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## Violation of Leggett inequalities in orbital angular momentum subspaces

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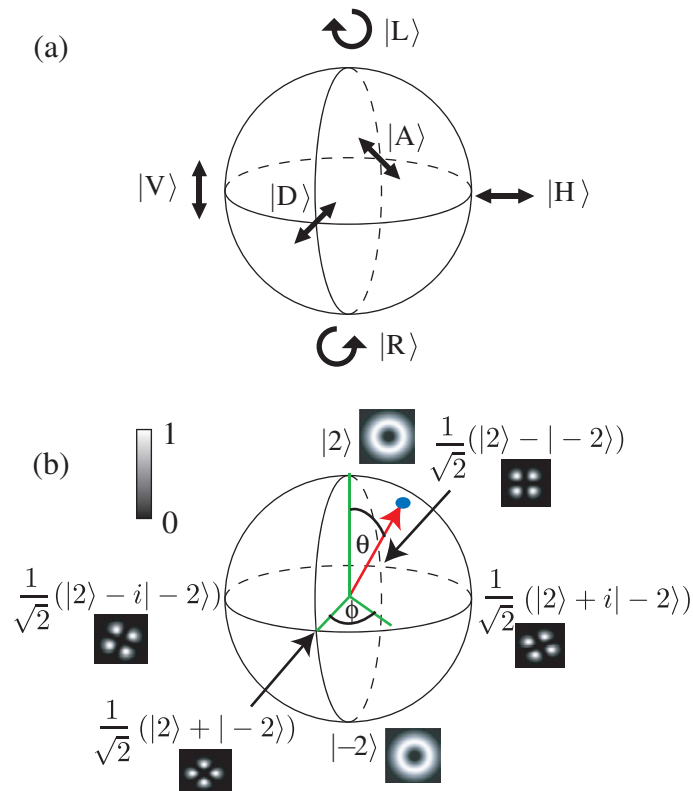
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**Abstract.** We report an experimental test of Leggett's non-local hidden variable theory in an orbital angular momentum (OAM) state space of light. We show that the correlations we observe are in conflict with Leggett's model, thus excluding a particular class of non-local hidden variable theories for the first time in a non-polarization state space. It is known that the violation of the Leggett inequality becomes stronger as more detection settings are used. The required measurements become feasible in an OAM subspace, and we demonstrate this by testing the inequality using three and four settings. We observe excellent agreement with quantum predictions and a violation of five and six standard deviations, respectively, compared to Leggett's non-local hidden variable theory.

Measurements on two spatially separated systems that have interacted in the past, such as two photons coming from the same source, manifest peculiar correlations clamouring for explanation [1]. Quantum mechanics offers one such explanation, but correlations can of course also be established in classical systems, for instance if two parties have a prior agreement on the results of specific measurements. Various families of realistic hidden-variable theories have been formulated, which maintain that the results of measurements are determined by the properties carried by each photon embodied in so-called hidden variables (i.e. the prior agreement). Realistic *local* hidden variable theories have been thoroughly tested and ruled out, most notably by violations of the Bell inequality [1]–[3]. More recently, Leggett introduced an inequality that

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**Figure 1.** Analogy between polarization and OAM. The Poincaré sphere that characterizes polarization states (a) is analogous to the OAM Bloch sphere (b). Any point (blue dot) on this sphere is a state given by equation (1). The amplitude and phase of these states can be encoded using an SLM (insets).

tests a family of realistic theories involving *non-local* hidden variables [4, 5], trying to explain non-local correlations while maintaining sharply defined individual properties of the particles. Experimental violations of the Leggett inequality have recently been demonstrated using polarization states of photons, thereby ruling out this class of hidden variable theories [6]–[10]. Here we show for the first time a violation of the Leggett inequality for parameters other than polarization, namely using the orbital angular momentum (OAM) states of photons. The simplicity of state manipulation in OAM space opens up the possibility of exploring more robust inequalities with a higher number of detector settings.

In addition to the spin angular momentum associated with a photon's polarization, light may also carry OAM, and photon pairs generated by spontaneous parametric down-conversion (SPDC) are known to be entangled in polarization as well as in their spatial profile, and in particular in their OAM [11, 12]. OAM eigenstates have helical phase fronts described by  $\exp(i\ell\phi)$ , where  $\phi$  is the azimuthal angle in the plane perpendicular to the beam axis,  $\ell$  is an integer and  $\ell\hbar$  is the OAM per photon [13]. While there are infinitely many OAM states, here we concentrate on a two-dimensional (2D) subspace encompassing all superpositions of  $|\pm\ell\rangle$ . For this subspace, we can devise a Bloch sphere similar to the Poincaré sphere for polarization, wherein the states at the poles are  $|\pm\ell\rangle$  (here we choose  $\ell = 2$ ), and all states on the Bloch sphere are complex superpositions of  $|\pm\ell\rangle$ , see figures 1(a)–(b) [14]. The states along the

equator are then the analogue of linear polarization states which, because of their intensity and phase structures, we call sector states. We have recently used this analogy between the polarization and OAM states to show that photon pairs emitted in SPDC violate a Bell inequality in 2D OAM subspaces [15, 16] and show entanglement of formation [17]. Similarly, we proceed to show that correlations of OAM states, just like polarization states, can be used to show a violation of the Leggett inequality.

We measure OAM states holographically by programming spatial light modulators (SLMs), which allows us to specify any OAM state without physically aligning any optical components. Measuring the violation of a Bell inequality [15] required measurement of the states on orthogonal great circles of the OAM sphere (e.g. along the equator, 0th and 180th meridians). In the case of the Leggett inequality, the use of SLMs is even more beneficial because it requires measurements of the states along non-orthogonal great circles (i.e. the measurement planes are not orthogonal). The SLMs enable us to measure quantum correlations between arbitrary states, positioned at any point of the Bloch sphere, conveniently and with high accuracy. On our Bloch sphere, a vector  $\mathbf{a}$  (red vector in figure 1(b)) corresponds to the state

$$|\mathbf{a}\rangle = \cos\left(\frac{\theta}{2}\right) |2\rangle + e^{i\varphi} \sin\left(\frac{\theta}{2}\right) |-2\rangle, \quad (1)$$

where the angles  $\theta$  and  $\varphi$  are the usual inclination and azimuth angles, respectively, defined such that  $0 \leq \theta < \pi$  and  $0 \leq \varphi < 2\pi$ .

Mathematically, a correlation can be defined by a conditional probability distribution  $P(\alpha, \beta|\mathbf{a}, \mathbf{b})$ , where  $\alpha$  and  $\beta$  are the outcomes of measurements  $\mathbf{a}$  and  $\mathbf{b}$  made on systems A and B, respectively. If the outcomes are predetermined by hidden variables  $\lambda$  (thus imposing realism) and if, in addition, these hidden variables are local (i.e. spatially separated measurements are independent of each other), the conditional probability becomes

$$P_\lambda(\alpha, \beta|\mathbf{a}, \mathbf{b}) = P_\lambda(\alpha|\mathbf{a})P_\lambda(\beta|\mathbf{b}). \quad (2)$$

Imposing locality on the hidden variables sets a limit on the correlations that can be achieved in experiments, expressed in Bell's famous inequality [1]. To date, the results of various experiments have been shown to violate the Bell inequality and its derivatives, such as the Clauser–Horne–Shimony–Holt inequality, leaving one to conclude that realism and locality cannot hold simultaneously [3, 18]. Whether one should abandon the notion of realism or locality is a question that has attracted much speculation [19, 20].

Leggett considered a different hidden variable model in which the condition of locality is relaxed. He analysed a family of non-local hidden variable theories and derived an inequality that would be satisfied by systems that abide by his model. In accordance with Leggett's model, the detection of photons emitted from SPDC has the following properties. (i) Each pair of photons has a characteristic set of hidden variables  $\lambda$ . (ii) The ensemble of photon pairs is determined by a statistical distribution of values of  $\lambda$ ,  $\rho(\lambda)$ , which depends only on the source, hence allowing one to write

$$P(\alpha, \beta|\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) P_\lambda(\alpha, \beta|\mathbf{a}, \mathbf{b}). \quad (3)$$

(iii) The outcome of a measurement on each photon,  $\alpha$ , may depend on  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\lambda$  and  $\beta$  (i.e. equation (2) is not necessarily satisfied, doing away with locality). (iv) Each photon of the pair, associated with the parameter  $\lambda$ , individually behaves as if it has well-defined properties (or OAM, in our case), and a measurement on it (conditioned on  $\lambda$ ) will show sinusoidal intensity

variations (known as Malus's law for polarization) [4]. This model is attractive because the properties of the individual photons are sharply defined, allowing one to make deterministic predictions locally on the measurement results of each photon. Moreover, the hidden variables are non-local and may depend on parameters outside the neighbourhood of the measurement apparatus. These properties lead to an incompatibility theorem, called the Leggett inequality. This inequality has been refined in recent work to be experimentally testable with a finite number of measurements and obviate the need for rotational invariance [7, 8]. Following these recent refinements, for  $N$  possible measurement settings in system A (figure 1(c)), the correlations of a photon pair that obeys Leggett's non-local hidden variable model are restricted by the inequality [9]

$$\frac{1}{N} \sum_{i=1}^N |C(\mathbf{a}_i, \mathbf{b}_i) + C(\mathbf{a}_i, \mathbf{b}'_i)| \equiv L_N(\chi) \leq 2 - 2\eta_N \left| \sin \frac{\chi}{2} \right| \quad (4)$$

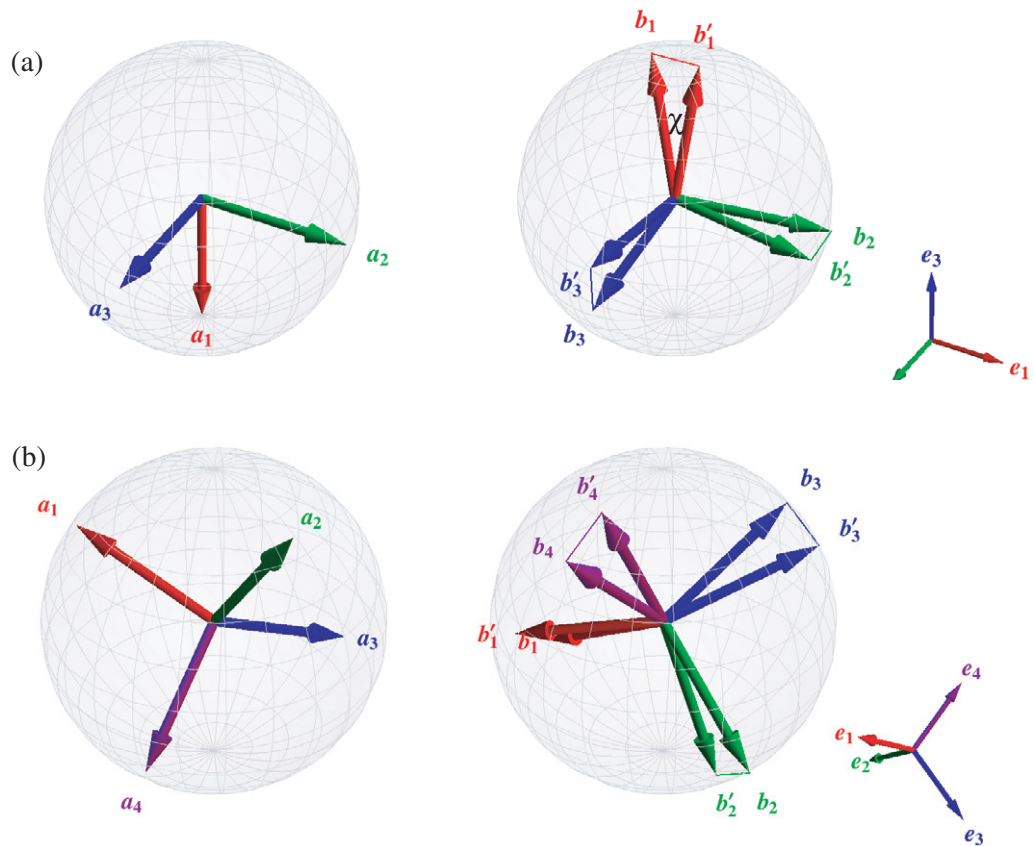
if

$$\frac{1}{N} \sum_{i=1}^N |\vec{v} \cdot \vec{e}_i| \geq \eta_N \quad (5)$$

holds for any vector  $\vec{v}$ . Here,  $C(\mathbf{a}, \mathbf{b})$  and  $C(\mathbf{a}, \mathbf{b}')$  are the correlation coefficients for measurement settings  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{a}$ ,  $\mathbf{b}'$  respectively.  $\mathbf{b}_i$  and  $\mathbf{b}'_i$  are separated by angle  $\chi$  and we define  $\mathbf{e}_i = \mathbf{b}_i - \mathbf{b}'_i$ . We choose  $\mathbf{a}_i$ ,  $\mathbf{b}_i$  and  $\mathbf{b}'_i$  such that we get maximum coincidence between  $\mathbf{a}_i$  and  $\mathbf{b}_i + \mathbf{b}'_i$ . This means that  $\mathbf{b}_i + \mathbf{b}'_i$  has the same azimuth angle as  $\mathbf{a}_i$  but reflected about the equator. The constant  $\eta_N$  depends on the geometry of the  $\mathbf{e}_i$ s, defined as in figure 2(a). Measurements with  $N = 3$  have been carried out previously in the polarization state space [8]–[10], with  $\eta_3 = \frac{1}{3}$  (calculated by minimizing the left-hand side of inequality (5)), and  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$  pointing to the coordinate axes (figure 2(a)). Increasing  $N$  leads to more robust inequalities, albeit with more sophisticated alignment requirements if working with polarization [10]. Increasing  $N$  becomes more feasible if the measurements are carried out holographically with SLMs because of the ability to specify any arbitrary state. We demonstrate this by showing a violation for  $N = 4$  where the largest violation is found for  $\mathbf{e}_i$ s being the vertices of a regular tetrahedron (see figure 2(b)). In this case  $\eta_4 = \frac{1}{\sqrt{6}}$ , and we choose  $\mathbf{a}_1$ ,  $\mathbf{a}_2$ ,  $\mathbf{a}_3$  and  $\mathbf{a}_4$  to be vertices of a tetrahedron, as shown in figure 2(b)), for maximal violation of the inequality. For both cases, quantum mechanics predicts that  $L_N(\chi) = 2 \cos \left| \frac{\chi}{2} \right|$ , violating the inequality (4) over a large range of angles,  $\chi$ .

Our experimental setup is shown in figure 3. We use a quasi-cw, mode-locked, 355 nm UV laser to pump a 3 mm long type I BBO crystal. The crystal is oriented in a collinear geometry, with the down-converted plane-polarized 710 nm signal and idler photons incident on the same beamsplitter. The exit face of the crystal is imaged to separate SLMs that display the holograms that specify the states we intend to measure. These SLMs are re-imaged to the input facets of single-mode fibres, which are themselves coupled to single-photon detectors, the outputs of which are fed into our coincidence counting circuit. The gate time for this coincidence measurement is 10 ns and we obtain typical count rates of  $200 \text{ s}^{-1}$ . The photon in the signal arm then represents system A and that in the idler arm system B.

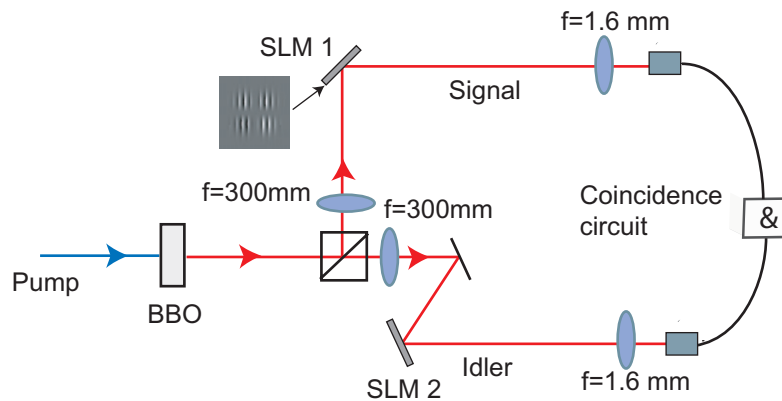
The results for  $N = 3$  and  $N = 4$  are shown in figure 4. Each data point in these plots corresponds to three settings,  $\mathbf{a}_i$ , for measurements on system A and  $\mathbf{b}_i$  and  $\mathbf{b}'_i$  separated by an angle  $\chi$  in system B, as indicated in figure 2, thus requiring  $2N^2$  per angle.



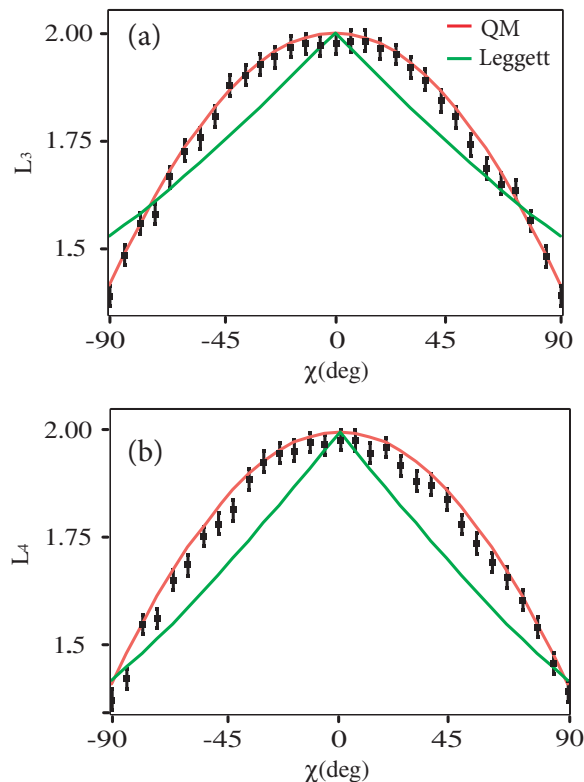
**Figure 2.** Measurements of the Leggett inequalities. (a) For  $N = 3$ , SLM A is set to measure the three mutually orthogonal states  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$ . SLM B is then set to measure coplanar states  $\mathbf{b}_i$ ,  $\mathbf{b}'_i$  separated by an angle  $\chi$  where  $\mathbf{b}_i - \mathbf{b}'_i$  is parallel to  $\mathbf{e}_i$ , which are mutually orthogonal. (b) For  $N = 4$ ,  $\mathbf{a}_1$ ,  $\mathbf{a}_2$ ,  $\mathbf{a}_3$  and  $\mathbf{a}_4$  are the vertices of a tetrahedron. The vectors  $\mathbf{e}_1$ ,  $\mathbf{e}_2$ ,  $\mathbf{e}_3$  and  $\mathbf{e}_4$  are then vertices of a regular tetrahedron. Violating the inequality requires measurements in four different non-orthogonal planes. In our case, we chose the planes defined by  $\mathbf{e}_1$  and  $\mathbf{e}_2$ ,  $\mathbf{e}_2$  and  $\mathbf{e}_3$ ,  $\mathbf{e}_3$  and  $\mathbf{e}_4$  and  $\mathbf{e}_4$  and  $\mathbf{e}_1$ .

We measure the coincidence as we vary the angle  $\chi$  and compare it to the maximal value of  $L_N$  allowed by Leggett's mode as defined in equation (4). Our results, depicted in figure 4, show that the experimental data follow the prediction of quantum mechanics closely, as expected, and that the inequality is violated over a large range of angles. For  $N = 3$ , we observe the maximum violation at  $\chi = -42^\circ$ , where  $L_3 = 1.8787 \pm 0.0241$ . For  $N = 4$ , the maximum violation occurs at  $\chi = -30^\circ$ , where  $L_4 = 1.9323 \pm 0.0239$ . These results imply that it is not possible to keep definite, individual OAM states of the photons while maintaining the observed OAM correlations.

In conclusion, we report the first experimental violation of the Leggett inequality outside a polarization state space. Our measurements in the OAM state space violate the Leggett inequality by 5 and 6 standard deviations, respectively, for  $N = 3$  and 4. The conclusions reached by performing a test of the Leggett inequality, whether in an OAM subspace or in



**Figure 3.** Experimental setup. We measure photons generated from SPDC by encoding holograms that define arbitrary OAM states to separate SLMs. The hologram (inset) imprints the phase and intensity of the first diffracted order [21].



**Figure 4.** Violation of the Leggett inequality. Experimentally measured correlations (black dots) for  $N = 3$  (a) and  $N = 4$  (b) violate the bound arising from Leggett's model (green line) and follow closely the predictions of quantum mechanics (red line). Maximum violation (boxed data points) occurs at  $-42^\circ$  and  $-30^\circ$ , respectively, for  $N = 3$  and 4, respectively.

polarization space, are the same, supporting quantum theory against a specific class of non-local hidden variable theory. However, the OAM space offers a much more accessible state space owing to the programmability of SLMs and less stringent alignment requirements. This is



exemplified by our measurements for  $N = 4$ . Experiments with higher  $N$  are practically possible and may prove to be more robust, as speculated on in [9].

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