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A simple method for enhanced vibration-based structural health monitoring

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A Guechaichia[12] and I Trendafilova[1]

[1]Department of Mechanical Engineering University of Strathclyde, James Weir Building, 75 Montrose street, Glasgow, G1 IXJ, United Kingdom

Abstract. This study suggests a novel method for structural vibration-based health monitoring for beams which only utilises the first natural frequency of the beam in order to detect and localise a defect. The method is based on the application of a static force in different positions along the beam. It is shown that the application of a static force on a damaged beam induces stresses at the defect which in turn cause changes in the structural natural frequencies. A very simple procedure for damage detection is suggested which uses a static force applied in just one point, in the middle of the beam. Localisation is made using two additional application points of the static force. Damage is modelled as a small notch through the whole width of the beam. The method is demonstrated and validated numerically, using a finite element model of the beam, and experimentally for a simply supported beam. Our results show that the frequency variation with the change of the force application point can be used to detect and in the same time localize very precisely even a very small defect. The method can be extended for health monitoring of other more complicated structures.

1. Introduction.

Vibration-based health monitoring (VHM) methods are global non-destructive testing methods, which are based on the fact that any changes introduced in a structure (including damage) change its physical properties, which in turn change the structural vibration response. VHM methods use the changes in the vibration response of a structure for the purposes of fault (damage) diagnosis. There are different strategies suggested for VHM purposes depending on the type of the structural response used: modal domain, frequency domain or time domain response. All of these have their advantages and disadvantages [1]. VHM methods that are based on the first several natural frequencies of the beam present a very attractive possibility since these are quite easy to obtain from experiment [2]. A confounding factor which limits the application of the lower natural frequencies is that damage is typically a local phenomenon while the first several modes capture the global structural response. Local response is captured by higher frequencies which are more difficult to excite and measure. Thus if one could increase the sensitivity of the lower frequency structural response to damage this can provide an excellent opportunity for the development of an easy to apply yet sensitive fault diagnosis method which is based on a simple assumption about the structural behaviour(the defect is caused by a loss of material and remains open during vibration.). This paper suggests the application of a static

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[2] Corresponding author. Email address: abdelhamid.guechaichia@strath.ac.uk

force in order to increase the sensitivity of the lower natural frequencies and especially the first one to damage. Thus a method for defect detection and localisation which only uses the first natural frequency of a beam is suggested. The rest of the paper is organised as follows. The next paragraph introduces the background of the suggested methodology. Then a simple method for defect detection and localisation is suggested. Section 4 introduces a beam model and the different damage cases. It also gives some results obtained using the FE model and experimental validation. Results and discussion are presented in section 5 followed by some conclusions in section 6.

2. Methodology

The method suggested is based on the findings that when damage is located in a structural part which is under higher stress, this results in a significant shift in the resonant frequencies, making the damage more easily detectable [3,4,6]. Thus if the damaged area is under low stress this will make its detection using the first several natural frequencies rather difficult and unreliable.

In this paper we suggest a possible way to increase the stresses in the damaged section of the structure by applying an additional transverse static force. Currently the method is developed for beams subjected to an additional static transverse force which is applied at different locations on the beam. This results in different stress values at the damaged area. The maximum bending moment occurs under the point of application of the static force, so the maximum stress at the damage area arises when the static force is applied on the defect itself. Thus if one moves the force position towards the damage, the stresses in the damaged region increase, resulting in a higher frequency shift as compared to the case when no force is applied. In this study we use a percentage frequency shift which is introduced as follows:

$$\Delta f = \frac{(f_f - f_{nf})}{f_{nf}} . 100 \qquad\qquad (1)$$

Where f_{nf} is the frequency with no force applied and f_f is the frequency with a static force applied.

The method suggested we take into account the fact that compression tends to decrease the natural frequencies while tension increases the natural frequencies of a structural element [5]. When a static transverse force is applied on an undamaged beam, bending and shear stresses will arise in the beam. These stresses vary from point to point along the beam. As a result the material on one side of the neutral axis of the beam will be under tension while the material on the other side of the neutral axis will experience compression, or vice versa depending on the curvature of the beam (see Figure 1). As the total axial forces acting on either side of the neutral axis are equal but act in opposite directions, they cancel each other and thus will have no effect on the natural frequencies of the beam. Hence a static force applied on an intact beam will have no influence on its vibratory behaviour and its natural frequencies. Now let's consider the situation when there is a defect in e.g. the part of the beam which is under compression (Fig.1). We shall regard the defect as a material loss. The existence of a defect will cause a decrease in the total internal compressive force for this half of the beam, as it depends on the volume of material under compression. This decrease depends on the stresses at the defect location as well as on the defect size. The reduction in compression will push the neutral axis to move down for equilibrium at the damaged area of the beam. Knowing that compression reduces the natural frequency, a reduction in compression will have the opposite effect, that is it will increase the natural frequency. The same but opposite scenario will apply when damage is in the lower part which is under tension. This will result in a decrease of the natural frequencies of the beam.
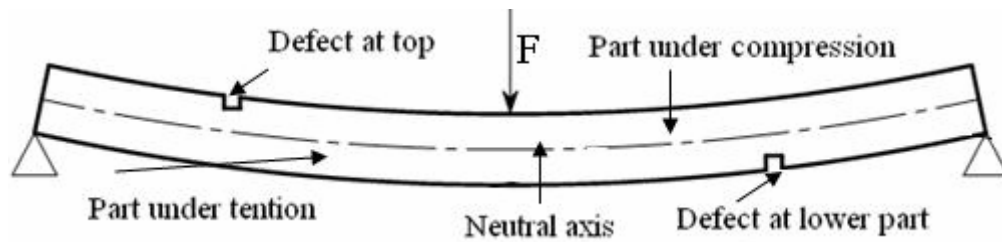
Figure 1: Schematic of beam under bending with defects.

3. Damage Detection and Localization

3.1.Detection

The presence of a fault can be detected by applying a static force at the middle of beam. The frequency shift is calculated for this position of the force according to equation (1). If there is frequency shift it means that there is a defect in the beam and the opposite is true as well. Since one does not know in advance where the defect is located, *vis.* to the left or to the right of the middle, the only logical choice for the application of the static force is the middle of the beam. At this position the stress at all the points along the beam and even close to the supports are substantial, which will result in a high frequency shift even when the defect is close to the supports. A positive shift will be caused by a defect located at the top of the beam, while a negative sign will be the result from a defect which is in the lower part of the beam (see Figure 1). Thus by performing a detection process as described above one already knows if the defect is in the upper half or in the lower half of the beam. This will help the process of the further localisation.

3.2. Localisation

As was explained in &2, applying a static force to a beam when there is a defect anywhere in the top half will increase its natural frequency, while the frequency will be reduced when the defect is at the bottom part. This is because applying a force on a beam with a defect causes an excess of compression or tension in the undamaged half of the beam. Thus when the defect is in upper half of the beam the total internal tension force in the lower part will be bigger than the total internal compressive force which corresponds to the upper half. We suggest that this causes an excess of tensile force around the defect which in turn increases the natural frequencies. In a similar way if the defect is in the upper half of the beam there will be an excess of compressive force in this part, which in turn will lower the natural frequency. This is also in agreement with some conclusions from [5] where the authors find that the application of tension or compression changes the natural frequencies of a structural element. In [5] it is also shown that the frequency shift is linearly proportional to the amplitude of the tensile or compressive force applied. In our case for each static force application position the magnitude of the total internal axial force which arises in the upper or in the lower half of the beam is different. And the amplitude of this internal force increases as the force position gets closer to the defect. The maximum value of the amplitude of this compressive/tensile is when the application point is at the defect. According to this the graph of the frequency shift versus the static force location along the beam will be linear. It will be comprised by two straight lines intersecting at the defect location x on the beam. Figures 3 and 4 present the graphs of the frequency shifts vs the force application point that correspond to different damage cases. Thus if one knows the frequency shifts that correspond to just two application points on either side of the defect, the position x of the defect can be calculated using the following equation (1):

$$x = \frac{\left(\frac{\Delta f_2}{(L-a_2)}\right)}{\left(\frac{\Delta f_2}{(L-a_2)}\right) + \left(\frac{\Delta f_1}{a_1}\right)} \qquad (2)$$

where a_1 and a_2 correspond to the distance of the static force from the left support and the right support, while Δf_1 and Δf_2 are the corresponding frequency shifts for the two application points respectively. In the case of a number of force application points, e.g. *n* to the left and *m* to the right of the defect, the location can be estimated by the average of the values calculated using all the combinations of points to the left and to the right of the defect:

$$x = \frac{1}{m*n} \sum_{i,j}^{n,m} \frac{\frac{\Delta f_j}{(L-a_j)}}{\left(\frac{\Delta f_j}{(L-a_j)}\right) + \left(\frac{\Delta f_i}{(a_i)}\right)} \qquad (3)$$
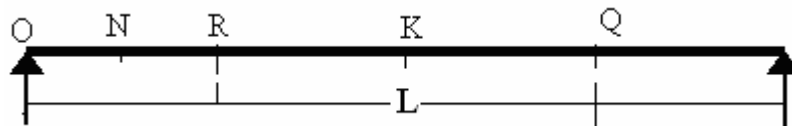
where $i = 1,2,...,n$ and $j = 1,2,..,m$

Figure 2: Visualisation of the locations of interest along the beam.

Let us denote with O and K the left hand support and the middle of the beam (see Figure 2). Let us also denote with R and Q two points to the left and to the right of the middle of the beam which are symmetric with respect to the middle (see Figure 2). The distance between the points R and Q should preferably be in the range 0.5L and 0.8L.

It is known that the bending moment at the defect is linearly proportional to the amplitude of the applied static force. Figure 6 represents the frequency shift versus the static force amplitude for different damage locations and the same static force application point while Figure 7 represents the frequency shift for different static force application points and amplitudes but for the same damage location. From these figures it can be seen that the frequency shift for each position of the static force is linearly proportional to the bending moment at the defect. This relation will be used for defect localization. Below we give notations for the frequency shift ratio and the bending moments ratio:

$$\frac{\Delta f(R)}{\Delta f(Q)} = c \qquad (4)$$

$$\frac{M_R}{M_Q} = d \qquad (5)$$

where $\Delta f(R)$ and $\Delta f(Q)$ are the frequency shifts when the static force is applied at points R and Q respectively. M_R and M_Q are the bending moments at the defect when the static force is applied at points R and Q respectively. If $\Delta f(R) > \Delta f(Q)$ this will indicate that the defect is left half of the beam, that is between left hand support and the middle of the beam and *vice versa*.

The moments M_R and M_Q are calculated at R in the case when $\Delta f(R) > \Delta f(Q)$ and at Q if $\Delta f(R) < \Delta f(Q)$. The localization process which follows will be done when the defect is situated in the half of the beam to the left of the middle.

1) We compare c and d from equation (4) and (5). If c is equal to d then the defect is at point R (see Figure 2).

2) If d is bigger than c, the defect is between points R and K (the middle of the beam) (see Figure 2).

3) If d is smaller than c, the defect is between points O (the left hand support) and R. In this case another point N between O and R is chosen, a frequency measurement is taken when the force is at point N and $\Delta f(N)$ is calculated. (see Figure 2)

The same process as above is applied this time comparing moment and frequency ratios for the new point N and another point on the beam. The process could be done between points N and R, N and K or N and Q. In all cases when the defect is located between two points on the beam, different from the beam ends, equation (2) is used to calculate the exact location of the defect, i.e. its distance x from the left hand side support of the beam.

4. Numerical and Experimental Validation of the Method

In this study we first use a numerical simulation based on a finite element model of the beam in order to validate the suggested detection and localisation method. Below we introduce some details about the beam and the model.

4.1 The beam, the model and the numerical verification of the method

In this study the beam is simply supported at both ends. (The method can be very well applied for other types of support and we have results for other cases as well.) The length of the beam is 1m and it has a rectangular cross section 20mm by 6mm. The material of the beam is stainless steel (Grade 304L) and the following characteristics were used for its properties: Yield tensile strength = 485MPa, Young's modulus = 193GPa, Poisson ratio = 0.29 and a density of 8000kg/m^3.

The beam was modelled using solid shell elements. Damage was modelled as a small notch on the beam with a width = 1.5mm and length equal to the width of the beam = 20mm, and depth of 50% of the thickness of the beam. Four damage cases with respect to the location of the defect were investigated: Case 1- damage located at $0.225L$ from the left hand support and at the upper part of the beam; Case 2- damage located at $0.45L$ from the left hand support and at the upper part of the beam; Case 3- damage located at $0.225L$ from the left hand support and at the bottom of the beam and Case 4- damage located at $0.45L$ from the left hand support and at the bottom of the beam; where L = length of the beam. For all these cases, a pre-stressed modal analysis was conducted and the first 10 modal frequencies were obtained.

In order to detect and localise the damage for each of these cases, nine equidistant points along the beam are chosen, the first one at 0.1m, the second one at 0.2m and so on, and a static force of 40N is applied was applied at each point consecutively. Pre-stressed model analysis was conducted for each position of the static force and the first 10 modal frequencies were obtained. Then the frequency shift in percent for each case is calculated using equation (1);

The results from the localisation are presented in Table 2, where for each of the four damage cases, the frequency shift and the obtained damage location are presented. The frequency shifts (%) are calculated for each of the nine positions of the static force as derived from equation 1 using the first natural frequency as obtained from the FEA for the beam with and without the static force. A negative value indicates reduction in natural frequency while a positive value indicates an increase. The damage location for the FEA models is calculated using equation (3). The first column of Table 2 gives the damage case (see Table 1); in the second column the frequency shifts for each of the nine location of the static force are shown. The third column is for the damage locations calculated using expression (3).

4.2 Experimental Validation

The same beam as the one in the numerical verification is used. The beam has the same dimensions as the one described above and is made of stainless steel. It was simply supported. A static force was

applied by tying a weight with a cord at a certain position on the beam. Cases 2 and 3 were tested experimentally using two beams with defects in different positions as given in Table 1.The same nine equidistant application points of the static force as the ones used in the FEA were used.   Three other damage cases with different defect positions were also tested experimentally using three beams, namely: *Case5*- defect located at 0.4*L* from the left hand support and in the lower part of the beam; *Case6*- defect located at 0.5*L* from the left hand support and in the upper part of the beam; *Case7*- defect located at 0.3*L* from the left hand support and in the upper part of the beam. Cases 5, 6 and 7 were used to detect and localise the defect by using three measurements points and cases 6, 8, and 9 - to verify that the magnitude of the static force is linearly proportional to the frequency shift. The natural frequencies were measured for all the damage cases (2, 3, 5, 6, 7, 8 and 9) with and without the static force. Then the percentage frequency shift for each of the seven cases was calculated using equation (1).The experimental results are summarised in Table 3, where column 1 represents the damage case (see Table 1), the second column shows the frequency shifts values in (%) for each static force position, while the predicted damage location and the error in (%) relative to the exact location are shown in column 3 and 4 respectively.

5. Results and discussion
It should be noted that all the values obtained using the FE modelling (see Table 2) give the exact damage position for all the damage cases. It can be observed that the biggest frequency shifts occur close to or at the position of the defect. For example for case 1 when the defect is close to the left-hand side of the beam at 0.225L the two biggest changes are at positions 0.2L and 0.3L, while for case 2 when the defect is close to the middle of the beam the biggest changes are at 0.4L and 0.5L.This makes it very easy to detect the damage and estimate its approximate position by just tracking the changes in the first natural frequency for the different application points of the static force.

Table 1: Damage case descriptions.

Case	Damage location	Description
1	X=0.225 L	50% At top
2	X=0.45L	50% At top
3	X=0.225L	50% At bottom
4	X=0.45L	50% At bottom
5	X=0.4L	50% At top
6	X=0.5L	50% At bottom
7	X=0.3L	50% At top
8	X=0.3L	50% At bottom
9	X=0.4L	50% At bottom

Table 2: Frequency shift for each static load position from FEA results.

Damage cases	Frequency shift (%)									Defect location
1	5.33	10.43	10.6	9.15	7.68	6.18	4.67	3.13	1.57	0.2249L
2	3.99	7.86	11.59	15.19	15.52	12.58	9.57	6.46	3.27	0.4497L
3	-5.67	-11.68	-11.88	-10.08	-8.32	-6.6	-4.91	-3.26	-1.62	0.2251L
4	-4.19	-8.56	-13.14	-17.98	-18.42	-14.42	-10.6	-6.95	-3.43	0.4498L
	0.1L	0.2L	0.3L	0.4L	0.5L	0.6L	0.7L	0.8L	0.9L	
				Static force position						

We shall now concentrate on the experimental validation. Table.3 shows the experimental values of the frequency shifts for damage cases (2,3 and 5,6,7). For cases (2, 3) the same number of application points for the static force as in the FEA are used for comparison purposes, while for cases 5, 6, and 7 only three points are used: at 0.5L for detection and at 0.2L and 0.8L for localization. Table.3 also shows the damage location and the error in *percent* in relation to the real location. For cases 2 , 3, 5, 6 and 7, when the static force is placed at the middle of the beam (point 0.5L) the frequency shifts measured are 14.35% , -7.73%, 12.5%,-19.68% and 10.17% respectively. These shifts are quite high which is an indication of damage. The signs of these shifts indicate that for cases 2, 5 and 7 the defect is in the top part of the beam while for cases 3 and 6 it is in the lower part, which corresponds to the real situations. From the measured frequency shifts at point 0.2L and 0.8L, we can conclude that the defect is between the middle and the left support for cases 5 and 7. In case 6 it can be seen that the frequency shifts when the force is at 0.2L and 0.8L are nearly equal while the shift is high for the position at the middle. From these observations we can conclude that for case 6 the defect is situated around the middle of segment AB. From (3) for case 5 one obtains c=2.3 and d=4 at A (0.2L) and at any point to the left of it. This indicates that the defect is between the points 0.2L and 0.5L.The same process applies to case 7. In case 6 we used just points 0.2L and 0.8L in expression (3) to calculate the location of the defect, while in cases 5 and 7 we used all the three points of application 0.2L to the left of the defect and 0.5L and 0.8L to the right of the defect to get the average value of the defect location. For each of the damage cases the damage detection was successful and the values of the defect location found were very close to the ones induced in the beams tested. The error in the damage location in relation to the real value in each case was found to be very small with values of 0.67%, 3.29%, 2.75%, 1%, and 0.66% respectively for cases 2, 3, 5, 6 and 7. Again this is an indication that the method is precise in detecting as well as locating damage.

Table 3: Frequency shifts and damage locations for cases (2,3, 5,6,7) from experiment

Damage cases	Frequency shift (%)									Defect location	Error (%)
2	4.16	6.8	10.96	13.78	14.35	11.82	8.49	6.14	3.01	0.447L	0.67
3	-5.45	-10.72	-11.1	-9.2	-7.73	-6.85	-4.72	-3.39	-1.92	0.233L	3.29
5		8.27			12.5			5.51		0.389L	2.75
6		-7.35			-19.68			-7.48		0.505L	1
7		9.47			10.17			4.12		0.302L	0.66
	0.1L	0.2L	0.3L	0.4L	0.5L	0.6L	0.7L	0.8L	0.9L		
					Static force position						

Figures 3 and 4 show how the frequency shift changes with the static force position. The results show that the curve on either side of the defect is linear. The absolute value of the frequency shift increases as the static force gets closer to the defect and decreases when the force position moves away from the defect. In these two damage cases, the defect position is found by the projection of the point P on the static force position axis. Where P is the intersection of the segments AB and CD (see Figures 3 and 4), the defect position was found to be equal to the real one. That is 0.225L for case 2 and 0.450L for case3.

From the results it can be seen that the frequency shift is bigger when the defect is in the lower part of the beam (part under tension). This makes it easy to detect even very small notches. This is in agreement with the results from [5]. They show that compression reduces the natural frequency by an amount higher than the increase caused by a tension of the same magnitude.
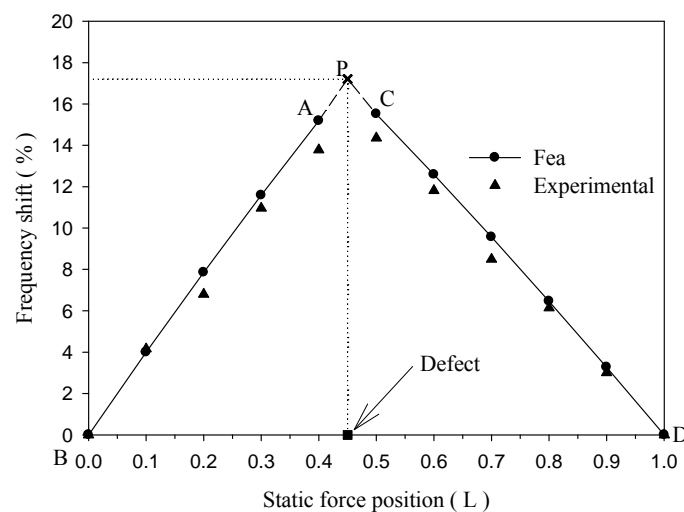

Figure 3: Frequency shift vs. static force position for numerical and experimental results for case 2.
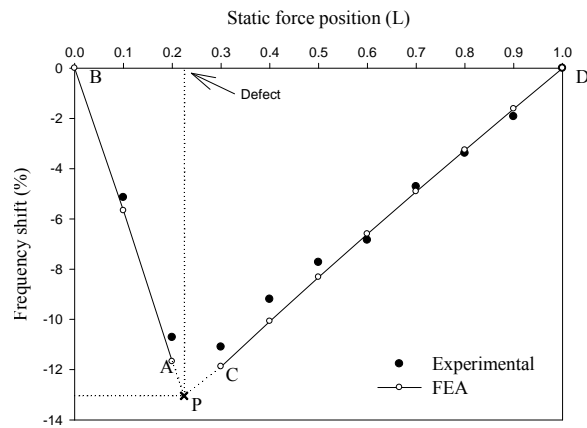
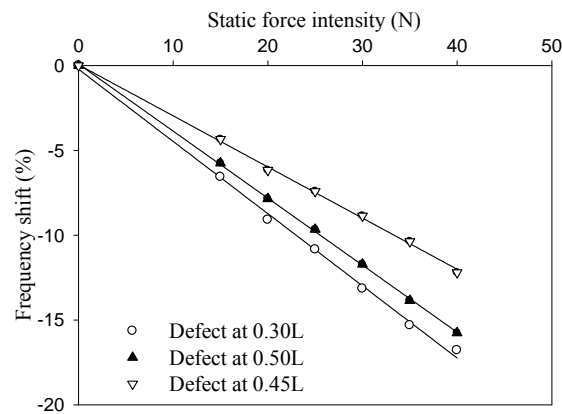Figure 4: Frequency shift vs. static force position for numerical and experimental results for case 3.

Figure 5: Experimental results for static force intensity vs. frequency shift, cases 6, 8 and 9, force applied at 0.4L.
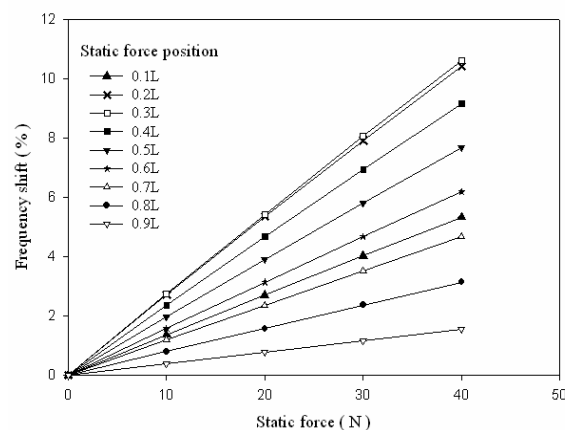

Figure 6: FEA results for frequency shift vs. static force intensity for each of the nine application points for case 2

Figure 5 shows the behaviour of the frequency shift in relation to the static force amplitude. The results are obtained experimentally for cases 6, 8 and 9 for one single position of the static force at

0.4L, with force amplitudes varying from 15N to 40N at an increment of 5N. Figure 6 shows the same results as Figure 5 for case 2 and with the static force applied at nine positions starting from 0.1L up to 0.9L. In this case the force amplitudes used for each position were 10N, 20N, 30N and 40 N. It can be seen from both figures that the relation between the frequency shift and the static force amplitude is linear. This relationship could be used to check if the frequency measurements readings are correct when looking for defects in beams. This is done by using another force amplitude. If the ratio of frequency shifts is equal to the ratio of the corresponding static forces amplitudes, then this means that the measurements are correct. Figure 6 shows that the frequency shifts increase as the static force location gets closer to the defect to its biggest value at 0.3L. The reason is that the bending moment at the defect when the force is at 0.2L is smaller to that at 0.3L.This justifies a bigger force at 0.3L even if the defect is at 0.225L is closer to 0.2L.

7. Conclusions

This paper suggests a very simple procedure for defect detection in beams which is based on the first natural frequency of the beam and no base model is needed. The procedure involves the application of a static force which is applied in different positions along the beam. The numerical and the experimental results show that applying the static force at the middle of the beam will not just detect damage but will also indicate in which side of the beam it is, *vis.* in the upper or in the lower half. An additional two points of application will locate the damage on the beam. It should be noted that the FE simulation as well as the experiment are successful to detect the presence of a defect in all the cases. When using the FE simulation for the purposes of defect localisation one obtains the exact locations of the defect for all the 4 cases. The damage location obtained using the FE model is the same as the real one while in the experimental results the predicted damage locations are very close to the real values with a maximum error of 3.29 %. The method is being developed for the case of multiple defects. In future inspections an increase in the value of the frequency shift, using the same force amplitude, and just one point of application which coincides to one of the previous application points, preferably the one closer to the defect, means the damage has grown.

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