# Determination of the Optimum Crown Pillar Thickness Between Open–Pit and Block Caving

**Ezzeddin Bakhtavar**, Assistant Professor
Mining Engineering
Urmia University of Technology
Urmia, Iran

**Kazem Oraee**, Professor Management
University of Stirling
Stirling, United Kingdom

**Kourosh Shahriar**, Professor
Mining and Metallurgical Engineering
Amirkabir University of Technology
Tehran, Iran

## ABSTRACT

In this paper, a relationship between dependant parameters and the crown pillar thickness is first introduced. This relationship defines geotechnical problems caused by thin pillars and economic considerations created by pillars that are thicker than the optimum size. For this purpose, a dimensional analysis as an effective physico-mathematical tool was used. This technique restructures the original dimensional variables of a problem into a set of dimensionless products using the constraints imposed upon them by their dimensions. A model is hence introduced that calculates the optimum pillar thickness. The relationship introduced here and the method applied can be used by mining engineers in all situations where a combined open-pit and block caving method is deemed to be the most appropriate mining method.

## INTRODUCTION

Many deposits can be mined entirely with the open-pit method; others must be worked underground from the very beginning. In addition, there are the near-surface deposits with considerable vertical extent. Although they are initially exploited by the open-pit method, there is often a "transition depth" where decision has to be made about changing to underground methods.

Some of the biggest open-pit mines worldwide will reach their final pit limits in the next 10 to 15 years. Furthermore, there are many mines planning to change from open-pit to underground mining due to increasing extraction depths and environmental requirements. In this way, it is likely that block and/or panel caving will enable the operations to continue achieving a high production rate at low costs as an underground method (Bakhtavar et al., 2009).

In these cases, it is often necessary to consider a crown pillar beneath the transition depth (open-pit floor) before starting an underground caving stope method (Figure 1).

There are generally two kinds of crown pillar: a "surface crown pillar" and "crown pillar between open-pit and underground mining." In general, they both play the similar role in mining. Since the primary purpose of a surface crown pillar is to protect surface land users, the mine, and those working in it from inflows of water, soil, and rock; it is vital that the surface crown pillars remain stable throughout their life. When a crown pillar could potentially remain between open-pit and underground mining it is specially proposed to prevent water entering from the open-pit floor into the stope, as well as to reduce open-pit wall and floor caving.

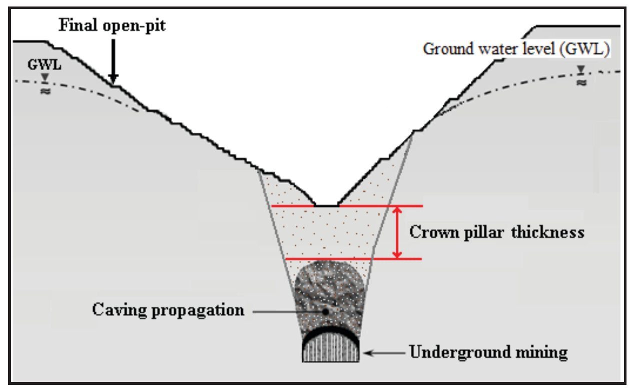


**Figure 1. General challenges in transition problem from open-pit to underground mining.**

In some cases, a period of simultaneous open-pit and underground mining could be required. Hence, at least for a certain period, a stable crown pillar must be maintained between the cave back and the open-pit bottom. This period must be defined after considering the stability of the crown pillar and the fact that its thickness would be reducing due to the ore drawn from the underground mine.

Determining the most adequate thickness of a crown pillar in a combined mining method using open-pit and block caving is one of the most interesting and useful problems faced by mining engineers today.

With increasing depth of the open-pit mines, a combination open-pit and block caving methods is gaining popularity and hence the importance of the problem is increasing. Leaving a pillar with adequate thickness will minimize detrimental interference between the two working areas, while maximizing ore recovery.

Limited studies have been undertaken over the years to determine the surface crown pillar thickness. However, because of the significant differences that exist in behavior between identified failure mechanisms (Carter, 1989; Betournay, 1987), most of these approaches have addressed specific failure characteristics (Goel and Page, 1982, Hoek, 1989). Others attempted to examine the resulting influence zone or the actual sinkhole geometry as a function of the collapse process (Szwedzicki, 1999).

The empirical scaled span approach has been in use for over a decade as a procedure for empirically dimensioning the geometry of crown pillars over near-surface mined openings based on precedent and experience (Carter, 1989 and 1992).

In this paper, on the basis of the scaled span approach and considering the effective parameters for properly dimensioning a crown pillar over the combined mining of open-pit and block caving, a relationship between dependant parameters and the crown pillar thickness is first introduced. Using a dimensional analysis a formula is established and it can be used as a useful tool in all similar mining situations for the mining design engineers to calculate an optimal crown pillar.

## DIMENSIONAL ANALYSIS

Dimensional analysis is a technique for restructuring the original dimensional variables of a problem into a set of dimensionless products using the constraints imposed upon them by their dimensions (Buckingham, 1914; Huntley, 1967; Vignaux, 1986; Vignaux and Jain, 1988). It is ultimately based on the simple requirement for dimensional homogeneity in any relationship between the variables.

There are two main systems: mass and force systems. In a mass system, three units are regarded as fundamental, namely, mass (M), length (L), and time (T), whereas force a system includes force (F), L, and T. Force system is termed base units in this paper. Any other physical unit is regarded as a derived unit, since it can be represented by a combination of these base units. Each base unit represents a dimension. For instance, the units of velocity and acceleration are derived ones and have two dimensions because they are defined by reference to two of the base units - length and time. In what follows, a variable whose unit is a base unit is called a base variable; otherwise the variable is called a derived variable.

The fundamental theorem of dimensional analysis is attributed to Buckingham, and is stated here without proof:

If Equation 1 is the only relationship among $x_i$, and if it holds for any arbitrary choice of the units in which $x_1, x_2, x_n$ are measured, then Equation 1 can be written in the form of Equation 2.

$$f(x_1, x_2, ..., x_n) = 0 \tag{1}$$

$$\phi(\pi_1, \pi_2, ..., \pi_m) = 0 \tag{2}$$

Where $p_1$, $p_2$, …, $p_m$ are independent dimensionless products of the $x$'s.

Further, if $k$ is the minimum number of primary quantities necessary to express the dimensions of the $x$'s, then Equation 3 is applicable.

$$m = n - k \tag{3}$$

Since $k > 0$, $m<n$, according to Equation 3, the number of dimensionless products is the number of dimensional variables minus the number of primary quantities. Another way of writing Equation 2 is:

$$\phi(\pi_1, \pi_2, ..., \pi_m; 1, ..., 1) = 0 \tag{4}$$

Where the number of 1s appearing in the argument list is $k$. Clearly the 1s carry no information about the functional relationship among the $p$'s, so we can just omit them, as was done in Equation 2. In Equation 4, the 1 clearly represent "extraneous" information, which entered the problem through extraneous units of the $x$'s.

The choice of the $x$'s can be made by inspection of the governing equations (if known) or by intuition.

The dimensions of the $x$'s can be determined in terms of chosen primary quantities. Although the primary quantities can be chosen arbitrarily, provided that their units can be assigned independently, we must be sure to choose enough of them so that we can complete the non-dimensionalization.

## MODELLING

Here, the most effective parameters should be first selected. For the purpose, it is essential to study and assess the available methods and the related parameters with emphasis on the scaled span approach. Then, on the basis of the selected parameters, a fundamental formula should be completely deduced by dimensional analysis.

### Available Methods and the Effective Parameters

The available methods for assessment of the stability of a crown pillar encompass a spectrum of techniques from empirical approaches to the application of sophisticated numerical modeling using computer codes such as UDEC, FLAC, PFC and PHASE 2. However, when determining a crown pillar thickness, there are limited semi-empirical procedures that can be only more applicable over certain limited regions (Carter and Miller, 1995).

Although various rule-of-thumb methods for the design of surface crown pillars have been applied in mining practice for well over a century, the research by Carter and Miller (1995) documented numerous failures that have occurred where the rules were simply inappropriate. Attempts have therefore been made to improve the existing rules by undertaking detailed checks of available data to establish rock mass characteristics and pre-failure geometry for as many failed and non-failed surface crown pillars as possible.

Early evaluation led to the development of an improved relationship of the form shown in Figure 2, whereby the thickness to span ratio was employed in the rule-of-thumb approach, and rather than being defined as a single value, was replaced by an expression related to rock mass quality (Carter and Miller, 1995):

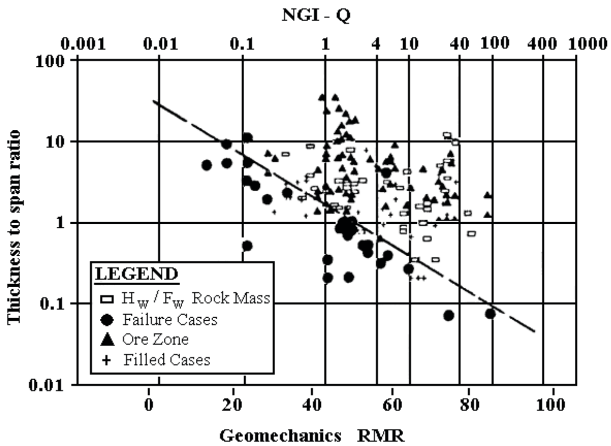


**Figure 2. Crown-pillar case records plotted as thickness to span ratios versus rock mass quality of weakest zone within crown geometry.**

$$\frac{t}{S} = 1.55Q^{-0.62} \tag{5}$$

where $t$, is thickness, $S$ is crown pillar span, and $Q$ is NGI-Q system.

In Fig. 2, $H_w$ and $F_w$ are hanging-wall and foot-wall, respectively.

Initially, it was considered that this method of evaluation would provide a simple guideline relationship setting similar to those for which assessments were held in the database. It was, however, quickly realized that since the relationship was not scale-independent, its use without calibration could very easily lead to significant errors.

The Canada Centre for Mineral and Energy Technology has developed an empirical method for assessing surface crown pillar stability based on an extensive database of crown pillar statistics from a number of countries (Carter and Miller, 1995). This method uses the concept of a critical span, which is a measure of the maximum scaled span for a given surface crown pillar in a particular quality of rock mass beyond which failure may occur. Figure 3 shows the main elements of this scheme.

Designing for stability of near surface crown pillars over excavated openings requires an understanding of many factors, including the excavation geometry, the characteristics of the rock

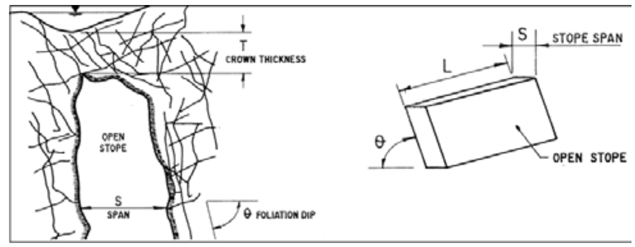


**Figure 3. Geometry definition for scaled span crown pillar stability assessment.**

mass, data on stress conditions, overburden loads, and ultimately, an understanding of safety factors associated with the planned near surface excavation (Hutchinson, 2000).

An extensive study was initiated to examine of the factors that controlled crown pillar stability, and various methods of structural analysis were examined as well (Betournay, 1987; Carter, 1989). These studies demonstrated that for any given rock quality, stability depended principally on geometry. The span, thickness, and weight of the rock mass comprising the crown were found to be the most critical characterizing parameters (Figure 3). This led to initial attempts at normalizing controlling parameters, recognizing the following:

$$C_s = f \cdot \frac{t \cdot \sigma_h \cdot \theta}{S \cdot L \cdot \gamma \cdot u} \tag{6}$$

where $C_s$ is the scaled crown span; $\sigma_h$, the horizontal in-situ stress; $\theta$, the dip (of the foliation or of the underlying stope walls); $L$, the overall strike length of the stope; $\gamma$, the mass (specific gravity) of the crown; and $u$, the groundwater pressure. Other parameters $t$ and $S$ are as defined earlier in Equations 5.

Here, it was evident that all parameters except σh and u were related solely to the geometry of the crown pillar. Therefore, in order to normalize the relationship to be only geometry and weight dependent, it was decided that both these terms should be handled as part of rock mass classification, because both the NGI-Q and the RMR systems take groundwater into consideration (Bieniawski, 1973; Barton et al., 1974), while the effects of in-situ confining stress are well-covered in the Q-system (Barton, 1976; Grimstad and Barton, 1993). Accordingly, the final empirical expression, termed the "Scaled Crown Span," was formulated as follows:

$$C_s = S \cdot \left\{ \frac{\gamma}{\mathrm{t} \cdot (1 + S_R) \cdot (1 - 0.4\cos\theta)} \right\}^{0.5} \tag{7}$$

where: $S_R$ = span ratio = S/L (crown pillar span/crown pillar strike length), and other parameters $C_s$, $S$, $\gamma$, and $\theta$ are as defined earlier in Equations 5 and 6.

Foliation dip in the above expression reflects the span controlling hanging-wall dip (Figure 3). Moreover, as the dip of the foliation, and hence the dip of the stope sidewalls, shallows from 90º to past 45º, the effective span of the stope is no longer the ore zone width but rather the hanging-wall dip length.

The "scaled crown span" expression can be effectively applied to provide a unique characterization of the three-dimensional geometry of a given surface crown pillar. In addition, the "scaled crown span" concept enables fairly reliable comparisons to be made of the stability of different pillars that have been excavated in different rock masses of different overall quality. The approach was based on a simple scaling expression of the form $C_S = S \times K_g$, where a geometric scaling factor, $K_g$, is used to modify the actual span, $S$, to take into account differences in three-dimensional pillar geometry. The scaling relationship was developed to consider all the critical dimensions of crown thickness as well as the dip and geometry of the rock forming the stope walls and surface crown pillar.

As shown in Figure 4, there are three power-law relationships for the assessment of maximum spans in different rock conditions. Although each was originally formulated for the definition of span only, they provide a useful framework for checking scaled spans on the premise that the scaling coefficient, $K_g$, incorporates all appropriate three-dimensional factors to ensure that $C_s$ is suitably scaled.

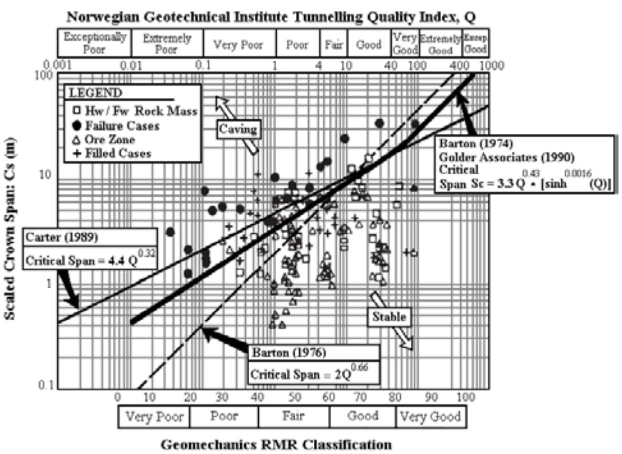


**Figure 4. Crown pillar case records in Golder-CANMET plotted as scaled crown spans versus Q or RMR.**

1. The line proposed by Barton in 1976 to define the maximum span of generally unsupported civil engineering openings (critical span, $S = 2Q^{0.66}$) tends to the conservative side for poor rock-quality conditions.
2. The power-law expression for average critical span proposed by Carter to fit the mean trend to the various mining engineering classifications (critical span, $S = 4.4\ Q^{0.32}$) tends, by contrast (mainly because it essentially addresses only short-term mining requirements), to underestimate the time-dependent influences on failure that are seen in some of the older case records at the poor-quality end of the scale.
3. When the shape of the original empirical "unsupported-span" curve outlined by Barton and co-workers in 1974 is plotted together with a non-linear tail to encompass Barton's various data points at the very good-quality end of the scale (these are not shown in Figure 4), the resulting curve tends not only to separate the case records in the crown pillar database better, but also tracks other available data for good and very good rock conditions more accurately. The following expression, termed the "critical span line", has therefore been developed to match the shape of Barton and co-workers' original curve:

$$S_c = 3.3Q^{0.43} * (\sinh^{0.0016} Q) \qquad (8)$$

Where $S_c$, m, provides a measure of the maximum scaled span for a given pillar beyond which failure may occur. It should be appreciated, however, that the hyperbolic sin *h* term in Equation 6 has been introduced simply to account for the non-linear trend to increasing stability at the very good quality and of the Q-RMR scale as indicated in Barton's original Q data and suggested by some of the case records in the Golder-CANMET crown-pillar database.

**Discussion and Selection of the Most Effective Parameters for Modeling**

Since the conditions and concept of "surface crown-pillar" and "crown-pillar between open-pit and underground mining" are immensely similar, the effective parameters considered in respect to "surface crown-pillar" can be also selected for the other crown-pillar.

In all open-pit mines where there is a risk of intersecting underground mine workings, appropriate studies must be carried out to determine the minimum stable crown pillar dimensions. The minimum crown pillar thickness is defined as the minimum rock cover, measured vertically, above the highest point of the underground workings which provides an acceptable factor of safety against crown pillar failure during all mining activities.

In general, decision-making is frequently complicated merely by the difficulty of determining a suitable thickness of the crown pillar between the open-pit and block caving methods.

The minimum surface crown pillar thickness requires approximately to 2 to 3 years of simultaneous open-pit and underground operations. This will allow for simultaneous mining of the final open-pit and initial underground panel cave (Arancibia and Flores, 2004).

This simultaneity implies an interaction between the open-pit and the underground mining which makes the problem more complex than the typical open-pit or underground mine designs, because the presence of the deep open-pit will affect the stress field in which the underground mine will be developed and, conversely, the propagation of the caving will affect the stability of the surface crown pillar that defines the bottom of the open-pit. Additionally, there are many other factors or potential hazards that could make the problem even more difficult if they are not identified prior to making the transition from open-pit to underground mining (Flores, 2004).

Some of the major questions to be answered in a transition from open-pit to underground cave mining are listed below (Flores, 2004):

- What is the optimum height of the ore column that can be mined safely from an economical/geotechnical/operational perspective?
- Will the cave propagate upward through the entire block height?
- What is the minimum thickness of the surface crown pillar that will allow simultaneous surface and underground operations?
- When is it no longer safe to be mining in the open-pit while caving is occurring? How long could both mines operate simultaneously?
- Will the subsidence generated by the underground mining affect the surface infrastructure surrounding the pit? When?
- What are the main geotechnical hazards, and how should they be dealt with?

In addition, many aspects of the transition problem are beyond the ranges of applicability of known solutions. For example, the simultaneous operation of the open-pit and underground mines by caving methods requires a stable surface crown pillar between the cave back and the pit bottom. However, at the same time, cave propagation requires the failure of this pillar to connect to ground surface, so the definition of crown pillar failure is not the usual. Furthermore, the span of this surface crown pillar is much larger than the maximum span of surface crown pillar used in open stope mining (Flores, 2004).

During a numerical analysis of interaction between block caving and open-pit mining and cavability assessment of the crown pillar, it was concluded that a weaker slope may impose higher stress in the crown pillar. This may, in turn, delay the cave propagation and, therefore, increase the risk of rapid crown pillar collapse. The open-pit rock mass quality may influence the crown pillar response and affect cave propagation behavior and, in turn, the caving-induced unloading of the open-pit influences open-pit slope stability (Vyazmensky et al., 2009).

The determination of the stable crown pillar thickness should be the result of a geotechnical engineering assessment in which specific attention is paid to the following (MOSHAB, 2000):

- Orebody geometry, particularly orebody dip and orebody width
- The likely modes of failure of the stope crown pillar, whether controlled by, or independent of, geological structure
- The likely modes of failure for the immediate hangingwall and footwall rocks whether controlled by, or independent of, geological structure
- The potential accumulation of water in the open-pit due to localized ponding via surface runoff from the surrounding catchment area and/or incident rainfall within the open-pit perimeter
- The loads imposed by equipment or stockpiles on the crown pillar
- Rock mass strength and/or general competence of pillar and wall rocks
- “worst-case” geotechnical conditions with particular emphasis on structural geological features (planes of weakness), groundwater, variations in rock strength and their likely impact on the stability of the crown pillar
- The influence of open-pit blasting on the integrity of the pillars
- The relationship of pillar thickness to the width and strike length of stoped areas

The adopted stable pillar thickness will vary both within an individual site and from site to site, to reflect the extent of the hazard, the variation in controls on pillar stability, and the range of geotechnical conditions together with the extent and dimensions of stopping (MOSHAB, 2000).

Here, considering the most important aspects of “crown pillars between open-pit and block caving” and the available methods in relation to “surface crown pillars” with emphasis on the scaled span approach, the most effective parameters have been selected, underlining the following notes and discussion:

- Examination of documented crown pillar failures in blocky rock mass suggests that failure most frequently develops where several adversely orientated discontinuities intersect or where a particular suit of major joints provides a release mechanism for gravity collapse. Similarly, in failures of significant areal extent, the geometry is often controlled by the orientations of major individual discontinuities. Most of discontinuities characteristics (such as discontinuity condition, spacing and orientation) are reflected in geomechanics RMR classification.
- The strength of a pillar depends on the following: geometrical parameters (the width-to-height ratio and the shape of the pillar), the strength of the rock mass, and the presence and orientation of joints and other weak zones (Kersten, 1984).
- Although correct characterization of the weakest part of the rock mass in the crown zone is the key to appreciation of the inherent strength of the pillar, accurate information on the geometry of the underground stope excavation is also essential to a proper assessment of stability.
- According to the concept of a critical span and the scaled crown span, which was developed by the Canada Centre for Mineral and Energy Technology, some parameters should be considered as: the *crown pillar or stope span*, the *overall strike length of the stope*, the *mass (specific gravity) of the crown*, and the *groundwater pressure*.
- Hutchinson (2000) noted that in order to design a crown pillar, some factors should be considered: the *excavation geometry*, the *characteristics of the rock mass*, *data on stress conditions*, *overburden loads*, and *safety factor*.
- On the basis of the study that was done in relation to evaluate the effective parameters that control crown pillar stability, it was demonstrated that for any given rock quality, the *span*, *thickness*, and *weight of the rock mass* are the most critical characterizing parameters.

For the purpose of reflecting the mentioned notes with the all aspects in order to determination of the optimum crown pillar thickness between open-pit and block caving, the most effective parameters (variables) are considered as the following.

- Stope span: It was considered in scaled crown span approach.
- Stope height: It should be considered as an important parameter affecting the height of cavable materials.

- RMR: In order to consider characteristics of discontinuities, groundwater condition, and some characteristics of the rock mass, such as uniaxial compressive strength (UCS).
- Cohesion strength: It should be better to consider as a critical rock mass character shows the rock cavability.
- Specific weight of rock mass: As a rock mass character, which was previously considered as a critical parameters in surface crown pillar assessment.

The considered variables with the meaning of the system variables are listed in Table 1.

**Table 1. Meaning of the considered variables.**

| Variable | Variable meaning |
|---|---|
| *t* | crown pillar thickness |
| *s* | stope span |
| *h* | stope height |
| *RMR* | rock mass rating |
| *C* | cohesion strength |
| *γr* | specific weight of rock |

**Dimensional Analysis for Crown Pillar Thickness Formulation**

In this section, on the basis of the considered variables, a fundamental equation should be established to determine the optimal crown pillar thickness. Therefore, the crown pillar thickness (*t*) is assumed to be a function of the variables as below:

$$t = f(s, h, RMR, C, \gamma_\gamma) \tag{9}$$

To specify the relationship among the independent and dependant variables of the problem, Equation 9 can be transformed into Equation 10.

$$f(t, s, h, RMR, C, \gamma_\gamma) = 0 \tag{10}$$

Here, adopting the force system for the expression of the dimensions, the dimensional values for each variable are worked out as shown in Table 2. Then, to make the dimensional matrix, the variables should be accurately arranged as in Table 3.

**Table 2. Dimensional values.**

| Variable | Dimension |
|---|---|
| t | [L] |
| s | [L] |
| h | [L] |
| RMR | [1] |
| C | [$FL^{-2}$] |
| $\gamma_r$ | [$FL^{-3}$] |

**Table 3. Dimensional matrix.**

| Dimension | Quantity | | | | | |
|---|---|---|---|---|---|---|
| | t | s | h | RMR | C | γr |
| F | 0 | 0 | 0 | 0 | 1 | 1 |
| L | 1 | 1 | 1 | 0 | -2 | -3 |
| T | 0 | 0 | 0 | 0 | 0 | 0 |

Now, in order to assign an appropriate degree of the matrix, determinant of right side of the dimensional matrix is calculated as the following:

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & -2 & -3 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Inasmuch as the determinant amount of this matrix is zero, on the basis of Buckingham theorem Equation 3 can be appropriately used. In this regard, there are two primary quantities, and six variables, so we should be able to eliminate 6 - 2 = 4 pieces of extraneous information.

The homogeneous linear algebraic equations (11 and 12) can be derived from the dimensional matrix.

$$K_5 + K_6 = 0 \tag{11}$$

$$K_1 + K_2 + K_3 - 2K_5 - 3K_6 = 0 \tag{12}$$

It should possibility be considered and allocated different amounts to $K_1$, $K_2$, $K_3$, $K_4$ and then to determine $K_5$, $K_6$. Then, Equations 11 and 12 can be solved. In this regard, matrix of responses can be made as shown in Table 4.

**Table 4. Matrix of responses.**

| | K1 | K2 | K3 | K4 | K5 | K6 |
|---|---|---|---|---|---|---|
| | t | s | h | RMR | C | γr |
| π1 | 1 | 0 | 0 | 0 | -1 | 1 |
| π2 | 0 | 1 | 0 | 0 | -1 | 1 |
| π3 | 0 | 0 | 1 | 0 | -1 | 1 |
| π4 | 0 | 0 | 0 | 1 | 0 | 0 |

There are obviously five independent dimensionless products, as the following: shows

$$\phi\left(\frac{t \cdot r_r}{C}, \frac{s \cdot r_r}{C}, \frac{h \cdot r_r}{C}, RMR\right) = 0 \quad (13)$$

After finding the relationship among the dimensionless products, it is essential to specify the best equation type, namely linear or non-linear. Here, linear and non-linear equations can be written as Equations 14 and 15, respectively.

$$\left(\frac{t \cdot \gamma_\gamma}{C}\right) = a + b_1 \cdot \left(\frac{s \cdot \gamma_\gamma}{C}\right) + b_2 \cdot \frac{h \cdot \gamma_\gamma}{C} + b_3 \cdot (RMR) \quad (14)$$

$$\ln\left(\frac{t \cdot \gamma_\gamma}{C}\right) = a + b_1 \cdot \ln\left(\frac{s \cdot \gamma_\gamma}{C}\right) + b_2 \cdot \ln\left(\frac{h \cdot \gamma_\gamma}{C}\right) + b_3 \cdot \ln(RMR) \quad (15)$$

On the basis of the problem nature and specification the non-linear relationship (Equation 15) seems to be more appropriate.

After making some simplifications, Equation 15 can be transformed into Equations 16 and 17, respectively. Equation 18 is finally achieved as the fundamental equation (formula) to determine the optimal thickness of the crown pillar between open-pit and underground mining, including the unknown coefficients.

$$e^{\ln(\frac{t \cdot \gamma_\gamma}{C})} = e^a + e^{b_1 \cdot \ln(\frac{S \cdot \gamma_\gamma}{C})} + e^{b_2 \cdot \ln(\frac{h \cdot \gamma_\gamma}{c})} + e^{b_3 \cdot \ln(RMR)} \quad (16)$$

$$t = e^a * \left(\frac{C}{\gamma_\gamma}\right) * \left(\frac{s \cdot \gamma_\gamma}{C}\right)^{b_1} * \left(\frac{h \cdot \gamma_\gamma}{C}\right)^{b_2} * (RMR)^{b_3} \quad (17)$$

$$t = e^a * C^{(1-b_1-b_2)} * S^{(b_1)} * h^{(b_2)} * RMR^{(b_3)} * \gamma_\gamma^{(b_1+b_2-1)} \quad (18)$$

The unknown coefficients of Equation 18 can be determined on the basis of a data set assembling from a number of case studies with similar conditions by using multiple regression. The assembled data of the four real cases are listed in Table 5.

Here, SPSS software (version 14) has been used for statistical and regression analyzing and determination of the unknown coefficients. In this relation, the coefficients are identified as below:

$a = 2.582$, $b_1 = 0.41$, $b_2 = 0.56$, $b_3 = -0.66$

**Table 5. The related data of the real case examples.**

| Case studies | Values | | | | | |
|---|---|---|---|---|---|---|
| | t | s | h | RMR | C | γr |
| **1** | 200 | 180 | 400 | 62.5 | 0.75 | 2.7 |
| **2** | 200 | 220 | 400 | 75 | 2.9 | 3.1 |
| **3** | 180 | 190 | 230 | 48 | 1 | 2.75 |
| **4** | 230 | 250 | 460 | 70 | 0.82 | 2.81 |

Therefore, Equation 19 can be derived; it is the best formula for determining a practical crown pillar thickness should be considered between open-pit and underground mining.

$$t = \frac{13.22 * C^{0.03} * S^{0.41} * h^{0.56}}{\gamma_\gamma^{\ 0.03} * RMR^{0.66}} \quad (19)$$

## CONCLUSIONS

Today, one of the most critical problems faced by mining engineers is determining the optimal thickness of a crown pillar in a combined mining method using open-pit and block caving. Therefore, the authors attempted to establish a formula for determining of optimal thickness of the crown pillar.

During the first step of modeling, "crown pillar thickness" has been considered as a function of the most effective variables such as stope span, stope height, cohesion strength, RMR, and specific weight of rock. Then, utilizing dimensional analysis, the fundamental equation was deduced which includes the unknown coefficients. The coefficients of the equation were determined based on a data set of combined mining case studies using the multiple regression and SPSS 14 software. The achieved formula can be practicable in all situations where a combined open-pit and block caving method is appropriately used.

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