# Strathprints Institutional Repository

Kelly, Mary E. and Phillips, Catriona and Scheurich, Frank and Kim, Hyo Wan and Fletcher, Timothy M. and Brown, R.E. (2009) *Applying rotorcraft modelling technology to renewable energy research.* In: 2nd AHS/KSASS International Forum on Rotorcraft Multidiscipllinary Technology, 2009-10-19 - 2009-10-20, Seoul, Korea.

Kelly, M.E. and Phillips, C. and Scheurich, F. and Kim, H.W. and Fletcher, T.M. and Brown, R.E. (2009) Applying rotorcraft modelling technology to renewable energy research. In: 2nd AHS/KSASS International Forum on Rotorcraft Multidiscipllinary Technology , 19-20 October 2009, Seoul, Korea.

http://strathprints.strath.ac.uk/27490/

# Applying Rotorcraft Modelling Technology to Renewable Energy Research

**Mary E. Kelly, Catriona Phillips, Frank Scheurich**[*]
**Hyo Won Kim, Timothy M. Fletcher**[†]
**Richard E. Brown**[‡]

*Rotor Aeromechanics Laboratory*
*Department of Aerospace Engineering*
*University of Glasgow*
*Glasgow, G12 8QQ, Scotland*

## Abstract

The perceived need to reduce mankind's impact on the global climate motivates towards a future society in which a significant proportion of its energy needs will be extracted from the winds and the tides of the planet. This paper shows several examples of the application of Brown's Vorticity Transport Model, originally developed to perform simulations of helicopter aeromechanics and wake dynamics, to the analysis of the performance of renewable energy devices and their possible impact on the environment. Prediction of the loading on wind turbines introduces significant additional challenges to such a model, including the need to account fully for the effects of radial flow on blade stall. The wake-mediated aerodynamic interactions that occur within a wind farm can reduce its power output significantly, but this problem is very similar to that where the aerodynamic unsteadiness of the coupled wake of the main and tail rotors of a helicopter can result in significantly increased pilot workload. The helicopter-related problem of brownout, encountered during operations in desert conditions, has its analogue in the entrainment of sediment into the wakes of tidal turbines. In both cases it may be possible to ameliorate the influence of the rotor on its environment by careful and well-informed design. Finally, calculations of the distortion and dispersal of the exhaust plumes of a helicopter by the wake of its rotor allow insight into how wind turbines might interfere with the dispersal of pollutants from nearby industrial sites. These examples show how cross-disciplinary information transfer between the rotorcraft field and the renewable energy community is helping to develop the technologies that will be required by our future society, as well as helping to understand the environmental issues that might need to be faced as these technologies become more prevalent.

## Notation

| | |
|---|---|
| $C_P$ | rotor power coefficient |
| $C_T$ | rotor thrust coefficient |
| $d$ | particle (effective) diameter |
| $R$ | rotor radius |
| $r$ | blade spanwise coordinate |
| $S$ | vorticity source |
| $S_p$ | source of particulates into the flow |
| $t$ | time |
| $u$ | flow velocity |
| $u_b$ | velocity of blade with respect to air |
| $u_g$ | particulate fallout velocity |
| $V_\infty$ | freestream velocity |
| $\lambda$ | tip-speed ratio, $(\Omega R)/V_\infty$ |
| $\mu$ | advance ratio, $V_\infty/(\Omega R)$ |
| $\mu^\star$ | thrust-normalized advance ratio, $\mu/\sqrt{C_T/2}$ |
| $\nu$ | kinematic viscosity of flow |
| $\nu_p$ | diffusion coefficient for particulates |
| $\rho$ | air density |
| $\rho_p$ | density of suspended particulates |
| $\rho_s$ | material density of particle |
| $\Omega$ | rotor rotational speed |
| $\omega$ | vorticity, $\nabla \times u$ |
| $\omega_b$ | vorticity bound to blade |

## Introduction

Mankind is slowly coming to a consensus that the continued emission of large quantities of carbon into the atmosphere is likely to alter significantly the climate on the earth through the mechanism known as the 'greenhouse effect'. One of the largest contributors to elevated levels of atmospheric carbon is the electricity generation industry. There is

---

[*] Postgraduate Research Students
[†] Postdoctoral Research Assistants
[‡] Mechan Professor of Engineering

Presented at the American Helicopter Society International Forum on Rotorcraft Multidisciplinary Technology, Seoul, Korea, 19–20 October, 2009.

significant interest within this community in methods that may be used to ameliorate its impact on atmospheric carbon levels. One of the most promising ways to do this appears to be through the exploitation of wind and tidal energy resources, using turbines to meet a significant proportion of the Earth's future electricity requirement.

Wind turbines are the fastest growing of the renewable energy technologies in Scotland. Most turbines in the European Union produce electricity at an average of 25% of their rated maximum power as a result of the intermittency of wind resources, but Scotland's particular wind regime enables an average of 40% or higher on the western and northern coasts (Ref. 1).

There are now numerous large on-shore wind farms in Scotland, including Black Law rated at over 96MW, Hadyard Hill, which is the first wind farm in the UK able to generate over 100MW, and Whitelee, a 322MW project under construction that is already the largest onshore wind farm in Europe. It is estimated that 11.5GW of onshore wind potential exists, enough to provide 45TWh of energy per year. More than double this capacity exists on offshore sites, where mean wind speeds are greater than on land. The total offshore potential is estimated at 25GW, which although more expensive to install than land-based systems, could be enough to provide almost half the total electricity used in Scotland (Ref. 2).

Unlike the winds in the atmosphere, the tides in the ocean provide an inherently predictable source of power. The few operational schemes for capturing energy from the flow of tides rely on underwater plant that is based quite closely on wind turbine technology. Tidal turbine technology is still in its infancy, however, with one of the few commercial-scale tidal turbine in the world being the Marine Current Turbines SeaGen 1.2MW device at Strangford Lough in Northern Ireland. The Pentland Firth between Orkney and mainland Scotland has been described as the "Saudi Arabia of tidal power," however, and may be capable of providing up to 10GW of power (Ref. 3). The overall aims of the Scottish Government are ambitious, with current plans to generate at least 50% of domestic energy requirements from renewable sources by the year 2020. Concurrent with these ambitions is a major programme of investment, both in the technologies and in the scientific basis that is required to meet this challenge.

The problems faced by those who would design and deploy tidal and wind turbines are, in some ways, very similar to those experienced by helicopter engineers. In addition, many recent developments in the rotorcraft world, such as the active control of rotors and the use of adaptive composite technology, are slowly finding their way into the turbine field. In this paper, several interesting applications of Brown's Vorticity Transport Model (VTM) to the analysis of wind and tidal turbines will be described. The VTM was originally developed to study rotorcraft aerodynamic and flight dynamic problems, and its exploitation in the renewable energy field shows some interesting examples of cross-disciplinary transfer of technology from the rotorcraft field.

Two types of turbine are most often encountered in practice. In the more common *horizontal-axis* configuration, the axis of the turbine is aligned, nominally, with the oncoming wind or water. The behaviour of the turbine bears some resemblance to that of a helicopter rotor in axial flight, particularly when operating under steady conditions in which case the rotor induces a component of velocity in opposition to the oncoming flow, and hence an expanding wake tube as in the rotor windmill brake state. In a *vertical-axis* turbine, the axis of rotation of the rotor is oriented transversely to the oncoming flow. Given that the orientation of the blades of the turbine is generally parallel to the axis of rotation, the connection of the behaviour of this type of turbine to that of helicopter rotors is somewhat more tenuous than that of a horizontal-axis device, although there are still some distinct similarities.

# Computational Model

The present formulation of the Vorticity Transport Model (VTM), developed by Brown (Ref. 4) and extended by Brown and Line (Ref. 5) couples a lifting-line model for the aerodynamics of the blade to an Eulerian representation of the vorticity in the flow field.

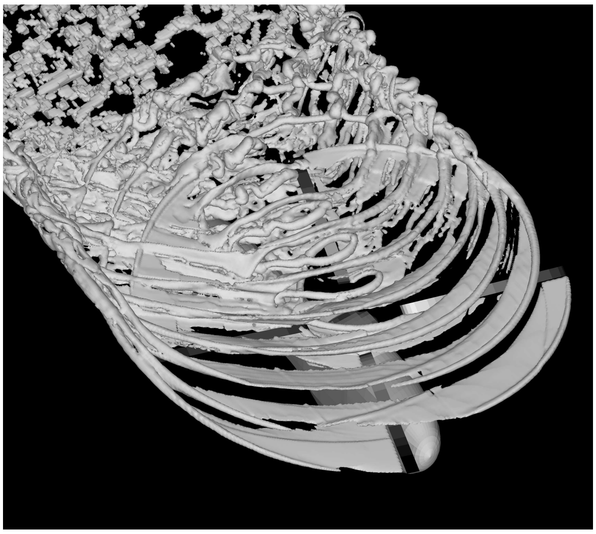


Figure 1: *VTM-predicted wake geometry of the HART-II rotor and drive enclosure. The system is in a descending flight condition at advance ratio $\mu$=0.151.*

The flow field is evolved by solution of the Navier-Stokes equations in vorticity-velocity form on a structured Cartesian grid. Assuming incompressible flow with velocity, $u$, the associated vorticity distribution $\omega = \nabla \times u$ evolves according to the unsteady vorticity transport equation

$$\frac{\partial}{\partial t}\omega + u \cdot \nabla\omega - \omega \cdot \nabla u = S + \nu\nabla^2\omega \qquad (1)$$

where $\nu$ is the kinematic viscosity of the fluid. The local rate of numerical diffusion is controlled very effectively by using a set of highly-compressive flux limiters within the particular implementation of Toro's Weighted Average Flux method (Ref. 6) that is used within the code to convect the solution through time. At each time step, the velocity at the cell faces is obtained from the vorticity distribution using a fast multipole technique to invert the differential form of the Biot-Savart equation

$$\nabla^2 u = -\nabla \times \omega. \qquad (2)$$

A semi-Lagrangian adaptive grid is used to track the evolving vorticity in such a way that cells only exist in regions of the computational domain where the vorticity is non-zero. As the vorticity moves to a new location, new cells are created and any cells that no longer contain vorticity are destroyed. Thus, the grid structure is free to follow the evolution of the wake, eliminating the requirement for explicit numerical boundary conditions at the edge of the computational domain and increasing the computational efficiency of the method. Moreover, a nested grid system allows for fine resolution close to the rotor and then a systematic decrease in resolution with distance from the rotor hub.

An extension of the Weissinger-L formulation of lifting-line theory is implemented on a series of discrete panels along the length of each rotor blade to yield the aerodynamic loading. A bound vortex is attached to the quarter-chord of each panel. The strength of the bound vorticity along the length of the blade is determined by enforcing, simultaneously, a condition of zero through-flow on a set of aerodynamic stations located at the 3/4 chord of each panel. As the computation is progressed through each time step, trailed and shed vorticity from each vortex panel is added to the near wake downstream of the blade as the local vorticity source,

$$S = -\frac{d}{dt}\omega_b + u_b \nabla \cdot \omega_b, \qquad (3)$$

where $\omega_b$ is the bound vorticity. The two-dimensional aerodynamic characteristics of the rotor blade sections are specified in a look-up table as a function of angle of attack and Mach number for a given Reynolds number. These characteristics can be used to precondition the boundary condition that is applied at the control points to allow the lifting line calculation to match closely the sectional aerodynamic characteristics, including stall, of the actual blade. As this approach is still essentially inviscid, the profile drag of the blade is calculated as a separate function of local angle of attack and is then added to the local aerodynamic force that is calculated from the lifting line model.

Fuselages or other solid bodies are represented using an unsteady vortex panel method, as described in Ref. 7. The surface of any body immersed in the flow-field is discretised into a system of panels, such that each panel edge is represented as a vortex filament with constant strength, forming a closed loop of vorticity. The velocity at the centroid of any panel is calculated as the sum of the influences from all vortex filaments on the body together with the velocity induced by all the other vorticity within the flow. To determine the strengths of the vortex loops, a boundary condition of zero through-flow is enforced simultaneously at the centroids of all panels.

The equations of motion for the blades, as forced by the aerodynamic load along their span, are derived by automatic differentiation of the Lagrangian of the system. No small-angle approximations are involved in this approach and the coupled flap-lag-feather dynamics of each of the blades, where appropriate, is fully represented. In all the examples presented in this paper, however, the blades themselves were assumed to be rigid.

The VTM has been applied extensively to the analysis of helicopter aerodynamic problems. The VTM has been used successfully to investigate the flight dynamics associated with penetration into the wakes of fixed-wing aircraft (Ref. 8), and for the study of helicopter main rotor - fuselage interaction (Ref. 7). It has been used to model the performance of rotors in axial flight (Ref. 9) and

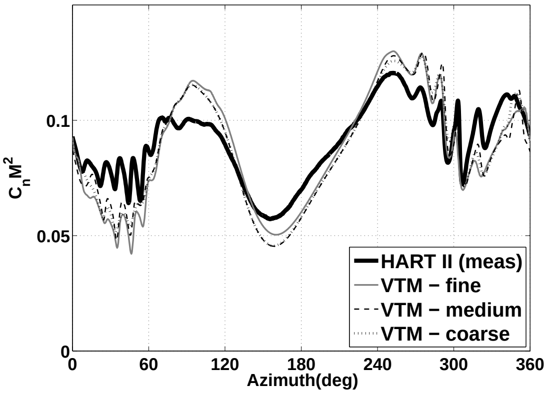
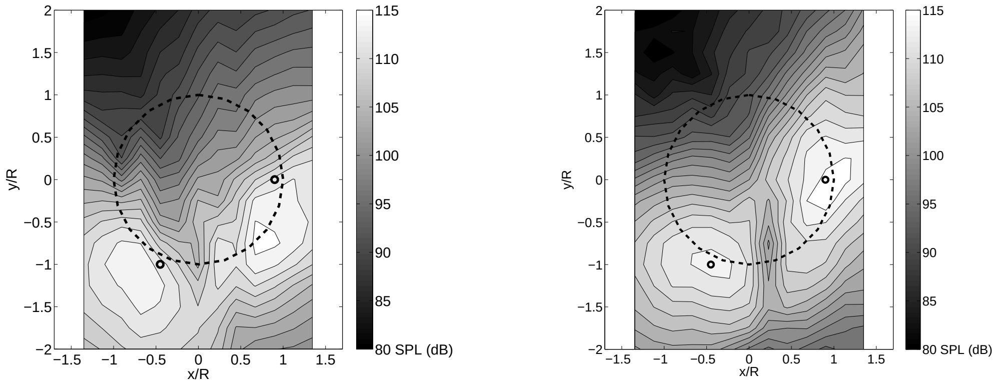


Figure 2: *Comparison of VTM-predicted and experimentally-measured sectional loading at 87% span of the HART-II rotor as a function of rotor azimuth.*



(a) *Predicted SPL*

(b) *Measured SPL*

Figure 3: *Comparison of VTM-predicted and experimentally-measured sound pressure level (SPL) on a plane one rotor radius below the HART-II rotor.*

to examine the fluid dynamics of the rotor vortex ring state (Ref. 10). It has also been validated against experimental data for co-axial helicopter rotors (Ref. 11) and used to explore the aerodynamic interactions that might occur within tightly-coupled helicopter configurations (Ref. 12).

## Loads Prediction

A variety of studies have contributed to faith in the VTM when applied to the analysis of the loads on helicopter rotor blades. Kelly and Brown (Refs. 13, 14) have shown very encouraging comparisons between VTM-predicted blade airloads and the experimental data that was gathered for the HART-II rotor system (Ref. 15). Figure 1 taken from Ref. 13 shows the VTM-predicted wake geometry of the HART-II rotor system, together with its drive enclosure, at an advance ratio of 0.151 during descending flight on a glide-slope of nominally 5.3°. At this flight condition, the wakes of the individual blades are convected up through the plane of the rotor, as can be seen clearly in the figure. Figure 2 shows the associated variation of blade loading, at 87% of the blade span, as a function of rotor azimuth, and reveals the effect of the very close interaction of the blades with individual vortices within the wake of the rotor in producing a series of localised spikes in the loading on the blades as they traverse the rear quadrants of the rotor disc. The high-frequency components of the airloads that arise from these blade-vortex interactions are primarily responsible for the objectionable acoustic characteristics of typical helicopter rotors as well as for the high vibration generated by many rotor systems under certain flight conditions. Methods such as the VTM are approaching a level of fidelity where they are able to predict these secondary characteristics of the rotor with reasonable confidence, as shown in Fig. 3 which compares the VTM-predicted pattern of acoustic radiation on a plane below the rotor to that measured during the HART-II experiment.

A rather different set of aerodynamic effects tends to concern the wind-turbine aerodynamicist, however. Blade-vortex interactions are important, but, particularly for horizontal-axis machines, are usually only encountered in a transient sense — for example as a consequence of a temporary shift in the wind direction under turbulent atmospheric conditions. The control system of most (horizontal-axis) wind turbine systems will rotate the axis of the device into the wind in order to eliminate any persistent edgewise flow condition from being experienced by the rotor. Nevertheless, recent studies using the VTM (Ref. 16) show that blade-wake interactions do play an important role in governing the load distribution along the length of the blades of *vertical-axis* wind turbines. In these devices, the axis of rotation of the rotor is oriented transversely to the oncoming wind, and thus the blades traverse through the wake of the turbine as they pass downwind of the centre of rotation of the device. Figure 4, taken from Ref. 16, shows the VTM-predicted wake structure that is produced by a typical, two-bladed vertical-axis wind turbine, and reveals the intricate, interwoven structure of the trailed and shed vorticity that is generated by the blades of the device. As shown in Fig. 5, again taken from Ref. 16, where the tangential loading at various spanwise stations near the tip of one of the blades of the turbine is plotted against its azimuth (by convention, 270° azimuth is immediately downwind of the axis of rotation), interactions between the blade and the vortices within the wake result in a high-frequency component to the induced loading on the system, as in the case of the helicopter rotor, over a small range of blade azimuths imme-

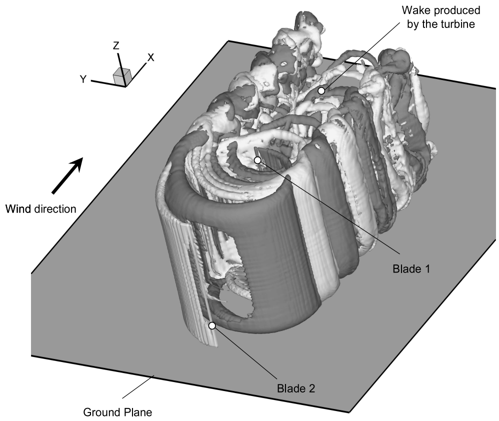


Figure 4: *VTM-predicted wake structure of a two-bladed vertical axis wind turbine. Vorticity from each blade is rendered in a separate shade.*

diately downwind of the axis of rotation. These loads have generally not been accounted for by the designers of vertical-axis wind turbines (who usually, but not always, adopt a relatively conservative approach to stress analysis compared to their helicopter counterparts), yet accurate quantification of the unsteady loads acting on the device is fundamental to correct assessment of the fatigue properties of such systems as well as their overall performance.

A very important consideration in the context of the aerodynamics of vertical-axis wind turbines is the onset of stall along a significant proportion of the length of the blades, particularly of any wind turbine that is operating in conditions where the wind speed is high compared to the tip speed of the blades. In a wind turbine, the centrifugally-driven flow along the length of the blades acts to suppress the onset of stall in high-wind conditions, even though parts of the blades, particularly near the roots, might experience very high angles of attack.

Blade stall is obviously also encountered in helicopter operation, and plays an important role in constraining the flight envelope of the vehicle. The centrifugal flows that act to suppress stall are most important near the blade roots and thus, in the helicopter situation, play an important role in the physics of retreating blade stall. Unlike in the case of the wind turbine, however, significant blade stall is not encountered, as a rule, during routine operation of the vehicle, and arguably this is the reason why these centrifugal effects have not received the same attention in the helicopter community as in the turbine community. Certainly the inherent unsteadiness of the flow near the root of the blade in the helicopter case poses additional problems to the extent that retreating blade stall is still very poorly understood. Indeed, no entirely believable model exists as yet that incorporates the dynamics of the stall process in conjunction with the three-dimensional fluid dynamic effects that are introduced by the nearby blade tip, let alone by spanwise flow along the length of the blade. Nevertheless, it seems a safe prediction that more attention will be devoted to understanding radially-pumped, separating flows in the helicopter context as current designs achieve ever higher advance ratios (Ref. 17).

The wind turbine community is some distance ahead in this respect since it is a long- and commonly-held belief that a stall delay model must be applied to the two-dimensional aerofoil data in lifting-line codes in order to obtain accurate predictions of the aerodynamic loading on the blades of any wind turbine. Indeed, wind tunnel measurements of the blade-loads on a wind turbine, conducted as part of the NREL Phase VI experimental campaign (Ref. 18), reveal that careful attention has to be paid to the development of radial flows along the length of the blade, and the subsequent effect that this has in suppressing the onset of stall. Unfortunately, a comprehensive comparison of applicable stall delay models that was conducted by Breton, Coton and Moe (Ref. 19) came recently to the conclusion that, although several of the stall delay models that have been developed do indeed offer large improvements in the accuracy of load predictions on wind turbines at intermediate wind speeds, there is currently no single stall delay model that enables comprehensive improvements over the full range of wind speeds that are encountered by these devices. As an example of what is currently possible, Figs. 6(a) and 6(b), taken from Ref. 20, show the experimentally-measured variation of normal and tangential force coefficient, respectively, along the length of the NREL blade when the rotor was operating in an

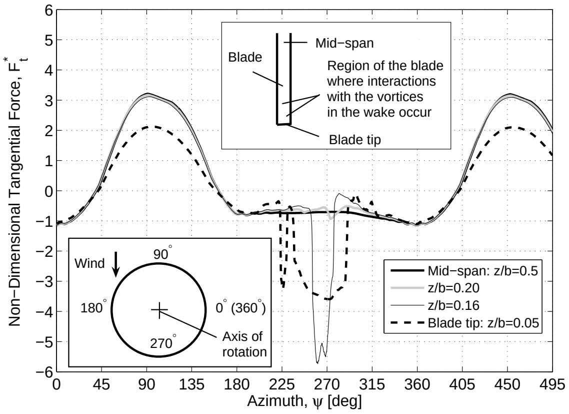
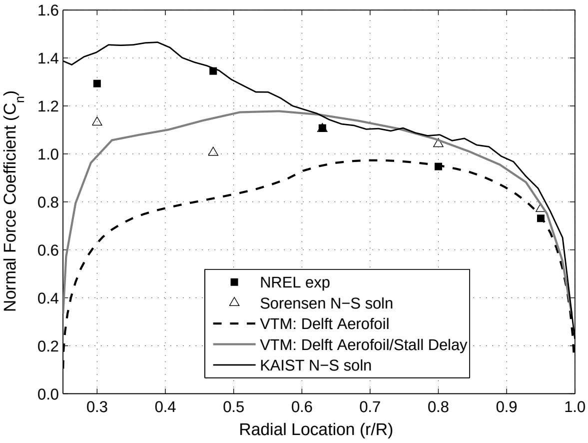


Figure 5: *VTM-predicted tangential blade loading at various stations near the tip of the two-bladed vertical axis wind turbine shown in Fig. 4.*

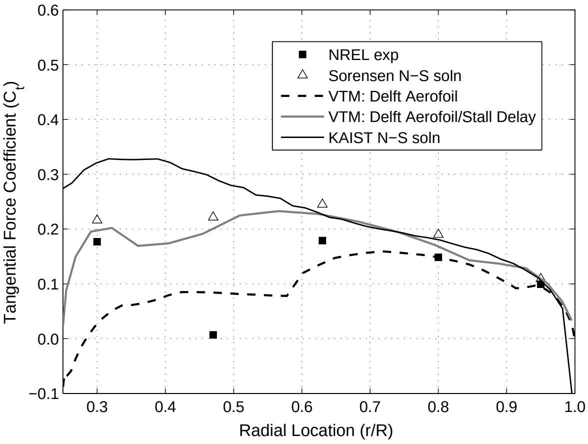
axial wind of 10m/s (the tip speed of the rotor was 37.9m/s). These data are compared to the Reynolds-Averaged Navier-Stokes simulations performed previously by Sørensen *et al.* (Ref. 21), and by Kwon's group at KAIST (Ref. 22), and the predictions of the Vorticity Transport Model (Ref. 22). To generate the data presented in Fig. 6, the lifting-line model of the VTM was used in conjunction with two-dimensional aerodynamic performance data for the S809 aerofoil that was measured during wind tunnel tests performed by Delft University of Technology (Ref. 23). Together with the predictions obtained using uncorrected aerofoil data, Figs. 6(a) and 6(b) also show the normal and tangential force calculated using the VTM following the application of the Corrigan and Schillings stall delay model (Ref. 24). When the VTM is used without the stall delay model, there is significant underprediction of the normal force coefficient on the inboard portion of the blade, despite excellent agreement with the experimental data at the $0.8R$ and $0.95R$ radial stations, as shown in Fig. 6(a).

A significant improvement in the correlation of normal force coefficient with the NREL experimental data is obtained along the inboard portion of the blade when the VTM is used in conjunction with the Corrigan and Schillings stall delay model. Unfortunately, a corresponding overprediction arises along the outboard portion of the blade. In comparison, the distribution of normal force coefficient predicted using the RANS codes represents a small over-prediction in normal force coefficient along both the inboard part of the blade and towards the tip of the blade. It is clear from Fig. 6(a) that accurate predictions of normal force coefficient along the outboard portion of the blade are often accompanied by poor predictions further inboard. This observation also applies to the prediction of the tangential force coefficient, as demonstrated by Fig. 6(b). Furthermore, despite the obvious advantages to applying a stall delay model to modify two-dimensional aerofoil data for the effects of three-dimensional flow, Fig. 6 also demonstrates the potential perils of chasing the experimental data when choosing a stall delay model, and specifically when identifying the coefficients that are to be used when applying semi-empirical models such as these.

Indeed, the development of a generalised model for stall delay, based on a fundamental physical understanding of the interplay between the centrifugal and Coriolis forces within the boundary layer on the blade (and somewhat less on blatant empiricism as has largely been the case up to now) appears still to be some way into the future. The development of a comprehensive model for three-dimensional stall onset would be of significant benefit to both the helicopter and wind turbine communities. A glimmer of light is offered by the fact that the predictions of the current generation of RANS codes do indeed seem to embody the correct physics (the data presented in Fig. 6 is suggestive, for instance), but the challenge to the operators of such codes is not to treat their models in 'black box' fashion but to digest their results into a form that provides useful input into simpler, more design-oriented approaches such as the lifting-line formalism of the VTM.



(a) *Normal force*



(b) *Tangential force*

Figure 6: *RANS and VTM-predicted spanwise distribution of blade loading on the NREL Phase VI rotor compared to experimental data for a wind speed of 10m/s.*

# Rotor Interactions

The aerodynamic interactions that occur within a wind farm have been known for some time to cause the constituent turbines to generate a lower power output than would be possible if each of the turbines were operated in isolation. Tightening of the constraints on the siting of wind farms is likely to increase the scale of this problem in the future. The problem is essentially one of wake interaction. In the process of extracting power from the wind, each individual turbine produces a momentum deficit in its wake. Should this wake impinge on another turbine located downwind, then the power output of this turbine will be curtailed as a result. The layout of turbines within a farm is invariably fixed, and the siting of individual turbines is calculated before construction in order to optimize the predicted power output of the farm under the prevailing wind conditions. Particularly for on-shore installations, the configuration of the turbines within the farm is constrained by the local topography and the allowable size and shape of the farm itself. The calculations required to ascertain the optimal placement of turbines within a farm are thus highly complex, involving as they do the interaction between the local topography and atmospheric conditions, and taking into account the modifying effect of the individual turbines on their local aerodynamic environment. The accuracy of these calculations is very heavily dependent on the fidelity with which the aerodynamic interactions between the constituent turbines can be modelled, and to date these interactions are usually calculated by the designers of wind farms using empirical models, many of which were based originally on bluff-body wake theory (see Ref. 25 for instance).

The validity of this approach is open to question, particularly given that the wake of a turbine has significantly more structure than that of the separated flow downstream of a bluff impediment to the flow. The VTM has been used to model the aerodynamic interaction between pairs of turbines in order to contribute to an improved insight into the physical processes that govern the power losses associated with turbines operating in close proximity. Figure 7, taken from Ref. 26, shows representative results for the power deficit experienced by a turbine operating directly downwind of another, as a function of the separation between the turbines and the wind speed (normalised by the tip speed of the turbine blades to yield the 'tip-speed ratio,' $\lambda$). For small separations, the power deficit is close to eighty percent, but for more practical separations (typically 10 rotor radii for on-shore turbines and 14 rotor radii for offshore turbines) the power deficit of the downwind turbine recovers to somewhat less than fifty percent.

Detailed calculations using the VTM reveal that the reason for this recovery is the disordering of the wake tube, with concomitant recovery in the dynamic head within the flow, as a result of the natural instability of the helical vortical filaments that are trailed from the individual blades of the upwind turbine. These results suggest strongly that a fundamentally different, and largely inviscid, mechanism is responsible for the recovery of the performance of the downwind turbine, in strong contrast to the essentially viscous dissipative mechanism that accompanies the bluff-body paradigm. The effect of wake destabilization on the aerodynamics of the coupled system is revealed in Fig. 8, again taken from Ref. 26, which shows how, for small separations, the downwind turbine is immersed within the structured part of the wake of the upwind turbine, whereas, for larger separations, the downwind turbine lies in the relatively unsteady and coarsely-structured region of the far-wake of the upwind turbine. The disruption to the wake of the downwind turbine as a result of the interaction is also clearly apparent.

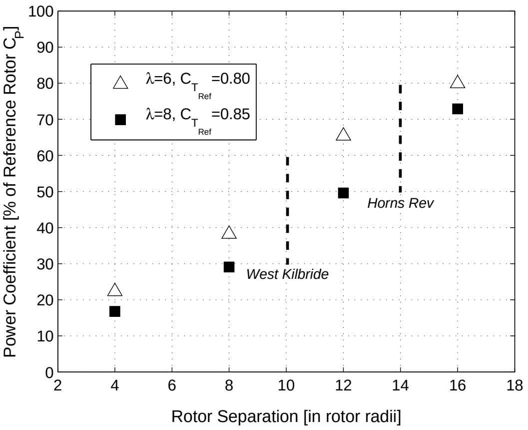


Figure 7: *Power coefficient of a wind turbine located downwind of another (as a percentage of the power coefficient of the upwind rotor) as a function of the separation between the rotors, for the upwind rotor operating at two different tip-speed ratios,* $\lambda$*=6 and* $\lambda$*=8.*

The aerodynamic interaction that is mediated by the dynamics of the wakes of the two turbines is responsible also for significant unsteadiness in the aerodynamic loading on the rotor blades of the downstream turbine. This unsteadiness is exacerbated should the wake of the upwind turbine impinge only partially on the face of the downwind turbine, as would necessarily be the case in certain wind conditions. This unsteadiness has considerable implications for the fatigue life of the blade structure and the rotor hub, as well as for the design of the turbine control systems.

This problem bears striking similarities to that associated with the interaction between the main and tail rotors of a conventional helicopter. In many helicopters, the aerodynamic interaction between the main and tail rotors can have a strong negative influence on the pilot workload. Significant unsteadiness in the loading on the tail rotor can be encountered under certain flight conditions, but the character of the unsteadiness seems to depend on the direction of rotation of the tail rotor of the helicopter. VTM simulations of the aerodynamic interaction between the main and tail rotors

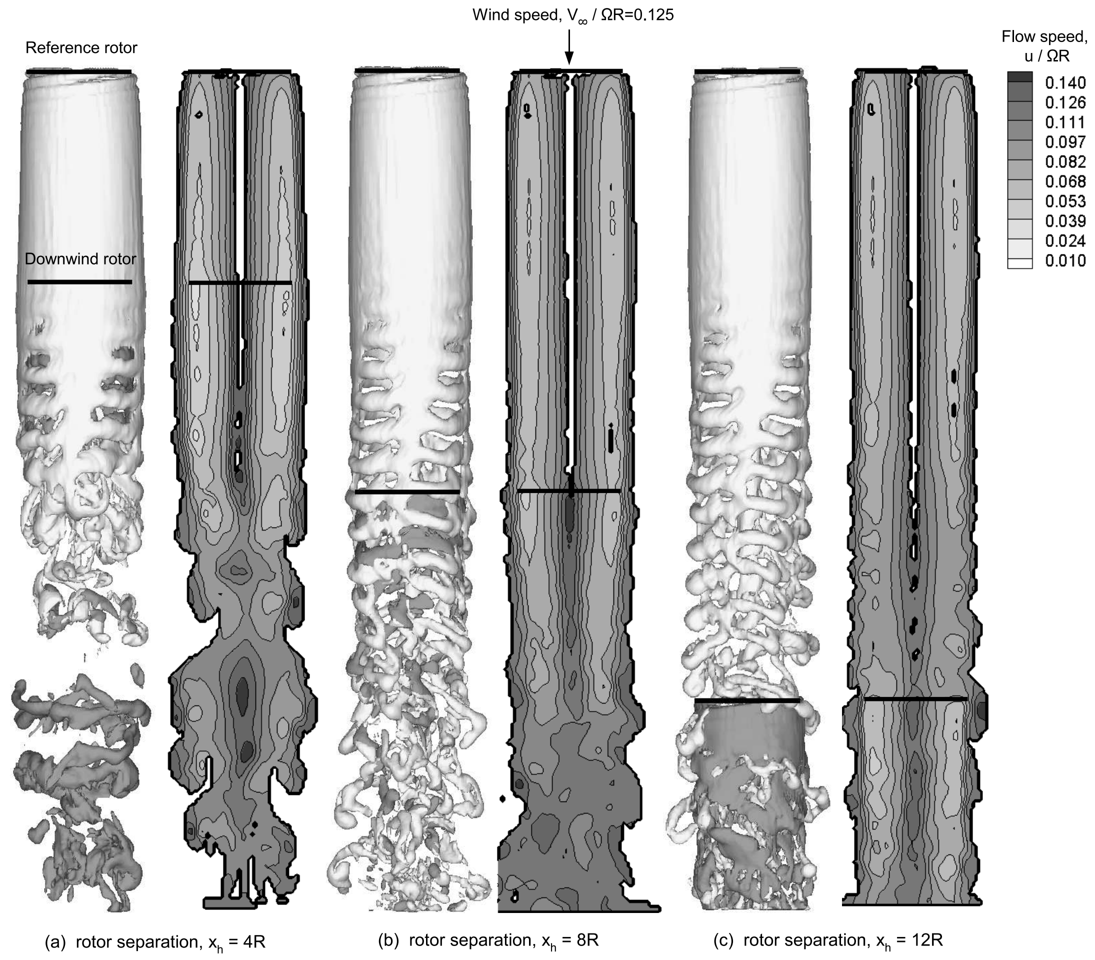


Figure 8: *Vorticity and velocity distributions in the flow surrounding two axially co-aligned rotors; At left of each sub-figure: instantaneous iso-surface of vorticity representing the wake (light grey: reference rotor, dark grey: downwind rotor). At right of each sub-figure: contours of instantaneous flow speed normalized using the rotor tip speed.*

of a helicopter show distinct differences in the behaviour of the system in left and right crosswind flight that are consistent with flight experience — the greatest fluctuations in loading or control input are required in left sideways flight (for a helicopter with counter-clockwise rotating rotor) and are generally more extreme for a system with tail rotor rotating top-forward than top-aft.

As an example, Fig. 9, taken from Ref. 27, shows VTM predictions of the torque generated by the main and tail rotors of a helicopter with conventional configuration in left rearwards (i.e. quartering) flight. To produce the figure, the torque that is generated by the two rotors has been sampled at once per main rotor revolution in order to expose the inherent low-frequency fluctuations in the loading on the system. The diagonal line on the diagrams thus represents the condition in which the net yaw moment on the system is zero, and it is clearly evident that the system spends very little time in this condition. The degree of scatter in the distribution of data points around this line is representative of the magnitude of the fluctuation about equilibrium of the yaw moment on the aircraft. The two types of symbols represent different data sets, one for the helicopter with top-forward rotating tail

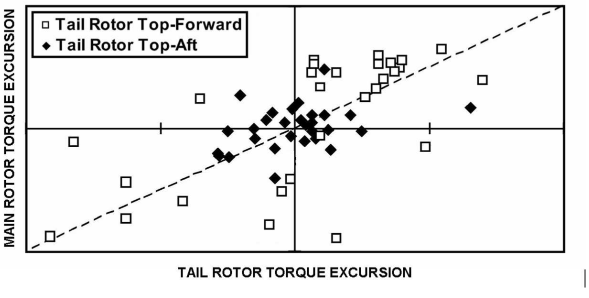


Figure 9: *Torque generated by the main and tail rotors of a conventional helicopter in left rearwards (i.e. quartering) flight. Data sampled at once per main rotor revolution.*

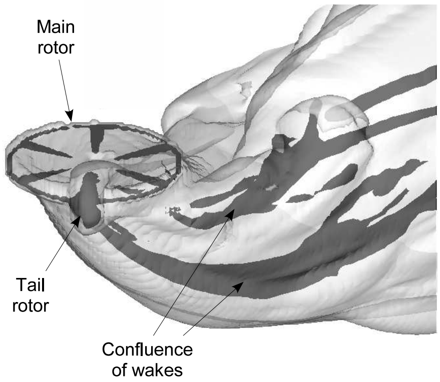


Figure 10: *Combined wake generated by the rotors of a helicopter with conventional main rotor – tail rotor configuration in left quartering flight (light grey: mean vorticity field, dark grey: root-mean-squared component of vorticity field).*

rotor and the other for the same helicopter but with top-aft rotating tail rotor. It is immediately apparent that the system with top-forward sense of tail rotor rotation is subject to significantly elevated fluctuations in yaw moment compared to the system with top-aft rotating tail rotor. Telling is the fact that both the main rotor *and* the tail rotor are subject to elevated fluctuations in their torque output in the top-forward case, since this implicates a non-local aerodynamic interaction within the system as the origin of the unsteadiness. This interaction is still not fully understood, but careful analysis of the dynamics of the wake of the coupled system suggests subtle differences in the way that the tail rotor wake is entrained into the wake of the main rotor in the two cases. This process is portrayed in Fig. 10, again taken from Ref. 27, which shows the vorticity field associated with the wake of the helicopter in left quartering flight after decomposition into the mean field plus a superimposed root-mean-square component. The confluence of the individual wakes of the main and tail rotors produces a region of significant unsteadiness in the flow, but the merging of the two wakes proceeds with somewhat greater unsteadiness in the case of the top-forward rotating tail rotor than the top-aft.

## Brownout and Sea-Bed Scouring

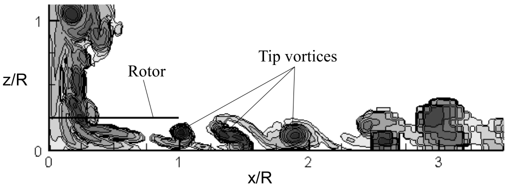
A particular concern to helicopter operators in desert or dusty conditions is the possibility of entrainment of dust from the ground into the air when the vehicle is operated close to the ground. This entrainment can cause large clouds of dust to form in the air surrounding the helicopter, and the possibility exists, under certain operational conditions, that these clouds might obscure the pilot's view, particularly while landing or taking off. A coupled VTM – sand transport model has been used to analyze the differences in the geometry and extent of the dust clouds that are produced by different helicopter configurations as they decelerate to land. It has been shown that the location of the ground vortex and the size of any regions of recirculatory flow, should they exist, play a primary role in governing the extent of the dust cloud that is created by the helicopter (Ref. 28). Anecdotal evidence suggests that various aspects of the design of the rotor may also have an influence on the size and shape of the dust cloud that is produced by a helicopter in brownout conditions, and calculations using the VTM (Ref. 29) bear out these observations to a certain extent (although perhaps not to the degree claimed by certain manufacturers).

Unlike most other current approaches to the modelling of brownout (see, for example, Refs. 30 and 31) which model the transport of particulates through the air by following the trajectory of a few individual particles that have been seeded into the flow, the VTM uses an alternative approach. A rigorous derivation, starting from the classical statistical mechanics of an ensemble consisting of a large number of particles, gives

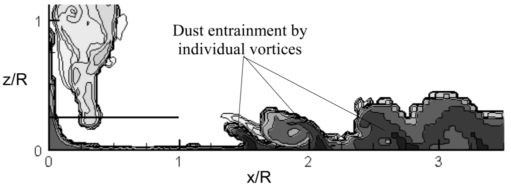
$$\frac{\partial}{\partial t}\rho_p + (u + u_g) \cdot \nabla \rho_p = S_p + \nu_p \nabla^2 \rho_p \qquad (4)$$

as the transport equation for particulate matter through the flow surrounding the helicopter. The equation is stricty valid when the particle drag parameter $18(\rho/\rho_s)(\nu/d^2)$ is very much smaller than the flow acceleration parameter $|\dot{u}|/|u|$, i.e. for very fine particles. If the dynamics of larger particles is to be modelled then additional terms can be appended to the equation to account for particle spin-out from vortex cores and other physical effects. The mathematical form of Eq. 4 bears a very close resemblance to the vorticity transport equation (Eq. 1) itself. This allows the model for sand transport to be advanced in parallel with the vorticity transport equation with very little additional computational overhead, yielding a very efficient approach to the modelling of helicopter brownout.

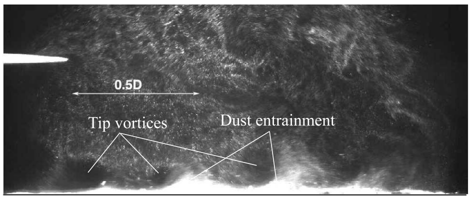
A major imponderable in current models for helicopter brownout, though, is the validity of the formalism that is used to entrain particulates from the ground into the airflow. For the moment an algebraic sublayer-type model, based on the concept of a threshold velocity below which particles will remain attached to the ground surface, is coupled to the VTM particulate transport model in order to simulate this process. This model is borrowed from the riverine sedimentology community, and significant effort is being devoted in some quarters in order to establish whether or not such models are entirely applicable to particulate entrainment



(a) *Contours of vorticity distribution*



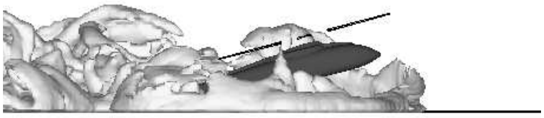
(b) *Contours of dust density distribution*



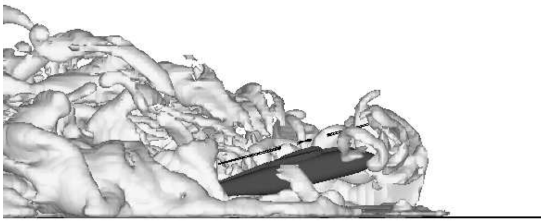
(c) *Experimental snapshot*

Figure 11: *VTM-predicted vorticity and corresponding dust density distributions compared to experimental measurements in the flow field below a rotor hovering in ground effect.*

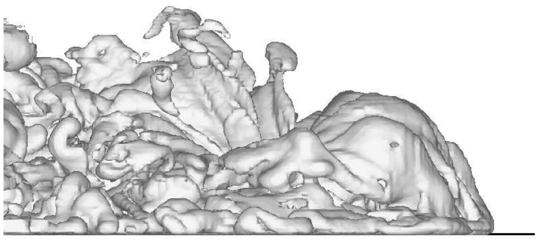
(a) *Thrust-normalized advance ratio* $\mu^\star = 0.80$



(b) *Thrust-normalized advance ratio* $\mu^\star = 0.47$



(c) *Thrust-normalized advance ratio* $\mu^\star = 0.29$



(d) *Thrust-normalized advance ratio* $\mu^\star = 0.12$

Figure 12: *VTM-predicted dust distribution in the air surrounding a representative large transport helicopter at various forward speeds above a desert surface.*

by the wake of a helicopter rotor as it impinges on the ground (Ref. 32). The contention arises from the fact that, in the context of riverine or wind-driven entrainment, particle suspension occurs through the action of a deep, relatively static shear layer above the particle bed, whereas entrainment in the helicopter context occurs through the interaction of very much more compact and unsteady vortical structures with the ground.

It is the view of the authors though that the algebraic character of the current entrainment model used within the VTM is essentially correct, even if not the exact value of every parameter. The model does indeed produce very similar physics to that observed in detailed experimental studies of the development of the brownout cloud. Figure 11, taken from Ref. 29, shows a comparison between experimental data (Ref. 33) and high-resolution VTM calculations of the interaction between the wake of a rotor hovering at low altitude and the ground. The numerics capture well the characteristic wedge-shaped zones of sediment uplift that are produced on the up-flow side of the individual vortices in the wake as they traverse across the ground plane. Hence the model is shown indeed to be sensitive to the small-scale vortical features within the flow that are thought to be primarily responsible for entrainment in the context of helicopter brownout.

As a consequence, some very compelling predictions of the evolution of the dust cloud that is responsible for the brownout phenomenon have been achieved using this entrainment model to date. Figure 12, taken from Ref. 28, shows the VTM-simulated development of the brownout cloud as a large transport helicopter decelerates close to the ground during its approach to land. When the helicopter is at relatively high forward speed above the ground, the dust cloud is relatively compact and forms well aft of the helicopter. As the helicopter decelerates, however, the size and extent of the cloud grows such that, at the lowest forward speeds, the helicopter is entirely enveloped in a cloud of recirculating particulate matter. The fu-

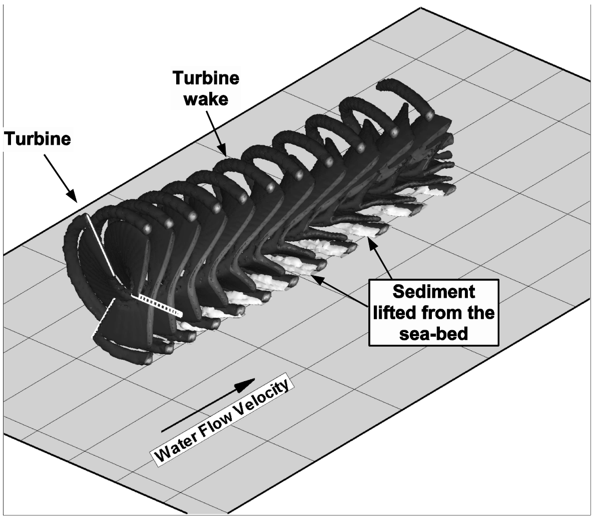


Figure 13: *VTM-predicted vorticity and sand distributions in the wake of a tidal turbine operating close to the sea-bed.*

ture challenge is to establish the key aerodynamic design parameters that influence most strongly the development of the dust cloud that is generated by the helicopter (tip shape and blade twist appear so far to be the strongest candidates, but the details of the trajectory that is followed to land would appear to be of overriding importance) and thus to inform the operation of such vehicles in desert conditions as well as possibly their future design. One possibility, for instance, is that interchangeable blades that are especially adapted for operation in desert conditions might be developed.

The helicopter brownout problem has its analogue in a related problem that is experienced when siting tidal turbines in shallow water. In this case, the wakes of the turbines can interact with the sea-bed to stir up sediment. This process has the potential to undermine the foundations of the turbine and also to increase water turbidity, possibly affecting marine life and water quality downstream of the turbines. Given the ecologically sensitive nature of many of the proposed sites for exploitation of tidal turbine technology, these are serious concerns — to the extent that significant public controversy exists regarding the benefits of such technology versus its perceived environmental impact.

VTM simulations show how, much as in the case of helicopter-induced brownout, it is the effect of the individual vortices on the sediment bed rather than the gross properties of the wake that is responsible for the most significant transfer of material from the ground into the fluid. Figure 13 shows a VTM-generated prediction of the wake and associated sediment cloud that is generated downstream of a three-bladed tidal turbine that is moored 1.1 rotor radii above the ocean floor, in a water stream that is 1/6 times the blade-tip velocity of the device. Since tidal devices operate continuously, it can be appreciated how even a moderate degree of persistent sediment uplift can, over time, result in significant erosion to the sea-bed unless measures are taken to ameliorate the ability of the wake to disturb any sediment layers that might be present downstream of the turbine. In this vein the technological challenges are rather similar to those encountered in the context of helicopter brownout.

Although understanding of the scouring problem is still in its infancy, high-resolution models such as the VTM will be essential if reliable estimates of the bearing capacity of specific proposed sites for undersea turbines are to be provided. The hope is also that, through appropriate design in order to balance the performance of individual turbines against their environmental impact, tidal turbine technology will be able to play an important and publicly-accepted role in providing a significant proportion of the energy needs of Scotland's future society.

## Plume Ingestion

The ingestion of exhaust plumes into the rotors of the helicopter has both good and bad effect. The rotor can help to disperse the plume, but this positive effect must be balanced against local heating of the blades and an increase in the size of the thermal footprint of the helicopter. Predicting the dynamics of the exhaust plume as it interacts with the wake of the helicopter is thus an important practical issue. The dynamics of exhaust plumes can be modelled fairly straightforwardly within the VTM. The geometry of the plume can be defined by the extent of its associated vorticity field, and

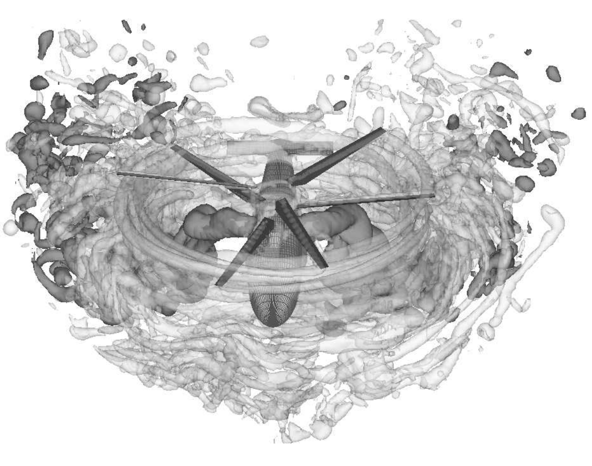


Figure 14: *VTM-predicted distortion of the exhaust plumes within the main rotor wake of a representative coaxial helicopter in forward flight at an advance ratio $\mu$=0.05 (light grey: rotor wake, dark grey: exhaust plume).*

this field can be evolved through time using the vorticity transport equation (Eq. 1). To model the influence on the dynamics of the plume of the temperature differential between the exhaust flow and the air surrounding the helicopter, an additional scalar transport equation for the plume temperature, of the form of Eq. 4, is evolved in parallel with the VTM, and the appropriate baroclinic vorticity source term is appended to Eq. 1. Figure 14 shows the results of a VTM calculation of the coupled dynamics of the rotor wake and the twin exhaust plumes of a representative coaxial helicopter in forward flight at an advance ratio of 0.05. The very strong influence of the rotor downwash in redirecting the trajectory of the plume is clearly evident in the figure. The effect of the individual vortices within the wake in mixing, dispersing and disordering the flow within the plume, and thus in reducing the local temperature differential of the plume with respect to the ambient flow, can also be inferred from the diagram.

In the environmental context, the dispersal of smokestack plumes if they should pass through the wake of a nearby wind turbine is a key concern, given the possibility that turbines located too close to any industrial plant might re-direct or otherwise interfere with the atmospheric pollutant load from the plant. The problem is two-fold. On a local scale, individual turbines might act to trap pollution from a nearby smokestack, thus allowing the secondary effect of particle fallout to produce a possibly very unwelcome deposition of pollutants on those properties located immediately downstream of the turbine. On the meso-scale, a major concern is that wind farms located too close to industrial sites might act as reservoirs of contaminants through their action in locally retarding the flow of air within the atmosphere.

The problem of predicting the interaction between a wind turbine and the plume emitted from a smokestack is very similar to that of predicting the dynamics of the exhaust plumes of rotorcraft, except that the problem involves longer ranges and is strongly influenced by atmospheric turbulence and large-scale changes in the thermal properties of the plume and the atmosphere — for example the presence or not of an inversion layer. Figure 15 shows the results of a VTM simulation of a model problem in which the interaction is modelled between a wind turbine, operating at a tip-speed ratio of six, and the plume that is generated by a smokestack located four rotor radii upstream that is producing a jet of effluent with a mean speed at its source of approximately 1.5 times the local wind speed. The smokestack is offset so that the centreline of the undisturbed plume would pass through the lateral edge of the turbine disc. This model problem is marginally unrealistic given the very close proximity of the smokestack to the turbine, but all the pertinent physics are fully represented. The ability of the VTM to act as an implicit large-eddy simulation is clearly apparent — indeed, a very convincing representation of the transition of the flow within the chimney plume from laminar to turbulent is produced. As with the helicopter exhaust plume, the interaction between the chimney plume and the wake of the turbine produces two effects. Firstly, the wake of the turbine acts to re-direct the trajectory of the plume — in this case the downwind convection of a significant proportion of the material within the plume is retarded. Secondly, the individual vortices in the wake act to mix and disperse the plume. The influence of the turbine wake in disordering and dispersing the chimney plume is particularly marked further downstream as the wake of the turbine succumbs to the inherent instability of its constituent vortex filaments. The results presented here hint at how a model, developed in the context of helicopter exhaust plume prediction, might be used effectively to provide information that is of use to environmental agencies and industrial planners in informing their strategies for future land-use. The utility of this particular incarnation of the VTM is expected to follow in the wake of steadily increasing environmental pressures to avoid the expansion of wind turbine sites into pristine natural estate. Indeed, the drive towards effective and concentrated use of appropriate land will increasingly be directed towards the location of small, localised turbine farms on recycled brownfield sites, possibly in the midst of pre-established industrial developments.

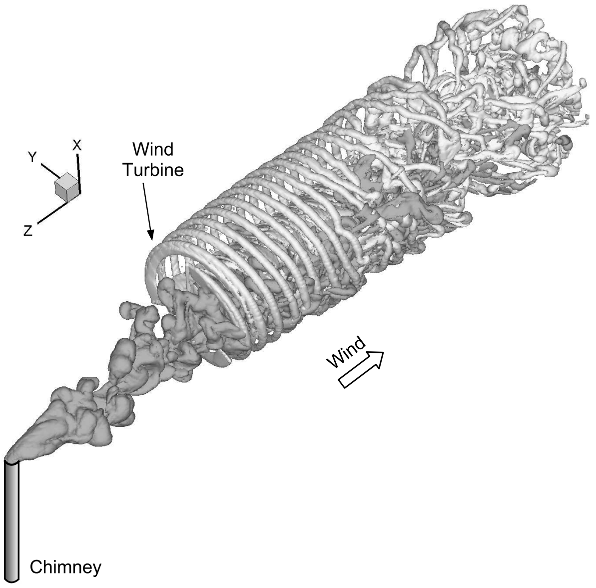


Figure 15: *VTM-predicted interaction between a wind turbine and the plume from a smokestack located four rotor radii upstream.*

# Conclusion

Several technical issues that arise in the field of renewable energy research have direct analogues in the field of helicopter engineering. This paper illustrates several areas where insights obtained from using the Vorticity Transport Model in the helicopter field have had, or are having, direct application in the area of wind and tidal turbine research.

A variety of studies have contributed to faith in the VTM when applied to the analysis of the loads on helicopter rotor blades. Comparisons of VTM predictions against the data set that was generated during the HART-II experimental campaign suggest that such computational methods are approaching a level of fidelity where they are able to predict the secondary characteristics of the rotor, such as the pattern of acoustic radiation that it generates, with reasonable confidence. Extension into the wind turbine field has exposed the VTM to a new set of aerodynamic challenges, however. Comparisons of VTM predictions against wind tunnel measurements of the blade-loads on a wind turbine, conducted as part of the NREL Phase VI campaign, reveal, for instance, that careful attention has to be paid to the development of radial flows along the length of the blade, and the subsequent effect that this has on the onset of stall on the turbine. Blade stall is an important issue in helicopter operation as well, since it places limits on the flight envelope of the vehicle. The influence of radial flow on the development of stall on the blades has not received the same attention in the helicopter community as in the wind turbine community, however. Nevertheless, as the advance ratio of operational helicopter systems continues to increase, there may be significant opportunity for technology transfer between the two fields.

The aerodynamic interactions that occur within a wind farm can cause the constituent turbines to generate a lower power output than would be possible if each of the turbines were operated in isolation. Tightening of the constraints on the siting of wind farms is likely to increase the scale of this problem in the future. VTM simulations show that the momentum deficit at a turbine operating within the wake developed by the rotor of a second turbine can limit substantially its mean power output. Also, wake induced unsteadiness in the aerodynamic loading on the rotor blades of the downstream turbine has considerable implications for the fatigue life of the blade structure and rotor hub, particularly in certain configurations and wind conditions. Detailed calculations show that the aerodynamic interaction between wind turbines is governed by the essentially inviscid physics of wake instability, rather than by viscous wake dissipation as is assumed in many of the current models used by the wind turbine community. This problem bears striking similarities to that associated with the interaction between the main and tail rotors of a conventional helicopter. This interaction can have a strong negative influence on the flight mechanics of the helicopter. Significant unsteadiness in the tail rotor loading is encountered under certain flight conditions, but the character of the unsteadiness can depend on the direction of rotation of the tail rotor. VTM simulations of the aerodynamic interaction between the main and tail rotors of a helicopter show distinct differences in the behaviour of the system in left and right crosswind flight that are consistent with flight experience — the greatest fluctuations in loading or control input are required in left sideways flight (for a counter-clockwise rotating rotor) and are generally more extreme for a system with tail rotor rotating top-forward than top-aft. Although not yet fully understood, the observed behaviour, in the helicopter context, appears to originate in a global unsteadiness in the flow field that arises in the process whereby the tail rotor wake merges with the wake of the main rotor.

A particular concern to helicopter operators in desert or dusty conditions is the possibility of entrainment of dust from the ground into the air when the vehicle is operated close to the ground. This process can cause large clouds of dust to form in the air surrounding the helicopter, and the possibility exists, under certain operational conditions, that these clouds might obscure the pilot's view, particularly while landing or taking off. A coupled VTM – sand transport model has been used to analyze the differences in the geometry and extent of the dust clouds that are produced by different helicopter configurations as they decelerate to land. This study has shown that the location of the ground vortex and the size of any regions of recirculatory flow, should they exist, play a primary role in governing the extent of the dust cloud that is created by the helicopter. This problem is very similar to that experienced when siting tidal turbines in shallow water. In this case, the wakes of the turbines can interact with the sea-bed to stir up sediment. This process has the potential to undermine the foundations of the turbine and also to increase water turbidity, possibly affecting marine life and water quality downstream of the turbines. VTM simulations show how, much as in the case of helicopter-induced brownout, it is the effect of the individual vortices on the sediment bed rather than the gross properties of the wake that is responsible for the most significant transfer of material from the ground into the fluid. The perceived environmental impact of large-scale marine turbine technology is a particularly important issue in its public acceptance as a significant contributor to future energy requirements. The observed sensitivity of the size and shape of the helicopter brownout cloud to details of the rotor configuration suggests that the environmental impact of tidal turbines may also be ameliorated by careful design of their rotors.

The ingestion of exhaust plumes into the rotors of the helicopter has both good and bad effect. The rotor can help to disperse the plume, but this positive effect must be balanced against local heating of the blades and an increase in the size of the thermal footprint of the helicopter. Predicting the dispersal of the thermal plume within the wake of the helicopter is thus an important issue. In the environmental context, the dispersal of smokestack plumes if they should pass through the wake of a nearby wind turbine is a key concern, given the possibility that turbines located too close to any industrial plant might re-direct or otherwise interfere with the atmospheric pollutant load from the plant. The problem of predicting this interaction is very similar to that of predicting the dynamics of the exhaust plumes of rotorcraft, except that the problem involves longer ranges and is strongly influenced by atmospheric turbulence and large-scale changes in the thermal properties of the plume and the atmosphere — for example the presence or not of an inversion layer. VTM simulations are presented that show how a model developed in the context of helicopter exhaust plume prediction can be used effectively to provide information that is of use to environmental agencies and industrial planners in informing their strategies for future land use.

The perceived need to reduce mankind's impact on his environment, and in particular on the global climate, is driving a very strong movement towards a future society in which a significant proportion of its energy needs will be extracted from the winds and the tides of the planet. This paper shows several examples of how cross-disciplinary information transfer between the rotorcraft field and the renewable energy community is helping to develop the technologies that will be required by this future society, as well as helping to understand the environmental issues that might need to be faced as these technologies become more prevalent.

# References

[1] de Noord, M., Beurskens, L.W.M., and de Vries, H.J., "Potentials and Costs for Renewable Electricity Generation — a Data Overview," Energy Research Centre of the Netherlands Report ECN-C-03-006, March, 2006.

[2] RSPB Scotland, WWF Scotland and FOE Scotland, "The Power of Scotland: Cutting Carbon with Scotland's Renewable Energy," February, 2006.

[3] Salmond, A, "Small Country Thinks Big," *Scottish Renewables Review*, No. 32, November 2006.

[4] Brown, R.E., "Rotor Wake Modeling for Flight Dynamic Simulation of Helicopters," *AIAA Journal*, Vol. 38, No. 1, 2000, pp. 57–63.

[5] Brown, R.E., and Line, A.J., "Efficient High-Resolution Wake Modeling Using the Vorticity Transport Equation," *AIAA Journal*, Vol. 43, No. 7, 2005, pp. 1434–1443.

[6] Toro, E. F., "A Weighted Average Flux Method for Hyperbolic Conservation Laws," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 423, No. 1865, 1989, pp. 401–418.

[7] Kenyon, A.R., and Brown, R.E., "Wake Dynamics and Rotor-Fuselage Aerodynamic Interactions," *Journal of the American Helicopter Society*, Vol. 54, No. 1, 2009, pp. 012003-1–18.

[8] Whitehouse, G.R., and Brown, R.E., "Modeling the Mutual Distortions of Interacting Helicopter and Aircraft Wakes," *Journal of Aircraft*, Vol. 40, No. 3, 2003, pp. 440–449.

[9] Green, R.B., Gillies, E.A., and Brown, R.E., "The Flow Field around a Rotor in Axial Descent," *Journal of Fluid Mechanics*, Vol. 543, No. 1, 2005, pp. 237–261.

[10] Ahlin, G.A., and Brown, R.E., "Wake Structure and Kinematics in the Vortex Ring State," *Journal of the American Helicopter Society*, Vol. 54, No. 3, 2009, pp. 032003-1–18.

[11] Kim, H.W., and Brown, R.E., "Coaxial Rotor Performance and Wake Dynamics in Steady and Manoeuvring Flight," 62nd American Helicopter Society Annual Forum, Phoenix, Arizona, USA, 9–11 May 2006.

[12] Kim, H.W., Kenyon, A.R., Duraisamy, K. and Brown, R.E., "Interactional Aerodynamics and Acoustics of a Hingeless Coaxial Helicopter with an Auxiliary Propeller in Forward Flight," *Aeronautical Journal*, Vol. 113, No. 1140, 2009, pp. 65–78.

[13] Kelly, M.E., and Brown, R.E., "The Effect of Blade Aerodynamic Modelling on the Prediction of High-Frequency Rotor Airloads," 65th American Helicopter Society Annual Forum, Grapevine, Texas, USA, 27–29 May, 2009.

[14] Kelly, M.E., and Brown, R.E., "The Effect of Blade Aerodynamic Modelling on the Prediction of the Blade airloads and the Acoustic Signature of the HART II Rotor," 35th European Rotorcraft Forum, Hamburg, Germany, 22–25 September, 2009.

[15] van der Wall, B. G., Junker, B., Burley, C., Brooks, T., Yu, Y., Tung, C., Raffel, M., Richard, H., Wagner, W., Mercker, E., Pengel, K., Holthusen, H., Beaumier, P., and Delrieux, Y., "The HART II test in the LLF of the DNW — a Major Step towards Rotor Wake Understanding," 28th European Rotorcraft Forum, Bristol, England, 17–20 September, 2002.

[16] Scheurich, F., Fletcher, T.M., and Brown, R.E., "Simulating the Aerodynamic Performance and Wake Dynamics of a Vertical-Axis Wind Turbine," *Wind Energy*, in press.

[17] Morris, J., "Sikorsky Calls X2 Shape of the Future," *Aviation Week*, 25 February, 2008.

[18] Hand, M.M., Simms, D.A., Fingersh, L.J., Jager, D.W., Cotrell, J.R., Schreck, S., and Larwood, S.M., "Unsteady Aerodynamics Experiment Phase VI: Wind Tunnel Test Configurations and Available Data Campaigns," National Renewable Energy Laboratory TP-500-29955, 2001.

[19] Breton, S., Coton, F.N., and Moe, G., "A Study on Rotational Effects and Different Stall Delay Models Using a Prescribed Wake Vortex Scheme and NREL Phase VI Experiment Data," *Wind Energy*, Vol. 11, No. 5, 2008, pp. 459–482.

[20] Fletcher, T.M, Brown, R.E., Kim, D.H., and Kwon, O.J., "Comparative Simulations of Wind Turbine Aerodynamics using Vorticity Transport and RANS Models," *Wind Energy*, in press.

[21] Sørensen, N.N., Michelsen, J.A., and Schreck, S., "Navier-Stokes Predictions of NREL Phase VI Rotor in the NASA Ames 80 ft×120 ft Wind Tunnel," *Wind Energy* Vol. 5, No. 23, 2002, pp. 151–169.

[22] Fletcher, T.M, Brown, R.E., Kim, D.H., and Kwon, O.J., "Predicting Wind Turbine Blade Loads using Vorticity Transport and RANS Methodologies," European Wind Energy Conference 2009, Marseille, France, 16–19 March, 2009.

[23] Somers, D.M., "Design and Experimental Results for the S809 Airfoil," Airfoils, Inc., State College, Pennsylvania, USA, 1989.

[24] Corrigan, J. J., and Schillings, J. J., "Empirical Model for Stall Delay due to Rotation," American Helicopter Society Aeromechanics Specialists Conference, San Francisco, California, USA, January 1994.

[25] Voutsinas, S.G., Rados, K.G., and Zervos, A., "On the Analysis of Wake Effects in Wind Farms," *Wind Engineering*, Vol. 14, No. 4, 1990, pp. 204–219.

[26] Fletcher, T.M, and Brown, R.E., "Simulating Wind Turbine Interactions using the Vorticity Transport Model," 28th ASME Wind Energy Symposium, Orlando, Florida, USA, January, 2009.

[27] Fletcher, T.M, and Brown, R.E., "Main Rotor – Tail Rotor Interaction and its Implications for Helicopter Directional Control," *Journal of the American Helicopter Society*, Vol. 53, No. 2, 2008, pp. 125–138.

[28] Phillips, C., and Brown, R.E., "Eulerian Simulation of the Fluid Dynamics of Helicopter Brownout," *Journal of Aircraft*, Vol. 46, No. 4, 2009, pp. 1416–1429.

[29] Phillips, C., Kim, H.W., and Brown, R.E., "The Effect of Rotor Design on the Fluid Dynamics of Helicopter Brownout," 35th European Rotorcraft Forum, Hamburg, Germany, 22–25 September, 2009.

[30] Lee, T.E., Leishman, J.G., and Ramasamy, M., "Fluid Dynamics of Interacting Blade Tip Vortices With a Ground Plane," 64th American Helicopter Society Annual Forum, Montréal, Canada, 29 April–1 May, 2008.

[31] Whitehouse, G.R., Wachspress, D.A., Quackenbush, T.R., and Keller, J.D., "Exploring Aerodynamic Methods for Mitigating Brownout," 65th American Helicopter Society Annual Forum, Grapevine, Texas, USA, 27–29 May, 2009.

[32] Johnson, B., Leishman, J.G., and Sydney, A., "Investigation of Sediment Entrainment in Brownout Using High-Speed Particle Image Velocimetry," 65th American Helicopter Society Annual Forum, Grapevine, Texas, USA, 27–29 May, 2009.

[33] Nathan, N.D., and Green, R.B., "Measurements of a Rotor Flow in Ground Effect and Visualisation of the Brown-out Phenomonon," 64th American Helicopter Society Annual Forum, Montréal, Canada, 29 April–1 May, 2008.