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A NOVEL APPROACH TO HYBRID PROPULSION TRANSFERS

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This paper introduces a hybrid propulsion transfer termed a Hohmann Spiral, incorporating low and high-thrust technologies, analogous to the high-thrust bi-elliptic transfer. To understand this transfer fully it is compared to a standard high thrust Hohmann and a bi-elliptic transfer. Two critical specific impulse ratios are derived independent of time that determine the point this novel transfer consumes the exact amount of fuel as the two compared transfer types. It is found that these ratios are valid for both a circular and elliptical starting orbit so long as the apogee of the elliptical orbit coincides with the target orbit radius. An expression representing the fuel mass fraction is derived dependent of time in order to allow a bound solution space. The final part of this paper investigates two orbit transfer case studies, one is a Geostationary Transfer Orbit to Geostationary Earth Orbit based on the Alphabus platform specification and the other is from Low Earth Orbit to an orbit near the Moon. It is found the thrust required to complete the former transfer in a specified duration of 90 days exceeds current technology and as such provides a technology requirement for future spacecraft. It is found however, for spacecraft of significantly smaller mass, in the region of 1000kg, compared to Alphabus (Max. mass at Launch =8100kg), the transfer consumes the same fuel mass as a standard high-thrust Hohmann transfer with realistic low-thrust propulsion values (150mN, 300mN and 450mN) within the set duration of 90 days. In addition, it is shown that utilising uprated thrusters (210mN, 420mN and 630mN) a fuel mass saving can be made. This could provide a potential transfer alternative for future smaller spacecraft. The second case study is bound to a maximum thrust of 150mN, but the mission duration is not specified to highlight the variation. It is found that the HST offers fuel mass savings of roughly 5% compared to a standard high-thrust transfer and approximately 1.5% compared to a bi-elliptic transfer for different scenarios.

I.   NOMENCLATURE

g – gravitational acceleration

μ - gravitational constant

m_{dry} – spacecraft mass without fuel

m_{wet} – spacecraft mass with total fuel

m_{highF} – high-thrust system fuel mass

m_{HSTF} – hybrid system fuel mass

m_{02} – spacecraft mass after phase 1 of the transfer

ΔV_{high} – high-thrust only system delta V

ΔV_H – high-thrust portion of hybrid system delta V

ΔV_L – low-thrust portion of hybrid system delta V

I_{spH} – high-thrust system specific impulse

I_{spL} – low-thrust system specific impulse

I_{spCHHS} – Hohmann Vs HST critical specific impulse ratio

I_{spCBHS} – bi-elliptic Vs HST critical specific impulse ratio

T – low-thrust system thrust value

r_i – initial orbit radius

r_t – target orbit radius

r_c – circular transfer orbit

a_1 – semi-major axis between r_i and r_c

$R1$ – target/initial orbit ratio

$R2$ – circular/initial orbit ratio

$R2^*$ – critical circular/initial orbit ratio

t_1 – hybrid transfer phase 1 duration (high-thrust)

t_2 – hybrid transfer phase 2 duration (low-thrust)

t_T – total hybrid transfer duration

II.   INTRODUCTION

This paper investigates an orbit transfer enabled through hybrid propulsion incorporating low and high-thrust technologies. The orbit transfer named Hohmann-Spiral (HST), is analogous to the high-thrust bi-elliptic transfer that involves three impulses to capture the target orbit; in the bi-elliptic transfer, the first impulse occurs at the initial orbit and sends the spacecraft into an elliptical orbit far beyond the target orbit. At the apoapsis a second impulse is applied increasing the orbit energy and initiating the return leg of the elliptical transfer. At the periapsis a third impulse is used to slow the spacecraft and capture into the final orbit. In a similar fashion the hybrid method uses two high-thrust impulses to firstly reach the apoapsis via an elliptical orbit and then to circularise at this radius. Subsequently the low-thrust system is activated and instead of following an elliptical orbit towards the target, a spiral trajectory is used until the final orbit is reached. To date, research in the area of hybrid propulsion transfers has focused on the use of a hybrid propulsion system where the low-thrust engine is activated at a point between the initial orbit and target orbit [1-3]. Figure 1 below shows this common transfer type.
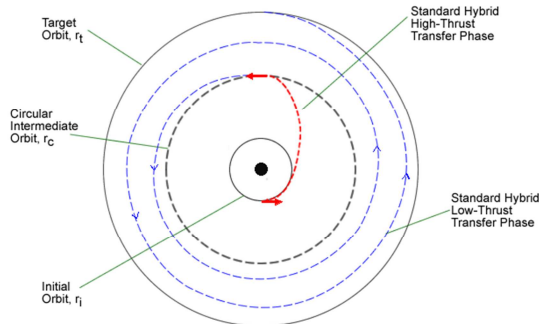

Figure 1 Standard Hybrid Transfer Technique

An important element of current research is determining the optimum specific impulse. This paper derives the critical specific impulse ratios in which a hybrid system is equivalent to that of a high-thrust system utilising a standard Hohmann and bi-elliptic transfer. Circular and elliptical starting orbits are considered in separate subsections. The analysis firstly omits time, which can be drastically increased

when considering a low-thrust system. It is then extended to consider this and thus determine the practicality for real mission scenarios through selected case studies. Figure 2 and 3 detail the different transfer type comparisons. The following assumptions are valid throughout the paper;

- orbits are co-planar
- finite burn losses are ignored
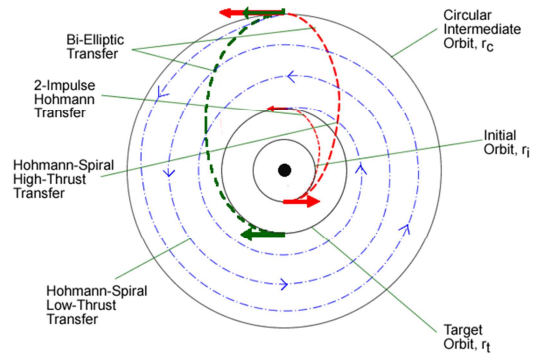- sphere of influence of the Earth is ignored

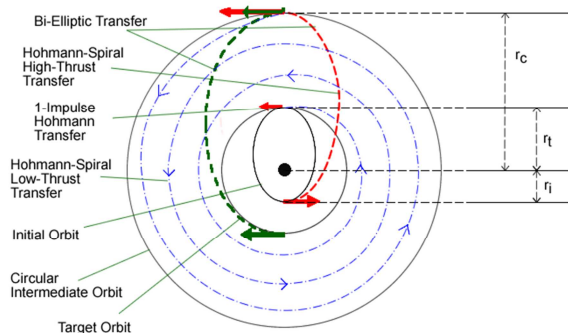Figure 2 HST compared with 2-Impulse Hohmann and bi-elliptic transfers

Figure 3 HST compared with 1-Impulse Hohmann and bi-elliptic transfer starting in elliptical orbit

III.   CRITICAL ISP DERIVATION

This section derives a critical specific impulse ratio which takes into consideration both the high and low thrust systems and determines the critical point at which a HST consumes the same amount of fuel as both a Hohmann and bi-elliptic transfer. The transfer starting in an elliptical orbit has an apogee at an altitude coinciding with the target orbit and in the high-thrust only case requires 1-impulse to capture.

This is also accounted for in this section. The critical ratio for the Hohmann and HST is referred to as I_{spCHHS} and for the bi-elliptic and HST is I_{spCBHS}. The generalised form is derived below before being applied to the different scenarios in the following subsections.

Firstly it is necessary to define the fuel mass fractions for the high and low thrust systems. The equations below represent the high thrust and hybrid system respectively.

$$\frac{m_{highF}}{m_{wet}} = 1 - exp^{\left(\frac{-\Delta V_{high}}{gI_{spH}}\right)} \qquad [1]$$

$$\frac{m_{HSTF}}{m_{wet}} = 1 - exp^{\left(\frac{-\Delta V_H}{gI_{spH}}\right)}exp^{\left(\frac{-\Delta V_L}{gI_{spL}}\right)} \qquad [2]$$

By comparing these equations the following condition is derived in which the hybrid system is equivalent or better in terms of fuel mass fraction.

$$exp^{\left(\frac{-\Delta V_{high}}{gI_{spH}}\right)} \leq exp^{\left(\frac{-\Delta V_H}{gI_{spH}}\right)}exp^{\left(\frac{-\Delta V_L}{gI_{spL}}\right)} \qquad [3]$$

This can be simplified to give the following condition

$$\frac{I_{spL}}{I_{spH}} \geq \frac{\Delta V_L}{\Delta V_{high} - \Delta V_H} \qquad [4]$$

This equation confirms that at the point the high thrust fuel consumption is equal to the hybrid fuel consumption the critical specific impulse ratio can be calculated. This ratio indicates that, for a given set of initial conditions, any value above this will show the hybrid system to be more fuel effective.

III.i. 2-Impulse Hohmann and HST

This section uses the generalised equations from the section above and derives the critical specific impulse for the case considering a high thrust system utilising the 2-Impulse Hohmann transfer and HST. Considering equation 4 and Figure 2 above, the following definitions are true for this scenario.

$$\Delta V_L = \sqrt{\frac{\mu}{r_t}} - \sqrt{\frac{\mu}{r_c}} \qquad [5]$$

where rc > rt, (for the case when rt > rc, the values in this equation are simply switched to correspond with Figure 1.)

$$\Delta V_{high} = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i+r_t}} - \sqrt{\frac{\mu}{r_i}} + \sqrt{\frac{\mu}{r_t}} - \sqrt{\frac{2\mu}{r_t} - \frac{2\mu}{r_i+r_t}} \qquad [6]$$

$$\Delta V_H = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i+r_c}} - \sqrt{\frac{\mu}{r_i}} + \sqrt{\frac{\mu}{r_c}} - \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_i+r_c}} \qquad [7]$$

It should be noted that ΔV_{high} represents the high thrust Hohmann transfer method from r_i directly to r_t whereas ΔV_H represents the high thrust part of the hybrid system.

By then introducing the orbit ratios of target to initial (R1 = r_t/r_i) and circular to initial (R2 = r_c/r_i), equation 4 for this scenario can be simplified to give the expression in equation 8.

$$\frac{I_{spL}}{I_{spH}} = I_{spCHHS} = \frac{\sqrt{\frac{1}{2R1}} - \sqrt{\frac{1}{2R2}}}{\sqrt{1 - \frac{1}{1+R1}} - \sqrt{1 - \frac{1}{1+R2}} - \sqrt{\frac{1}{R1} - \frac{1}{1+R1}} + \sqrt{\frac{1}{R2} - \frac{1}{1+R2}} + \sqrt{\frac{1}{2R1}} - \sqrt{\frac{1}{2R2}}} \qquad [8]$$

The above equation is now only dependent on two variables, R1 and R2. In the case where the initial and target orbits are known, the critical ratio is simply dependent on the r_c value. Varying this will give a range of transfer orbits with a given critical ratio defining the point where the hybrid system is equivalent in terms of fuel mass fraction.

From the equations it is clear that, for the condition when the high thrust section of the HST equals that of the pure high thrust system, a singularity exists. This singularity signifies the region in which the HST requires more fuel than the Hohmann transfer and consequently would be required to add mass rather than remove it to operate. Due to this singularity, the graphs below are bound to regions that are deemed feasible for both current and near future propulsion capabilities.[4-8]

Figure 4 and Figure 5 represent the scenario where r_c < r_t, whereas Figure 6 and Figure 7 represent the scenario where r_c > r_t. For the 2D plots a value of R1=6.4(r_t = 42,164km) is used and represents a standard LEO (Low Earth Orbit) to GEO (Geostationary earth Orbit) transfer. A LEO altitude of 200km is used.[9]
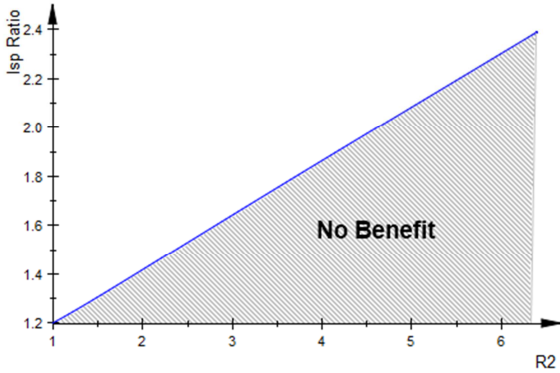

Figure 4 Hohmann and Hybrid Critical Specific Impulse Ratio with Varying R2 ($r_c \leq r_t$)

Figure 5 Hohmann and Hybrid Critical Specific Impulse Ratio with Varying R1 and R2 ($r_c \leq r_t$)

It can be seen from the above figures that values of R2 smaller than 6, for a standard LEO – GEO transfer, represent realistic I_{spCHHS} ratios and therefore certain missions may benefit, in terms of fuel mass fraction alone, from using this approach. For example, considering the new 500N Bipropellant European Apogee Motor (EAM) developed by

Astrium, with a minimum specific impulse of 325s[*], a low-thrust specific impulse range is identified as anything above 780s.

The following two figures represent r_c > r_t and represent the main focus of this paper. Figure 6 below indicates that as the transfer orbit increases and thus R2 increases, I_{spCHHS} decreases. This suggests that depending on the mission characteristics and the system configuration used, there is once again a potential fuel mass saving.
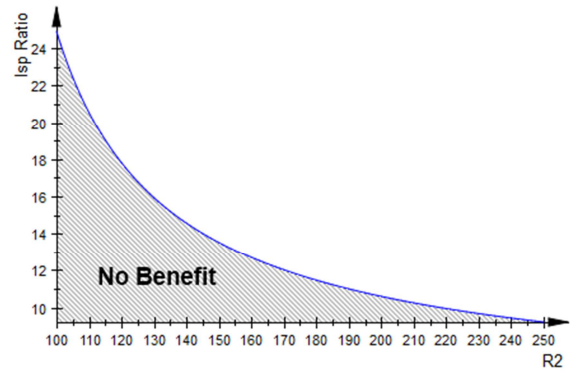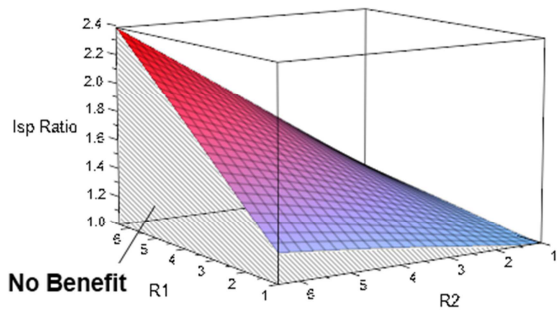

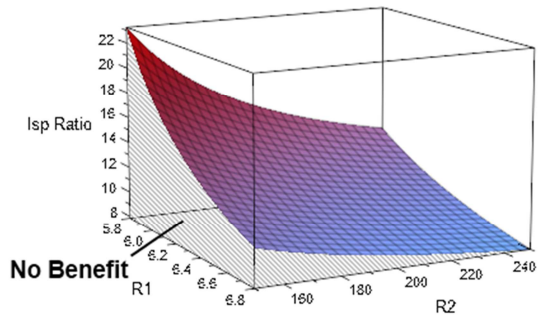Figure 6 Hohmann and Hybrid Critical Specific Impulse Ratio with Varying R2 ($r_c > r_t$)

Figure 7 Hohmann and Hybrid Critical Specific Impulse Ratio with Varying R1 and R2 ($r_c > r_t$)

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[*] Astrium, E. *500 N Bipropellant European Apogee Motor (EAM)*. 2011; Available from: http://cs.astrium.eads.net/sp/spacecraft-propulsion/apogee-motors/500n-apogee-motor.html.

III.ii. 3-Impulse Bi-elliptic and HST

Using the generalised equations as done previously, this section focuses on the case comparing the 3-Impulse bi-elliptic transfer with the HST. However due to previous work on bi-elliptic transfers determining that the critical ratio where this transfer outperforms a Hohmann transfer is R1>15.58[10], no smaller orbit ratio is considered in this section. A transfer to the Moon from a low Earth orbit with an altitude of 250km is considered giving an approximate value of R1 = 59[†](r_t = 0.391x10^6km). This is based on the assumption that the lunar orbit is circular.

Using Equation 4 and Figure 2 the following definition is used for the bi-elliptic scenario. The low-thrust spiral and high-thrust sections of the HST remain unchanged.

$$\Delta V_{high} = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i+r_c}} + \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_t+r_c}} -$$
$$\sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_i+r_c}} + \sqrt{\frac{2\mu}{r_t} - \frac{2\mu}{r_t+r_c}} - \qquad [9]$$
$$\sqrt{\frac{\mu}{r_t}} - \sqrt{\frac{\mu}{r_i}}$$

For this scenario ΔV_{high} represents a 3 Impulse bi-elliptic transfer whereas before it represented a Hohmann transfer. By then using the orbit ratios defined in the previous section, the critical specific impulse ratio can be calculated. This is given below.

$$\frac{I_{spL}}{I_{spH}} = I_{spCBHS} = $$
$$\frac{\sqrt{\frac{1}{2R1}} - \sqrt{\frac{1}{2R2}}}{\sqrt{\frac{1}{R1} - \frac{1}{R1+R2}} + \sqrt{\frac{1}{R2} - \frac{1}{R1+R2}} - \sqrt{\frac{1}{2R1}} - \sqrt{\frac{1}{2R2}}} \qquad [10]$$

As discussed in the previous section, this equation is now only dependent on the variables R1 and R2 which are a function of the initial, target and circular intermediate orbit. If the initial and target orbits are defined the only variable undefined is R2. By then varying the circular intermediate orbit, a range of I_{spCBHS} values can be obtained detailing the point at

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[†] NASA. *Moon Fact Sheet.* 2010; Available from; http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html

which the HST consumes exactly the same amount of fuel mass as the bi-elliptic transfer. Figure 8 and Figure 9 show the nature of this function. Similar to the case of the Hohmann transfer comparison, it can be seen that with increasing R2 there is a decrease in I_{spCBHS}.
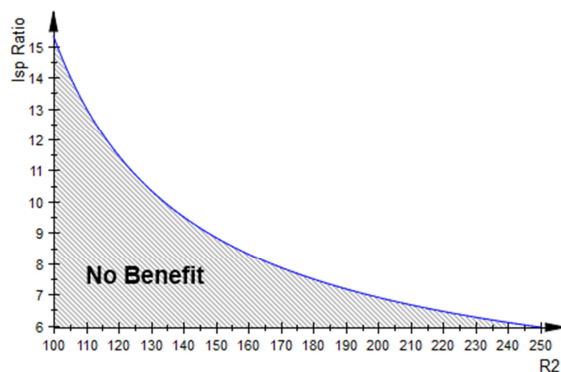
Figure 8 Bi-elliptic and Hybrid Critical Specific Impulse Ratio with Varying R2 ($r_c > r_t$)
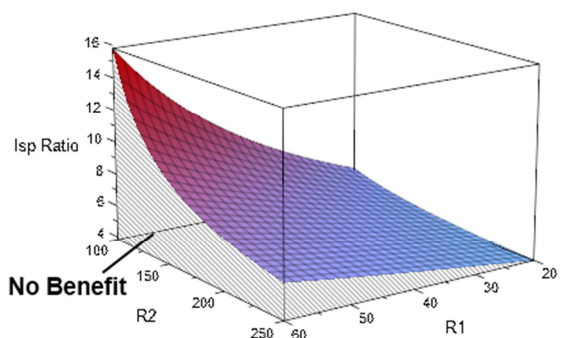
Figure 9 Bi-elliptic and Hybrid Critical Specific Impulse Ratio with Varying R1 and R2 ($r_c > r_t$)

III.iii. Elliptical Initial Orbit

Using the generalised equations as done in the previous two sections this case considers a highly elliptical starting orbit with an apogee at the same altitude of the target orbit so as the high-thrust only transfer requires 1-impulse to capture. Both the Hohmann transfer and bi-elliptic transfer types are considered in this section. Using Equation 4 the following definitions are used for the Hohmann transfer. The low-thrust spiral section remains unchanged.

$$\Delta V_{high} = \sqrt{\frac{\mu}{r_t}} - \sqrt{\frac{2\mu}{r_t} - \frac{2\mu}{r_i + r_t}} \qquad [11]$$

$$\Delta V_H = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i + r_c}} - \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i + r_t}} - \\ \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_i + r_c}} + \sqrt{\frac{\mu}{r_c}} \qquad [12]$$

After substituting these values into equation 4 it is found that with a little simplification it reduces to give the same equation as described in the Hohmann Transfer section earlier. As such, equation 8 is used as a representation of both a circular and elliptical initial orbit.

For the bi-elliptic transfer, also starting in an elliptical orbit, it is again found that the equation can be shown to equal equation 10. The different delta V requirements are given below to highlight the variance with the previous bi-elliptic comparison. The low-thrust spiral section remains unchanged.

$$\Delta V_{high} = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i + r_c}} - \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i + r_t}} + \\ \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_t + r_c}} - \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_i + r_c}} + \\ \sqrt{\frac{2\mu}{r_t} - \frac{2\mu}{r_t + r_c}} - \sqrt{\frac{\mu}{r_t}} \qquad [13]$$

$$\Delta V_H = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i + r_c}} - \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_i + r_t}} - \\ \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_i + r_c}} + \sqrt{\frac{\mu}{r_c}} \qquad [14]$$

III.iv. Comparing Critical Specific Impulse Ratios

As described by the previous two sections, there are two critical ratios related to using the hybrid orbit raising method and as such it is necessary to compare these ratios to determine how they interact with each other. This section investigates this interaction and highlights the range of critical ratios where the hybrid technique outperforms both the Hohmann and bi-elliptic transfer methods. Figure 10 compares the two critical ratios considering the lunar transfer as in the previous bi-elliptic section. It can be seen that both I_{spCHHS} and I_{spCBHS} tend to zero with increasing R2 as expected. It is noted that I_{spCBHS} is always higher than I_{spCHHS} which is expected, especially in

this case as R1>15.58 which was previously defined as the critical point at which a bi-elliptic transfer is more efficient than the Hohmann transfer type.
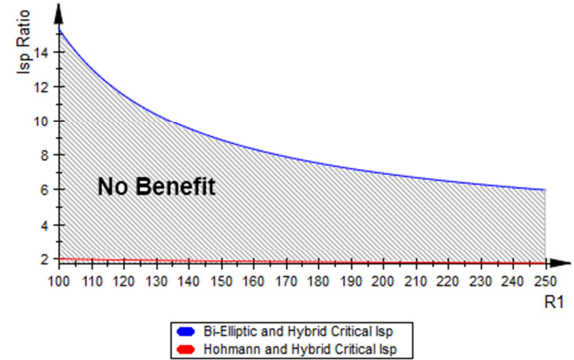

Figure 10 Hohmann and HST Critical Specific Impulse Ratio compared with bi-elliptic and HST (R1 = 59)

Figure 11 gives a 3-dimensional representation of the two critical ratios. It highlights the region in which the critical ratios intersect and indicates where one transfer type assumes control of the system i.e. the critical specific impulse ratio has to exceed that value for the HST to become more fuel-efficient than both the Hohmann and bi-elliptic transfers.
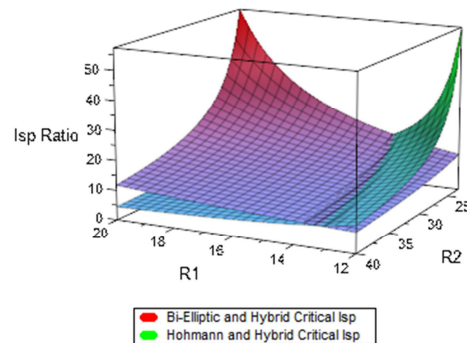
Figure 11 Hohmann and HST Critical Specific Impulse Ratio compared with bi-elliptic and HST

It can be seen that any value of R1 $<\approx$ 12 gives the HST and Hohmann critical ratio control of the system and any value of R1 $>\approx$ 15 ensures the HST and bi-elliptic critical ratio has control. This leaves a region of uncertainty where the two critical ratios intersect in which it is difficult to distinguish which has control. As validation for this work it was decided to consider this region of intersection, as it was found previously by Escobal that when comparing the Hohmann and bi-elliptic transfer types there is a

similar region of uncertainty when $11.93876 < R1 < 15.58176$.[10] It was found that a test is required to determine which transfer is superior and discovered that as $R1 \rightarrow 11.93876$, $R2 \rightarrow \infty$, whereas for the upper bound when $R1 \rightarrow 15.58176$, $R2 \rightarrow R1$. This means that for this novel transfer it can be determined, depending on the value of $R2$, which critical ratio must be considered to ensure the HST manoeuvre is superior by performing a similar test to Escobal. The test for this novel transfer can be demonstrated, and hence provide validation, by equating the critical ratios for each transfer type, in this case equation 8 and 10. It follows that

$$\sqrt{\frac{R2}{R2+1}} - \sqrt{\frac{R1}{R1+1}} + \sqrt{\frac{R1}{R2(R1+R2)}} +$$
$$\sqrt{\frac{R2}{R1(R1+R2)}} + \sqrt{\frac{1}{R1(R1+1)}} - \qquad [15]$$
$$\sqrt{\frac{1}{R2(R2+1)}} - \sqrt{2}\sqrt{\frac{1}{R1}} = 0$$

This equation can be solved for $R2$, corresponding to the zero of equation 15, within the range $11.93876 < R1 < 15.58176$. Table 1 below shows the $R2_0$ ratio at several $R1$ values.

R1 Value	R2$_0$ Value
11.93876	∞ (1x10^{30})
12	815.82
13	48.90
14	26.10
15	18.19
15.58545	15.58176

Table 1 R1 and R2$_0$ Values within Region of Uncertainty

It should be noted that the value of $R1 = 15.58545$ is used as the function failed to provide any result when $R1=15.58176$ was considered. This is due to the complex nature of the function when $R1=R2_0$. From the naturally decreasing form of the function it can be said that any $R2$ value greater than $R2_0$ will ensure the critical ratio, comparing a HST to bi-elliptic, has control of the system. Anything smaller will result in the critical ratio comparing HST to a Hohmann transfer having control. This coincides with the work of Escobal who drew a similar conclusion when determining the most efficient transfer in this region of uncertainty.

IV. FUEL MASS FRACTION ANALYSIS (TIME DEPENDENT)

This section derives an equation that describes the fuel mass fraction consumed by the HST manoeuvre when a time constraint is included.

$$\frac{m_{HSTF}}{m_{wet}} = 1 - exp^{\left[\left(\frac{-\Delta V_H}{gIsp_H}\right)+\left(\frac{-\Delta V_L}{gIsp_L}\right)\right]} \qquad [16]$$

where

$$\Delta V_L = \frac{T}{m_{02}}(t_2) \qquad [17]$$

$$t_T = t_1 + t_2 \qquad [18]$$

$$t_1 = \pi\sqrt{\frac{a_1^3}{\mu}} \qquad [19]$$

$$a_1 = \frac{r_t}{2R1}(1 + R2) \qquad [20]$$

Substituting these into Equation 16 yields

$$\frac{m_{HSTF}}{m_{wet}} = 1 -$$
$$exp^{\left[\left(\frac{-\Delta V_H}{gIsp_H}\right)+\left(\frac{-\left(\frac{T}{m_{02}}\left(t_T-\frac{\pi}{\sqrt{\mu}}\sqrt{\left(\frac{rt(1+R2)}{2R1}\right)^3}\right)\right)}{gIsp_L}\right)\right]} \qquad [21]$$

The above equation now has the functionality to include an overall mission duration, allowing a design space to be created.

V. CASE STUDIES

V.i. Alphabus Mission Scenario

This section considers a transfer of the new ESA satellite platform, Alphabus, from GTO to GEO using a HST compared to a standard Hohmann transfer. Table 2[‡] provides the specification for the

transfer. It is assumed that the launch vehicle places the spacecraft in GTO with zero inclination to coincide with the assumption that orbits are co-planar.

Transfer Specification	Property
Initial Orbit GTO Perigee Radius, r_i (km)	6628
Initial Orbit GTO Apogee Radius, r_t (km)	42,164
Target Orbit Radius GEO, r_t (km)	42,164
Mission Duration Limit,t_T(days)	90
European Apogee Motor Specific Impulse, I_{spH} (s)	325
T6 Thruster Specific Impulse, I_{spL} (s)	4500
Gravitational Constant, μ (m^3/s^2)	3.986x10^{14}
Gravity, g (m/s^2)	9.81
Alphabus Maximum Launch (Wet) Mass , m_{wet} (kg)	8100
Calculated Parameters	
R1(r_t/r_i)	6.36
I_{spCHHS}	13.846

Table 2 Alphabus GTO - GEO Specification

Firstly it is necessary to determine the point at which the HST transfer consumes the exact amount of fuel as the Hohmann transfer. This is done by first determining R1 which allows the correct critical ratio to be considered. As this is a GTO-GEO transfer R1 is found by using the target orbit radius and initial orbit perigee radius. As shown in the table above, R1=6.36(r_t = 42,164km) which by then considering the critical ratio, also in the table above, equation 8 can be rearranged to calculate R2. Upon doing this it is found that R2 = 150.39 which represents an intermediate orbit roughly 23 times greater than GTO. This can then be used in association with equation 21 which can be re-arranged to calculate the desired thrust based on the information above. The re-arranged equation is

$$T = \frac{-m_{02}\left[gI_{spL}log\left(1-\frac{m_{fuel}}{m_0}\right)+\Delta V_H\frac{I_{spL}}{I_{spH}}\right]}{t_T-\frac{\pi}{\sqrt{\mu}}\sqrt{\left(\frac{rt(1+R2)}{2R1}\right)^3}} \qquad [22]$$

It is noted that in this case m_{fuel} has been set to equal that of a standard 2-impulse Hohmann transfer. Using this, it is found that the required thrust for this mission specification is 2193.5mN. Table 3 below summarises the required performance to enable this mission. It can be seen that to equal the fuel mass consumption of a 2-impulse Hohmann transfer the HST requires 15 T6 thrusters rated at 150 mN, which is a standard value within the T6 operating range[5], and 11 thrusters for the maximum thrust demonstrated under experimental conditions.[§]

T6 Thrust (mN)	Thrusters Required to meet Thrust Value
150	15
210	11

Table 3 T6 Thrusters required for Alphabus Mission

In order to introduce any mass benefit to the system, the following changes can be made;

- Increase low-thrust engine specific impulse
- Increase transfer duration and hence R2
- Increase thrust of system

It is well understood that this technology requirement is not readily available and is unlikely to be at any point in the near future. It should be noted however that this study is based on the maximum launch mass of the Alphabus platform. As the transfer duration is known to vary with the spacecraft mass it can be shown that for a realistic thrust range, as shown in Figure 12, there is potential application for this transfer when considering smaller spacecraft. The figure below and Table 4 highlight the spacecraft mass required at launch for a system fitting the Alphabus specification detailed in Table 2, which can deliver the satellite to GEO in the defined transfer duration of 90 days. With this initial mass and thrust range, the HST consumes the exact amount of fuel as the Hohmann transfer. Table 4 details the mass breakdown of these different transfers. It is known that most GEO platforms have a dry mass much greater than this, but with the increased interest in smaller spacecraft this could provide a viable alternate orbit transfer solution.

[§] Qinetiq. T6 Gridded Ion Engine. Available from: http://www2.qinetiq.com/home/markets/related_markets/space/electric_propulsion/electric_propulsion0/t6_gridded_ion_engine.html
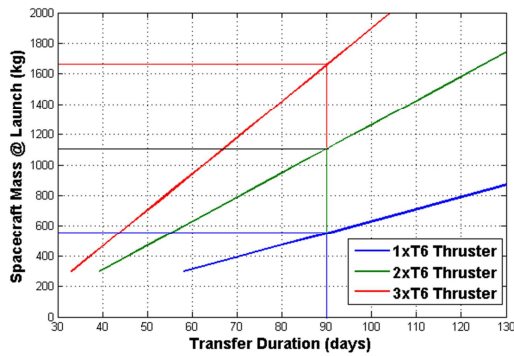
Figure 12 Spacecraft Mass at Launch against Transfer Duration

Thrust, T (mN)	Launch (Wet) Mass, m_{wet} (kg)	Dry mass, m_{dry} (kg)
1 x T6 (150)	554	350
2 x T6 (300)	1108	699
3 xT6 (450)	1662	1048

Table 4 Mass Breakdown of Spacecraft at Fuel Breakeven Point

It can be shown that if the same initial mass is used but the T6 thruster is uprated to its maximum thrust as previously defined then a mass saving is possible. The mission duration remains unchanged but the R2 value differs. This is now defined at R2=223 which represents an intermediate orbit approximately 35 times greater than GTO. Table 5 below shows the potential mass saving if the uprated T6 thruster is used on the spacecraft in the different configurations shown.

Thrust, T (mN)	Launch (Wet) Mass, m_{wet} (kg)	Dry mass, m_{dry} (kg)	Mass Saving (kg)
1 x T6 (210)	554	357	7
2 x T6 (420)	1108	714	15
3 x T6 (630)	1662	1071	23

Table 5 Mass Saving with Increased Thrust

It should be noted that the low-thrust system acceleration is based on the spacecraft mass after Phase 1 of the transfer. As such it can be assumed that as the spacecraft expels mass the acceleration will increase resulting in the spacecraft taking less than 90 days to reach the target orbit.

V.ii. Lunar Mission Scenario

This section of the paper focuses on a Lunar Mission Scenario where there is no set specification. This is to demonstrate the potential of this transfer when considering distant targets. As there is no specification for this, the same propulsion system values used for the Alphabus study are considered. The transfer has a starting orbit in LEO and for simplicity the target orbit is considered to be the distance between the Earth and Moon, In this case, similar to the analysis on page 5, which compared the bi-elliptic and HST, R1=59. The table below provides a more detailed specification of the transfer.

Transfer Specification	**Property**
Initial Orbit LEO Radius, r_i (km)	6628
Target Orbit Radius, r_t (km)	391,052
Initial Spacecraft Mass, m_{wet} (kg)	1000
European Apogee Motor Specific Impulse, I_{spH} (s)	325
T6 Thruster Specific Impulse, I_{spL} (s)	4500
1xT6 Thrust, T (mN)	150
Calculated Parameters	
R1(r_t/r_i)	59
I_{spCBHS}	13.846

Table 6 Lunar Mission Specification

As R1>15.58 this analysis only considers the HST and bi-elliptic critical ratio but does offer a comparison to the standard 2-impulse Hohmann transfer when analysing the results. Using the values specified in the table it is found that, in order for the HST to equal the fuel consumption of the bi-elliptic transfer, R2 = 106.62. This is approximately 1.8 times the lunar orbit of the Earth. Based on this number and the thrust specified, the figure below highlights the potential fuel mass saving with increased R2. It also plots the transfer duration to show the relationship between R2 and duration.
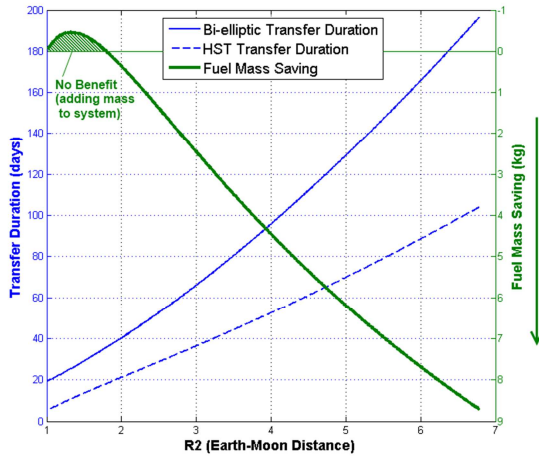
Figure 13 Bi-elliptic and HST Comparison

Figure 13 suggests that there is a time benefit in addition to the fuel mass benefit by using the HST method and going beyond the fuel mass equilibrium point. If R2= 300 (5.08 Earth-Moon Distance) the mass saving compared to the Bi-elliptic transfer is approximately 7kg. Compared to a 2-impulse Hohmann transfer the mass saving is 30 kg. The duration of the HST is 71 days compared to 132 days for the Bi-elliptic Transfer. Table 7 shows the comparison between the HST, bi-elliptic and Hohmann transfers for a range of R2 values. The duration of the chemical transfer is excluded as it is always shorter than the HST.

R2 (Earth-Moon Distance)	Fuel Mass Saving Compared To (kg)		Transfer Duration (days) [Saving Compared to Bi-elliptic]
	Chemical	Bi-elliptic	
5.08	30	7	71 [61]
7.63	34	9	122[109]
10.17	38	12	180[168]

Table 7 HST Lunar Transfer Comparison

This table highlights that with a greater R2 ratio this transfer can offer real benefits for realistic technologies. Although the mass saving is relatively small, it could extend the spacecraft lifetime if it is used as fuel for simple on orbit manoeuvres.

VI. CONCLUSION

This paper has derived a critical specific impulse ratio for the Hohmann Spiral Transfer (HST) compared to the standard Hohmann and bi-elliptic transfers. These ratios determine the exact point at which the HST consumes the same amount of fuel as the standard transfer. It is shown that a fuel mass saving can be achieved if the spacecraft enters a circular orbit way beyond the target orbit, before initiating its spiral return with the low-thrust propulsion system. It is found that the critical ratios are valid for elliptical starting orbits so long as the apogee of the starting orbit coincides with the target orbit radius.

Upon comparing the two critical ratios, for values of R1(r_t/r_i) > 15.58 the bi-elliptic and HST critical ratio assumes control of the system and thus defines the minimum technology requirements to ensure the HST out-performs the standard transfers. Similarly, it is shown that for R1 < 11.938 the Hohmann and HST critical ratio assumes control of the system. Through this analysis, a region of uncertainty, where 11.938 < R1 < 15.58, is identified. This work coincides with that of Escobal[10] when comparing the Hohmann and Bi-elliptic transfers. Using the test described in Section III it is shown as R1 → 11.93876, R2(r_c/r_i) → ∞ , whereas for the upper bound as R1 → 15.58176, R2 → R1. These findings again tie in with the work of Escobal and provide validation for the analysis within this paper.

In the first case study, investigating a GTO – GEO transfer based on the specification of the new Alphabus platform at maximum launch mass (8100kg), it is found that to complete the transfer in 90 days a thrust of 2193.5mN is required. This is the equivalent of 15 T6 thrusters rated at 150mN each, thus proving this transfer unrealistic at present but providing a technology requirement for future spacecraft. When considering current technology (1, 2 and 3xT6 thrusters), an initial launch mass is derived which can offer a fuel mass saving compared to a standard 2-impulse Hohmann manoeuvre. Table 4 and 5 show the different available wet masses at launch and dry masses delivered on orbit. The second case study investigates a potential transfer from LEO to the Moon. The transfer has no fixed time duration

but is limited to one T6 thruster and an initial spacecraft mass of 1000kg. It is found that for different R2 values the HST outperforms the bi-elliptic transfer and can offer small fuel mass savings in the region of 1.5%. When compared to the high-thrust transfer a saving of approximately 35kg (5%) is achievable.

VII. REFERENCES

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