CHANNEL CODING FOR POWER LINE COMMUNICATION BASED ON OVERSAMPLED FILTER BANKS

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Abstract. Oversampled filter banks (OSFB) can be applied as channel coding and have shown benefits in correlated and impulsive noise. Therefore, in this paper, we consider such channel coders for a power line communications scenario, where such conditions create a hostile channel environment. The proposed channel coder assumes the knowledge of the noise covariance matrix. The transmission is accomplished over the N weakest eigenmodes of the noise, which are identified by means of a broadband eigenvalue decomposition. We demonstrate by simulations that this approach can lead to an enhancement of the SNR in the receiver and to improved performance over standard channel coding approaches.

1. INTRODUCTION

Due to the increasing importance of networking in homes, offices and other buildings, power lines are being considered as one of the main media for high speed data transmission. Unfortunately, the power line communication (PLC) environment is hostile and subject to highly correlated and impulse, non-Gaussian channel noise [1].

Potential solutions in order to mitigate the nature of the channel noise include advanced channel coding methods, and orthogonal frequency division multiplexing, whereby different subcarriers experience different noise corruption and will subsequently be encoded using e.g. different levels of quadrature amplitude modulations according to their individual signal-to-noise ratio [2]. Oversampled filter banks (OSFBs) have recently been proposed for channel coding in environments where the channel noise is correlated and impulse in nature [3]. The OSFB coding approach exploits the structure of the noise by transmitting the signal over a low-noise subspace. Additionally, this coding approach has been shown to be robust towards impulsive noise [3], which appears attractive for PLC where impulsive noise has also been noted [1].

Therefore, we here investigate OSFB channel coding for PLC, utilizing the design approach in [6], whereby the OSFBs are designed to be paraunitary by means of a broadband singular value decomposition [4]. In contrast to the original FFT based encoding [3], the chosen method is optimized based on the noise properties only. The later ensures an enhanced noise suppression, while paraunitarity guarantees that the transmit power is strictly bound, and that the inverse of the filter bank is trivial to obtain.

The paper is organized as followed. in Sec. 2 we review the PLC model on which we rely prior to introducing the OSFB channel coder. The OSFB design for the proposed channel coder is discussed in Sec. 3.

2. SYSTEM MODEL

2.1. Power Line Model

A power line suffers from sever noise distortion and dispersion. We assume that the dispersive effect of the PLC channel could be sufficiently mitigated by an equaliser in the receiver, and therefore only concentrate on the channel noise. PLC channel noise is generally modelled as a mixture of correlated additive Gaussian noise and impulsive components. Although OSFB channels coders have been shown to work well in mitigating the effects of impulsive noise [3], we focus on the Gaussian noise components, whose power spectral density can be modelled as [1]

$$S_n(f) = a + b|f|^c \quad \text{dBm/Hz} \quad . \tag{1}$$

The values for a, b, and c vary depending on the harshness of the environment. For our experiments further below, we have adopted the parameters $a = 0$, $b = 38.75$, and $c = -0.720$ corresponding to a worst case scenario described in [1].

2.2. Channel Coding Setup

The general channel coding system using OSFBs is outlined in Fig. 1. In the transmitter, a symbol stream denoted $X(z)$ is demultiplexed into N polyphase components $X_n(z)$ with $n = 0, 1, \ldots N-1$. The latter are passed to a polyphase

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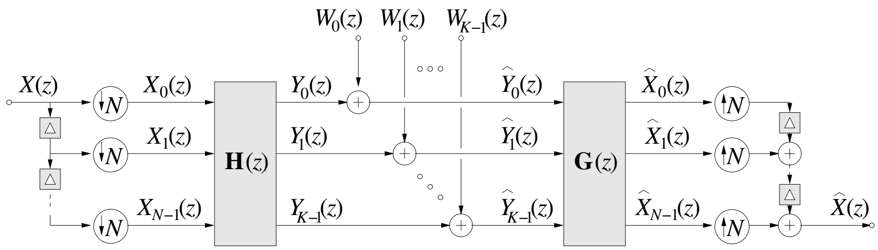

Fig. 1. General setup of a channel coder based on K channel analysis and synthesis filter banks, arranged around the transmission over K additive Gaussian noise channels.

analysis matrix $\mathbf{H}(z)$ of an oversampled filter bank, yielding a K element output vector $\underline{Y}(z)$,

$$\underbrace{\begin{bmatrix} Y_0(z) \\ \vdots \\ Y_{K-1}(z) \end{bmatrix}}_{\underline{Y}(z)} = \underbrace{\begin{bmatrix} H_{0,0}(z) & \dots & H_{0,N-1}(z) \\ \vdots & \ddots & \vdots \\ H_{K-1,0}(z) & \dots & H_{K-1,N-1}(z) \end{bmatrix}}_{\mathbf{H}(z)} \underbrace{\begin{bmatrix} X_0(z) \\ \vdots \\ X_{N-1}(z) \end{bmatrix}}_{\underline{X}(z)}. \tag{2}$$

Oversampling implies that $K > N$, i.e. redundancy has been introduced into the coded signal vector $\underline{Y}(z)$.

The polyphase components $\hat{X}_n(z)$ of the received signal in Fig. 1 can be collected similar to $\underline{X}(z)$ in (2) in a vector $\hat{\underline{X}}(z)$, which is given by

$$\hat{\underline{X}}(z) = \mathbf{G}(z)\left(\underline{Y}(z) + \underline{W}(z)\right) \tag{3}$$

whereby $\underline{Y}(z) = \mathbf{H}(z)\underline{X}(z) \in \mathbb{C}^K(z)$ and $\underline{W}(z) \in \mathbb{C}^K(z)$ contain the subband signal components of the transmitted data and the noise, respectively. Selecting perfect reconstruction filter banks such that $\mathbf{G}(z)\mathbf{H}(z) = \mathbf{I}_N$,

$$\underline{E}(z) = \underline{X}(z) - \hat{\underline{X}}(z) = -\mathbf{G}(z)\underline{W}(z) \tag{4}$$

is obtained.

To assess the total received noise variance σ_e^2 in $\hat{X}(z)$, let the N-element vector $\mathbf{e}[m]$ contain the N time series associated with the z-domain quantities in $\underline{E}(z) \bullet\!\!-\!\!\circ\ \mathbf{e}[m]$, which depend on the time index m in the decimated domain. Thus we have

$$\sigma_e^2 = \frac{1}{N}\mathrm{tr}\{\mathcal{E}\{\mathbf{e}[m]\ \mathbf{e}^{\mathrm{H}}[m]\}\} \quad , \tag{5}$$

where $\mathrm{tr}\{\cdot\}$ denotes trace and $\mathcal{E}\{\cdot\}$ is the expectation operator. Defining the auto-correlation matrix as $\mathbf{R}_{ee}[\tau] = \mathcal{E}\{\mathbf{e}[m]\mathbf{e}^{\mathrm{H}}[m-\tau]\}$ and its z-transform $\mathbf{R}_{ee}(z) \bullet\!\!-\!\!\circ\ \mathbf{R}_{ee}[\tau]$ denoting the power spectrum of the process $\mathbf{e}[m]$ [5], the noise variance is given by [6]

$$\sigma_e^2 \quad = \quad \frac{1}{N}\mathrm{tr}\{\mathbf{G}(z)\mathbf{R}_{ww}(z)\tilde{\mathbf{G}}(z)\}\big|_{z=0} \quad , \tag{6}$$

Note that (4) has been exploited to trace the noise variance back to the power spectrum $\mathbf{R}_{ww}(z)$, which is the z-transform of the covariance matrix of the channel noise,

$$\mathbf{R}_{ww}[\tau] = \mathcal{E}\{\mathbf{w}[m]\ \mathbf{w}^{\mathrm{H}}[m-\tau]\} \tag{7}$$

with $\mathbf{w}[m] \circ\!\!-\!\!\bullet\ \underline{W}(z)$ as defined in Fig. 1.

3. CHANNEL CODER AND FILTER BANK DESIGN

Based on the idea of the channel coder outlined in Sec. 3.1, this section considers a suitable factorisation of the power spectrum at the decoder output in Sec. 3.2, admitting a useful coder design in Sec. 3. An algorithm to construct filter banks achieving this design is reviewed in Sec. 3.4.

3.1. Proposed Coding Approach

It is the quantity σ_e^2 in (5) which is generally minimised in some sense in channel coders. In [3], for a given $\mathbf{H}(z)$, the degrees of freedom (DOFs) in the design of $\mathbf{G}(z)$ are exploited to minimise σ_e^2 in the MSE sense. Note however that this approach limits the DOFs that can be dedicated to fit the synthesis matrix to the spatio-temporal structure of the noise.

Therefore, we proposed to minimise (5) by optimising $\mathbf{G}(z)$ without restriction by a specific $\mathbf{H}(z)$. The only condition placed on $\mathbf{G}(z)$ is that it admits a right inverse $\mathbf{G}^{\dagger}(z)$ such that $\mathbf{G}(z)\mathbf{G}^{\dagger}(z) = z^{-\Delta}$. A stronger restriction than simple invertibility placed on $\mathbf{G}(z)$ is paraunitarity, which however has two important advantages: (i) the analysis filter banks is immediately given by $\mathbf{H}(z) = \tilde{\mathbf{G}}(z)$, and (ii) paraunitarity provides a minimum norm solution such that the transmit power is limited. As a counter example, an invertible $\mathbf{G}(z)$ might elicit an ill-conditioned $\mathbf{H}(z)$ which may attempt to transmit highly powered signals over subspaces associated with near rank deficiency.

3.2. Factorisation of the Noise Covariance Matrix

We approach the minimisation of (6) via a factorisation of the power spectrum

$$\mathbf{R}_{ww}(z) = \mathbf{U}(z)\mathbf{\Gamma}(z)\tilde{\mathbf{U}}(z) \tag{8}$$

such that $\mathbf{U}(z) \in \mathbb{C}^{K\times K}(z)$ is paraunitary and strongly decorrelates $\mathbf{R}_{ww}(z)$, i.e.

$$\mathbf{\Gamma}(z) = \mathrm{diag}\{\Gamma_0(z),\ \Gamma_1 z,\ \dots\ \Gamma_{K-1}(z)\} \tag{9}$$

is a diagonal matrix with polynomial entries $\Gamma_k(z)$. This factorisation presents a broadband eigenvalue decomposition, which can be further specified by demanding $\mathbf{\Gamma}(z)$ to be spectrally majorised [7, 4] such that the power spectral density of the kth noise component $\Gamma_k(e^{j\Omega}) = \Gamma_k(z)\big|_{z=e^{j\Omega}}$ evaluated on the unit circle obeys

$$\Gamma_k(e^{j\Omega}) \geq \Gamma_{k+1}(e^{j\Omega}) \qquad \forall\Omega \text{ and } k = 0(1)K-2, \quad (10)$$

similar to the ordering of the singular values in a singular value decomposition. Note that paraunitarity of $\mathbf{U}(z)$ conserves power, i.e. $\mathrm{tr}\{\mathbf{\Gamma}(z)\}|_{z=0} = \mathrm{tr}\{\mathbf{R}_{ww}(z)\}|_{z=0}$.

3.3. Channel Coder Design

Using the redundancy $N < K$ due to oversampling, we can construct $\mathbf{G}(z)$ from $\mathbf{U}(z)$ to select the lower (and therefore smallest) N elements on the main diagonal of $\mathbf{\Gamma}(z)$. Let

$$\mathbf{U}(z) = [\underline{U}_0(z) \;\; \underline{U}_1(z) \;\; \cdots \;\; \underline{U}_{K-1}(z)] \quad , \qquad (11)$$

then

$$\mathbf{G}(z) = \begin{bmatrix} \underline{\tilde{U}}_{K-N}(z) \\ \underline{\tilde{U}}_{K-N+1}(z) \\ \vdots \\ \underline{\tilde{U}}_{K}(z) \end{bmatrix} \quad \in \mathbb{C}^{N\times K}(z) \quad , \qquad (12)$$

such that $\mathbf{G}(z)\mathbf{U}(z) = [\mathbf{0}_{N\times K-N} \quad \mathbf{I}_N]$. Thus the noise power at the decoder output becomes [6]

$$\sigma_e^2 \;=\; \frac{1}{N}\sum_{i=K-N}^{K-1}\int_0^{2\pi}\Gamma_i(e^{j\Omega})\,d\Omega \,. \qquad (13)$$

Therefore, the spectral majorisation in the broadband eigenvalue decomposition (8) is essential to the success of the proposed channel coder design.

3.4. Sequential Best Rotation Algorithm

In order to achieve the factorisation in (8) fulfilling spectral majorisation according to (10), we use the second order sequential best rotation (SBR2) algorithm [4, 8]. SBR2 is an iterative broadband singular value decomposition technique, which finds a sequence of paraunitary matrix operations, each eliminating the largest off-diagonal element of the remaining power spectrum matrix. In extensive simulations, SBR2 has proven very robust and stable in achieving both a diagonalisation and spectral majorisation of any given covariance matrix, whereby the algorithm is stopped either after reaching a certain measure for suppressing off-diagonal terms or after exceeding a defined number of iteration [4, 8].

4. SIMULATIONS AND RESULTS

4.1. Simulation Setup

To illustrate the proposed channel coder design, we assume the PLC model outlined in Sec. 2.1. The colouring described in (1) is achieved by identifying an innovations filter $p[n] \circ\!\!-\!\!\bullet P(e^{j\Omega})$ by means of an iterative least squares design such the squared magnitude response matches the noise PSD. Thus the desired PLC noise process $w[n]$ is simulated by filtering a complex zero mean white Gaussian noise process with unit variance by $p[n]$, therefore having a autocorrelation sequence $r[\tau] = \sum_n p[n]p^*[n-\tau] \circ\!\!-\!\!\bullet R(z)$.

After demultiplexing into K channels in the receiver, the resulting noise power spectrum $\mathbf{R}_{ww}(z)$ can be shown to be given by the pseudo-circulant matrix

$$\mathbf{R}_{ww}(z) = \begin{bmatrix} R_0(z) & R_1(z) & \dots & R_{K-1}(z) \\ z^{-1}R_{K-1}(z) & R_0(z) & & R_{K-2}(z) \\ \vdots & \ddots & & \dots \\ z^{-1}R_1(z) & z^{-1}R_{K-1}(z) & & R_0(z) \end{bmatrix}$$

containing the K polyphase components $R_k(z)$, $k = 0(1)K-1$, of $R(z)$,

$$R(z) = \sum_{k=0}^{K-1} R_k(z^K)z^{-k} \quad . \qquad (14)$$

SBR2 can be applied to the noise power spectrum $\mathbf{R}_{ww}(z)$, yielding the basis for our channel coder design according to (12).

4.2. Coding Gain

Applying SBR2 to the noise covariance matrix R, an example for the resulting spectral majorisation is given in Fig. 2 for a decomposition into $K = 20$ channels. For this case, a single strong eigenmode of the noise is clearly visible. Therefore, if oversampling is applied and the strongest eigenmodes of the noise subspace can be deselected form transmission, the noise power in the decoded signal in the receiver can be reduced according to (13). If the filter bank is oversampled due to the omission of $K - N$ channels from transmission, a code rate of $N/K < 1$ results. This reduces the data throughput, but increases the SNR in the receiver. We therefore define a coding gain σ_w^2/σ_e^2, which also measures the noise reduction applied to the received and OSFB decoded signal as compared to the absence of any coding. This coding gain for the PLC simulation model in (1) is given in Fig. 3 for various selection of channels K, and compared to a maximum ratio combining by repeated transmission of symbols over an otherwise uncoded channel.

Fig. 3 suggests that the OSFB approach can provided considerable coding gain at a high coding rate close to unity

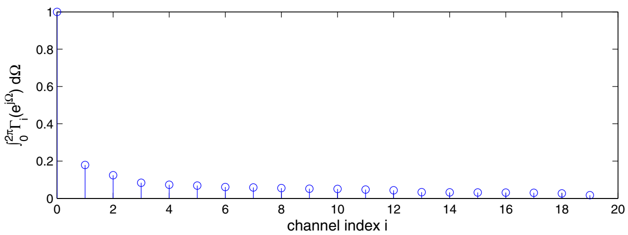

Fig. 2. Spectral majorisation in the decomposition of the noise power spectral matrix $R(z)$ by SBR2 for $K = 20$ channels.

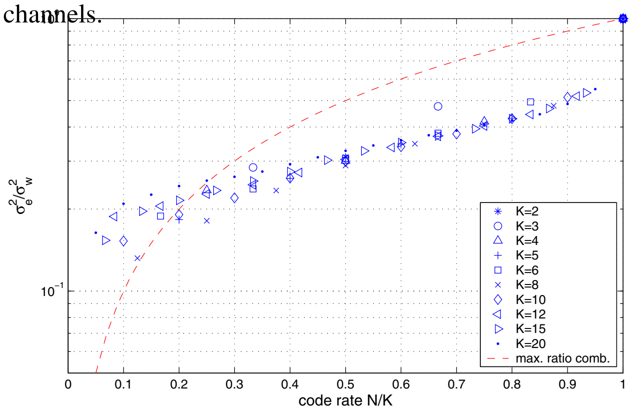

Fig. 3. Coding gain of the OSFB coder applied to the PLC channel defined in (1) for various values of K and different code rates.

for the case of highly correlated noise. In order to exploit this, K has to be chosen sufficiently high in order to offer sufficiently fine steps in the possible code rates and separate the strong eigenmodes of the noise from the weaker modes.

4.3. BER Comparison

In the following we consider transmitting quadrature amplitude modulated (QAM) symbols over the OSFB coded PLC channel. For a channel SNR of 3dB, Fig. 4 presents results for different code rates for a QPSK / 4-QAM and a 16-QAM based transmission.

As a comparison, we also present results for a (63,N_{BCH}) BCH coder, where N_{BCH} is varied to achieve various code rates. The BCH encoded bit stream is M-QAM mapped and transmitted over the PLC channel. In the receiver, after slicing and demapping, a BCH decoder aims to recover the original bit stream. An (37,20) matrix interleaver, imposing the same processing delay as the OSFB coder, is set to assist in breaking up noise correlation and burst-type errors. It is clear that the OSFB coder provides superior protection against correlated channel noise, and almost enables the use of 16-QAM rather than QPSK as opposed to a BCH coder, thus almost doubling the data throughput without sacrificing error protection.

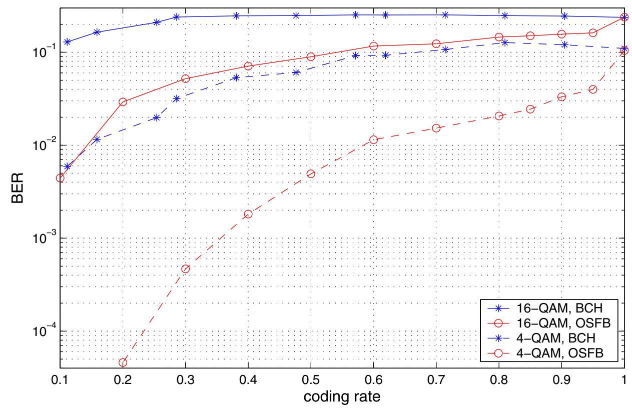

Fig. 4. BER for coding using M-QAM and OSFB and BCH channel coding, in dependency of the code rate.

5. CONCLUSIONS

We have presented a channel coder based on OSFBs which is designed to transmit over the weakest eigenmodes of the channel noise. This design is based on the knowledge of the noise's power spectral matrix when demultiplexed into a number of subchannels; subsequently the OSFB coder is extracted by means of a broadband eigenvalue decomposition approach. While the potential robustness of OSFB decoding towards impulsive noise has not been addressed, we have concentrated on and demonstrated the enhanced performance in correlated channel noise over conventional coding approaches in simulations.

6. REFERENCES

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