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Vertical-Axis Wind Turbines in Oblique Flow: Sensitivity to Rotor Geometry

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Abstract

Increasing interest is being shown worldwide in the application of vertical-axis wind turbines for decentralised electricity generation within cities. The distortion of the onset air flow by buildings within the urban environment might however, under certain conditions of wind speed or direction, cause vertical-axis wind turbines to operate in oblique flow – in other words in conditions in which the wind vector is non-perpendicular to the axis of rotation of the turbine. Little is known about the effect on the operation of a vertical-axis wind turbine when the wind is perturbed from supposedly optimal conditions. In the present study, the Vorticity Transport Model has been used to simulate the aerodynamic performance and wake dynamics, both in normal and in oblique flow, of three different vertical-axis wind turbines: one with a straight-bladed configuration, another with a curved-bladed configuration and another with a helically twisted configuration. The results partly confirm previous experimental measurements that suggest that a straight-bladed vertical-axis wind turbine that operates in oblique flow might produce a higher power coefficient compared to when it is operated in normal flow. The simulations suggest, however, that significantly higher power coefficients in oblique flow are obtained only at higher tip speed ratios, and indeed only if the height of the turbine is not large compared to its radius. Furthermore, it is shown that a vertical-axis wind turbine with blades that are helically twisted around its rotational axis produces a relatively steady power coefficient in both normal and oblique flow when compared to that produced by turbines with either a straight- or a curved-bladed configuration.

Keywords: vertical-axis wind turbine; oblique flow; numerical modelling; Vorticity Transport Model

Nomenclature

A	swept area
AR	aspect ratio, b/c
b	blade span
c	blade chord
C_P	power coefficient, $P/\frac{1}{2}\rho A V_\infty^3$
$C_{P,Normal}$	power coefficient in normal flow
$C_{P,Oblique}$	power coefficient in oblique flow
ΔC_P	unsteady component of the power coefficient, $C_P - C_{P,Mean}$
F_t	sectional force - tangential to the chord
H	turbine height
P	power
R	radius of the rotor at blade mid-span
Re	blade Reynolds number, $Re = \Omega Rc/\nu$
S	vorticity source
u	flow velocity
u_b	flow velocity relative to the blade
V_∞	wind speed
β	oblique flow angle, i.e. angle between wind vector and horizon
λ	tip speed ratio, $\Omega R/V_\infty$
ν	kinematic viscosity
ψ	azimuth angle
ρ	density
ω	vorticity, $\nabla \times u$
ω_b	vorticity bound to rotor blades
Ω	angular velocity of the rotor

1. Introduction

In recent years, there has been increasing interest in the deployment of wind turbines within cities. In contrast to current plans for providing electricity at the scale of the national grid by concentrating *large-scale* wind turbines in on- or offshore wind farms, another potentially effective strategy for harvesting wind energy, particularly in the urban environment, relies on using *small-scale* wind turbines, with rotor diameters of only several metres. These devices can be distributed within the urban environment and thus contribute to a decentralised energy supply in which the urban energy requirement is provided by on-site energy production equipment. The design of a wind turbine that operates efficiently within a city poses a significant challenge, however, since the wind in the built environment is characterised by frequent and often rapid changes in direction and speed. In these wind conditions, *vertical-axis* wind turbines might offer several advantages over *horizontal-axis* wind turbines. This is because vertical-axis turbines do not require a yaw control system, whereas horizontal-axis wind turbines have to be rotated in order to track changes in wind direction. In addition, the gearbox and the generator of a vertical-axis turbine can be situated at the base of the turbine, thereby reducing the loads on the tower under unsteady wind conditions, and facilitating the maintenance of the system. The principal advantage of these features of a vertical-axis configuration is to enable a somewhat more compact design that alleviates the material stress on the tower and requires fewer mechanical components compared to a horizontal-axis turbine.

A concern though is that the local distortion to the air flow that is caused by the presence of buildings in urban environments might, under certain conditions of wind speed and direction, cause vertical-axis wind turbines to operate in oblique flow – in other words in conditions in which the wind vector is non-perpendicular to the axis of rotation of the turbine. Little is known about the effect on the operation of a vertical-axis wind turbine when the wind is perturbed from supposedly optimal conditions. In what appears to be one of the only experimental measurements of the behaviour of vertical-axis wind turbines in oblique flow, Mertens *et al.* [1] and Ferreira *et al.* [2] showed that their straight-bladed vertical-axis wind turbine produced a higher power coefficient when operated in oblique flow compared to when it was operated in normal flow – in other words in conditions in which the wind vector was perpendicular to the axis of rotation of the turbine.

In the present study, the Vorticity Transport Model has been used to simulate the aerodynamic performance and wake dynamics, both in normal and in oblique flow, of three different vertical-axis wind turbines: one with a straight-bladed configuration, another with a curved-bladed configuration and another with a helically twisted configuration. The results partly confirm the observations described in Refs. [1] and [2] but also suggest that significantly higher power coefficients in oblique flow are obtained only at higher tip speed ratios and indeed only if the ratio between the height of the turbine and the radius of the rotor is sufficiently low. The results that will be presented in this paper also indicate that straight- and curved-bladed vertical-axis wind turbines produce power coefficients that vary significantly within one revolution of the rotor irrespective of whether the turbine is operated in normal flow or in oblique flow. A turbine with blades that are helically twisted around its rotational axis produces a relatively steady power coefficient, in contrast, under the same flow conditions.

2. Computational Aerodynamics

The Vorticity Transport Model (VTM), developed by Brown [3], and extended by Brown and Line [4], simulates the aerodynamics of wind turbines by providing a high-fidelity representation of the dynamics of the wake that is generated by the turbine rotor. In contrast to more conventional CFD techniques in which the flow variables are pressure, velocity and density, the VTM is based on the vorticity-velocity form of the unsteady incompressible Navier-Stokes equation

$$\frac{\partial}{\partial t}\omega + u \cdot \nabla\omega - \omega \cdot \nabla u = S + \nu\nabla^2\omega \tag{1}$$

The advection, stretching, and diffusion terms within Equation (1) describe the changes in the vorticity field, ω, with time at any point in space, as a function of the velocity field, u, and the kinematic viscosity, ν. The physical condition that vorticity may neither be created nor destroyed within the flow, and thus may only be created at the solid surfaces that are immersed within the fluid, is accounted for using the vorticity source term, S. The vorticity source term is determined as the sum of the temporal and spatial variations in the bound vorticity, ω_b, on the turbine blades, so that

$$S = -\frac{d}{dt}\omega_b + u_b\nabla \cdot \omega_b \tag{2}$$

In the Vorticity Transport Model, Equation (1) is discretised in finite-volume form using a structured Cartesian mesh within a domain that encloses the turbine rotor, and then advanced through time using an operator-splitting technique. The numerical diffusion of vorticity within the flow field surrounding the wind turbine is kept at a very low level by using a Riemann problem technique based on the Weighted Average Flux method developed by Toro [5] to advance the vorticity convection term in Equation (1) through time. This approach permits many rotor revolutions to be captured without significant spatial smearing of the wake structure and at a very low computational cost compared with those techniques that are based on the pressure-velocity-density formulation of the Navier-Stokes equations. Dissipation of the wake does still occur, however, through the proper physical process of natural vortical instability. The bound vorticity distribution on the blades of the rotor is modelled using an extension of lifting-line theory. The lifting-line approach has been modified appropriately by the use of two-dimensional experimental data in order to represent the real performance of any given aerofoil. The effect of dynamic stall on the aerodynamic performance of the blades is accounted for in the VTM by using a semi-empirical dynamic stall model that is based on the method that was proposed by Leishman and Beddoes [6].

Scheurich *et al.* [7] have validated VTM predictions against the experimental measurements of the performance of a straight-bladed vertical-axis wind turbine that were made by Strickland *et al.* [8]. Scheurich *et al.* [9] have also used the VTM to study the influence of blade curvature and helical blade twist on the performance of vertical-axis wind turbines in normal flow. These works have shown the VTM to be an effective tool for resolving the effects of blade geometry, aerofoil performance and wake dynamics on the behaviour of vertical-axis wind turbines.

3. Turbine Model

The geometry of each of the three vertical-axis wind turbines that are investigated in this paper is illustrated in Figure 1. The blades of each turbine are separated by 120° azimuth. Starting from the straight-bladed configuration, shown in Figure 1(a), the radial location of each blade section was displaced using a hyperbolic cosine distribution in order to yield the troposkien shape of the curved-bladed configuration shown in Figure 1(b). The helically twisted configuration, shown in Figure 1(c), was then obtained by twisting the blades around the rotor axis.

The maximum radius, R, of each rotor is at the mid-span of the reference blade (‘blade 1’) of the turbine, and is identical for each of the three different configurations. This radius is used as the reference radius of the rotor when presenting non-dimensional data for the performance of the turbine. The tip speed ratio, λ, is defined as the ratio between the circumferential velocity at the mid-span of the blade, ΩR, and the wind speed, V_∞. The orientation of the blades and specification of the azimuth angle of the turbine with respect to the mid-span of blade 1 is shown in Figure 2. The key parameters of the rotor, common to all three turbines, are summarised in Table 1.

Table 1: Rotor parameters.

Number of blades	3
Aerofoil section	NACA 0015
Blade Reynolds number at mid-span	800,000
Chord-to-radius ratio at mid-span	0.115

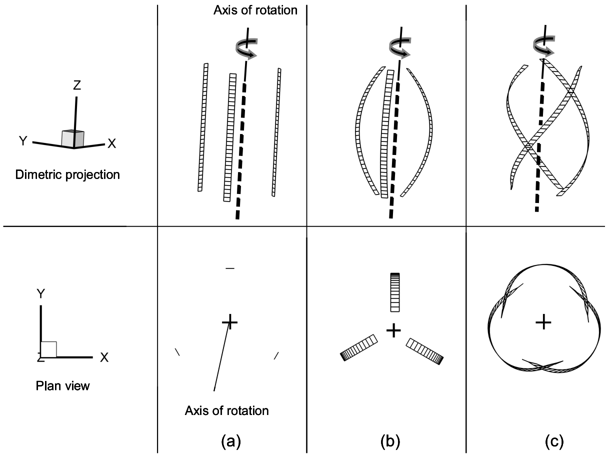


Figure 1: *Geometry of the vertical-axis wind turbines with (a) straight, (b) curved, and (c) helically twisted blades.*

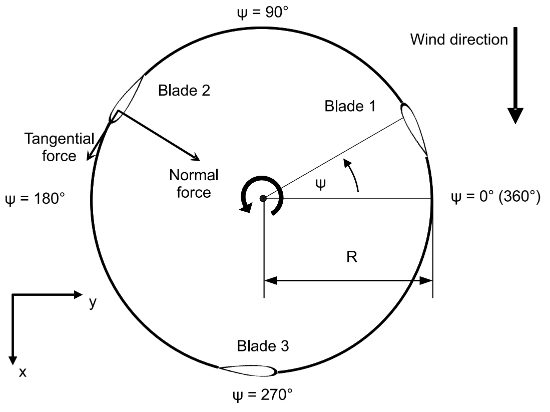


Figure 2: *Diagram showing the wind direction, the definition of rotor azimuth and direction of positive normal and tangential forces, as well as the relative positions of the rotor blades.*

Figure 3 shows the definition of the oblique flow angle, β, together with a representation of the wake that is produced by the turbine with helically twisted blades in oblique flow, as visualised by plotting one of the surfaces within the flow field surrounding the rotor on which the vorticity has a uniform magnitude.

The influence of the ratio between the height of the turbine and the radius of the rotor, henceforth called simply the height-to-radius ratio, on the performance of the straight-bladed turbine both in normal and in oblique flow will be discussed in the following section, whereas an analysis of the influence of blade curvature and helical blade twist on turbine performance is presented in section 5.

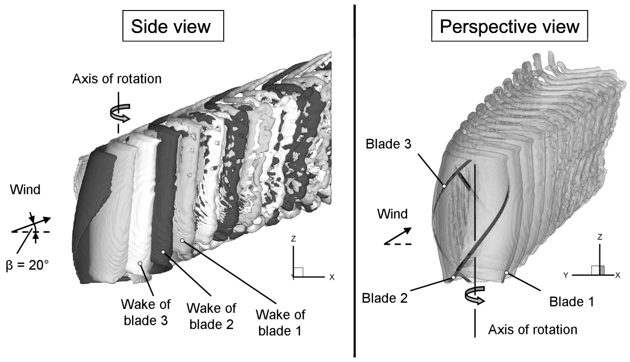


Figure 3: *VTM-predicted flow field of the vertical-axis wind turbine with helically twisted blades at a tip speed ratio* $\lambda = 3.5$ *and at an oblique flow angle* $\beta = 20°$*, visualised by plotting an isosurface of vorticity.*

4. Height-to-Radius Ratio

The power coefficient produced by the straight-bladed vertical-axis wind turbine for three different height-to-radius ratios is shown in Figure 4 when the turbine is operated at three different tip speed ratios in normal flow. The rotor radius was kept constant for all the turbine configurations that were investigated. Thus, a lower turbine height-to-radius ratio results in a lower aspect ratio of the blades, and consequently, in a slightly lower aerodynamic efficiency of the turbine. This is the principal reason for the reduction in performance with decreasing height-to-radius ratio seen in Figure 4. This figure is fundamental in understanding the behaviour of the turbine in oblique flow.

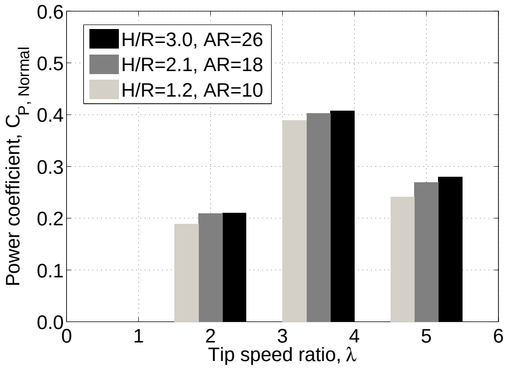


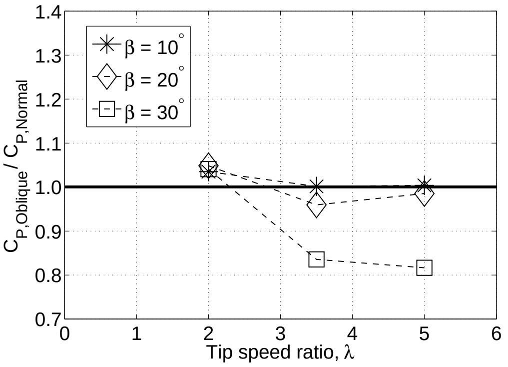
Figure 4: *Influence of turbine height-to-radius ratio (H/R) on the VTM-predicted power coefficient of a straight-bladed vertical-axis wind turbine in normal flow at tip speed ratios* 2.0*,* 3.5 *and* 5.0*.*

The simplest theory for the performance of a vertical-axis wind turbine in oblique flow would consider the rotor from the perspective of swept-wing theory [10]. According to this theory, the component of velocity parallel to the turbine axis would have no effect on the aerodynamics of the turbine and thus, for the same wind speed V_∞, the performance of the turbine in oblique flow would be the same as that of the same turbine in normal flow when operated at wind speed $V_\infty \cos\beta$ and thus at an increased effective tip speed ratio $\lambda/\cos\beta$. In other words

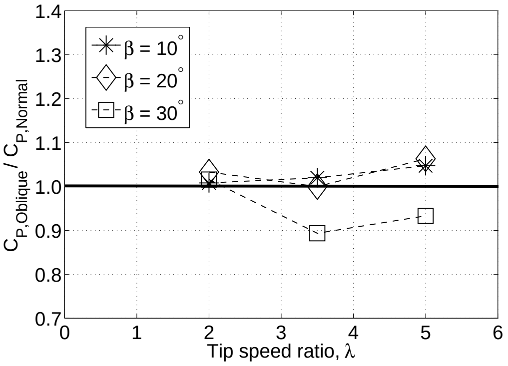
$$C_p(\lambda, \beta) = C_p(\lambda/\cos\beta, 0)\ \cos^3\beta \tag{3}$$

According to this simple theory, whether the power produced by the turbine would increase or decrease as the flow became more oblique would depend on the gradient of the C_P vs. λ curve at the particular operating point of the turbine: for operating points to the right of the maximum in the $C_P - \lambda$ curve the effective increase in tip speed ratio in oblique flow should result in a tendency for the power coefficient to decrease, and vice versa. In all cases though the power coefficient would tend to reduce at higher oblique flow angles as a result of its dependency on $\cos^3\beta$.

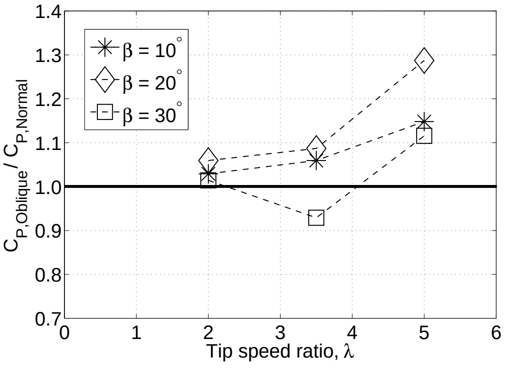
Figure 5 shows the power coefficient produced by the straight-bladed vertical-axis wind turbine, as predicted by the VTM for the same three different height-to-radius ratios at the three different tip speed ratios as shown in Figure 4, but when the turbine is operated with various angles of oblique flow. The predictions of the simulation are consistent with the simple theory, to the extent that the power produced by the three turbines under most of the simulated operating conditions remains relatively constant or even reduces somewhat as the flow becomes increasingly oblique. At the lowest tip speed ratio, the performance of all the turbines is predicted to be marginally enhanced in oblique flow, but this is to be expected from the simple theory given that the operating points of the turbines under the simulated conditions all lie in the regime where the gradient of the $C_P - \lambda$ curve is positive (see Figure 4). Interestingly, though, Figure 5 shows a marked anomaly in the predicted performance of the turbine with the smallest height-to-radius ratio when compared to the predictions of the simple theory. Indeed, this turbine is predicted to develop a significantly higher power coefficient in oblique flow than in normal flow – at least when operated at high tip-speed ratio. These results are consistent with the experimental study documented in Refs. [1] and [2]. When Figure 5 is taken in its entirety, however, it shows that the increase in the power output of the turbine in oblique flow that was observed in the experiments should by no means be taken as evidence of a general characteristic of vertical-axis wind turbines.



(a) $H/R = 3.0$



(b) $H/R = 2.1$



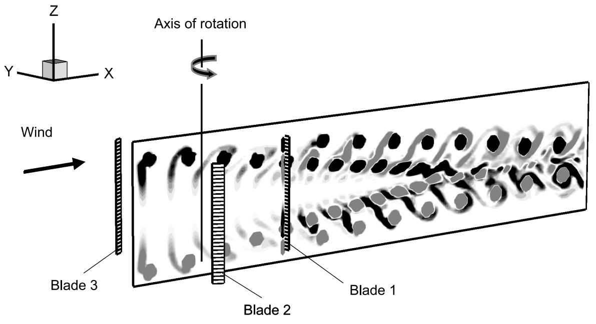
(c) $H/R = 1.2$

Figure 5: *Influence of turbine height-to-radius ratio on the VTM-predicted power coefficient of a straight-bladed vertical-axis wind turbine at three different oblique flow angles.*

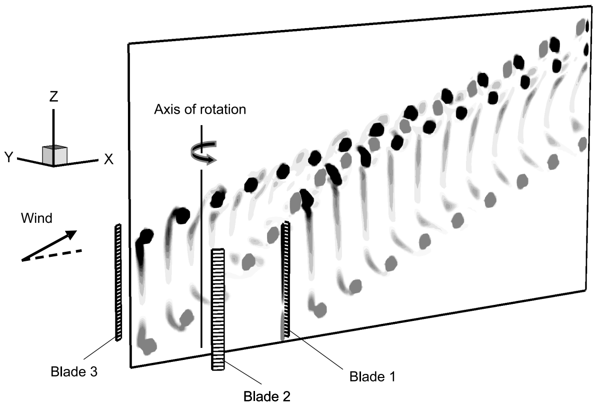
The reason why vertical-axis wind turbines with certain characteristics might produce higher power coefficients in oblique flow than in normal flow is revealed in Figure 6. This figure shows the VTM-predicted vorticity distribution on a plane that is immersed within the wake that is produced by the turbine with the smallest height-to-radius ratio that was simulated. This plane is aligned with the wind direction and contains the axis of rotation of the turbine. The vorticity distribution is depicted at the instant when blade 1 is located at 270° azimuth. The flow field is represented using contours of the component of vorticity perpendicular to the plane, thereby emphasising the vorticity that is trailed by the blades. The dark rendering corresponds to vorticity with a clockwise sense of rotation, and the light rendering to vorticity with a counter-clockwise sense of rotation.

As the turbine rotates, its blades encounter the wake of the turbine as they traverse the downwind part of their azimuthal cycle (i.e. between $\psi = 180°$ and $\psi = 360°$). In normal flow, the entire length of each blade of the rotor is immersed in the wake of the turbine during the downwind portion of its cycle – as shown in Figure 6(a). In oblique flow, however, the convection of the wake is skewed, as shown in Figure 6(b), thereby allowing a portion of each blade to operate, over its entire azimuth, in a flow regime in which the influence of the wake is reduced significantly compared to the situation in normal flow. The resultant effect on the tangential force, and, consequently, on the power that is produced by the turbine, is illustrated in Figure 7. In normal flow, the interaction between the wake and the blades acts effectively to suppress the loading on each blade during the downwind portion of its cycle – see Figure 7(b) – and hence the tangential force that is produced during the upwind segment of the blade cycle is the major contributor to the power that is produced by the turbine. In oblique flow, however – see Figure 7(d) – the contribution to the tangential force from the loading that is produced on the undisturbed portion of the blade during the downwind portion of its cycle contributes to an overall increase in the power that is produced by the turbine. This is despite the fact that the peak loading on the blades is reduced somewhat by the sweep effect alluded to earlier, in other words as a result of the reduction of the component of the wind vector that is perpendicular to the blade.

A simple geometric argument suggests then that the larger the height-to-radius ratio of the turbine, the smaller the proportion of the downwind blade that is exposed to relatively undisturbed flow by the effects of wake skew – and hence the smaller the proportion of the blade that develops significant tangential force during the downwind portion of its trajectory. This observation suggests thus that turbines with larger height-to-radius ratio should be less susceptible to the effects of wake skew than those with smaller. When the wake skew effect is considered together with the basic effects of oblique flow that are captured by the simple sweep-based theory, it can then be understood why only those turbines with smaller height relative to their radius (in other words those that are more susceptible to the effects of wake sweep) should exhibit an increase in power output as the oncoming wind becomes increasingly oblique.



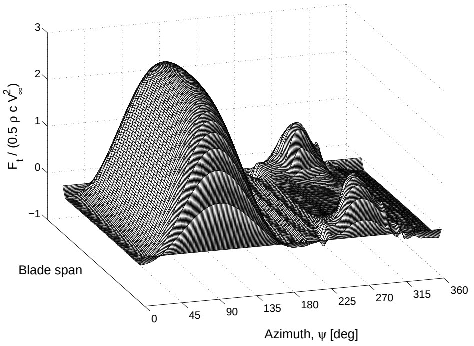
(a) *Normal flow* ($\beta = 0°$)



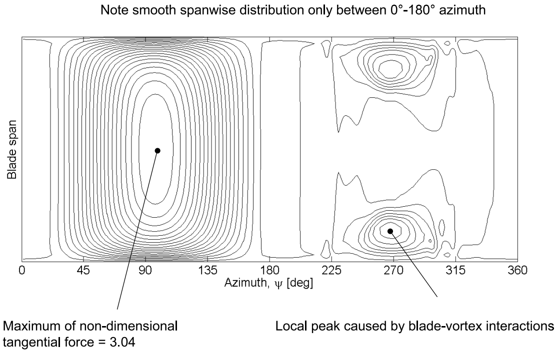
(b) *Oblique flow at* $\beta = 20°$

Figure 6: *Computed vorticity field surrounding the straight-bladed vertical-axis wind turbine with* $H/R = 1.2$ *at* $\lambda = 3.5$*, represented using contours of the vorticity component that is perpendicular to a vertical plane containing the axis of rotation of the turbine. Blade 1 is located at* $270°$ *azimuth.*

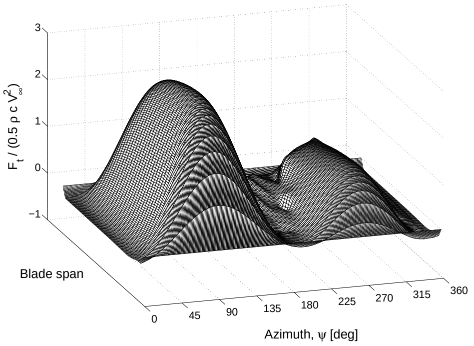
The results presented in Figure 5 suggest also that the effect of wake skew on the power produced by the rotor becomes increasingly important the higher the tip speed ratio of the turbine. This can be explained in terms of the detailed mechanics of the vorticity distribution in the wake of the turbine. A previous study by the present authors [9] has shown the subsequent mutual interaction between the vortex filaments, once created behind the blades, to result in a significant coalescence of vorticity in the immediate vicinity of the blades during the downwind part of their cycle. The effect of this coalescence appears to be to enhance the influence of the wake in ameliorating the load produced by the blades during their passage downwind of the rotational axis of the turbine. At higher tip speed ratios, the vortical structures in the turbine wake convect more slowly downstream relative to the motion of the blades than at low tip speed ratios and the coalescence of the vorticity is more pronounced. Conversely, at lower tip speed ratios the wake is swept away from the rotational trajectory of the blades before signifi-



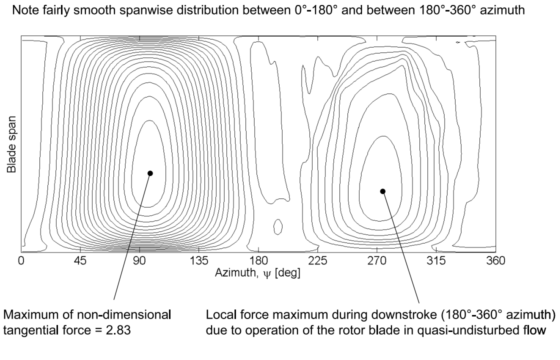
(a) *Normal flow* ($\beta = 0°$)



(b) *Normal flow* ($\beta = 0°$)



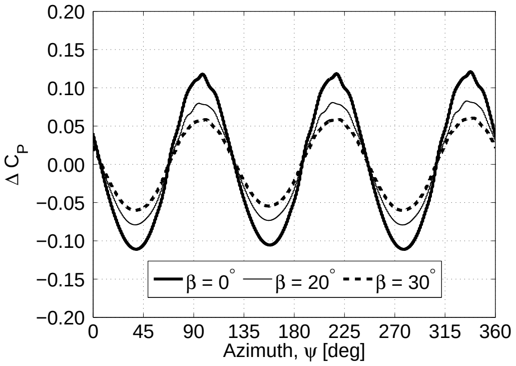
(c) *Oblique flow* ($\beta = 20°$)



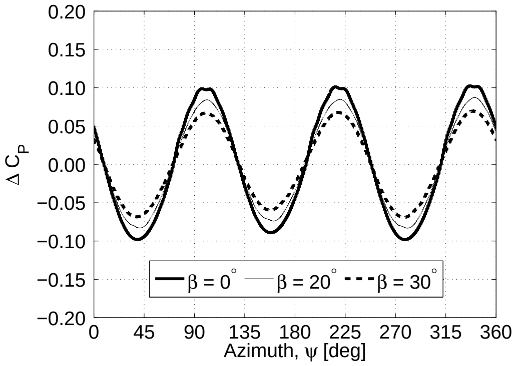
(d) *Oblique flow* ($\beta = 20°$)

Figure 7: *VTM-predicted variation with azimuth of the non-dimensional tangential force along the blade span of the straight-bladed vertical-axis wind turbine with* $H/R = 1.2$ *when operated at* $\lambda = 3.5$ *in normal flow* ($\beta = 0°$) *and in oblique flow with* $\beta = 20°$.

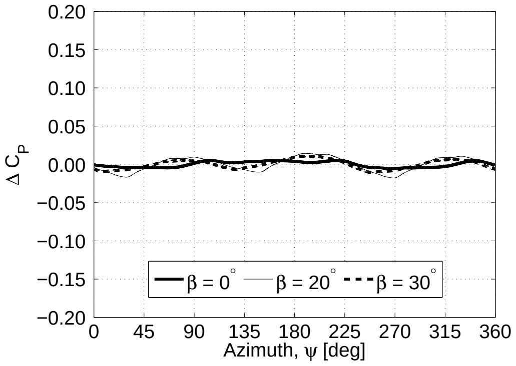
cant coalescence can take place, and consequently the influence of the wake on the loads produced on the blades is enhanced to a lesser extent by this mechanism. Note too that the mutual interaction between the vortex filaments seems to be responsible for a nonlinearity in the trajectory of the wake that precludes overt use of a simple geometric model to account for the effects of wake skew on the performance of the turbine. This nonlinearity can be seen quite clearly in the trajectory of the vortices originating from the bottom of the rotor that are shown in Figure 6(b).



(a) *Straight − bladed*



(b) *Curved − bladed*



(c) *Helically twisted*

Figure 8: *Influence of rotor geometry on the unsteady component of the VTM-predicted power coefficient of three different turbine configurations with* $H/R = 3.0$ *in normal flow and in oblique flow at* $\lambda = 3.5$.

5. Rotor Geometry

In normal flow, the power coefficients that are produced by both a straight-bladed and a curved-bladed vertical-axis turbine as well as the forces that act on the rotor vary cyclically within one revolution of the turbine, as shown by Scheurich *et al.* [9]. These cyclic loads cause vibrations that can generate noise and that can also lead to material fatigue of the turbine structure. The variation with azimuth of the power coefficient is reduced slightly in oblique flow conditions, as shown in Figures 8(a) and 8(b). This is because the larger second local peak in tangential force that is generated between 180° and 360° azimuth in oblique flow, as shown in Figure 7, results in a greater symmetry in the forces between the blades on the advancing and on the retreating sides of the rotor. The power coefficient that is developed by the helically twisted configuration is relatively steady over the entire azimuth, in comparison, irrespective of whether the turbine is operated in normal or oblique flow, as depicted in Figure 8(c).

6. Conclusions

A vertical-axis wind turbine can generate a higher power coefficient in oblique flow, compared to that developed in normal flow, if its height-to-radius ratio is sufficiently small. This is because oblique flow skews the convection of the wake, thereby allowing a significant portion of the blade to operate, over its entire azimuth, in a flow region in which the influence of the wake is significantly reduced compared to the situation in normal flow. This effect becomes more dominant at higher tip speed ratios since oblique flow causes the vortical structures in the turbine wake, which otherwise convect relatively slowly downstream, to be swept away more efficiently from the rotational trajectory of the blades than at lower tip speed ratios. The variation of the power coefficient within one revolution of the rotor that is observed both for straight- and for curved-bladed vertical-axis wind turbines in normal flow is shown to be reduced, to some extent, in oblique flow. The power coefficient that is produced by a vertical-axis wind turbine with helically twisted blades is fairly steady over the entire azimuth, by comparison, irrespective of whether the turbine is operated in normal or in oblique flow. As such, the results presented here show that, in contrast to inferences drawn from limited previous experimental data, the behaviour of a vertical-axis wind turbine in oblique flow conditions is influenced by its tip speed ratio as well as its geometry - more specifically the configuration of its blades and its height-to-radius ratio.

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