\delta(T_e^x - x)}\right] \qquad \text{if } x > \kappa,$$

as the worker never quits to an outside offer. Differentiating with respect to T_e^x and some re-arranging establishes the following result.

CLAIM 3: *At any experience* $x \in (0, T_u^{0*})$ *and for* $T_e^x \in (\underline{T}, T_u^{0*})$:

$$\frac{\partial \Pi_e}{\partial T_e^x} = e^{-(\lambda+\delta)(\kappa-x)}\left[-\frac{\lambda\frac{p}{\delta}\left[1 - e^{-\delta(T_e^x-\kappa)}\right]}{dT_e^*(\kappa)/dx} + pe^{-\delta(T_e^x-\kappa)}\right] \text{ for } x < \kappa,$$

$$\frac{\partial \Pi_e}{\partial T_e^x} = pe^{-\delta(T_e^x - x)} \text{ for } x > \kappa,$$

where κ is defined by $T_e^*(\kappa) = T_e^x$.

A marginal increase in T_e^x increases expected profit by the marginal revenue product of the worker at experience T_e^x, p, multiplied by the probability that he remains employed at the firm until promotion date T_e^x. The loss, however, is that the worker is more likely to quit. In this case the firm (conditional on the worker receiving an outside offer) looses the profits that otherwise would have obtain from delaying the worker's promotion date.

Step 2: Fix an $x \in (0, T_u^{0*})$ and recall a firm's steady state flow profit by offering contract $T_e^x \in (\underline{T}, T_u^{0*})$ is given by (5). Note that neither marginal profit $\partial \Pi_e / \partial T_e^x$ nor the density function $\partial G / \partial T_e^x$ are continuous at $T_e^x = T_e^*(x)$. Hence, in what follows consider left and right differentiation.

(a) Right differentiation: consider $T_e^x \in (T_e^*(x), T_u^{0*})$. Claims 2 and 3 imply

$$\begin{aligned}
\frac{\partial \Omega_e^x}{\partial T_e^x} &= -\lambda \frac{\partial G}{\partial T_e^x} dN(x) \Pi_e(x, T_e^x) + \lambda[1 - G(T_e^x \mid x)] dN(x) \frac{\partial \Pi_e}{\partial T_e^x} \\
&= \lambda e^{-\lambda x} e^{-(\lambda+\delta)(\kappa - x)} \left[-\frac{\lambda}{\delta} \frac{p\left[1 - e^{-\delta(T_e^x - \kappa)}\right]}{dT_e^*(\kappa)/dx} + p e^{-\delta(T_e^x - \kappa)} \right] dN(x).
\end{aligned}$$

Hence a necessary condition for optimality is that $\lim_{\varepsilon \to 0+} [\partial \Omega_e(T_e^* + \varepsilon, x) / \partial T_e^x] \leq 0$, otherwise offering a $T_e^x > T_e^*(x)$ is optimal. This implies

$$-\frac{\lambda}{dT^*(x)/dx} \frac{p}{\delta} \left[1 - e^{-\delta(T_e^*(x) - x)}\right] + p e^{-\delta(T_e^*(x) - x)} \leq 0.$$

(b) Left differentiation: consider $T_e^x \in (\underline{T}, T_e^{x*}(x))$. Claims 2 and 3 imply

$$\begin{aligned}
\frac{\partial \Omega_e^x}{\partial T_e^x} &= -\lambda \frac{\partial G}{\partial T_e^x} dN(x) \Pi_e(x, T_e^x) + \lambda[1 - G(T_e^x \mid x)] dN(x) \frac{\partial \Pi_e}{\partial T_e^x} \\
&= \lambda e^{-\lambda \kappa} \left[-\frac{\lambda}{\delta} \frac{p\left[1 - e^{-\delta(T_e^x - x)}\right]}{dT_e^*(\kappa)/dx} + p e^{-\delta(T_e^x - x)} \right] dN(x).
\end{aligned}$$

Hence a necessary condition for optimality is that $\lim_{\varepsilon \to 0+} [\partial \Omega_e(T_e^* - \varepsilon, x) / \partial T_e^x] \geq 0$, otherwise offering a $T_e^x < T_e^*(x)$ is optimal. This implies

$$-\frac{\lambda}{dT_e^*(x)/dx} \frac{p}{\delta} \left[1 - e^{-\delta(T_e^*(x) - x)}\right] + p e^{-\delta(T_e^*(x) - x)} \geq 0.$$

It follows that these conditions are satisfied if and only if T_e^* is the solution to the non-autonomous differential equation

$$\frac{dT_e^*(x)}{dx} = \frac{\lambda}{\delta}\frac{p\left[1-e^{-\delta(T_e^*(x)-x)}\right]}{pe^{-\delta(T_e^*(x)-x)}} = \frac{\lambda}{\delta}\left[e^{\delta(T_e^*(x)-x)}-1\right] \tag{6}$$

for all $x \in (0, T_u^{0*})$, subject to the initial condition $T_e^*(0) = \underline{T}$. The fundamental theorem of differential equations imply such a T_e^* exists, is continuously differentiable in x and strictly increasing if $x < T_e^*(x)$. Note that $A1$ also requires $\lim_{x\to T_u^{0*}} T_e^*(x) = T_u^{0*}$. As in any initial value problem, the stability of T_e^* and hence the existences of a fixed point T_u^{0*} depends on the value of the initial condition, $T_e^*(0) = \underline{T}$.

Recall $z_e^*(x) = T_e^*(x) - x$ denote the corresponding promotion tenure offered to a worker with experience x. In this case (6) can be expressed as,

$$\frac{dz_e^*(x)}{dx} = \frac{\lambda}{\delta}\left[e^{\delta z_e^*(x)}-1\right]-1,$$

which must be solved subject to the boundary condition $z_e^*(0) = \underline{T}$. The corresponding phase diagram implies that

(a) if $\underline{T} < (1/\delta)\log[1+\delta/\lambda]$, then z_e^* is strictly decreasing for all x. Since $z_e^*(x) \to 0$, $T_e^*(x) \to T_u^{0*}$ as $x \to T_u^{0*}$ and $T_u^{0*} > 0$ is determined where $z_e^*(T_u^{0*}) = 0$ (which exists).

(b) if $\underline{T} \geq (1/\delta)\log[1+\delta/\lambda]$, then z_e^* is non-decreasing and a T_u^{0*} where $z_e^*(T_u^{0*}) = 0$ does not exist.

Hence we have established the following Proposition.

PROPOSITION 2: *A necessary condition for a market equilibrium with* T_e^* *satisfying* $A1$ *requires:*

(a) $\underline{T} \in (0, \underline{T}_1)$, *where* $\underline{T}_1 = (1/\delta)\log[1+\delta/\lambda]$.

(b) conditional on such a $\underline{T}$, T_e^* *is the solution to the differential equation (6) with initial value* $T_e^*(0) = \underline{T}$,*and*

(c) T_u^{0*} is determined where $dT_e^*(x)/dx = 0$.

Note (6) describes how outside offers for employed workers must vary with experience in a market equilibrium. Under the conditions stated in Proposition 2 these offers become more generous with experience. The offered promotion tenure, $z_e^*(x) = T_e^*(x) - x$, decreases with x. Since $z_e^*(x) > 0$ for all $x \in (0, T_u^{0*})$, (6) implies T_e^* is strictly increasing in x for $x < T_u^{0*}$. Hence a solution to the conditions of Proposition 2 yields a T_e^* which satisfies $A1$. However note that there exists many T_e^* that satisfy this requirement, each of them indexed by an initial condition $\underline{T} \in (0, \underline{T}_1)$.

The next claim shows that a T_e^* satisfying Proposition 2 implies firms are indifferent to increase (marginally) their promotion date $T_e^*(x)$ at any market $x \in (0, T_u^{0*})$. Thus, when posting a contract $T_e^*(x)$ at any market $x > 0$ firms trade off a longer period during which they make positive profit with a higher chance the worker will quit in the future.

CLAIM 4: *Given a T_e^* satisfying Proposition 2, then*

$$\begin{aligned} \frac{\partial \Omega_e^x}{\partial T_e^x} &= 0 \text{ for all } T_e^x \in (T_e^*(x), T_u^{0*}); \\ \frac{\partial \Omega_e^x}{\partial T_e^x} &> 0 \text{ for all } T_e^x \in (\underline{T}, T_e^*(x)). \end{aligned}$$

Proof: See Appendix.

7.2 Workers' Expected Payoffs

Given that at market $x = 0$ it is optimal for firms to offer T_u^{0*} and that at markets $x > 0$ firms' optimal contract offers are described by any T_e^* satisfying Proposition 2, we now compute the worker's expected payoffs. First, fix a step-contract $T_e^*(\kappa) \in (\underline{T}, T_u^{0*})$. Only workers that where hired from unemployment who quit with experience κ are employed on this contract. Consider such a worker and with no loss of generality consider experience $x \geq \kappa$. Noting that $T_e^*(x) > T_e^*(\kappa)$ for all $x > \kappa$ implies the worker never quits, the worker's expected lifetime payoff is:

$$V_e(x, T_e^*(\kappa)) = e^{-\delta(T_e^*(\kappa)-x)} \int_{T_e^*(\kappa)}^{\infty} e^{-\delta(s-T_e^*(\kappa))} p ds = e^{-\delta(T_e^*(\kappa)-x)} \frac{p}{\delta},$$

as the worker receives a zero wage until promoted, and earns marginal revenue product thereafter.

Now consider the expected payoff of a worker employed under contract T_u^{0*}, the least generous contract. The expected lifetime payoff at experience $x < T_u^{0*}$ is

$$\begin{aligned} V_u(x, T_u^{0*}) &= \lambda \int_x^{T_u^{0*}} e^{-(\lambda+\delta)(s-x)} V_e(s, T_e^*(s)) ds + e^{-(\lambda+\delta)(T_u^{0*}-x)} \int_{T_u^{0*}}^{\infty} e^{-\delta(s-T_u^{0*})} p ds, \\ &= \frac{p}{\delta} \left[\lambda \int_x^{T_u^{0*}} e^{-(\lambda+\delta)(s-x)} e^{-\delta(T_e^*(s)-s)} ds + e^{-(\lambda+\delta)(T_u^{0*}-x)} \right], \end{aligned} \quad (7)$$

where $V_e(s, T_e^*(s)) = (p/\delta)e^{-\delta(T_e^*(s)-s)}$ is the starting payoff offered by a contract T_e^* at market $s \in (x, T_u^{0*})$. Under this contract the worker also gets paid a zero wage until promotion and marginal revenue product thereafter, but before promotion arrives (which happens after $T_u^{0*} - x$ units of time) he might quit to a contract $T_e^*(s)$ and receive $V_e(s, T_e^*(s))$.

Finally, consider an unemployed worker of experience x. In general, the expected value of unemployment is given by

$$\delta U(x) = b + \lambda \max \left[V_u(x, T_u^*(x)) - U(x), 0 \right] \qquad \text{for all } x \geq 0, \quad (8)$$

where $T_u^*(x)$ is the equilibrium contract firms offer to unemployed workers of experience x.

7.3 Outside Offers for Unemployed Workers

Given workers' expected payoff, we now determine $T_u^*(x)$ for all $x < T_u^{0*}$. Fix a T_e^* satisfying Proposition 2. First consider the market $x = 0$ and recall that $T_u^*(0) = T_u^{0*}$. Conditional on a hire, a firm's expected profit is then described by

$$\Pi_u(0, T_u^{0*}) = \int_0^{T_u^{0*}} e^{-(\lambda+\delta)s} p ds = \frac{p}{\lambda+\delta} \left[1 - e^{-(\lambda+\delta)T_u^{0*}} \right]. \quad (9)$$

Note that if at market $x = 0^+$ is optimal to set $\underline{T}$, then at market $x = 0$ firms can always deviate from T_u^{0*} by posting a contract $T_u^0 = \underline{T}$ and retain all the workers. Since firms hiring unemployed workers have a constant hiring rate, (4) implies optimality of T_u^{0*} requires that $\Pi_u(0, T_u^{0*}) \geq \Pi_u(0, \underline{T})$. However, at $x = 0^+$ firms can still deviate by posting a contract $T_e^{0+} = T_u^{0*} - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small, and attract workers with probability one. Since Claim 4 implies $\Pi_e(0^+, \underline{T}) = \Pi_e(0^+, T_u^{0*} - \varepsilon)$ for any $\varepsilon > 0$, by continuity $\Pi_u(0, T_u^{0*}) = \Pi_u(0, \underline{T})$ must hold in equilibrium.

CLAIM 5: *Given a T_e^* satisfying Proposition 2, then*

$$\Pi_u(0, T_u^0) = \overline{\Pi}_u \text{ for all } T_u^0 \in [\underline{T}, T_u^{0*}].$$

Proof: See Appendix.

Furthermore, equilibrium requires that an unemployed worker with no experience must be indifferent to accept a T_u^{0*} contract. Otherwise, equations (4) and (9) imply firms could increase steady state flow profits by offering $T_u^{0*} + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. (8) then implies $U(0) = V_u(0, T_u^{0*}) = b/\delta$, where $V_u(0, T_u^{0*})$ is given by (7) at $x = 0$. Note that the latter result and Claim 5 ensures that all starting offers $T_u^0 > T_u^{0*}$ are rejected by unemployed workers (and so make zero profit), while excessively generous starting offers, $T_u^0 < \underline{T}$, yield a lower profit.

Since equilibrium requires $V_e(x, T_e^*(x)) > V_u(x, T_u^{0*})$ for all $x \in \left(0, T_u^{0*}\right)$, an important corollary of this condition is that $T_e^*(0^+)$ must satisfy $V_e(0^+, T_e^*(0^+)) > b/\delta$. Using the expression for $V_e(s, T_e^*(s))$ the following result guarantees that workers employed in a T_u^{0*} contract quit to jobs offering a T_e^* contract.

CLAIM 6: *Given a T_e^* satisfying Proposition 2, a necessary condition for a market equilibrium is that $T_e^*(0) = \underline{T} < \underline{T}_2$, where $\underline{T}_2$ is defined by*

$$\underline{T}_2 = (1/\delta) \log(p/b).$$

Proof: See Appendix.

Note that these conditions uniquely determine T_e^*, and hence T_u^{0*}, as the solution of (6) that satisfies $V_u(0, T_u^{0*}) = b/\delta$ subject to $\underline{T} < \underline{T}_1$ and $\underline{T} < \underline{T}_2$.

Given such a T_e^*, now consider a worker of experience $0 < x < T_u^{0*}$ employed in contract T_u^{0*} and let $\underline{T} < \underline{T}_2$. If this worker decided to quit into unemployment, firms would optimally offer a contract $T_u^*(x)$ such that it extracts the entire match rents, $V_u(x, T_u^*(x)) = U(x)$. As before, (4) implies firms have no incentive to improve this offer since they have a constant hiring rate and the worker accepts any contract that gives him at least $U(x)$ and quit as soon as a T_e^* contract arrives. As this is true for any experience, (8) implies $T_u^*(x)$ is determined by $V_u(x, T_u^*(x)) = U(x) = b/\delta$ for all $x > 0$. Hence we have established the following Proposition.

PROPOSITION 3: *Given a T_e^* satisfying Proposition 2, a necessary condition for a market equilibrium is that outside offers for unemployed workers, $T_u^*(x)$, satisfy*

$$\frac{p}{\delta}\left[\lambda \int_x^{T_u^*(x)} e^{-(\lambda+\delta)(s-x)} e^{-\delta(T_e^*(s)-s)} ds + e^{-(\lambda+\delta)(T_u^*(x)-x)}\right] = \frac{b}{\delta} \tag{10}$$

for all $x \geq 0$.

Note that under such a T_u^*, no worker employed in a T_u^{0*} quits into unemployment since in equilibrium $V_u(x, T_u^{0*})$ is strictly increasing in $x < T_u^{0*}$. Moreover, given $V_u(x, T_u^{0*}) > U(x)$ for $x > 0$, Claim 6 implies this is also the case for a worker employed in a $T_e^*(x)$ contract. In equilibrium then $\mu(x) = 0$ for $x > 0$ and firms' expected steady state flow profits in the unemployed workers' market are given by $\Omega_u^x(T_u^*(x)) = \overline{\Omega}_u^x = 0$ for all $x > 0$.[14]

8 Existence of a Market Equilibrium

We now consider existence of a market equilibrium. The next claim gives necessary and sufficient conditions such that the definition of a market equilibrium is satisfied.

[14] Note that firms can also obtain $\overline{\Omega}_u^x = 0$ with a contract T_u^x such that $U(x) \in [b/\delta, V_u(x, T_u^{0*}))$. Conditional on $\underline{T} < \underline{T}_2$, such a contract would imply no worker will quit into unemployment and firms would obtain $\Omega_u^x(T_u^x) = \overline{\Omega}_u^x$.

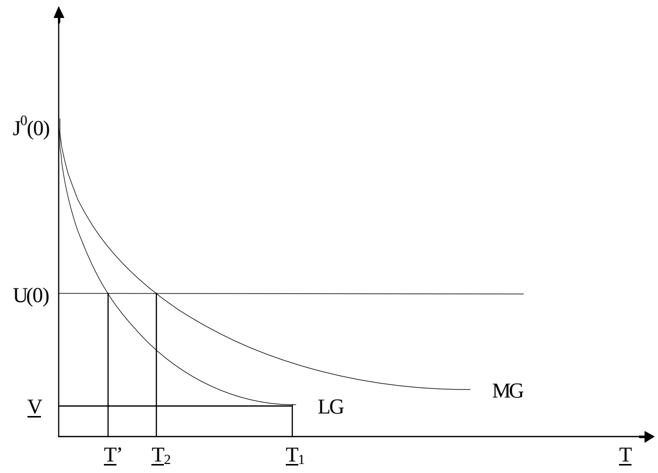

Figure 1: Existence of T_e^*.

CLAIM 7: *Necessary and sufficient conditions for a market equilibrium are:*

(a) a function T_e^* *such that (6) and*

$$\frac{p}{\delta}\left[\lambda\int_0^{T_u^{0*}} e^{-(\lambda+\delta)s}e^{-\delta(T_e^*(s)-s)}ds + e^{-(\lambda+\delta)T_u^{0*}}\right] = \frac{b}{\delta}, \tag{11}$$

are simultaneously satisfied subject to $\underline{T} < \underline{T}_1$ *and* $\underline{T} < \underline{T}_2$.

(b) Given such a T_e^**, a function* T_u^* *such that (10) is satisfied for all* $x > 0$.

Proof: See Appendix.

Existence of equilibrium can then be shown by solving Claim 7. Figure 1 shows the conditions for existence of a T_e^* satisfying part (a) of Claim 7.[15]

In particular, the locus LG describes $V_u(0, T_u^{0*})$ -the LHS of (11)- as a function of $\underline{T}$. Note that $V_u(0, T_u^{0*})$ is continuous and strictly decreasing in $\underline{T} < \underline{T}_1$, since (6) implies $T_e^*(x)$ increases continuously with $\underline{T}$ for all $x \in [0, T_u^{0*}]$. Let $\underline{V}$ denote the limit of $V_u(0, T_u^{0*})$ as $\underline{T} \to \underline{T}_1$ and note that $V_u(0, T_u^{0*})$ converges to $J^0(0)$ as $\underline{T} \to 0$. On the other hand, the locus MG describes $V_e(0, T_e^*(0))$ as a function of $\underline{T}$. $V_e(0, T_e^*(0))$ is also continuous and strictly decreasing in $\underline{T}$ and converges to $J^0(0)$ as $\underline{T} \to 0$. Since $T_u^{0*} > \underline{T} > 0$, $V_e(0, T_e^*(0))$ is

[15] Note that (11) is just (10) at $x = 0$.

strictly greater than $V_u(0, T_u^{0*})$ for any given $\underline{T} \in (0, \underline{T}_1)$. It then follows that for any value of $b/\delta \in (\underline{V}, J^0(0))$ there exists a unique $\underline{T}'$ satisfying $\underline{T}' < \underline{T}_1$ and $\underline{T}' < \underline{T}_2$ such that the corresponding solution, $T_e^{*\prime}$, to (6) solves (11).

Given such a T_e^*, Claim 7 then requires that $T_u^*(x)$ satisfies (10) for all $x > 0$. In what follows lets consider $T_u^*(x) > T_u^{0*}$ for all $x > 0$. As shown below, this ensures that $T_u^*(x) > x$ for all $x \leq T_u^{0*}$. Hence, using (7), equation (10) can be expressed as

$$\frac{b}{\delta} = V_u(x, T_u^{0*}) + \frac{p}{\delta}\left[\frac{\lambda}{\lambda+\delta}\left[1 - e^{-(\lambda+\delta)(T_u^*(x) - T_u^{0*})}\right] + e^{-(\lambda+\delta)(T_u^*(x)-x)} - e^{-(\lambda+\delta)(T_u^{0*}-x)}\right],$$

for all $x > 0$. Solving for $T_u^*(x)$ we obtain

$$\begin{aligned} T_u^*(x) &= \frac{1}{\lambda+\delta}\ln\Phi + T_u^{0*}, \qquad \text{where} \\ \Phi &= \left[\frac{(p/\delta)\left(e^{-(\lambda+\delta)(T_u^{0*}-x)} - \frac{\lambda}{\lambda+\delta}\right)}{(p/\delta)\left(e^{-(\lambda+\delta)(T_u^{0*}-x)} - \frac{\lambda}{\lambda+\delta}\right) - (V_u(x, T_u^{0*}) - (b/\delta))}\right]. \end{aligned} \tag{12}$$

It follows that T_u^* is uniquely determined if and only if $\Phi > 1$ for all $x > 0$. However, as (6) cannot be solved explicitly for T_e^* and $\Phi > 1$ cannot be verified analytically, we must analyse the model numerically to show existence of equilibrium.

Table 1 shows existence of T_e^* for several values of b and λ when $p = 18$ and $\delta = 0.0175$.[16],[17] Note that Figure 1 implies for any $b \in (\delta\underline{V}, p)$, b and T_e^* are inversely related. As b decreases (and $U(0)$ decreases), firms at $x = 0$ market are able to attract unemployed workers by offering them a contract with a longer time-to-promotion period. Since poaching firms at any market $x > 0$ will also increase their promotion dates, a decrease (increase) in b shifts outwards (inwards) T_e^* for all $x \in (0, T_u^{0*})$.

[16] These parameter values are chosen only as an example. Existence is not resticted to these set of values. However, the values of the transition parameter δ and λ are in line with the estimates of Sulis (2003) who uses matched employer-employee data based on the Italian National Social Security Institute Administrative Archives.

[17] All the results presented in Table 1 are approximated to 2 decimal point. More accurate estimates are availiable upon request.

Similarly, when search frictions increase (λ gets smaller), equation (6) implies that dT_e^*/dx decreases for all $x \in (0, T_u^{0*})$ and promotion dates offered by good jobs at positive experience markets converge to the ones offered by bad jobs; i.e. $T_e^*(x) \to T_u^{0*}$ for all $x \in (0, T_u^{0*})$. On the other hand, as frictions disappear $\underline{T}_1$ decreases. This implies that $\underline{T}$ must also decrease. At the same time workers hired in bad jobs have more opportunities to get a better job before promotion, more markets open and hence T_u^{0*} increases. However, an increase in λ also increases $\underline{V}$ and hence reduces the set of values of b for which T_e^* can exists. For values of λ high enough there is no $b < p$ such that T_e^* exists.[18]

For these set of parameters, the highlighted results in Table 1 shows the subset of parameters for which $T_u^*(x) > T_u^{0*}$ for all $x > 0$ exists.[19] Note that (12) implies $\Phi > 1$ for all x is satisfied if and only if

$$(p/\delta)\left(e^{-(\lambda+\delta)(T_u^{0*}-x)} - (\lambda/(\lambda+\delta))\right) > V_u(x, T_u^{0*}) - (b/\delta) \qquad \text{for all } x > 0.$$

Table 1 shows this inequality is satisfied only when $(p-b)$ and/or when $(\lambda-\delta)$ are sufficiently small. Otherwise, Φ becomes a decreasing function of x in the neighborhood of $x = 0$ until it reaches zero at $x = \widetilde{x} < T_u^{0*}$, where it fails to exist. This implies T_u^* is decreasing in $x < \widetilde{x}$ and does not exist at $\widetilde{x}$. In terms of $z_u^*(x)$, the numerical solutions show that when T_u^* exists the promotion tenure is increasing for low values of x and then decreases. Outside offers for unemployed workers worsen with experience at early stages and then improve.

Hence when $p = 18$ and $\delta = 0.0175$ a market equilibrium exists for those parameters for which both T_e^* and T_u^* exist. As an example, Figures 2 and 3 show the functions T_e^* and T_u^* and Figure 4 shows the corresponding solutions for $V_u(x, T_u^{0*})$ and $V_e(x, T_e^*(x))$ given $U(x)$ and $J^0(x)$ when $b = 14$ and $\lambda = 0.02$.

[18] Given the values of b shown in Table 1, T_e^* fails to exist for the corresponding values of $\lambda = 0.1$, 0.12, 0.14, 0.16, 0.19, 0.23, 0.29, 0.4, 0.61 and 1.3.

[19] Although not shown in Table 1, for $b = 17.5$ T_u^* also exists when $\lambda = 0.09$.

Table 1: Existence of Equilibrium T_e^* and T_u^* for $p = 18$, $\delta = 0.0175$

λ		b = 13	13.5	14	14.5	15	15.5	16	16.5	17	17.5
0.02	$T_e^*(0)$	15.50	**14.00**	**12.45**	**10.93**	**9.40**	**7.80**	**6.28**	**4.70**	**3.16**	**1.58**
	$T_u^*(0)$	18.97	**16.71**	**14.50**	**12.45**	**10.48**	**8.52**	**6.73**	**4.94**	**3.27**	**1.61**
	$\underline{T}_1$	35.92	**35.92**	**35.92**	**35.92**	**35.92**	**35.92**	**35.92**	**35.92**	**35.92**	**35.92**
	$\underline{T}_2$	18.60	**16.44**	**14.36**	**12.36**	**10.42**	**8.54**	**6.73**	**4.97**	**3.27**	**1.61**
0.03	$T_e^*(0)$	14.38	13.05	11.74	10.38	8.98	**7.56**	**6.10**	**4.62**	**3.10**	**1.57**
	$T_u^*(0)$	19.49	17.01	14.76	12.62	10.57	**8.63**	**6.76**	**4.98**	**3.26**	**1.61**
	$\underline{T}_1$	26.26	26.26	26.26	26.26	26.26	**26.26**	**26.26**	**26.26**	**26.26**	**26.26**
	$\underline{T}_2$	18.60	16.44	14.36	12.36	10.42	**8.54**	**6.73**	**4.97**	**3.27**	**1.61**
0.04	$T_e^*(0)$	13.40	12.27	11.10	9.87	8.60	7.28	5.92	**4.52**	**3.05**	**1.56**
	$T_u^*(0)$	20.19	17.52	15.09	12.81	10.69	8.68	6.79	**5.00**	**3.26**	**1.61**
	$\underline{T}_1$	20.74	20.74	20.74	20.74	20.74	20.74	20.74	**20.74**	**20.74**	**20.74**
	$\underline{T}_2$	18.60	16.44	14.36	12.36	10.42	8.54	6.73	**4.97**	**3.27**	**1.61**
0.05	$T_e^*(0)$	12.55	11.55	10.52	9.40	8.25	7.03	5.75	4.42	**3.00**	**1.54**
	$T_u^*(0)$	21.17	18.13	15.49	13.03	10.83	8.75	6.83	5.01	**3.26**	**1.61**
	$\underline{T}_1$	17.15	17.15	17.15	17.15	17.15	17.15	17.15	17.15	**17.15**	**17.15**
	$\underline{T}_2$	18.60	16.44	14.36	12.36	10.42	8.54	6.73	4.97	**3.27**	**1.61**
0.06	$T_e^*(0)$	11.79	10.93	10.00	9.00	7.94	6.80	5.60	4.33	**2.96**	**1.53**
	$T_u^*(0)$	22.54	19.02	16.02	13.37	11.53	8.86	6.88	5.04	**3.27**	**1.60**
	$\underline{T}_1$	14.62	14.62	14.62	14.62	14.62	14.62	14.62	14.62	**14.62**	**14.62**
	$\underline{T}_2$	18.60	16.44	14.36	12.36	10.42	8.54	6.73	4.97	**3.27**	**1.61**
0.07	$T_e^*(0)$	11.10	10.35	9.52	8.62	7.65	6.59	5.45	4.24	2.93	**1.52**
	$T_u^*(0)$	24.49	20.14	16.66	13.76	11.24	8.99	6.93	5.05	3.28	**1.60**
	$\underline{T}_1$	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	12.75	**12.75**
	$\underline{T}_2$	18.60	16.44	14.36	12.36	10.42	8.54	6.73	4.97	3.27	**1.61**

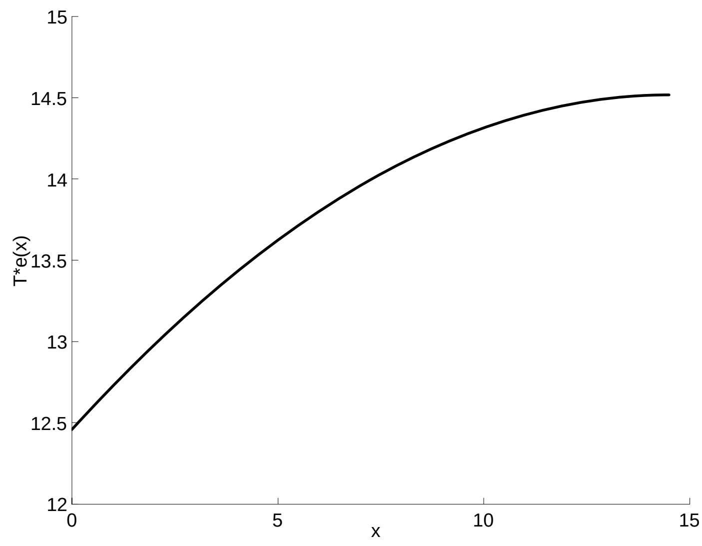

Figure 2: Equilibrium T_e^* when $p = 18$, $b = 14$, $\lambda = 0.02$ and $\delta = 0.0175$.

Before interpreting these results lets make a brief comparison with Stevens (2004). Recall that in her framework firms are not able to contract upon workers' initial experience nor employment status. Equilibrium is characterised by a contract offer distribution that is degenerate at T_s, such that $V(0, T_s) = U$. This contract is offered to all unemployed workers and hence experience and tenure are equal in this case. Since an employed worker never quits to a different employer, the optimal promotion tenure (experience) in this framework is given by

$$\frac{b}{\delta} = e^{-\delta T_s} \int_{T_s}^{\infty} e^{-\delta(t-T_s)} p dt. \tag{13}$$

Solving for T_s implies that $T_s = \underline{T}_2$, where $\underline{T}_2$ is described in Claim 6.

Now consider firms' total steady state profit flows in both frameworks. Given that $\underline{T} < T_s$, Claim 5 implies firms' steady state flow profits at the market of unemployed workers with zero experience is lower than the one obtain in Stevens. However, by using experience to segment markets between $\left(0, T_u^{0*}\right)$, firms are able to increase their total profits and match that of Stevens. The following equations describe total steady state flow profits in both

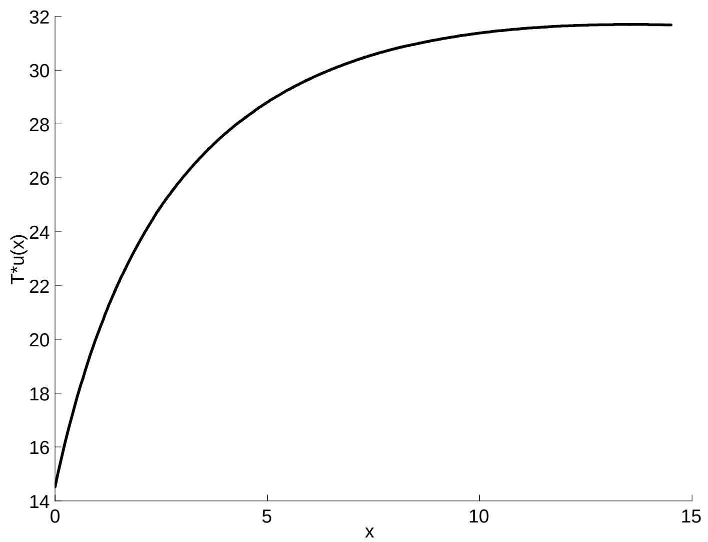

Figure 3: Equilibrium T_u^* when $p = 18$, $b = 14$, $\lambda = 0.02$ and $\delta = 0.0175$.

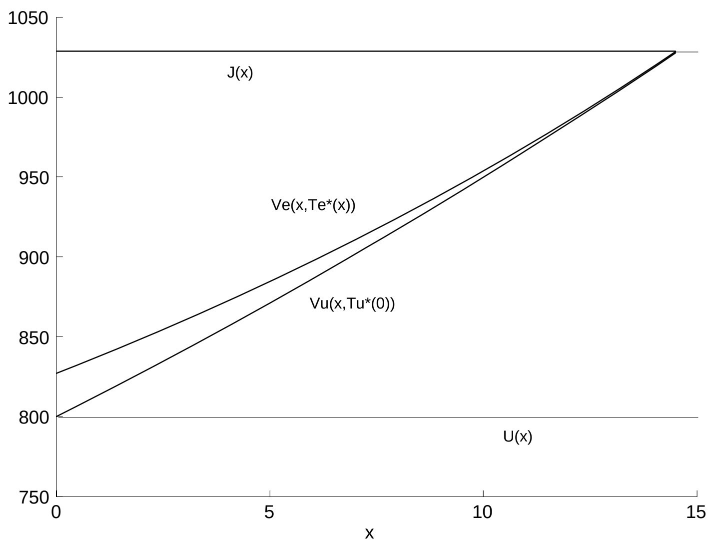

Figure 4: Equilibrium Expected Value of Employment, $J^0(x)$, $V_e(x, T_e^*(x))$, $V_u(x, T_u^{0*})$ and $U(x)$.

frameworks

$$\overline{\Omega}_s = \frac{\lambda\delta}{(\lambda+\delta)}\frac{p}{\delta}\left[1-e^{-\delta T_s}\right],$$

$$\overline{\Omega} = \frac{\lambda\delta}{(\lambda+\delta)}\frac{p}{\delta}\left[1-\left(\lambda\int_0^{T_u^{0*}} e^{-(\lambda+\delta)x}e^{-\delta(T_e^*(x)-x)}dx + e^{-(\lambda+\delta)T_u^{0*}}\right)\right],$$

where $\overline{\Omega}_s$ denotes Stevens' steady state profit flows. Noting (13) and (10) imply that in both cases the second term inside the square brackets equals b/p, we obtain that $\overline{\Omega} = \overline{\Omega}_s$.

Moreover, as in Stevens, the full information and offer-matching case described by Postel-Vinay and Robin (2002) imply firms extract the entire match rents from the worker when agents are homogeneous. With full information and offer-matching firms' total steady state profit flow is given by

$$\overline{\Omega}_{PV-R} = \frac{\lambda}{\delta+\lambda}(p-b).$$

It follows by inspection that in all three cases firms acheive equal total steady state flow profits, $\overline{\Omega} = \overline{\Omega}_s = \overline{\Omega}_{PV-R}$.[20]

A similar result was obtained in Carrillo-Tudela (2004b). In that paper I extend a version of Burdett and Mortensen (1998) model by allowing firms to contract upon employment status. Unemployed workers are offered their reservation wage and firms obtain the same profits as in Postel-Vinay and Robin homogenous agents case. This suggest that the equal profit result is driven by the ability of firms to offer unemployed workers contracts such that they are indifferent to accept them.[21] In the cases of Stevens and Postel-Vinay and Robin, firms extract the entire match rents with one contract. In the present case and as in Carrillo-Tudela (2004b), firms are able to extract total match rents in several markets. Although there is a positive probability a worker hired from unemployment might quit in

[20] A crucial difference, however, is that in Postel-Vinay and Robin's case workers are not liquidity constraint. The reservation wage of unemployed workers is in fact negative. Imposing the liquidity constraint $\underline{w} = 0$ would reduce firms profits as they would not longer be able to extract the entire match rents from the workers.

[21] Recall that contracts T_u^{0*} and T_s yield the same starting expected payoff, b/δ.

the future, firms compensate that loss by hiring employed workers of positive experience.

9 Interpretation and Conclusions

I argue that the framework derived in this paper is useful to understand wage variation within a firm. Doeringer and Piore (1971) and Saint-Paul (1996) present evidence that firms segment the labour market internally offering two types of jobs. In the upper tier of their labour market firms create an internal labour market offering high wages, employment stability and promotions. The lower tier is characterised by low wages and a high degree of turnover. Medoff and Abraham (1980) and Baker, Gibbs and Holmstrom (1994a) document a significant amount of wage variation both within and between a firm's job levels for managerial and professional categories.

The results suggest that search frictions alone can give an alternative explanation of why this "dual" labour market might appear within a firm. In the model, profit maximising firms exploit their monopsony power and offer a low paying job to an unemployed worker with no previous experience and a high paying one to an employed with positive experience because it knows the former does not have an outside option while the latter does. This is in contrast is the typical explanations based on efficiency wages and the existence of monitoring cost.

Moreover, Medoff and Abraham (1980) show that wages among jobs of the same difficulty are positively correlated with workers' experience after controlling by tenure and productivity. In the present paper, this result is solely determined by search frictions. As the experience-earnings profile of workers hired under a T_u^{0*} contract increases steeply during the period in which these workers earn less than p, firms at markets $x \in (0, T_u^{0*})$ must offer contracts with increasing starting values if they are going to successfully recruit them before promotion arrives.

Since more experienced workers are offered contracts with higher starting values, the model predicts cohort effects within a firm. The expected value of employment of two workers hired at experiences x and x' follow a common pattern that is independent of outside offers. Hence, much of the variation between cohorts' expected value of employment implied by the earnings distribution G (see Claim 2) comes from the difference in starting payoff described by $V(x, T_e^*(x))$ and persists until promotion arrives. Through simulations I have further derived the impact of changes in market conditions on T_e^* and hence on how these cohort effects behave when frictions and the opportunity cost of employment change.

Baker, Gibbs and Holmstrom (1994b) find some empirical support for this prediction. Using personnel data of a mayor US corporation during the period 1969-1988 they find strong evidence of cohort effects that depend on the year in which workers where hired. In their study, employee's wages of different cohorts follow a common pattern (increasing and convex with tenure) and move in parallel. New entrant wages, however, follow a more idiosyncratic and erratic path which is described by external market conditions. Not surprisingly, they argue that these wage patterns are consistent with wage policies found in the incentive/agency theory.

Furthermore, the theory present in this paper implies that inside a firm, holding tenure constant, workers that where hired with more pre-company experience (experience gained outside the firm) are higher in the earnings ladder and when controlling for experience workers with more tenure have higher earnings. These predictions are also consistent with the empirical findings of Medoff and Abraham (1980). Their analysis shows that in both cases for managerial and professional employees there exists positive returns to outside firm (pre-company) experience and positive returns to tenure when controlling for tenure and experience, respectively. Interestingly, they find that these effects explain nearly 40% of wage differentials found within a job level.

In conclusion, the results presented in this paper seem to suggest that allowing for more complex firm wage policies in search equilibrium type of environments can proof useful to further understand the interaction between search frictions and the worker's wage pattern inside the firm. The empirical evidence of Baker, Gibbs and Holmstrom (1994a) and Lazear (1995) indicates that the wage variation observed within a firm is not only influenced by workers' characteristics but also by external market forces. If search frictions are important in shaping labour markets as recent empirical evidence suggest, there is a strong case to further analysis the role search frictions play in determining a firm's internal wage structure.

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Appendix

Proof of Claim 1:

For a given $i = u, e$, fix an x, an initial starting payoff $V_i^x \in [U(x), J^x(0))$ and a profile $F_u(. \mid \varkappa)$ and $F_e(. \mid \varkappa)$ for $\varkappa \geq x$.

(i) Suppose there exists a $t' > 0$ such that $V_i^x(t'; w_i^{x*}) < U(x + t')$. In this case the worker quits and the firm obtains a continuation payoff of zero. However, this firm can retain the worker and increase profits by offering a constant wage contract $w_i^{x+t'}(\tau) = w_\varepsilon = p - \varepsilon > b$ for all $\tau \geq 0$, where $\varepsilon > 0$ is sufficiently small. In this case ε is chosen such that $(p/\delta) - V_i^{x+t'}(0; w_\varepsilon) = \beta(\varepsilon) > 0$, where $\beta(\varepsilon) > 0$ is sufficiently small. Given $p/\delta > U(x)$ for all $x \geq 0$ there always exists a $\epsilon-$neighborhood around p/δ, $\Lambda_\epsilon(p/\delta)$, such that $U(x) \notin \Lambda_\epsilon(p/\delta)$ for all x. However, by choosing ε appropriately $V_i^{x+t'}(\tau; w_\varepsilon) \in \Lambda_\epsilon(p/\delta)$ such that $(p/\delta) > V_i^{x+t'}(\tau; w_\varepsilon) > U(x + t' + \tau)$ for all $\tau \geq 0$. Hence the worker does not quit into unemployment while the firm is able to makes strictly positive profits. Since continuity implies such a contract always exists, the above argument contradicts the optimality of the original contract and $V_i^x(t; w_i^{x*}) \geq U(x + t)$ for all $x, t \geq 0$.

(ii) Next, suppose there exists a tenure $t' > 0$ such that $V_i^x(t'; w_i^{x*}) > J^x(t')$. Since $V_i^x(0; w_i^{x*}) < J^x(0)$ and $V_i^x(.; w_i^{x*})$ is continuous over t there exists an $s \in (0, t')$ such that $V_i^x(s; w_i^{x*}) = J^x(s)$. However, at that tenure the optimal contract implies $w_i^x(t) = p$ for all $t \geq s$ and therefore $V_i^x(.; w_i^{x*}) = J^x(.)$ for all $t \geq s$ contradicting the optimality of w_i^{x*}. $\|$

Proof of Proposition 1:

For any employment status $i = u, e$ and initial experience x fix a $V_i^x \in [U(x), J^x(0))$ and a profile $F_e(. \mid \varkappa)$ for $\varkappa \geq x$. Consider the following step-contract, w_i^{zx},

$$
\begin{aligned}
w_i^{zx}(t) &= \underline{w} && \text{for } t < z, \\
w_i^{zx}(t) &= p && \text{for } t \geq z;
\end{aligned}
$$

where z is chosen such that $V_i^x(0; w_i^{zx}) = V_i^x$. Note that $V_i^x(t; w_i^{zx}) = J^x(t)$ for $t \geq z$ and (2) implies $V_i^x(t; w_i^{zx})$ solves

$$\delta V_i^x(t; w_i^{zx}) = \underline{w} + \dot{V}_i^x(t; w_i^{zx}) + \lambda \varphi_e(V_i^x\,(t; w_i^{zx})\,; x+t), \tag{14}$$

for all $t < z$, where

$$\varphi_e(V_i^x\,(t)\,; x+t) \equiv \int_{V_i^x(t)}^{\overline{V}_e^{x+t}} [V_e^{x+t} - V_i^x(t)] dF_e(V_e^{x+t} \mid x+t)$$

denotes the "surplus function" for any market $x+t$ and is continuous and non increasing in $V_i^x(t) \in [U(x+t), J^x(t)]$.

Step 1: I will show that (i) the step-contract w_i^{zx} has a unique starting payoff $V_i^x(0; w_i^{zx})$; and (ii) $V_i^x(0; w_i^{zx})$ is strictly decreasing in z. Let the step-contracts, w_i^{zx} and $w_i^{\widetilde{z}x}$, be such that the promotion tenure $\widetilde{z} = z + \varepsilon$ for any $\varepsilon > 0$.

(i) Suppose that $V_i^x(0; w_i^{zx}) = V_i^x(0; w_i^{\widetilde{z}x})$. Using (14) it follows that $\dot{V}_i^x(0; w_i^{zx}) = \dot{V}_i^x(0; w_i^{\widetilde{z}x})$. Continuity then implies $\dot{V}_i^x(t; w_i^{zx}) = \dot{V}_i^x(t; w_i^{\widetilde{z}x})$ for all $t \in (0, z]$ and hence $V_i^x(z; w_i^{zx}) = J^x(z) = V_i^x(z; w_i^{\widetilde{z}x})$. However, $w_i^{\widetilde{z}x}$ implies $V_i^x(t; w_i^{\widetilde{z}x}) < J^x(t)$ for all $t \in [0, \widetilde{z})$ and $V_i^x(t; w_i^{\widetilde{z}x}) = J^x(t)$ at $t = \widetilde{z}$. Hence $V_i^x(0; w_i^{zx}) \neq V_i^x(0; w_i^{\widetilde{z}x})$.

(ii) Now suppose that $V_i^x(0; w_i^{zx}) < V_i^x(0; w_i^{\widetilde{z}x})$. Since $V_i^x(z; w_i^{zx}) > V_i^x(z; w_i^{\widetilde{z}x})$, continuity implies that there exists a $t' \in [0, z)$ such that $V_i^x(t'; w_i^{zx}) = V_i^x(t'; w_i^{\widetilde{z}x})$. Using the same arguments as in (i) we have $V_i^x(t; w_i^{zx}) = V_i^x(t; w_i^{\widetilde{z}x})$ for all $t \in (t', z]$. Hence $V_i^x(0; w_i^{zx}) > V_i^x(0; w_i^{\widetilde{z}x})$.

Note that as $z = 0$ implies $V_i^x(0; w_i^{zx}) = J^x(0)$, any $z > 0$ must then correspond to a $V_i^x(0; w_i^{zx}) < J^x(0)$. Using (i) and (ii), continuity then implies that for each $V_i^x \in [U(x), J^x(0)]$, there exists a unique step-contract with corresponding promotion tenure $z \geq 0$.

Step 2: Let $\widetilde{w}_i^x$ denote any other contract such that $V_i^x(0; \widetilde{w}_i^x) = V_i^x$. Subtracting the corresponding continuation payoffs we obtain

$$\begin{aligned}\dot{V}_i^x(t;w_i^{zx})-\dot{V}_i^x(t;\widetilde{w}_i^x) &= \widetilde{w}_i^x(t)-\underline{w}+\delta[V_i^x(t;w_i^{zx})-V_i^x(t;\widetilde{w}_i^x)] \\ &\quad -\lambda[\varphi_e(V_i^x(t;w_i^{zx});x+t)-\varphi_e(V_i^x(t;\widetilde{w}_i^x);x+t)];\end{aligned} \tag{15}$$

for all $t<z$. Since $V_i^x=V_i^x(0;w_i^{zx})=V_i^x(0;\widetilde{w}_i^x)$ by assumption, it follows that $\dot{V}_i^x(0;w_i^{zx})\geq\dot{V}_i^x(0;\widetilde{w}_i^x)$. Continuity of $V_i^x(.)$ and the properties of φ_e then imply $\dot{V}_i^x(t;w_i^{zx})\geq\dot{V}_i^x(t;\widetilde{w}_i^x)$ and $V_i^x(t;w_i^{zx})\geq V_i^x(t;\widetilde{w}_i^x)$ for all $t<z$. Furthermore, as $V_i^x(t;w_i^{zx})=J^x(t)$ for all $t\geq z$, it follows that $V_i^x(t;w_i^{zx})\geq V_i^x(t;\widetilde{w}_i^x)$ for all $t\geq 0$.

Step 3: Let $J_i^{mx}(0;w_i^x)=V_i^x(0;w_i^x)+\Pi_i^x(0;w_i^x)$ denote the total expected value of the match and note that $J^x(0)\geq J_i^{mx}(0;w_i^x)$ as J^x denote the maximum expected value of a match. Let $\Pi_i^x(t;w_i^x)$ denote the continuation payoff of a firm offering w_i^x such that

$$\delta\Pi_i^x(t;w_i^x)=p-w_i^x(t)+\frac{d\Pi_i^x(t;w_i^x)}{dt}-\lambda(1-F_e(V_i^x(t;w_i^x)\mid x+t)\Pi_i^x(t;w_i^x) \tag{16}$$

Since the objective of a firm is to chose a w_i^x to maximise $\Pi_i^x(0;w_i^x)$ subject to $V_i^x(0;w_i^x)=V_i^x$, the problem is equivalent to chose a w_i^x such that it maximises $J_i^{mx}(0;w_i^x)$.

Consider the step-contract w_i^{zx} and a contract $\widetilde{w}_i^x$. It follows that $J_i^{mx}(t;w_i^{zx})=J^x(t)\geq J_i^{mx}(t;\widetilde{w}_i^x)$ for all $t\geq z$, where (14) and (16) characterise $J_i^{mx}(t;w_i^x)$. Subtracting the corresponding J_i^{mx} for w_i^{zx} and $\widetilde{w}_i^x$ and using the results of Step 2 we obtain that

$$[\dot{J}_i^{mx}(t;\widetilde{w}_i^x)-\dot{J}_i^{mx}(t;w_i^{zx})]\geq(\delta+\lambda(1-F_e(V_i^x(t;w_i^{zx})\mid x+t))[J_i^{mx}(t;\widetilde{w}_i^x)-J_i^{mx}(t;w_i^{zx})]$$

for all $t\geq 0$. Since $J_i^{mx}(t;w_i^{zx})\geq J_i^{mx}(t;\widetilde{w}_i^x)$ at $t=z$, continuity implies $J_i^{mx}(0;w_i^{zx})\geq J_i^{mx}(0;\widetilde{w}_i^x)$. Hence, conditional on $V_i^x(0;\widetilde{w}_i^x)=V_i^x(0;w_i^{zx})=V_i^x\in[U(x),J^x(0))$, it follows that $\Pi_i^x(0;w_i^{zx})\geq\Pi_i^x(0;\widetilde{w}_i^x)$.‖

Proof of Claim 2:

Consider first T_u^{0*}. As all starting offers imply T_u^{0*} then $1-G(T_u^{0*}\mid 0)=1$. Further, as

$T_e^* < T_u^{0*}$ for $x > 0$, then conditional on remaining in the labour market, workers quit to $T_e^* < T_u^{0*}$ at rate λ. Hence $1 - G(T_u^{0*} \mid x) = e^{-\lambda x}$ for $0 < x < T_u^{0*}$.

Now fix a $x \in (0, T_u^{0*})$ and a $T_e^x \in (\underline{T}, T_u^{0*})$. Define κ where $T_e^*(\kappa) = T_e^x$. If $x < \kappa$, then all offers $T_e^*(s) < T_e^x$ for all experiences $s \in [0, x]$, and so $\partial G / \partial T_e^x = 0$. It then follows from the first part of the proof that $1 - G(T_e^x \mid x) = e^{-\lambda x}$.

Suppose instead $x > \kappa$. Steady state turnover implies for dx arbitrarily small

$$G(T_e^*(\kappa + dx) \mid x) - G(T_e^*(\kappa) \mid x) = \lambda dx[1 - G(T_e^*(\kappa + dx) \mid \kappa)] + O(dx^2),$$

where conditional on remaining in the labour market, the proportion of workers on contract $T_e^x \in [T_e^*(\kappa), T_e^*(\kappa + dx)]$ at experience x are those who at experience κ and on contract T_u^{0*} received an outside offer and so quit to a contract $T_e^\kappa \in [T_e^*(\kappa), T_e^*(\kappa + dx)]$. But at experience κ, $1 - G(T_e^*(\kappa + dx), \kappa) = e^{-\lambda\kappa}$. Dividing by dx and taking the limit $dx \to 0$ implies the condition stated in the Claim.

Finally, note that $A1$ implies that workers under contracts no greater than $T_e^*(\kappa)$ will never quit to an outside offer and that at experience κ, $G(T_e^*(\kappa), \kappa) = 1 - e^{-\lambda\kappa}$. Hence, conditional on those workers remaining in the labour market at experience x, $1 - G(T_e^*(\kappa), x) = e^{-\lambda\kappa}$. ||

<u>Proof of Claim 4</u>:

First consider $T_e^x > T_e^*(x)$. Then

$$sign[\frac{\partial \Omega_e^x}{\partial T_e^x}] = sign[e^{-\lambda x} e^{-(\lambda+\delta)(\kappa - x)} \left[-\lambda \frac{p}{\delta} \left[1 - e^{-\delta(T_e^x - \kappa)}\right] + [dT_e^*(\kappa)/dx]\, pe^{-\delta(T_e^x - \kappa)}\right]].$$

Substituting out $dT_e^*(\kappa)/dx$ from (6) and using Proposition 2 implies the first part of the claim.

Now consider $T_e^x < T_e^*(x)$,and so

$$sign[\frac{\partial \Omega_e^x}{\partial T_e^x}] = sign[-\lambda \frac{p}{\delta} \left[1 - e^{-\delta(T_e^x - x)}\right] + [dT_e^*(\kappa)/dx]\, pe^{-\delta(T_e^x - x)}].$$

Substituting out $dT_e^*(\kappa)/dx$ gives

$$sign[\frac{\partial\Omega_e^x}{\partial T_e^x}] = sign[-\lambda\frac{p}{\delta}\left[1 - e^{-\delta(T_e^x - x)}\right] + \lambda\frac{p}{\delta}\left[e^{\delta(T_e^x - \kappa)} - 1\right]e^{-\delta(T_e^x - x)}].$$

$$sign[\frac{\partial\Omega_e^x}{\partial T_e^x}] = sign[\lambda\frac{p}{\delta}[-1 + e^{-\delta(\kappa - x)}]] > 0$$

as $\kappa < x$. ||

Proof of Claim 5:

Fix $x = 0$. Suppose a firm offers a contract $T_u^0 \in [\underline{T}, T_u^{0*}]$ to an unemployed worker with no experience. Define κ where $T_u^0 = T_e^*(\kappa)$ such that T_e^* satisfies Proposition 2. Conditional on a hire, the firm makes expected profit

$$\Pi_u(0, T_u^0) = \frac{p}{\lambda + \delta}\left[1 - e^{-(\lambda+\delta)\kappa}\right] + e^{-(\lambda+\delta)\kappa}\frac{p}{\delta}\left[1 - e^{-\delta(T_u^0 - \kappa)}\right]$$

as the worker quits for experience $s < \kappa$, does not quit thereafter, and the firm makes zero profits for experiences $s > T_u^0$. Differentiating the above equation with respect to T_u^0 yields

$$\frac{\partial\Pi_u(0, T_u^0)}{\partial T_u^0} = e^{-(\lambda+\delta)\kappa}\left[-\frac{\lambda}{\delta}\frac{p\left[1 - e^{-\delta(T_u^0 - \kappa)}\right]}{dT_e^*(\kappa)/dx} + pe^{-\delta(T_u^0 - \kappa)}\right].$$

Using (6) to substitute out $dT_e^*(\kappa)/dx$ such that T_e^* satisfies Proposition 2 leads to $\partial\Pi_u(0, T_u^0)/\partial T_u^0 = 0$ which implies the condition stated in the claim.||

Proof of Claim 6:

Recall $V_e(x, T_e^*(x)) = (p/\delta)e^{-\delta(T_e^*(x) - x)}$ and $V_u(0, T_u^{0*}) = b/\delta$. Since both $V_u(x, T_u^{0*})$ and $V_e(x, T_e^*(x))$ are increasing in x and converge to p/δ as $x \to T_u^{0*}$, continuity implies that a necessary condition for equilibrium is that

$$V_e(0, T_e^*(0)) = (p/\delta)e^{-\delta T_e^*(0)} > b/\delta = V_u(0, T_u^{0*}).$$

Otherwise workers employed in a T_u^{0*} contract will not quit to jobs offering a T_e^* contract. For this inequality to hold, $T_e^*(0) = \underline{T} < \underline{T}_2$, where $\underline{T}_2$ is such that $(p/\delta)e^{-\delta\underline{T}_2} = b/\delta$. Solving for $\underline{T}_2$ implies the condition stated in the claim.||

Proof of Claim 7:

Necessary: Proposition 2 implies that a necessary condition for $A1$ to be satisfied in a market equilibrium such that the outside offers of employed workers with experience $x \in (0, T_u^{0*})$ are optimal is that T_e^* describes the solution of (6) subject to $\underline{T} < \underline{T}_1$. Moreover, optimality of T_u^{0*} implies equilibrium requires $V_u(0, T_u^{0*}) = U(0)$. Using (7) and (8) we obtain that such a T_e^* must also satisfy

$$V_u(0, T_u^{0*}) = \frac{p}{\delta}\left[\lambda \int_0^{T_u^{0*}} e^{-(\lambda+\delta)s} e^{-\delta(T_e^*(s)-s)} ds + e^{-(\lambda+\delta)T_u^{0*}}\right] = \frac{b}{\delta} = U(0).$$

Optimal worker turnover then implies Claim 6 must be satisfied and hence T_e^* must also be solve subject to $\underline{T} < \underline{T}_2$. Conditional on such a T_e^* and on an unemployed worker of positive experience, optimality of outside offers for unemployed workers requires that T_u^* must satisfy Proposition 3 and $U(x) = b/\delta$ for all $x > 0$.

Sufficient: Now consider a T_e^* and T_u^* such that Proposition 2, Claim 6 and Proposition 3 are simultaneously satisfied. Claim 4 implies firms will not have the incentive to offer a $T_e^x \in (\underline{T}, T_e^*(x))$ and a $T_e^x \in (T_e^*(x), T_u^{0*})$ at any experience market $x \in (0, T_u^{0*})$. Moreover, no firm will offer a $T_e^x > T_u^{0*}$ at experience market $x \in (0, T_u^{0*})$ as no worker will accept the offer. Claim 5 and the fact that unemployed workers of zero experience will reject any contract T_u^0 with $V_u(0, T_u^0) < U(0)$ imply that no firm will offer a $T_u^0 > T_u^{0*}$ or a $T_u^0 < \underline{T}$. Finally, conditional on an unemployed worker of positive experience $x \in (0, T_u^{0*}]$, firms will not have incentives to deviate from offering $T_u^*(x)$ as a $T_u^x < T_u^*(x)$ will imply less profits and unemployed workers will reject a $T_u^x > T_u^*(x)$. $\|$