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Discrete rate maximisation power allocation with enhanced bit error ratio

W. Al-Hanafy[1] *S. Weiss*[2]

[1]*Electronics and Communication Engineering Department, Faculty of Electronic Engineering, Menoufia University, Menouf, Egypt*
[2]*Centre for White Space Communications/CeSIP, Department of EEE, University of Strathclyde, Glasgow, UK*
E-mail: waleed_alhanafy@yahoo.com

Abstract: This study aims to maximise the rate over a multiple-in multiple-out (MIMO) link using incremental power and bit allocation. Two different schemes, greedy power allocation (GPA) and greedy bit allocation (GBA), are addressed and compared with the standard uniform power allocation (UPA). The design is constrained by the target bit error ratio (BER), the total power budget and fixed discrete modulation orders. The authors demonstrate through simulations that GPA outperforms GBA in terms of throughput and power conservation, whereas GBA is advantageous when a lower BER is beneficial. Once the design constraints are satisfied, remaining power is utilised in two possible ways, leading to improved performance of GPA and UPA algorithms. This redistribution is analysed for fairness in BER performance across all active subchannels using a bisection method.

1 Introduction

The adaptation of transmission resources in terms of bit and power allocation according to channel conditions in multichannel systems has been proven to significantly enhance the overall system performance provided that channel state information (CSI) is known at the transmitter [1, 2]. This includes the achievement of either higher data rates or lower power requirements under one or more practical design constraints known, respectively, in the literature as rate maximisation [3, 4], or margin maximisation [5, 6]. Multiple transmission channels arise, for example, in multicarrier systems such as orthogonal frequency division multiple access (OFDM) and for multiple-in multiple-out (MIMO) systems using spatial multiplexing based on, for example, the singular value decomposition (SVD) [7]. In both cases a number of subchannels with different gains is obtained over which reliable communication is to be established. The parameters to be considered in such loading problems are the bit error ratio (BER), the data rate and the total expended transmit power. The sum rate of a multichannel system with different subchannel gains is of particular interest from a system design point of view, and can be optimised using bit and/or power loading schemes [8, 9].

Existing research in resource (power and bit) allocation usually assumes optimal standard water-filling (WF) [10] solutions, WF-based solutions [11] or a modified WF algorithm [12]. However, WF algorithms assume infinite modulation orders and real-valued data rates which is realistically infeasible and leads to a final rounding remedy step that degrades the overall performance [3]. Alternatively, so-called incremental or greedy approaches [5, 13–16] have been proved to be optimal in this sense. Incremental algorithms in [17, 18] use all the power budget but yield poor bit distribution as they do not consider the power gains on subchannels, and instead assume uniform power allocation. On the contrary, in [4, 9, 19], the authors use integer bit loading, but attain poor power utilisation because of the integer bit constellation constraint and therefore lift some undistributed power. A remedial step to redistribute the excess power claiming for further bit allocation in practical OFDM loading systems is considered in [20]; however, power budget is still not fully consumed. Moreover, the final bit allocation does not achieve fair BER among all subchannels which is important for, for example, digital subscriber line applications.

Power and bit allocation problems are usually phrased as closed-form expressions with respect to either channel capacity [10] or bit error probability [21, 22]. In particular, optimising sum rate using power [4] and bit loading schemes [18] can achieve higher rates at the expense of computational complexity.

In this paper, data rate maximisation is considered for multicarrier or SVD-based decoupled MIMO systems using both power and bit loading schemes. Moreover, the aim is to establish fair, – that is, balanced – BER performance among active subcarriers/subchannels. Two different greedy approaches are examined and compared, both are trying to maximise the overall rate with the same set of constraints. However, one of these algorithms considers greedy power allocation (GPA) which achieves the target BER to its maximum desirable value. The algorithm in [18] uses the greedy approach for bit loading, whereas power is

uniformly distributed among all subchannels. The latter approach is here referred to as greedy bit allocation (GBA), and its main task is to ensure that the average BER does not exceed the target BER. Both GPA and GBA approaches will be compared with the standard uniform power allocation (UPA) scheme. While achieving the target BER, both GPA and UPA schemes would save some unused (excess) power, which can be redistributed for BER improvements. Two power redistribution algorithms are considered, whereby the first approach simply allocates power equally among all active subchannels whereas the second method aims to achieve fair BER across these subchannels.

The remainder of this paper is organised as follows. In Section 2 the rate maximisation problem of our system model is formulated, whereas the greedy approach solutions are outlined in Section 3. BER improvement algorithms using excess power redistribution are proposed in Section 4. Simulation results evaluating system performance are highlighted in Section 5 and conclusions are drawn in Section 6.

2 Problem formalisation

We consider the problem of maximising the throughput of a narrowband MIMO system with N_{T} transmit and N_{R} receive antennas characterised by an $N_{\mathrm{R}} \times N_{\mathrm{T}}$ channel matrix $\boldsymbol{H}$ under the constraints of: a fixed total transmit power budget P_{budget}, a specified target BER $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$ and fixed quadrature amplitude modulation (QAM) modulation orders

$$M_k = \begin{cases} 2^{b_k}, & 1 \le k \le K \\ 0 & k = 0 \end{cases} \tag{1}$$

where the maximum constellation size $M_K = 2^{b^{\max}}$, with $b_k \in \{0, 1, 2, \ldots, b^{\max}\}$, is limited.

By means of an SVD, the channel matrix $\boldsymbol{H}$ can be decoupled into N independent subchannels with gains σ_i, $1 \le i \le N$, organised in descending order, where $N = \mathrm{rank}(\boldsymbol{H}) \le \min(N_{\mathrm{R}}, N_{\mathrm{T}})$ and σ_i are the singular values of $\boldsymbol{H}$. This leads to the formulation of a rate maximisation problem

$$\max \sum_{i=1}^{N} b_i \tag{2a}$$

subject to the constraints

$$\sum_{i=1}^{N} P_i \le P_{\mathrm{budget}} \quad \text{and} \quad \mathcal{P}_{\mathrm{b}} = \mathcal{P}_{\mathrm{b}}^{\mathrm{target}} \tag{2b}$$

or

$$\sum_{i=1}^{N} P_i = P_{\mathrm{budget}} \quad \text{and} \quad \mathcal{P}_{\mathrm{b}} \le \mathcal{P}_{\mathrm{b}}^{\mathrm{target}} \tag{2c}$$

where b_i and P_i are, respectively, the number of bits and amount of power allocated to the ith subchannel. The average BER is defined as

$$\mathcal{P}_{\mathrm{b}} = \frac{\sum_{i=1}^{N} b_i \mathcal{P}_{\mathrm{b},i}}{\sum_{i=1}^{N} b_i} \tag{3}$$

with $\mathcal{P}_{\mathrm{b},i}$ being the BER of the ith subchannel. The aim of this paper is to explore the effect of these two different constraints on the overall data rate by using greedy algorithms that perform power or bit allocation, respectively. In this work, we only aim to improve BER once the maximum system throughput is achieved.

The channel-to-noise ratio of the ith subchannel is given by

$$\mathrm{CNR}_i = \frac{\sigma_i^2}{\mathcal{N}_0} \tag{4}$$

where $\mathcal{N}_0$ is the total noise power at the receiver, whereas its signal-to-noise ratio (SNR) is

$$\gamma_i = P_i \times \mathrm{CNR}_i \tag{5}$$

Closed-form expressions and solutions of the throughput in (2a) are extensively considered in the literature, see for example [23, 24] for a review. Based on the concept of the SNR-gap approximation [25], a closed form for b_i is given by [24]

$$b_i = \log_2\left(1 + \frac{\gamma_i}{\Gamma}\right) \tag{6}$$

where Γ denotes the SNR-gap that signifies the loss in SNR of a particular transmission scheme when compared with the theoretical channel capacity. For QAM modulation schemes, this SNR-gap is given by

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{\mathcal{P}_{\mathrm{s},i}}{4}\right)\right]^2 \tag{7}$$

where Q^{-1} is the inverse of the well-known Q-function $Q(x) = (1/\sqrt{2\pi})\int_x^{\infty} \mathrm{e}^{-u^2/2}\,\mathrm{d}u$, and $\mathcal{P}_{\mathrm{s},i}$ is the symbol error ratio (SER) of the ith subchannel. It is clear from (7) that Γ is not fixed for all subchannels but depends on the subchannel SER which in turn depends on b_i and γ_i of (6). This dependence has to be taken into account whenever the rate or the gain in (6) is changed. Nevertheless, this approximation is valid only for very low BER, typically 10^{-6}, and higher QAM orders which is not usually the case for realistic applications [3].

Direct optimisation of (6) under the constraints in (2b) or (2c) leads to the well-known WF solution [10]. However, the resultant bit allocation obtained by WF is real valued and requires rounding off to the nearest integer value. This quantisation leads to an overall loss in performance.

We assume M-ary QAM modulation where the BER is given by [26] (see (8))

By allocating the power equally among all subchannels,

$$\mathcal{P}_{\mathrm{b},i} = \mathcal{F}(\gamma_i, M_{k_i}) = \begin{cases} Q(\sqrt{2\gamma_i}), & \text{for BPSK} \\ \left\{1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{M_k}}\right) Q\left(\sqrt{\frac{3\gamma_i}{M_k - 1}}\right)\right]^2\right\} (\log_2 M_k)^{-1}, & \text{for } M_k - \text{QAM} \end{cases} \tag{8}$$

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the subchannels SNR γ_i in (8) are given by

$$\gamma_i = P_i \times \mathrm{CNR}_i = \frac{P_{\mathrm{budget}}}{N} \times \mathrm{CNR}_i \tag{9}$$

According to (8) and by assuming the existence of the inverse of $\mathcal{F}$ w.r.t. γ_i,

$$\gamma_k^{\mathrm{QAM}} = \mathcal{F}^{-1}(\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}, M_k) \tag{10}$$

represents the minimum SNR that is required to achieve a throughput $b_k = \log_2 M_k$ with a target BER of $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$.

3 Incremental bit and power loading

3.1 Incremental bit loading

In [18], an incremental bit loading approach is proposed to maximise the throughput and efficiently fulfil the quality-of-service (QoS) in terms of the mean BER, that is, the constraints in (2c). However, in order to achieve this, a power allocation scheme has to be predefined across all subchannels which was chosen to be a simple UPA. The algorithm starts with filling all subchannels with the highest modulation order M_K and iteratively removes bits from the worst subchannels in order to achieve the mean BER of (3) in order not to violate the constraint $\mathcal{P}_{\mathrm{b}} \le \mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$. This solution can be described as a GBA scheme, which distributes power equally among all subchannels. In the following, we will therefore introduce an efficient greedy power allocation scheme.

3.2 GPA scheme

By adjusting the transmit power to exactly fulfil the target BER $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$ across all subchannels $\mathcal{P}_{\mathrm{b},i} = \mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$, the GPA algorithm is trying to maximise the throughput with the constraints in (2b). In order to achieve this, a UPA initialisation step is performed to load all subchannels with QAM orders M_{k_i} according to their γ_i in (9) and by using (10), where the index k_i is obtained such that

$$k_i\colon \quad \gamma_i \ge \gamma_k^{\mathrm{QAM}} \quad \text{and} \quad \gamma_i < \gamma_{k+1}^{\mathrm{QAM}} \tag{11}$$

with $\gamma_0^{\mathrm{QAM}} = 0$ and $\gamma_{K+1}^{\mathrm{QAM}} = +\infty$ as depicted in Fig. 1. The throughput of this UPA scheme is therefore

$$B_{\mathrm{upa}} = \sum_{i=1}^{N} b_i^{\mathrm{upa}} = \sum_{i=1}^{N} \log_2 M_{k_i} \tag{12}$$

whereas the difference between the allocated and budgeted power is

$$P_{\mathrm{d}}^{\mathrm{upa}} = \sum_{i=1}^{N} \frac{\gamma_i - \gamma_{k_i}^{\mathrm{QAM}}}{\mathrm{CNR}_i} = P_{\mathrm{budget}} - \sum_{i=1}^{N} \frac{\gamma_{k_i}^{\mathrm{QAM}}}{\mathrm{CNR}_i} \tag{13}$$

The procedure of the GPA algorithm based on the UPA initialisation is illustrated in Fig. 1 and algorithmically characterised in Fig. 2. Thereafter, the power difference $P_{\mathrm{d}}^{\mathrm{upa}}$ is iteratively allocated to subchannels that have not yet reached their maximum allowable QAM level K. The throughput of this algorithm B_{gpa} and its final power difference from P_{budget}, $P_{\mathrm{d}}^{\mathrm{gpa}}$, can be evaluated. The used power for both UPA and GPA algorithms is given by,

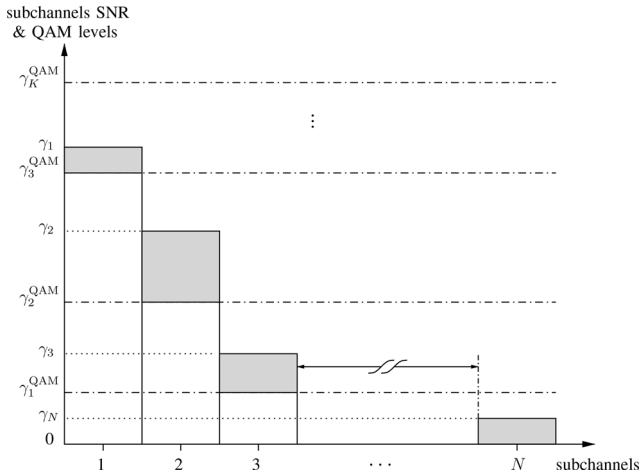

Fig. 1 *Subchannels residing in different QAM levels according to their SNRs*

Excess power after UPA is shown by shading

Initialisation:
Initiate GPA with $P_d^{\mathrm{gpa}} = P_d^{\mathrm{upa}}$ in (13)
For each subchannel i do the following:
  Set $b_i^{\mathrm{gpa}} = b_i^{\mathrm{upa}}$ and k_i using (12) and (11), respectively
  Cal. the min required upgrade power $P_i^{\mathrm{up}} = \frac{\gamma_{k_i+1}^{\mathrm{QAM}} - \gamma_{k_i}^{\mathrm{QAM}}}{\mathrm{CNR}_i}$
Recursion:
while $P_d^{\mathrm{gpa}} \ge \min(P_i^{\mathrm{up}})$ **and** $\min(k_i) < K$
  $j = \underset{1 \le i \le N}{\operatorname{argmin}}(P_i^{\mathrm{up}})$
  Update $k_j = k_j + 1$, $P_d^{\mathrm{gpa}} = P_d^{\mathrm{gpa}} - P_j^{\mathrm{up}}$
  if $k_j = 1$
    $b_j^{\mathrm{gpa}} = \log_2 M_1$, $P_j^{\mathrm{up}} = \frac{\gamma_2^{\mathrm{QAM}} - \gamma_1^{\mathrm{QAM}}}{\mathrm{CNR}_j}$
  elseif $k_j < K$
    $b_j^{\mathrm{gpa}} = b_j^{\mathrm{gpa}} + \log_2\left(\frac{M_{k_j}}{M_{k_j-1}}\right)$, $P_j^{\mathrm{up}} = \frac{\gamma_{k_j+1}^{\mathrm{QAM}} - \gamma_{k_j}^{\mathrm{QAM}}}{\mathrm{CNR}_j}$
  else
    $b_j^{\mathrm{gpa}} = b_j^{\mathrm{gpa}} + \log_2\left(\frac{M_{k_j}}{M_{k_j-1}}\right)$, $P_j^{\mathrm{up}} = +\infty$
  end
end
Evaluate $B_{\mathrm{gpa}} = \sum_{i=1}^{N} b_i^{\mathrm{gpa}}$ and P_d^{gpa}

Fig. 2 *Bit loading using GPA – constraint (2B)*

respectively

$$P_{\mathrm{used}}^{\mathrm{upa}} = P_{\mathrm{budget}} - P_{\mathrm{d}}^{\mathrm{upa}} \tag{14a}$$

$$P_{\mathrm{used}}^{\mathrm{gpa}} = P_{\mathrm{budget}} - P_{\mathrm{d}}^{\mathrm{gpa}} \tag{14b}$$

The terms in (14a) and (14b) provide useful measures of how efficient both the UPA and GPA algorithms perform in terms of power utilisation. Note that this quantity does not need to be defined for the GBA scheme as it uses the total power budget by definition.

4 BER improvement via excess power redistribution

The UPA and GPA algorithms presented in Section 3.2 cannot guarantee the total usage of the budgetted power because of integer-valued and limited modulation orders imposed via the constraint equation. Also, BER has to be tied to a given target value $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$ to ensure mathematical tractability. Therefore this section proposes to utilise the remaining excess power, – that is, the difference between

budgeted and allocated power – to provide additional BER performance enhancement. This is achieved by redistribution of this excess power of both the UPA and GPA algorithms, for which two different algorithms are presented below.

4.1 Uniform power redistribution (UPR)

The most straightforward way to redistribute the remaining excess power left unused by the UPA and GPA algorithms is to uniformly allocate these powers across all active subchannels regardless of how much BER improvement is attained by each subchannel. We refer to this method as the UPR algorithm. The excess powers $P_{\mathrm{d}}^{\mathrm{upa}}$ and $P_{\mathrm{d}}^{\mathrm{gpa}}$ are utilised for BER improvement of both UPA and GPA, respectively.

The algorithm can be described for the UPA by the following steps:

1. Determine the active subchannels i: $b_i^{\mathrm{upa}} \neq 0$ and their respective modulation orders M_{k_i} allocated by the UPA algorithm, where k_i, as above, is the index into the QAM order M_k assigned to the ith subchannel.
2. Calculate the minimum required SNR to achieve $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}$ across these active subchannels using (10) as $\gamma_{k_i}^{\mathrm{QAM}} = \mathcal{F}^{-1}(\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}, M_{k_i})$.
3. Uniformly allocate the excess power $P_{\mathrm{d}}^{\mathrm{upa}}$ among all active subchannels and compute the subchannels' new SNRs as

$$\gamma_i^{\mathrm{u}} = \gamma_{k_i}^{\mathrm{QAM}} + \frac{P_{\mathrm{d}}^{\mathrm{upa}}}{N_{\mathrm{a}}^{\mathrm{upa}}} \times \mathrm{CNR}_i \tag{15}$$

where $N_{\mathrm{a}}^{\mathrm{upa}}$ is the number of active subchannels under the UPA algorithm.
4. Calculate the subchannels' new BERs using (8) as $\mathcal{P}_{\mathrm{b},i}^{\mathrm{upa}} = \mathcal{F}(\gamma_i^{\mathrm{u}}, M_{k_i})$ and then the mean BER $\mathcal{P}_{\mathrm{b}}^{\mathrm{upa}}$ using (3).

The same procedures are used for the case of the GPA algorithm to redistribute $P_{\mathrm{d}}^{\mathrm{gpa}}$ and obtain $\mathcal{P}_{\mathrm{b}}^{\mathrm{gpa}}$ but the redistribution in this case should include all active subchannels under the GPA scheme, $N_{\mathrm{a}}^{\mathrm{gpa}}$.

Note that, in general, $P_{\mathrm{d}}^{\mathrm{gpa}} \leq P_{\mathrm{d}}^{\mathrm{upa}}$ owing to the improvement in power allocation gained by the GPA algorithm and $N_{\mathrm{a}}^{\mathrm{gpa}} \geq N_{\mathrm{a}}^{\mathrm{upa}}$ as a result of the chance to upgrade more inactive subchannels to be involved for transmission as a result of applying GPA. Consequently, $(P_{\mathrm{d}}^{\mathrm{gpa}}/N_{\mathrm{a}}^{\mathrm{gpa}}) \ll (P_{\mathrm{d}}^{\mathrm{upa}}/N_{\mathrm{a}}^{\mathrm{upa}})$ is the most likely outcome and accordingly by substituting in (15), the subchannels' new SNRs in case of GPA γ_i^{gpa} will result in less improvement in the mean BER $\mathcal{P}_{\mathrm{b}}^{\mathrm{gpa}}$ as compared to what can be attained via UPA.

4.2 Fairness-BER power redistribution (FPR)

The UPR presented above equally allocates the excess power among all active subchannels and therefore results in unequal subchannel BERs which depend on subchannels CNR_i and their allocated modulation orders M_{k_i}. Therefore the expected mean BER $\mathcal{P}_{\mathrm{b}}^{\mathrm{upa}}$ or $\mathcal{P}_{\mathrm{b}}^{\mathrm{gpa}}$ is likely to be dominated by the worst individual subchannel BER. Moreover, it is desirable to achieve an approximately uniform BER performance across all subchannels for reasons of fairness in QoS and link reliability. Therefore below we adapt the power redistribution to an algorithm that can achieve QoS fairness across all active subchannels for both UPA and GPA algorithms, to which we here refer to as BER FPR algorithm. Compared to the UPR algorithm, a new factor $\alpha_i \in \mathbb{R}$, $1 \leq i \leq N_{\mathrm{a}}$, $\sum_i \alpha_i = 1$ is introduced to the last term of the r.h.s. of (15) to adjust the power redistribution conditions for balanced BERs across all active subchannels. This can be mathematically formulated as

$$\begin{aligned} &\text{solve for} \quad \boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_{N_{\mathrm{a}}}] \\ &\text{that results in} \quad \gamma_i^{\mathrm{F}} = \mathcal{F}^{-1}(\mathcal{P}_{\mathrm{b}}^{F}, M_{k_i}) \quad \forall i \end{aligned} \tag{16}$$

where

$$\gamma_i^{\mathrm{F}} = \gamma_{k_i}^{\mathrm{QAM}} + \alpha_i P_{\mathrm{d}} \times \mathrm{CNR}_i \tag{17}$$

are the new subchannel SNRs and the constant $\mathcal{P}_{\mathrm{b}}^{\mathrm{F}}$ is the fair BER across all active subchannels. From (16) and (17), the entries of the unknown vector α are given by

$$\alpha_i = \frac{\mathcal{F}^{-1}(\mathcal{P}_{\mathrm{b}}^{\mathrm{F}}, M_{k_i}) - \gamma_{k_i}^{\mathrm{QAM}}}{P_{\mathrm{d}}\mathrm{CNR}_i}, \quad 1 \leq i \leq N_{\mathrm{a}} \tag{18}$$

Since $\sum_{i=1}^{N_{\mathrm{a}}} \alpha_i = 1$ and by defining the function

$$f(\mathcal{P}_{\mathrm{b}}) \stackrel{d}{=} \sum_{i=1}^{N_{\mathrm{a}}} \frac{\mathcal{F}^{-1}(\mathcal{P}_{\mathrm{b}}, M_{k_i}) - \gamma_{k_i}^{\mathrm{QAM}}}{P_{\mathrm{d}}\mathrm{CNR}_i} - 1 \tag{19}$$

it is possible to find a solution $\mathcal{P}_{\mathrm{b}}^{\mathrm{F}}$ of $f(\mathcal{P}_{\mathrm{b}})$ such that $f(\mathcal{P}_{\mathrm{b}})|_{\mathcal{P}_{\mathrm{b}}=\mathcal{P}_{\mathrm{b}}^{\mathrm{F}}} \simeq 0$. Here, the bisection method is used to iteratively solve this root finding problem. The complete FPR algorithm is given as follows:

1. Given the active subchannels i: $1 \leq i \leq N_{\mathrm{a}}$ and their respective M_{k_i} as well as CNR_i and P_{d} for either UPA or GPA algorithm calculate $\gamma_{k_i}^{\mathrm{QAM}} = \mathcal{F}^{-1}(\mathcal{P}_{\mathrm{b}}^{\mathrm{target}}, M_{k_i})$.
2. Locate two appropriate BER points that return small values of opposing signs for the function $f(\mathcal{P}_{\mathrm{b}})$ in (19). These $\mathcal{P}_{\mathrm{b}}$ points exists in the domain $(0, \mathcal{P}_{\mathrm{b}}^{\mathrm{target}} - \varepsilon)$, where $\varepsilon \to 0^+$.
3. Use the bisection method to find the root $\mathcal{P}_{\mathrm{b}}^{\mathrm{F}}$ that returns $f(\mathcal{P}_{\mathrm{b}}^{\mathrm{F}}) \to 0$. This BER solution is denoted by $\mathcal{P}_{\mathrm{b}}^{F,\mathrm{upa}}$ for the UPA algorithm and by $\mathcal{P}_{\mathrm{b}}^{\mathrm{F,gpa}}$ for the GPA algorithm.

Note that the complexity of this algorithm is dominated by the root finding search method. Although less complex methods may exist in the literature, the bisection approach has been chosen because of its relative simplicity and stable operation.

Fig. 3 demonstrates a simulated example of the function $f(\mathcal{P}_{\mathrm{b}})$ against its BER argument $\mathcal{P}_{\mathrm{b}}$ for both UPA and GPA under the FPR algorithm when applied to an SVD-decoupled 6×6 MIMO channel $\boldsymbol{H}$ with entries $h_{ij} \in \mathcal{CN}(0,1)$, $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}} = 10^{-3}$ and an SNR of 30 dB. Obviously, the BER improvement of the UPA is much better than that of the GPA as the function root $\mathcal{P}_{\mathrm{b}}^{\mathrm{F,upa}} \leq \mathcal{P}_{\mathrm{b}}^{\mathrm{F,gpa}}$ as discussed in Section 4.1. This is again owing to the good expenditure of power attained by the GPA that is used to maximise the sum-rate. It is also clearly noted that $f(\mathcal{P}_{\mathrm{b}})$ reaches its solution $\mathcal{P}_{\mathrm{b}}^{\mathrm{F,upa}}$ faster than $\mathcal{P}_{\mathrm{b}}^{\mathrm{F,gpa}}$ and its values for both UPA and GPA intersect at $\mathcal{P}_{\mathrm{b}}^{\mathrm{target}} = 10^{-3}$.

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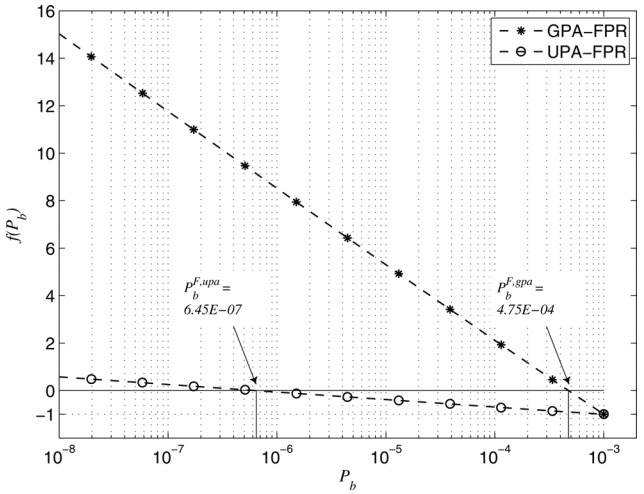

Fig. 3 *Function $f(\mathcal{P}_b)$ defined in (19) for both UPA and GPA algorithms of a 6×6 MIMO system at SNR = 30 dB and $\mathcal{P}_b^{target} = 10^{-3}$*

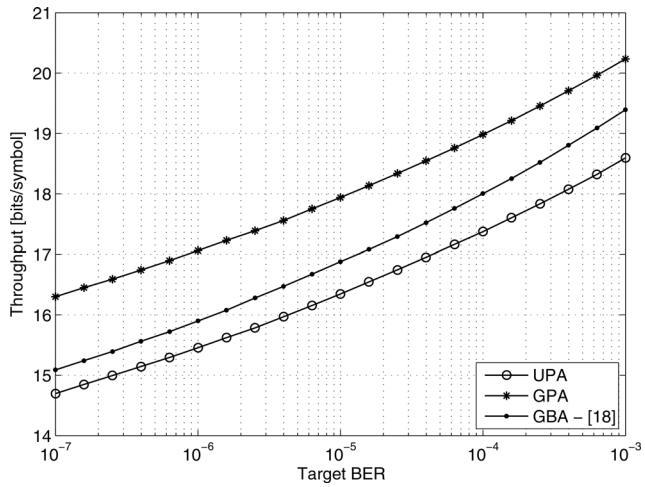

Fig. 5 *Throughput results for a 4×4 MIMO system at SNR = 25 dB and varying $\mathcal{P}_b^{target}$*

5 Simulations, results and discussion

An ensemble of 10^4 frequency-flat 4×4 MIMO systems characterised by a channel matrix $\boldsymbol{H} \in \mathbb{C}^{N_R \times N_T}$ with entries $h_{ij} \in \mathcal{CN}(0,1)$ is considered in this simulation, with results averaged across all ensemble probes. A target BER of $P_b^{target} = 10^{-3}$ is to be achieved through the bit loading schemes presented in this paper. Fixed QAM modulation orders of $\{2^1, 2^2, \ldots, 2^{b^{max}}\}$, where $b^{max} = 6$ bits, are constraining the system under consideration. Both the GBA algorithm of Wyglinski *et al.* [18] and our proposed GPA algorithm presented in Section 3.2 along with the UPA scheme are evaluated in this simulation.

It is shown from the throughput results in Figs. 4 and 5 that the GPA algorithm outperforms both the GBA and UPA algorithms in terms of system throughput. Since the power allocation of the GBA algorithm is based on the UPA with its inefficient power allocation scheme, power is wasted for unnecessarily improving the mean BER $\mathcal{P}_b < \mathcal{P}_b^{target}$ rather than just fulfilling the requirement $\mathcal{P}_b^{target}$. On the other hand, the GPA algorithm efficiently utilises the total power budget P_{budget}, which is allocated according to the greedy approach, in order to maximise the overall throughput thereby achieving the specified target BER $\mathcal{P}_{b,i} = \mathcal{P}_b^{target}$, $\forall i$. Therefore the GPA algorithm provides a better investment of the total power towards the rate maximisation problem compared to the GBA and UPA methods.

In Fig. 6, the power usage of the UPA and GPA algorithms are compared. In conjunction with the achieved rate according to Figs. 4 and 5, the GPA algorithms demonstrated better performance in terms of power usage compared to the UPA algorithm. Note that the GBA algorithm spends the entire power budget by definition, and therefore its power usage is identical to the curve P_{budget} in Fig. 6. As will be shown later in Fig. 7, the GBA invests all excess power into improving the achieved average BER. Once the throughput reaches its expected maximum of 4(subchannels) $\times$ 6 bits = 24 bits, extra power is no longer required. Therefore the effectively used power for both UPA and GPA algorithms in (14a) and (14b), respectively, starts to saturate and asymptotically approach the minimum power that is theoretically required to achieve the maximum bit loading b^{max} for all subchannels, that is, $\sum_i (\gamma_K^{QAM}/\mathrm{CNR}_i)$, which for the simulation setting used in this paper is found to be $\simeq$38.17 dB and highlighted by the dashed line in Fig. 6.

As proposed in Section 4 and demonstrated by Fig. 6, the UPA and GPA algorithms redistribute the excess power

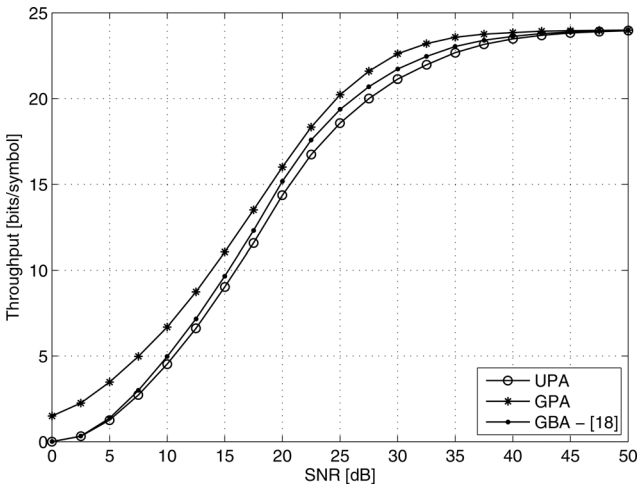

Fig. 4 *Throughput results for a 4×4 MIMO system with $\mathcal{P}_b^{target} = 10^{-3}$ and varying SNR*

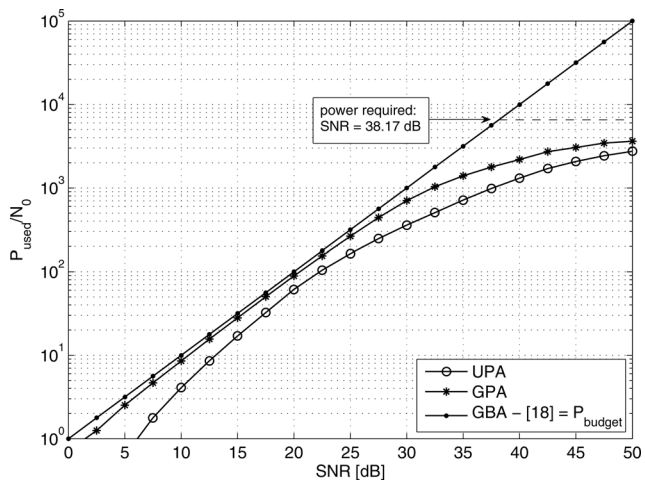

Fig. 6 *Power usage for a 4×4 MIMO system with $\mathcal{P}_b^{target} = 10^{-3}$ and varying SNR*

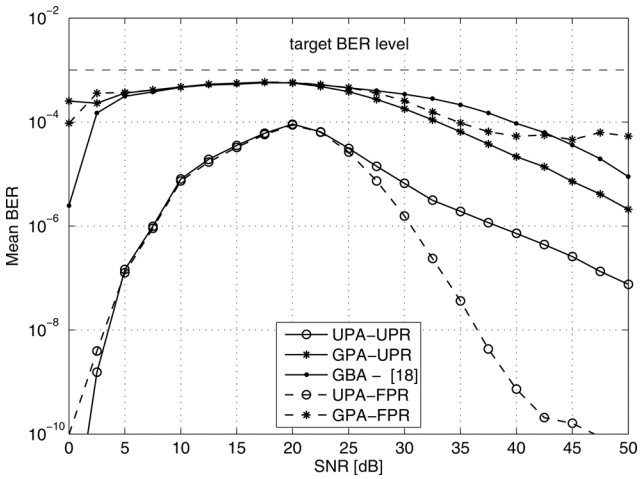

Fig. 7 *BER improvements of UPA and GPA algorithms*

in order to improve the BER performance. Fig. 7 shows these improvements for both UPR and FPR power redistribution algorithms compared to the actual achieved BER of the GBA algorithm. Mean BER is investigated against varying SNR showing BER improvements compared to the target BER of 10^{-3}. Interestingly, in conjunction with the achieved rates in Fig. 4, both UPA and GPA algorithms with excess power redistribution can achieve better BER performance than the GBA algorithm of [18]. It is also noted that FPR performs better than UPR if applied to the UPA, whereas the situation is inverted for the GPA algorithm. This can be attributed to the fact that since the excess power of the UPA algorithm is greater than that of the GPA, it is most likely that the mean BER of UPA-UPR is dominated by subchannels with poor CNR_i. In contrast, the FPR algorithm is advantageous in this case because of it is inherent fair w.r.t. BER. On the other hand, since for the GPA algorithm the excess power is relatively small and the BER constraint is to be balanced across all active subchannels, most of the redistributed power will be allocated to subchannels in lower QAM levels, leading to lower BER performance compared to that obtained by the UPR algorithm.

6 Conclusions

The inefficient UPA scheme is well known to result in suboptimal throughput performance of multichannel systems with constrained loading parameters. In this paper, we have investigated and introduced methods to perform rate maximisation based on both power and bit allocation, in particular the GPA and GBA schemes. However, since the GBA approach sacrifices power utilisation by adopting UPA for BER improvements, a degradation in the achieved data rate is expected as a result. By optimising power allocation, GPA demonstrates optimal performance in the rate maximisation sense. Another aspect of the UPA and GPA schemes is that power can be saved in achieving the target BER, which can be redistributed for better BER using different design aspects. We have suggested and analysed two redistribution approaches to allocate this excess power and achieve fair BER performance across all active subchannels. Simulation results show that GPA can achieve better BER performance compared to the GBA scheme.

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