University of Strathclyde
Glasgow

Strathprints Institutional Repository

Trendafilova, Irina (2011) *A method for vibration-based structural interrogation and health monitoring based on signal cross-correlation.* Journal of Physics Conference Series, 305 (1). ISSN 1742-6588

http://strathprints.strath.ac.uk/

A Method for Vibration-Based Structural Interrogation and Health Monitoring Based on Signal Cross-Correlation

A Method for Vibration-Based Structural Interrogation and Health Monitoring Based on Signal Cross-Correlation.

I Trendafilova

Department of Mechanical Engineering, The University of Strathclyde, 75 Montrose street, Glasgow, G1 1XJ

Irina.Trendafilova@strath.ac.uk

Abstract. Vibration-based structural interrogation and health monitoring is a field which is concerned with the estimation of the current state of a structure or a component from its vibration response with regards to its ability to perform its intended function appropriately. One way to approach this problem is through damage features extracted from the measured structural vibration response. This paper suggests to use a new concept for the purposes of vibration-based health monitoring. The correlation between two signals, an input and an output, measured on the structure is used to develop a damage indicator. The paper investigates the applicability of the signal cross-correlation and a nonlinear alternative, the average mutual information between the two signals, for the purposes of structural health monitoring and damage assessment. The suggested methodology is applied and demonstrated for delamination detection in a composite beam.

1. Introduction

Maintenance and operation costs are usually among the largest expenditures for most structures - civil, aerospace, and military. An ageing structure may reduce profits with increased maintenance costs and down time and it can become a hazard for its users. The ability to access the integrity of a structure and discover a fault at a rather early stage, before it has developed so that it can cause damage to the structure, can significantly reduce these costs. A large class of the structural health monitoring (SHM) methods are vibration-based methods where the state of the structure is assessed using its vibration response. Among the most common features are the ones extracted from the modal properties, like resonant frequencies, damping and mode shapes [1,2,3]. All of them have their advantages and disadvantages, the main problems being lack of sensitivity to damage, noise sensitivity and difficulties to estimate from measured data. Such methods assume structural linearity. A large group of monitoring methods, the model-based methods assume and use a model for the structure under interrogation [1]. A lot of the model-based methods use a linear structural model. Contrary to the model-based methods, methods that are based on the measured vibration data only do not assume any structural model or linearity [1,2]. These methods have seen quite a development during recent years. Some of them utilise signal analysis and statistical methods. Monitoring methods based on the time-domain vibration signatures represent a relatively new paradigm in SHM [3,4]. These methods are mostly based on non-linear signal analysis and non-linear dynamics tools. They represent a very attractive alternative since they only require the measured structural vibration time-domain signals in the current and possibly in the baseline (undamaged) state especially when the suggested features are easy to estimate from data. Possible problems might be lack of damage sensitivity and noise

(imperfection) robustness. Several studies suggest to use the idea of signal comparison, correlation and dissimilarity measures for the purposes of structural health monitoring [5-8]. All of them share the general concept of comparison of signals coming from different structural states – the healthy and possibly damaged one. The idea is that if two signals (measured in a certain point on the structure) come from the same structural state they will be highly correlated, while this correlation will decrease if damage is introduced in the structure. The authors suggest several criteria for damage detection and localization [6-8].

We are taking a different view thus suggesting a novel concept for damage assessment which is to compare an input signal and an output signal for the case of random excitation. The main idea behind the method suggested is that for a linear system the output signal will be highly correlated to the input. For a nonlinear structure the correlation between the input and the output will go down. The introduction of a fault introduces a nonlinearity in the system and this will accordingly decrease the correlation between the two signals. Thus the correlation of a structure will go down, as compared to the one of the original (intact) structure, when a fault is introduced and when the fault grows.

We consider random excitation as the natural excitation created by passing traffic (on e.g. bridges), wind and any other vibrations (including seismic) to which most civil and mechanical engineering structures are subjected.

The method suggested can use any signal as an input- it could be the displacement, the velocity or the acceleration. The input and the output signals are measured in different points so that the output signal is captured after propagating through the structural member under interrogation. In this case we consider a beam subjected to vertical vibration and a displacement signal measured at the bottom of the beam is considered an input while the output is an acceleration signal measured on the top (see Figure 1). The proposed method is based only on the measured input and output signals and does not assume any model or structural linearity.

In this study the method is applied and demonstrated for delamination detection in a composite beam. Composite materials are inherently nonlinear and composite beams are known to demonstrate well expressed nonlinear vibratory behaviour [3].

2. The method and the characteristics used.

The damage assessment method proposed is based on two time domain vibration signals measured in two different points on the structure. The method is based on the fact that for an ideal linear system the input and the output signals will be highly correlated, while if there is damage (or nonlinearity) present in the structure then the system will no longer be neither linear nor ideal and the correlation will go down. As an alternative for the case of nonlinear structures, for which the cross correlation between the two signals will to be lower in their initial intact state, another metric based on the mutual information between the signals is suggested.

2.1. *Signal cross correlation and damage assessment.*

In signal processing cross-correlation is a measure of similarity of two signals as a function of a time-lag applied to one of them. Let $x(t)$ is an input signal and $y(t)$ is the output signal measured on the structure. The cross correlation between $x(t)$ and $y(t)$ is defined as follows [9]:

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [x(t) - \mu_x].[y(t+\tau) - \mu_y]dt \qquad (1)$$

where μ_x and μ_y are the mean values of $x(t)$ and (t) respectively. Or for a discrete signal

$$R_{xy}(m) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^N [x(n) - \mu_x].[y(n+m) - \mu_y] \qquad (2)$$

The cross correlation is a signal as well. It will have a maximum when the two signals are aligned. It examines linear relationship between the signals x and y. If y is the same signal as x the cross-correlation will have a maximum for 0. If y is a shifted and amplified/attenuated version of x then the cross correlation will have a maximum for the shift between the two signals. The normalized cross-correlation function between two signals is defined as [9]:

$$\rho_{xy}(m) = \frac{R_{xy}(m)}{\sqrt{R_{xx}(0).R_{yy}(0)}}$$ (3)

where R_{xx} and R_{yy} are the autocorrelations of x and y respectively and $\left|\rho_{xy}(m)\right| \leq 1$ for all m.

For a linear structure the output $y(n)$ will be linearly related to the input $x(n)$ in the sense that $y(n)$ will be a shifted and attenuated version of $x(n)$. The normalized cross correlation (3) between the input and the output of a linear system/structure (for long enough signals x and y) will have a maximum of 1. On the other hand the normalized cross correlation will be 0 or close to 0 for all time lags m if two signals are completely uncorrelated. When the maximum normalized cross correlation between an input and an output signal of a structure is less than 1, then this is due to noise and any nonlinearities present in the system. For a real structure with close to linear behaviour when there is not a lot of noise interference the maximum normalised cross correlation is expected to be close to 1.

In this study we use the maximum normalized cross correlation (4) as a damage metric:

$$\Omega_{xy} = \max_{m} \rho_{xy}(m)$$ (4)

2.2. The average mutual information and damage assessment

A lot of vibrating systems cannot be considered linear especially at high amplitude vibrations and/or at high frequencies, rather than using the linear cross-correlation we suggest a nonlinear alternative for signal correlation, the mutual information. The mutual information can be regarded as a nonlinear analogue to the cross correlation. The mutual information is a theoretic idea that connects two sets of measurements and it determines the amount of information that one of the sets "learns" from the other, or in other words, it determines their mutual dependence in terms of information [4,5].

The mutual information between two signals x and y is defined as:

$$I(x, y) = \log_2 \frac{P_{xy}(x(i), y(j))}{P_x(x(i))P_y(y(j))}$$ (5)

where P_{xy} is the joint probability density function of the signals $x(i)$ and $y(j)$ and P_x and P_y are the individual probability densities of $x(i)$ and $y(j)$ respectively. The average over all measurements, the average mutual information between $x(i)$ and $y(j)$ is

$$I_{xy} = \sum_{x(i), y(j)} P_{xy}(x(i), y(j)).\log_2 \frac{P_{xy}(x(i), y(j))}{P_x(x(i))P_y(y(j))}$$ (6)

The average mutual information (AMI) between two signals is 0 if they are completely independent and their joint probability density is equal to the product of their individual probability densities. On the contrary if two signals are highly correlated then their AMI will tend to 1. If x and y are the input and the output signals to a structure respectively then if the structure is linear the AMI for such structure will be one or close to 1. For a nonlinear structure the signals x and y will be nonlinearly

related in the sense that the structure's transfer (impulse response) function [9] will be nonlinear. The AMI is supposed to measure nonlinear relation between two signals. Thus for some nonlinear dependencies between x and y the AMI will tend to 1 as well. For an arbitrary nonlinear structure in its intact state the AMI will have a certain value which might or might not be close to 1. But it will keep on the same level (and we prove that later for our composite beam) as long as the no changes are introduced in the structure and thus the mutual dependence between two signals, an input and an output, is kept the same. But the AMI will change when damage is introduced in the structure since damage will introduce an additional nonlinearity which in turn will affect the relation between x and y and hence their mutual information.

In this study the average mutual information I_{xy} is used for the purposes of signal comparison and as a damage metric.

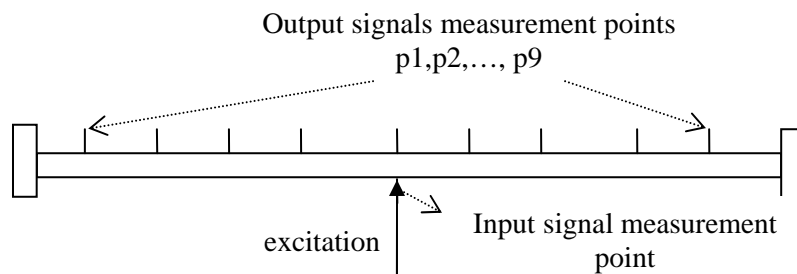## 3. Our structure and the delamination scenarios.

Figure 1. The beam, the excitation and measurement points

In this paper we demonstrate the method for a composite laminate beam made of carbon fibre. The beam dimensions are as follows: length 1m, width 0.06 m and thickness 0.008m. It is made of 10 layers. Delamination is introduced between two layers. It is introduced in three different positions along the beam thickness, vis. between the upper two layers (up) between the layers 9 and 10 (down) and in the middle between layers 5 and 6 (middle) and in three different positions along the length of the beam, vis. 100mm from the left end (left), in the middle (centre) and 100 mm from the right end (right). The delamination is over the whole width of the beam and has different lengths, namely 0.01m (small), 0.02m (medium) and 0.03m (large).

The beam is clamped at both sides and it is excited at the bottom at 0.4 m from the left end with a random Gaussian broadband signal. The input signal is the displacement measured at the bottom in the middle of the beam. The output signals are the accelerations measured in nine equidistant points at the top of the beam (Figure 1).

4. Robustness of the characteristics to noise and to changes in the measurement points and the excitation signal.

In this paragraph we present the results from a test carried out to check the robustness of the average correlation and the AMI to signal changes, measurement point changes as well as to noise. This is to confirm that our damage features do not change when the excitation signal or the measurement position are changed and are not affected by noise. The tests are done for an intact beam and thus they are used to prove the robustness of the damage metrics for the case of no damage.

The following test was carried out in order to check for the sensitivity of Ω_{xy} and I_{xy} to changes in the input signal as well as to changes in the measurement point. The applied force is generated by a signal generator. Ten different j=1,2,…,10 normally distributed signals with frequency range between 0-1 kHz are used as an excitation force. The corresponding input signal is measured and the output signals are measured at i=1,2,…,9 nine different points along the beam length at the measurement

points p1,p2,.., p9 which are at 0.1m, 0.2m, 0.3m and 0.4m from each end and in the middle. Each test is conducted 20 times. We then calculate Ω_{xy} and I_{xy} for the input and the response signals for all the nine positions. The results are presented in the Tables below. Tables 1a) and 1b) give the mean values and standard deviations of the maximum normalized correlation for the different measurement points and for the different signals respectively. In the first case each of the statistics is calculated for a single measurement point over the different signals and experimental realizations (Table 1a)) while in the second case each statistic is calculated for a single excitation signal over the 9 measurement points and 20 realizations for each point (Table (1b)). It can be seen that the mean values of Ω_{xy} are very much the same: they change between 0.755 and 0.836 in the first case and the standard deviations do not exceed 1.8%, and between 0.795 and 0.812 with maximum standard deviation of 3.7% for the second case. Thus it can be appreciated that there is bigger variability in the cross correlation if the measurement points are varied, while changing the excitation signal causes a rather small variability. Similar conclusions can be made for the AMI- the mean values of the AMI keep very much on the same level: they change between 0.863 and 0.919 with standard deviations less than 1.7% for the different measurement points and between 0.893 and 0.906 with standard deviations of up to 3.1%.

Table 1. Maximum normalized cross correlation mean values and standard deviations **a)** for the different measurement points (calculated over the 10 signals) and **b)** for the different signals (calculated over the 10 points)

point	Mean value	Standard deviation	signal	Mean value	Standard deviation
1	0.774	0.015	1	0.804	0.025
2	0.804	0.013	2	0.804	0.037
3	0.827	0.014	3	0.800	0.032
4	0.806	0.006	4	0.802	0.036
5	0.784	0.013	5	0.800	0.026
6	0.812	0.014	6	0.795	0.023
7	0.836	0.018	7	0.812	0.032
8	0.755	0.009	8	0.802	0.018
9	0.828	0.013	9	0.800	0.030
			10	0.801	0.027

a)   b)

Table 2. Mean value and standard deviations of AMI **a**) for the different measurement points (calculated over the 10 signals) and **b)** for the different signals (calculated over the 9 measurement points)

point	Mean value	Standard deviation	signal	Mean value	Standard deviation
1	0.899	0.009	1	0.900	0.020
2	0.916	0.017	2	0.906	0.031
3	0.863	0.010	3	0.894	0.029
4	0.892	0.010	4	0.897	0.026
5	0.865	0.011	5	0.900	0.028
6	0.919	0.010	6	0.901	0.024
7	0.928	0.008	7	0.898	0.024
8	0.902	0.015	8	0.897	0.025
9	0.901	0.010	9	0.893	0.020
			10	0.897	0.024

a)   b)

These results are similar to the results obtained for the cross correlation namely changing the measurement points results in a bigger variability of the AMI than the change in the excitation signal.

The next two Tables 3a) and 3b) give information about the noise sensitivity of the method. In this case the method is verified experimentally and thus the only way to judge about its noise sensitivity is to test it experimentally. Tables 3a) and 3b) give the mean values and the standard deviations of both quantities that are suggested a damage metrics over 20 test realizations that are carried out for each tests. Since each experiment is noise contaminated then if the standard deviations over a number of realizations are low then this is proof of the robustness of the method to noise contamination. Moreover if the mean values of the quantities investigated do not change a lot for different measurement points and for different excitation signals then this also implies robustness of the method with respect to noise. It can be seen from the tables below that the standard errors of the cross correlation are quite low- for most cases they are below 2% and the biggest value is 2.5%. The mean values keep between 0.789 and 0.806 so their variability is 0.55% only and thus it can be concluded that they practically do not change.

Table 3a) Mean values and standard deviations of the maximum normalized cross correlation over the 20 experimental realizations for different signals (vertically) and different measurement points (horizontally)

Measurement point	1		2		3		4	
signal	Mean value	Standard deviation%	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
1	0.799	2.5	0.801	1.0	0.800	1.5	0.790	2.0
2	0.804	1.2	0.795	1.9	0.810	1.6	0.800	2.1
3	0.800	1.3	0.789	1.4	0.802	1.5	0.800	0.8
4	0.802	0.9	0.802	1.1	0.803	1.9	0.806	1.2
5	0.795	1.9	0.803	1.7	0.804	2.0	0.789	1.2

The results for the AMI are very much similar again the mean values practically do not change or exhibit very small changes. Overall they vary between 0.889 and 0.906 and thus their variability (standard error) is very low *vis.* 0.55%. This proves the robustness of this characteristic to noise for this particular case.

Table 3b) Mean values and standard deviations of the AMI over the 20 experimental realizations for different signals (vertically) and different measurement points (horizontally)

Measurement point	1		2		3		4	
signal	Mean value	Standard deviation %	Mean value	Standard deviation %	Mean value	Standard deviation %	Mean value	Standard deviation %
1	0.899	1.5	0.901	2.0	0.900	0.8	0.890	2.9
2	0.904	2.2	0.895	2.7	0.910	2.6	0.900	2.1
3	0.900	2.5	0.889	2.4	0.902	1.9	0.900	2.8
4	0.902	1.9	0.902	2.1	0.903	1.8	0.906	2.5
5	0.895	1.5	0.903	1.6	0.904	2.2	0.889	1.2

5. Some results and conclusions.

Below some results obtained for the damage scenarios given in &3 are presented. The following experiment was performed in order to test the detection abilities of the two characteristics. We tested 10 specimens identical with respect to size and material– one without delamination and nine others with each type of delamination with respect to the location along the length and the thickness of the beam. And we also made experiments with the three different delamination sizes, namely small, medium and large delamination. Each experiment was conducted 10 times and measurements were taken on the input and the output signals to get a representative sample of measured signals.

We introduce two damage indexes based on the two metrics, which give their relative percentage changes. The one below ω_{xy} represents the percentage change in the maximum normalised cross-correlation:

$$\omega_{xy} = \frac{(\Omega_{xy}^{in} - \Omega_{xy}^{dam})}{\Omega_{xy}^{in}}.100 \qquad (7)$$

where Ω_{xy}^{in} is the maximum normalized cross-correlation (calculated according to equation (4)) corresponding to the initial state, which is assumed undamaged and Ω_{xy}^{dam} is the maximum normalized cross-correlation corresponding to the current possibly damaged state. In a similar way a delamination index based on the AMI is introduced, which represents the relative percentage change in the AMI between the baseline (undamaged) condition and the current possibly damaged one:

$$i_{xy} = \frac{I_{xy}^{in} - I_{xy}^{dam}}{I_{xy}^{in}}.100 \qquad (8)$$

Tables 4a) and 4b) represent the changes in the corresponding damage indexes. The tables give the mean values of the changes (over the number of experiments conducted) and their standard deviations in *per cent*.

Table 4a) Cross correlation-based index ω_{xy} mean value and standard deviation with delamination

Delamination position/ size		small		medium		large	
		mean	Standard deviation (%)	mean	Standard deviation (%)	mean	Standard deviation (%)
left	upper	7.70	2.11	32.11	0.29	62.11	1.22
	middle	7.20	2.09	30.02	2.11	56.00	3.44
	lower	7.40	0.98	34.00	3.06	54.21	3.12
centre	upper	8.00	2.99	33.23	1.35	61.34	2.11
	middle	6.60	3.10	33.41	1.23	51.00	3.09
	lower	7.90	1.98	34.10	1.67	50.44	3.13
right	upper	7.60	1.45	25.00	1.78	57.12	1.89
	middle	6.30	2.09	29.40	1.87	50.67	1.09
	lower	7.20	1.34	34.09	1.99	49.01	1.91

It can be observed from Table 4a) that if we choose a threshold of 5% for ω_{xy} so that $\omega_{xy} < 5$ indicates no delamination and $\omega_{xy} \geq 5$ indicates delamination, all delaminations regardless of their size and position will be correctly detected using the above correlation-based index. In a similar way it can be observed from Table 4b) that most delaminations except for three cases of small delamination will be correctly detected using the same threshold for i_{xy} . All the medium and the large delamination cases are correctly detected using both indexes. Thus one can conclude that both delamination indexes can be successfully used for delamination detection.

Table 4b) AMI-based index i_{xy} mean value and standard deviation with delamination

Delamination position/ size		small		medium		large	
		Mean value	Standard deviation (%)	Mean value	Standard deviation (%)	Mean value	Standard deviation (%)
left	upper	5.39	3.12	11.11	2.11	18.42	3.01
	middle	4.29	4.01	9.70	2.07	16.07	0.45
	lower	5.63	3.12	8.98	1.89	17.36	3.23
centre	upper	6.38	2.45	9.22	3.11	23.12	4.10
	middle	6.13	2.21	11.11	3.09	20.16	2.61
	lower	4.77	2.09	10.91	3.45	19.19	2.05
right	upper	5.02	2.13	10.83	2.06	17.19	2.53
	middle	4.17	2.34	9.79	1.23	20.00	3.09
	lower	6.13	2.45	10.62	0.11	20.00	2.22

Both delamination indexes, ω_{xy} and i_{xy}, seem to possess the capability for size estimation of
the delamination in the sense that a bigger index corresponds to bigger delamination. It can be observed from Tables 4a) and 4b) and from Figures 2 and 3 that the indexes corresponding to small delamination are the smallest ones. The cross-correlation indexes increase quite a big deal for medium delamination (to about 30%) and they grow to about 50-60% for large delamination (see Table 4a) and Figure 2). The changed in the AMI-based indexes are smaller but still can be used to distinguish between the three delamination size groups (Table 4b) and Figure 3).

At first glance the delamination indexes introduced do not seem very helpful for purposes of delamination localisation along the beam length and/or thickness (Figures 2 and 3). It is difficult to find a common property/quantity in order to distinguish between delaminations in different locations. In general the upper delamination tends to cause bigger changes as compared to middle and lower ones for most locations along the beam length. But there is an exception for the case of medium delamination when the centre and the right delaminations tend to produce bigger cross correlation changes when they are in the lower part of the beam. Thus it can be said that the method is can be used to detect and quantify delamination.

It is expected that for certain materials and certain structures one of the indexes might work better than the other one. It is the author's opinion that the AMI-based index is more appropriate for well-expressed nonlinear dynamic/vibratory behaviour, while the cross correlation-based index is expected to work better for structures with behaviour which is close to linear. But this still has to be verified with more experiments and numerical simulations.
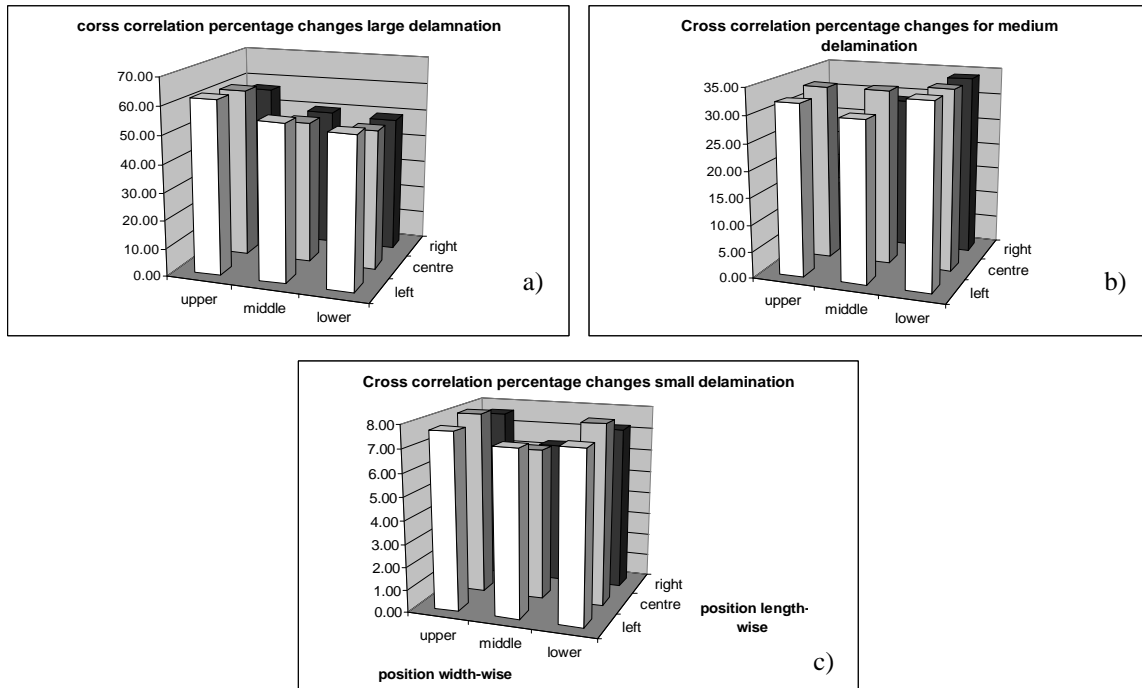
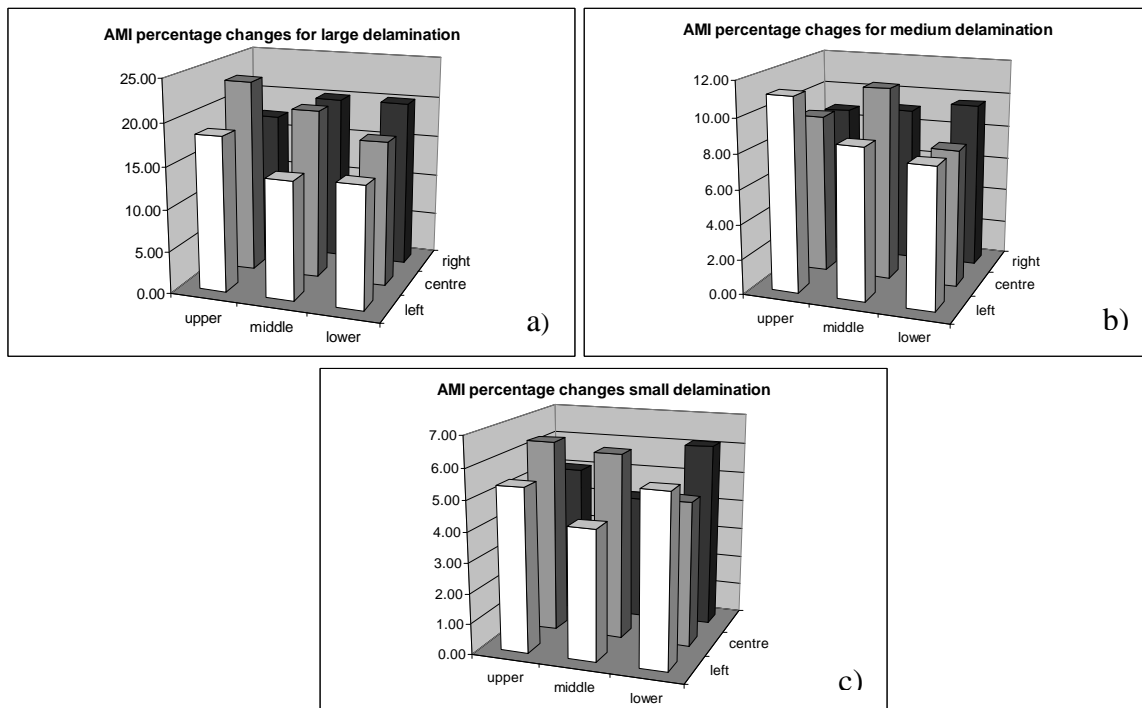**Figure 2.** Cross correlation percentage changes with delamination a) large, b) medium, c) small delamination

Figure 3. AMI percentage changes with delamination a)large, b)medium, c) small delamination

6.   References

[1]   Doebling S.W., Farrar C.R. 1998 A *The Shock and Vibration Digest* **30** (2) 91

[2]   Worden, K., Farrar, C.R., Haywood, J., Todd, M. 2008 *Structural Control and Health Monitoring*, **15** (4) 540

[3]   Palazetti R., Trendafilova I. , Zuccheli A., Minak G. 2010 *Mechanics of composite materials,* **4** 412

[4]   Trendafilova I., Heylen W. and Van Brussel H. 2001 *Smart Materials and Structures* **10** 528

[5]   Roshni V.S, Revathy K. 2008 *Journal of Theoretical and Applied technology*, **4** (6) 474

[6]   Yang Z. , Yu Z., Sun H. 2007 *Mechanical Systems and Signal Processing* **21** 2918

[7]   Overbey L.A. and Todd M.D. 2008 *Structural Health Monitoring* **7** 347

[8]   Yu, Z., Yang, Z. 2007 *Key Engineering Materials* **353-358** (4) 2317

[9]   Bendat J.S. and Piersol A. 2010 Random data: Analysis and measurement Procedures, IV edition, J. Wiley.

Acknowledgments