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Abstract: What is very often overlooked in the literature is that the Harrod's Post-Keynesian growth model is more to do with the problem of instability in a market economy which is caused by the role of expectations of the investors. The neoclassical model of growth due to Solow achieves stability not due to its assumption of smooth twice differentiable production function but assuming away the role of uncertainty.

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Writing in the 1930s, Keynes thought that the prevailing world depression and the problem of unemployment, in a world full of wants, were caused by "a new disease … namely, *technological unemployment*" (Keynes, 1972: 325; emphasis in original) which means unemployment caused by the labour saving technical progress. Keynes, however, envisaged it as a temporary phase of maladjustment and was convinced that the problem would be solved within a hundred years which "…means that the economic problem is not – if we look into the future – *the permanent problem of the human race*."(ibid: 326; emphasis in original). We must not forget that the 'economic problem' Keynes was referring to was the struggle for subsistence, which "always has been hitherto the primary, most pressing problem of the human race.." (*ibid*: 327). However, to even a casual reader of The General Theory of Employment, Interest and Money, the importance Keynes attached to the problem of unemployment should be obvious; particularly when he writes that, "(w)e need to throw away the second postulate of the classical doctrine and to work out the behaviour of a system in

which involuntary unemployment in the strict sense is possible" (Keynes, 1936: 16-17). Later, in Chapter 22 he states that a theory on determinants of volume of employment "must be capable of explaining the phenomena of trade cycle" (*ibid*; 313).

Though Harrod (1937) found that Keynes's contributions on economics constitute a genuine revolution in many fields the only criticism that he had of Keynesian economic system is that it *is still static*. After acknowledging the fact that Keynes has laid great stress on the role of anticipations in determining current equilibrium, he goes on to say

> (b)ut reference to anticipation is not enough to make a theory dynamic. For it is still static equilibrium which the anticipations along with other circumstances serve to determine; we are still seeking to ascertain what amounts of the various commodities and factors of production will be exchanged or used and what prices will obtain, so long as the conditions, including anticipations, remain the same. But in the dynamic theory, as I envisage it, one of the determinands will be the rate of growth of these amounts. Our question will then be, what rate of growth can continue to obtain, so long as the various surrounding circumstances, including the propensity to save remain the same? (Harrod, 1937: 86).

Thus the dynamic equilibrium, as Harrod saw it, would be concerned with such question as "what rate of growth of certain magnitudes is consistent with the surrounding circumstances" (Harrod, *ibid*: 86).

Harrod's dynamic theory has developed through a number of years (see for example, Harrod, 1939; 1948; 1973). In what follows we present the basic features of Harrod's model of economic growth. We start from Harrod's contribution since much of the subsequent literature on growth economics have Harrod's instability problem (to be explained bellow) as their starting point[1].

The axiomatic basis of Harrod's theory, as presented in Harrod (1939: 14) are (i) the level of a community's income is the most important determinant of its supply of saving; (ii) the rate of increase of its income is an important determinant of its demand for saving and (iii) that demand is equal to supply. These set of three axioms "consists in a marriage of the 'accelerator principle' and the 'multiplier' theory" (*ibid*: 14). Thus, as Harcourt (1972: 15) puts it, Harrod's work added "the capacity creating effects of investment" to Keynes's "employment-creating aspects of investment". We now turn our attention to the Harrod's model of economic growth.

The instability problems due to Harrod

We start from, what is known in the literature as, the *First Harrod Problem* – that although steady state growth at full employment is possible in a model of economic growth, such a '*Golden Age'* (Robinson, 1969: 99-100) is highly improbable since the constituents variables determining the actual rate of growth G_a, *warranted* rate of growth, G_w, (defined as 'that overall rate of advance which, if executed, will leave entrepreneurs in a state of mind in which they will be prepared to carry on a similar advance' Harrod (1948: 82)), the natural rate of growth G_n (the rate of growth of labour) are determined independently of each other. *First Harrod*

[1] Though the literature some times refers to the Harrod-Domar problem, the problem of instability addressed in it is due to Harrod.

Problem – that although steady state growth at full employment is possible in a model of economic growth, such a '*Golden Age'* (Robinson, 1969: 99-100) is highly improbable since the constituents variables determining the actual rate of growth G_a, *warranted* rate of growth, G_w, (defined as 'that overall rate of advance which, if executed, will leave entrepreneurs in a state of mind in which they will be prepared to carry on a similar advance' Harrod (1948: 82)), the natural rate of growth G_n (the rate of growth of labour) are determined independently of each other. The First Harrod Problem is the first step to what Harrod believes is the central issue in a free market economy; that "sooner or later we shall be faced once more with the problem of stagnation, and that it is this problem that economists should devote their main attention" (Harrod, 1948: v). This central issue known as the Second Harrod Problem is deviation of the actual rate of growth from the warranted rate for from being self-correcting are cumulative in effect. As Hahn and Matthews (1969: 27) commented:

> It is important to distinguish clearly between the two quite separate obstacles to steady growth that were considered by Harrod in his pioneering contribution. (1) The warranted rate may be unequal to the natural rate. (2) The warranted rate may itself be unstable, even without reference to the natural rate. The second of these problems is the "knife-edge" properly so-called, though the term is sometimes used confusingly to refer to the first problem as well.

The Second Harrod Problem arises due to the fact that planned (or ex-ante) investment may not always be equal to actual (ex-post) saving. From an accounting point of view ex-post investment is always equal to ex-post saving – it is a tautology. However, to reach equilibrium ex-ante investment needs to be equal to ex-

post saving. And this is the context in which the roles of uncertainty and expectation in an economy become vital; where the Second Harrod Problem stems from. This Problem states that the warranted rate of growth is fundamentally unstable in the sense that divergence of actual rate of growth, G_a, from the warranted rate, G_w, is not only not self-correcting but, *left to itself*, would produce even larger divergences over time (Harrod, 1948).

In Chapter 2 of the *General Theory* Keynes made the statement that the savings and investment decisions in an economy are made independently of each other and there is no connection guaranteeing savings and investment equality; and the investors lack the power of equating real rate of return with the marginal disutility of the factor of production and thereby elicit full employment. However, "(w)hat is missing from this chapter, and from the whole *General Theory,* is an explanation of how these two propositions interlock" (Shackle, 1967: 139). Though in his *Treatise on Money*, Keynes said that if "if investment exceed saving, the system would be stimulated to expand and conversely." (Harrod, 1939: 19) he seems to have later abandoned any further attempt to explore the implication of the concept. According to Harrod, Keynes's proposition of Treatise may still be a useful aid to thinking, "if for the definitions on which that proposition was based, we *substitute* (for investment) the definition of *ex-ante* investment" (Harrod, *ibid*: 19).

Commenting on the instability problem due to Harrod Joan Robinson (1961:360) pointed out that:

> 'As the statement of *ex ante* equilibrium conditions, it (the familiar formula $g = s/v$) fails to isolate independent variables; s, the ratio of annual net saving to annual net income, is strongly influenced by the ratio of profits to income,

> which in turn is strongly influenced by the ratio of annual net investment to the value of capital, that is, by *g* itself; *v*, the ratio of value of capital to annual net income, is influenced both through the prices of capital goods and through the choice of technique, by the ratio of profit, which is a function of *s* and *g*. All the formula can say is that, if growth is going on *under equilibrium conditions* at the rate *g*, then *s/v* is equal to it.' (emphasis added).

She went on to add:

> 'Harrod…….. did not want to throw away the *General Theory* and make savings govern investment…What he shows is that, if we write down a function for the inducement to invest (whether in terms of the accelerator, or of expected profits, of the supply of finance, or just of the animal spirits of the managers of firms) generating a desired rate of growth, and a set of identical conditions (the labour supply, the flow of new investment and so forth) providing a 'natural' or better, a physically possible rate of growth, and, furthermore, postulate equilibrium with full employment, we have overdetermined our system.' (Robinson, 1961:360-61).

What are the ways out of this problem of an over determined system? Robinson (1961) suggested three ways out;

(i) Give up the idea of equilibrium and exhibit an economy blundering on from one situation to another (as happens in the history of the world we live in) following no simple predictable path. In other words, learn to live with the problem envisaged by Harrod.

(ii) Introduce a functional connection between the desired and the possible growth rate so that the one determines the other

(iii) Give up the desired rate of growth and simply assume that actual growth goes on, in equilibrium conditions, with continuous full employment of available labour

The Neo-classical Solution

Solow (1956) is an example of the third way out of Harrodian instability problem suggested by Joan Robinson above. He claimed that the Harrod-Domar[2] model studied long run behaviour of the economy, which is the domain of neoclassical analysis ('the land of margin'), with tools of short run analysis such as the multiplier, accelerator and fixed capital coefficient. He proposed a 'model of long-run growth which accepts all the Harrod-Domar assumptions *except that of fixed proportions*' (Solow, 1956:66; emphasis added). In this model Solow could demonstrate that the steady state growth with full employment can be achieved in a model of economic growth, constructed using the neoclassical general equilibrium methodology.

Solow (1956), however, implicitly assumed away existence of uncertainty as presented by equation 1 of his model, as well as his arguments leading up to the equation, that savings – a proportion of income, is always equal to the investment at every instant of time. This assumption of investment as an accommodating variable *not* that of a flexible capital-labour ratio and an exogenously given rate of growth of labour guaranteed long run stable equilibrium. Not surprisingly, growth in this general equilibrium model comes from the exogenously determined rate of growth of the efficiency of labour.

[2] Though Solow (1956) refers to Harrod-Domar, the problem of instability addressed in it is due to Harrod.

As Hahn and Matthews (1964:789-790) have pointed out, 'In its basic form the neo-classical model depends on the assumption that it is always possible and consistent with equilibrium that investment should be undertaken of an amount equal to full-employment savings. The mechanism that ensures this is as a rule not specified.' The rate of interest that ensures planned investment is equal to full employment saving is adjusted in one of the three possible interventions: (a) through the operation of Say's Law, in the absence of money or when demand for money is interest inelastic; (b) through adjustment of the price level to influence the rate of interest via its effect on the real money balances or (c) through the use of appropriate monetary policy (Hahn and Matthews, 1964, p.790). The last of these three options is due to Meade (1961). The implication of this policy suggestion, due to Meade, is that there has to be some form of *intervention by the monetary authorities* to ensure the stability of the neoclassical models of economic growth. The implication of this last observation is that the neoclassical growth model can no longer be considered as a closed system. Otherwise, following Hahn and Matthews (1964:790), 'The familiar Keynesian difficulties therefore arise….'

What will happen if we reintroduce the role of an independent investment function in a neoclassical growth model while retaining all the other assumptions of the model – constant returns to the scale, a smooth twice-differentiable production function that satisfies the Inada conditions (Inada, 1964), the marginal productivity theory of distribution as well as the *flexible capital labour ratio*? Sen (1970) did just that. He started with the well behaved neoclassical production function

$$Y = e^{mt} K^{\alpha} L^{1-\alpha} \qquad (1)$$

(where, Y, K and L stand for output, capital and labour respectively.)

Which leads us to $\frac{Y}{K} = \frac{r}{\alpha}$ (1′)

where *r*, commodity (assumed = money) rate of interest is equal to $\frac{\delta Y}{\delta K}$ or marginal productivity of capital. Since *r* and α are given, *Y* and *K* must grow at the same proportional rate. Therefore Harrod's warranted rate, $G_w = s/v$, turns out to be

$$G_w = \frac{rs}{\alpha} \qquad (2)$$

Sen (1970) then introduced an independent investment function based on an expected rate of growth, which is not necessarily equal to the warranted rate. A set of neoclassical entrepreneurs, given the expected (exponential) rate of growth of *j* over time plan to invest enough to make an expected rate of profit equal to the own rate of interest.

Using (1′) we get:

$$r = \frac{\bar{Y} e^{jt} \alpha}{K_t} \qquad (3)$$

$\bar{Y}$ = current income

From (3)

$$K_t = \frac{\bar{Y} e^{jt} \alpha}{r}$$

From the multiplier relationship, we have

$$Y_t = \frac{1}{s}\frac{dK_t}{dt} = \bar{Y} e^{jt} j \frac{\alpha}{rs} \qquad (4)$$

But from (2) $G_w = \frac{rs}{\alpha}$

So if $j = \frac{rs}{\alpha}$ (i.e., expected rate = warranted rate) actual income is equal to the expected income. Let the actual rate of growth is given by $Y_t = \bar{Y} e^{Gt}$. Now, if $j > \frac{rs}{\alpha}$ we have $j < G_a$ and if $j < \frac{rs}{\alpha}$ we have $j > G_a$

Thus, in whatever direction one might err in, one would feel that the error lies in the other direction causing the second Harrod problem to come back.

Before we end this section, it will be interesting to point out that Solow himself was not completely unaware of the possibility that investment may not always be an accommodating variable. In a small section, entitled 'Uncertainty etc.', at the end of his 1956 article he wrote:

> 'No credible theory of investment can be built on the assumption of perfect foresight and arbitrage over time. There are only too many reasons why net investment should be at times insensitive to current changes in the real return to capital, at other times oversensitive. All these *cobwebs* and some others have been brushed aside throughout this essay. *In the context, this is perhaps justifiable*.' (Solow, 1956:93-94; emphasis added).

It would be interesting to speculate about the 'context' which 'justifies' these 'cobwebs' to 'be brushed aside'.

Open System

What makes Harrod's post-Keynesian growth model different from Solow's orthodox neoclassical growth model? We have already seen that the one crucial difference between them is the role of uncertainty in these models. From another perspective the differences between these models can be the underlying economic systems of these two models. As Loasby (2003: 291) observed, '[a]ny system consists of elements of connections between them … If every element is connected to every other element, the system (real or conceptual) exists in integral space' and he cites general equilibrium models in economics as an example. Thus the neoclassical general equilibrium model due to Solow is an example of a closed system – in this model it is

not possible to incorporate uncertainty; hence uncertainty is assumed away from this kind of model. However, as opposed to a close system, '(a)n open system, …….has interactions with the outside world. In the real world instances of perfect isolations are rare' (Chick, 2004: 5). And this becomes only more relevant in the presence of uncertainty. In real world, since the elements of social life go through many changes and transformations over time, their interaction with it implies that the underlying economic system will also undergo many changes as time passes. The arguments presented in this paper are based upon the understanding that the economic system in Harrod is an open system in the sense described by Chick (2004) and Loasby (2003).

In closed systems like the neoclassical general equilibrium model, history or social interactions do not matter. As Kaldor (1972) writes, '(t)he very notion of 'general equilibrium' carries the implication that it is legitimate to assume that the operation of economic forces operate in an environment that is 'imposed' on the system in a sense other than being just a heritage of the past – one could almost say an environment which in its most significant characteristics is independent of history.' (Kaldor, 1972:.1244). He then goes on to observe (in the same paragraph) that 'Continuous economic change on these assumptions can only be conceived as some kind of 'moving equilibrium' through the postulate of an autonomous (and unexplained) *time rate of change in the exogenous variables* of a kind that is consistent with 'continuous equilibrium' through time…'.

One of the main objections made by Kaldor is to the underlying assumption of the general equilibrium analysis of the automatic equality between planned saving and investment. He pointed out that Keynes postulated that '…in one particular market, the market for savings, the price is not, or need not be, 'market clearing' (owing to liquidity preference), and if it is not, there is another mechanism that of the multiplier,

to bring about equality in that market …..But that mechanism operates by varying the amount of production in *general*. It leads to a situation that is *not* resource constrained.' (Kaldor, 1975: 350; emphasis and parentheses in the original). Looked at from this point of view, the condition which is relevant for our discussion relates to the assumptions of the economic agents' behaviour in the post-Keynesian and neoclassical growth models.

In an open system, agents and their interactions may change and the structure and agency are interdependent. In closed systems, the nature of atomistic economic agents is treated as if constant (Chick and Dow, 2005: 366-367). Such an assumption of agents' behaviour is what guarantees equality between planned savings and investment. In the context of Keynesian uncertainty, however, an assumption of economic agents running on an 'auto pilot' is hard to justify. Planned saving need not and does not equalise with planned investment all the time.

Going back to Joan Robinson's suggestion (i) listed above, in the presence of instability which is; "[g]ive up the idea of equilibrium and exhibit an economy blundering on from one situation to another (as happens in the history of the world we live in) following no simple predictable path. This is relevant for policy making in a real world. As Loasby (2003: 294) has observed "[p]artial closure is necessary for any exploration of openness; we have to close our mind to many possibilities in order to pay attention to a few. However, our choice of partial closure in a real world will be dictated by the political economy of the time as shaped by the economy's history. The economic policies which were relevant during the 1930s depression are not strictly relevant during the current economic downturn of a globalized world of off balance sheet items and sub-prime collateral debt obligation.

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