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A note on "New travelling wave solutions to the Ostrovsky equation"

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Abstract

In a recent paper by Yaşar [E. Yaşar, New travelling wave solutions to the Ostrovsky equation, Appl. Math. Comput. 216 (2010), 3191–3194], 'new' travelling-wave solutions to the transformed reduced Ostrovsky equation are presented. In this note it is shown that some of these solutions are disguised versions of known solutions.

Key words: Travelling-wave solutions; reduced Ostrovsky equation; tanh-function method.
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Over the past two decades or so several methods for finding travelling-wave solutions to nonlinear evolution equations have been proposed, developed and extended. The solutions to dozens of equations have been found by one or other of these methods. References [1–5] and some of the references therein mention some of this activity. Unfortunately, one unwanted consequence of this work is the large number of papers in which authors claim to have found 'new' solutions which, in truth, are just disguised versions of previously known solutions. Recently, in a series of enlightening papers [6–12], Kudryashov and co-workers have warned researchers and referees of the danger of not recognizing that apparently different solutions may simply be different forms of the same solution. In these papers, numerous examples are given to illustrate this phenomenon. Some other recent examples are given in [13–18].

In [14] we discussed 'disguised' solutions of an equation that we have dubbed the 'transformed reduced Ostrovsky equation', namely

$$uu_{xxt} - u_x u_{xt} + u^2 u_t = 0. \tag{1}$$

In this note we reveal yet more such solutions, this time as presented by Yaşar [1].

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In passing we note that Eq. (1) is a transformed form of the Vakhnenko equation [19] which, in turn, is a transformed version of the reduced Ostrovsky equation [20]. Misleadingly, in [1–4], Eq. (1) is referred to as the Ostrovsky equation.

First we summarize a derivation of some distinct solutions to Eq. (1). For this purpose we have used the extended tanh-function method for finding travelling-wave solutions which we summarized in [16]. A tedious but routine application of the method to (1) gives

$$u = 6k^2 - 6Y^2 \quad \text{or} \quad u = 2k^2 - 6Y^2, \tag{2}$$

where Y is one of five possible functions of $\eta := x - ct - x_0$, and k, c and x_0 are constants. If $k^2 > 0$ (so that k is real), then $Y := k\tanh(k\eta)$ or $Y := k\coth(k\eta)$; if $k^2 < 0$ (so that k is imaginary), let $k = iK$ (so that K is real) and then $Y := -K\tan(K\eta)$ or $Y := K\cot(K\eta)$; if $k = 0$, then $Y := 1/\eta$. These results lead to the following solutions to (1):

$$u_{11} = 6k^2 - 6k^2\tanh^2(k\eta) = 6k^2\,\mathrm{sech}^2(k\eta), \tag{3}$$
$$u_{12} = 6k^2 - 6k^2\coth^2(k\eta) = -6k^2\,\mathrm{cosech}^2(k\eta), \tag{4}$$
$$u_{13} = -6K^2 - 6K^2\tan^2(K\eta) = -6K^2\sec^2(K\eta), \tag{5}$$
$$u_{14} = -6K^2 - 6K^2\cot^2(k\eta) = -6K^2\,\mathrm{cosec}^2(K\eta), \tag{6}$$
$$u_{21} = 2k^2 - 6k^2\tanh^2(k\eta) = -4k^2 + 6k^2\,\mathrm{sech}^2(k\eta), \tag{7}$$
$$u_{22} = 2k^2 - 6k^2\coth^2(k\eta) = -4k^2 - 6k^2\,\mathrm{cosech}^2(k\eta), \tag{8}$$
$$u_{23} = -2K^2 - 6K^2\tan^2(K\eta) = 4K^2 - 6K^2\sec^2(K\eta), \tag{9}$$
$$u_{24} = -2K^2 - 6K^2\cot^2(k\eta) = 4K^2 - 6K^2\,\mathrm{cosec}^2(K\eta), \tag{10}$$
$$u_3 = -6/\eta^2. \tag{11}$$

We note that all these solutions may also be derived via the basic tanh-function method. The basic tanh-function method (also summarized in [16]) delivers the solutions u_{11} and u_{21}. These may be obtained by hand or with minimal effort by use of the automated tanh-function method [21] which uses `ATFM`, a Mathematica package designed to take the drudgery out of applying the tanh-function method by hand. We may obtain u_{12} and u_{22} by replacing kx_0 by $kx_0 + i\pi/2$ in u_{11} and u_{21}, respectively, and then using (A.1). We may obtain u_{13} and u_{23} by replacing k by iK in u_{11} and u_{21}, respectively, and then using (A.2). We may obtain u_{14} and u_{24} by replacing k by iK in u_{12} and u_{22}, respectively, and then using (A.3). We may obtain u_3 from u_{12} or u_{22} by taking the limit $k \to 0$.

In [19], we derived u_{11} via Hirota's method. This solution played an important role in the investigation of N loop-soliton solutions of the Vakhnenko equation [19,22]. In [2], the solutions u_{11} and u_{21} with $x_0 = 0$ were derived by both the basic tanh-function method and the equivalent exponential rational function method. In [14], we derived u_{11}, u_{21}, u_{12} and u_{22} via the basic tanh-function method. In [3], the authors derived twenty eight solutions by using the Sirendaoreji auxiliary equation method. In [14], we pointed out that all these solutions are just disguised versions of one or other of u_{11} or u_{12} with appropriate choices of x_0. In [4], two solutions were derived via the Exp-function method. In [14], we showed that these are just disguised versions of u_{11} and u_{21}, respectively, with an appropriate choice of x_0. In [5], the (G'/G)-expansion method is applied to the modified generalized Vakhnenko equation. In the notation of [5], with $p = q = 1$ and $\beta = 0$, the solutions for u expressed as a function of X and T are solutions to the transformed reduced Ostrovsky equation. By using the results in [16], we may show that, in [5], (26) corresponds to u_{11} or u_{12}, (27) corresponds to u_{21} or u_{22}, (32) corresponds to u_{23} or u_{24}, (33) corresponds to u_{13} or u_{14}, and (35) corresponds to u_3.

In [1], the author derives thirteen solutions by using an 'improved tanh-function method'. The ones given by (19), (20), (23), (24) and (25) in [1] are respectively u_{11}, u_{12}, u_{13}, u_{14} (all with $x_0 = 0$) and u_3. The remaining eight solutions are claimed to be 'new' and 'important'; they are as follows:

$$u(x,t) = \frac{3}{2}\alpha^2\left(1 - [\tanh(\alpha\zeta) \pm i\,\mathrm{sech}(\alpha\zeta)]^2\right), \tag{12}$$

$$u(x,t) = \frac{3}{2}\alpha^2\left(1 - [\coth(\alpha\zeta) \mp \mathrm{cosech}(\alpha\zeta)]^2\right), \tag{13}$$

$$u(x,t) = -\frac{3}{2}\alpha^2\left(1 + [\sec(\alpha\zeta) \pm \tan(\alpha\zeta)]^2\right), \tag{14}$$

$$u(x,t) = -\frac{3}{2}\alpha^2\left(1 + [\mathrm{cosec}(\alpha\zeta) \mp \cot(\alpha\zeta)]^2\right), \tag{15}$$

where α is an arbitrary constant and $\zeta := x - ct$. However, on using (A.4), we find that the two solutions in (12) are just u_{11} and u_{12} with $k = \alpha/2$ and $kx_0 = -i\pi/4$. Similarly, with (A.5), the two solutions in (13) are just u_{11} and u_{12} with $k = \alpha/2$ and $x_0 = 0$; with (A.6), the two solutions in (14) are just u_{13} and u_{14} with $k = \alpha/2$ and $kx_0 = -\pi/4$; with (A.7), the two solutions in (15) are just u_{13} and u_{14} with $k = \alpha/2$ and $x_0 = 0$. Note that the solutions u_{21}, u_{22}, u_{23} and u_{24} are not given in [1].

Appendix: Identities

$$\tanh(\theta - i\pi/2) = \coth(\theta), \tag{A.1}$$

$$\tanh(i\theta) = i\tan(\theta), \tag{A.2}$$

$$\coth(i\theta) = -i\cot(\theta), \tag{A.3}$$

$$\tanh(\theta) + i\,\mathrm{sech}(\theta) = \tanh\left[\frac{1}{2}\left(\theta + \frac{i\pi}{2}\right)\right], \quad \tanh(\theta) - i\,\mathrm{sech}(\theta) = \coth\left[\frac{1}{2}\left(\theta + \frac{i\pi}{2}\right)\right], \tag{A.4}$$

$$\coth(\theta) - \mathrm{cosech}(\theta) = \tanh(\theta/2), \quad \coth(\theta) + \mathrm{cosech}(\theta) = \coth(\theta/2), \tag{A.5}$$

$$\sec(\theta) + \tan(\theta) = \tan\left[\frac{1}{2}\left(\theta + \frac{\pi}{2}\right)\right], \quad \sec(\theta) - \tan(\theta) = \cot\left[\frac{1}{2}\left(\theta + \frac{\pi}{2}\right)\right], \tag{A.6}$$

$$\mathrm{cosec}(\theta) - \cot(\theta) = \tan(\theta/2), \quad \mathrm{cosec}(\theta) + \cot(\theta) = \cot(\theta/2). \tag{A.7}$$

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