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Reduced Complexity Schemes to Greedy Power Allocation for Multicarrier Systems

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Abstract—**Discrete bit loading for multicarrier systems based on the greedy power allocation (GPA) algorithm is considered in this paper. A new suboptimal scheme that independently performs GPA on groups of subcarriers and therefore can significantly reduce complexity compared to the standard GPA is proposed. These groups are formed in an initial step of a uniform power allocation (UPA) algorithm. In order to more efficiently allocate the available transmit power, two power re-distribution algorithms are further introduced by including a transfer of residual power between groups. Simulation results show that the two proposed algorithms can achieve near optimal performance in two separate and distinctive SNR regions. We demonstrate by analysis how these methods can greatly simplify the computational complexity of the GPA algorithm.**

I. INTRODUCTION

In multicarrier systems, e.g. OFDM, a number of independent subcarriers arise for transmission, which differ in SNR. Data throughput maximisation over such systems under the constraint of limited transmit power leads to the well-known water-filling solution [1]. However, water-filling is generally followed by a rounding-off step to allocate an integer number of bits to the transmitted QAM symbols across all subcarriers, thus lowering the overall throughput. In addition, unbounded modulation orders in the case of infinite SNR are required to efficiently utilise the transmit power but are unfeasible. Moreover, the dependence of water-filling on SNR-gap approximation which is poor in the operating regions of wireless systems [2] motivates for other efficient loading solutions.

Pure water-filling-based solutions have been reported in [3], [4], [2], leading to some of the above stated problems. Alternatively, allocation of the transmit power when realising the target bit error ratio (BER) across all subcarriers i given that $b_i \in \mathbb{Z}$, where b_i is number of bits allocated to the ith subcarrier, has lead to a rate-optimal algorithm known as the greedy power allocation (GPA) algorithm [5], [6], of which a number of different variations have emerged constraining either the average BER [7] or the total power [8]. For a good review of greedy algorithms, please refer to [9].

While achieving rate optimality, the family of greedy algorithms is also known to be greedy in terms of computing requirements. Therefore, reduced complexity schemes are either water-filling based only [3] or aim at simplifications [10]. In this paper we propose a novel suboptimal greedy algorithm, whereby the power re-allocation is performed in groups of subcarriers. Different from our previous work in [11], the interest of this paper is focusing on the simplification achievements of our proposed power allocation scheme compared to the standard greedy approach by further elaborating on the complexity analysis of both algorithms. We show that with simple overall power re-distribution between groups, two different methods in terms of approximate overall optimisation can be proposed. These suboptimal schemes, while greatly simplifying complexity, hardly sacrifice any performance compared to the full GPA algorithm, provided that the proper algorithmic version is selected for specific SNR regions.

The rest of the paper is organised as follows. In Sec. II, the standard greedy approach is first reviewed including the initialisation step of uniform power allocation (UPA). Our proposed reduced-complexity schemes are presented in Sec. III, where computational complexity is analysed and evaluated in Sec. IV. Simulation results are discussed in Sec. V and conclusions are drawn in Sec. VI.

II. THE GREEDY APPROACH

In this Section greedy approach for the power allocation problem to maximise the transmission rate over a multicarrier system is introduced.

A. Constrained Optimisation Problem

A MIMO-OFDM system is considered, whereby the ISI MIMO channel $\mathbf{H}$ can be converted into an N-subcarrier system with different gains $|H_i|, i = 1 \cdots N$. The ith subcarrier experiencing the gain $|H_i|$ will be used to transmit b_i bits per symbol. The maximisation of the sum-rate

$$\max \sum_{i=1}^{N} b_i, \qquad (1)$$

constrained by: the total power budget, the target bit error ratio (BER), and the maximum permissible QAM modulation order. These constraints can be formulated as

$$\sum_{i=1}^{N} P_i \leq P_{\text{budget}}, \quad \mathcal{P}_{b,i} = \mathcal{P}_b^{\text{target}}, \quad \text{and} \quad b_i \leq b^{\text{max}}, \forall i \qquad (2)$$

where P_i is the amount of power allocated to the ith subcarrier to achieve a BER $\mathcal{P}_{b,i}$, and b^{max} is the maximum number of permissible bits allocated to a subcarrier. Note that the target BERs are assumed to be equal, i.e. $\mathcal{P}_{b,i} = \mathcal{P}_b^{\text{target}}$ in (2) for all subcarriers $i = 1 \cdots N$ and therefore the subscript i will be dropped from the BER notation.

The carrier-to-noise ratio of the ith subcarrier can be defined as

$$\text{CNR}_i = \frac{|H_i|^2}{\mathcal{N}_0}, \qquad (3)$$

where $\mathcal{N}_0$ is the total noise power at the receiver, whereas the SNR of this subcarrier is

$$\gamma_i = P_i \times \text{CNR}_i . \qquad (4)$$

We consider rectangular M-QAM modulation of order $M_k, 1 \leq k \leq K$, where M_K is the maximum QAM constellation that is permissible by the transmission system, i.e., $M_K = 2^{b^{\text{max}}}$. The BER of this modulation scheme is given by [12]

$$\mathcal{P}_b = \frac{1 - \left[1 - 2 \left(1 - \frac{1}{\sqrt{M_k}} \right) Q \left(\sqrt{\frac{3\gamma_i}{M_k - 1}} \right) \right]^2}{\log_2 M_k} . \qquad (5)$$

Assuming availability of channel state information (CSI) at the transmitter, symbols of b_k-bits, $b_k = \log_2 M_k$ can be loaded to a subcarrier with minimum required SNR to achieve $\mathcal{P}_b^{\text{target}}$ obtained from (5) as

$$\gamma_k^{\text{QAM}} = \frac{M_k - 1}{3} \left[Q^{-1} \left(\frac{1 - \sqrt{1 - \mathcal{P}_b^{\text{target}} \cdot \log_2 M_k}}{2 \left(1 - 1/\sqrt{M_k} \right)} \right) \right]^2 , \quad (6)$$

where Q^{-1} is the inverse of the well-known Q function.

Based on (6), the bit loading problem is solved in two steps, (i) a uniform power allocation (UPA) initialisation step and (ii) the greedy algorithm step, both described below.

B. UPA Algorithm

The uniform power allocation is performed by the following steps:
1) Calculate γ_k^{QAM} for all $M_k, 1 \leq k \leq K$ using (6).
2) Equally allocate P_{budget} among all subcarriers $1 \leq i \leq N$,

$$\gamma_i = P_i \times \text{CNR}_i = \frac{P_{budget}}{N} \times \text{CNR}_i . \quad (7)$$

3) Reside subcarriers according to their SNR γ_i into QAM groups $G_k, 0 \leq k \leq K$ bounded by QAM levels γ_k^{QAM} and $\gamma_{k+1}^{\text{QAM}}$ with $\gamma_0^{\text{QAM}} = 0$ and $\gamma_{K+1}^{\text{QAM}} = +\infty$ such that

$$\gamma_i \geq \gamma_k^{\text{QAM}} \quad \text{and} \quad \gamma_i < \gamma_{k+1}^{\text{QAM}} . \quad (8)$$

4) For each group G_k, load subcarriers within this group with QAM constellation M_k and compute the group's total allocated bits

$$B_k^{\text{u}} = \sum_{i \in G_k} b_{i,k}^{\text{u}} = \sum_{i \in G_k} \log_2 M_k \quad (9)$$

with $B_0^u = 0$. Notice that from step (3), subcarriers reside in QAM groups with SNR levels that are below their actual SNRs, $\gamma_k^{\text{QAM}} \leq \gamma_i$, therefore leaving some unused (excess) power

$$\begin{aligned} P_k^{\text{ex}} &= \sum_{i \in G_k} \frac{\gamma_i - \gamma_k^{\text{QAM}}}{\text{CNR}_i} = \sum_{i \in G_k} P_i - \frac{\gamma_k^{\text{QAM}}}{\text{CNR}_i} \\ &= N_k \frac{P_{budget}}{N} - \sum_{i \in G_k} \frac{\gamma_k^{\text{QAM}}}{\text{CNR}_i} \end{aligned} , \quad (10)$$

where $N_k, 1 \leq k \leq K$ is the number of subcarriers that occupies the QAM group G_k.

5) Overall, the allocated bits and the used power for the uniform power allocation scheme are therefore,

$$B_{\text{u}} = \sum_{k=1}^{K} B_k^{\text{u}} \quad (11a)$$

$$P_{\text{u}}^{\text{used}} = P_{\text{budget}} - \sum_{k=0}^{K} P_k^{\text{ex}} = P_{\text{budget}} - P^{\text{ex}}, \quad (11b)$$

where P^{ex} is the total excess power that remains unallocated under the UPA scheme.

C. Full Greedy Power Allocation (GPA) Algorithm

The second step towards the GPA is described next. Based on the initialisation step described in the UPA, the full GPA algorithm [8] performs an iterative re-distribution of the unallocated power of the UPA algorithm P^{ex} by applying the algorithmic steps detailed in Table I. At each iteration, this algorithm tries to increase bit loading by upgrading (to the next higher QAM level) the subcarrier of the least power requirements through an exhaustive search, by performing step (4) in Table I for all subcarriers N. When either i) the remaining power cannot support any further upgrades or ii) all subcarriers appear in the highest QAM level K, the algorithm stops resulting in the system allocating B_{gpa} bits.

Table I
FULL GPA ALGORITHM

Initialisation:
1. Set power available for GPA to $P_a^{\text{gpa}} = P^{\text{ex}}$ in (11b)
For each subcarrier i do the following:
2. Set $b_i^{\text{gpa}} = b_i^{\text{u}}$ in (9) and index $k_i = k$ in (8)
3. Cal. the min required upgrade power: $P_i^{\text{up}} = \frac{\gamma_{k_i+1}^{\text{QAM}} - \gamma_{k_i}^{\text{QAM}}}{\text{CNR}_i}$
Recursion:
while $P_a^{\text{gpa}} \geq \min(P_i^{\text{up}})$ and $\min(k_i) < K, 1 \leq i \leq N$
4. $j = \underset{1 \leq i \leq N}{\text{argmin}}(P_i^{\text{up}})$
5. $k_j = k_j + 1, P_a^{\text{gpa}} = P_a^{\text{gpa}} - P_j^{\text{up}}$
if $k_j = 1$
6. $b_j^{\text{gpa}} = \log_2 M_1, P_j^{\text{up}} = \frac{\gamma_2^{\text{QAM}} - \gamma_1^{\text{QAM}}}{\text{CNR}_j}$
elseif $k_j < K$
7. $b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 \left(\frac{M_{k_j}}{M_{k_j-1}} \right), P_j^{\text{up}} = \frac{\gamma_{k_j+1}^{\text{QAM}} - \gamma_{k_j}^{\text{QAM}}}{\text{CNR}_j}$
else
8. $b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 \left(\frac{M_{k_j}}{M_{k_j-1}} \right), P_j^{\text{up}} = +\infty$
end
end
9. $B_{\text{gpa}} = \sum_{j=1}^{N} b_j^{\text{gpa}}$

III. PROPOSED REDUCED-COMPLEXITY GPA

Given B_k^{u} as defined in (9) and P_k^{ex} in (10), three low-cost greedy algorithms are proposed to efficiently utilise the total excess power of the uniform power allocation in (11b) using the QAM grouping concept. More precisely, GPA is separately accomplished for each QAM group G_k aiming to increase the total bit allocation of this group and therefore the overall system allocated bits. Based on the way of utilising P_k^{ex}, we propose three different algorithms, which below are referred to as (i) grouped GPA (g-GPA), (ii) power Moving-up GPA (Mu-GPA) and (iii) power Moving-down GPA (Md-GPA).

A. g-GPA Algorithm

As discussed in Sec. II, the optimum discrete bit loading constrained by total power and maximum permissible QAM order can be performed by the GPA approach. However, the direct application of the GPA algorithm is computationally very costly due to the fact that at each iteration an exhaustive sorting of all subcarriers is required as evident from Table I.

A simplification of the GPA algorithm can be achieved if subcarriers are firstly divided into QAM groups $G_k, 0 \leq k \leq K$ according to their SNRs (step (3) of the UPA). GPA algorithm is therefore independently applied to each group G_k, trying to allocate as much of the excess power P_k^{ex} within this QAM group as possible. This excess power is iteratively allocated to subcarriers within this group according to the greedy concept with the aim of upgrading as many subcarriers as possible to the next QAM level. The pseudo code for the kth QAM group G_k of the g-GPA algorithm is given in Table II.

Notice that different from the standard GPA, this algorithm permits upgrades to the next QAM level only (P_j^{up} is set to $+\infty$ in steps (5) and (6) of Table II). Accordingly, g-GPA may leave some left-over (LO) power P_k^{LO} for each QAM group G_k, resulting in a total LO power of

$$P_{\text{g}}^{\text{LO}} = \sum_{k=0}^{K-1} P_k^{\text{LO}} + P_K^{\text{ex}} . \quad (12)$$

Intuitively, for the overall performance of the g-GPA algorithm, the algorithm in Table II has to be executed K times, once for each QAM

Table II

G-GPA ALGORITHM FOR SUBCARRIERS IN THE kTH QAM GROUP G_k

---

1. $\forall i \in G_k$, calculate $P_i^{\mathrm{up}} = (\gamma_{k+1}^{\mathrm{QAM}} - \gamma_k^{\mathrm{QAM}})/\mathrm{CNR}_i$
2.   Initiate $b_{i,k}^{\mathrm{g}} = b_{i,k}^{\mathrm{u}}$ and $P_k^{\mathrm{LO}} = P_k^{\mathrm{ex}}$
     while $P_k^{\mathrm{LO}} \geq \min(P_i^{\mathrm{up}})$
3. $j = \underset{i \in G_k}{\arg\min}(P_i^{\mathrm{up}})$
4. $P_k^{\mathrm{LO}} = P_k^{\mathrm{LO}} - P_j^{\mathrm{up}}$
     if $k = 0$
5. $b_{j,k}^{\mathrm{g}} = \log_2 M_1$, $P_j^{\mathrm{up}} = +\infty$
     else
6. $b_{j,k}^{\mathrm{g}} = b_{j,k}^{\mathrm{g}} + \log_2 \frac{M_{k+1}}{M_k}$, $P_j^{\mathrm{up}} = +\infty$
     end
     end
7. $B_k^{\mathrm{g}} = \sum\limits_{i \in G_k} b_{i,k}^{\mathrm{g}}$

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group, from G_0 to G_{K-1}, resulting in the system allocating bits

$$B_{\mathrm{g}} = \sum_{k=0}^{K-1} B_k^{\mathrm{g}} + B_K^{\mathrm{u}} . \tag{13}$$

B. Mu-GPA Algorithm

The g-GPA algorithm results in unused LO power P_k^{LO} for each QAM group. This residual power can be exploited by a second stage, whereby it is proposed to move power upwards starting from the lowest QAM group, as outlined in Fig. 1(a). This modifies the g-GPA algorithm by considering the LO power P_0^{LO} of the QAM group G_0 after running the g-GPA algorithm on that group, and assign this power for re-distribution to group G_1. Any LO power after running g-GPA on G_1 is then passed further upwards to G_2, and so forth. At the kth algorithmic iteration, the Mu-GPA algorithm is working on G_k and tries to allocate the sum of the excess power missed by the UPA algorithm of that group along with the LO power of the previous group G_{k-1}, i.e., $P_k^{\mathrm{ex}} + P_{k-1}^{\mathrm{LO}}$. Finally, the LO power resulting from the QAM group G_{K-1} is added to the excess power of the Kth QAM group P_K^{ex} representing the final LO power of the Mu-GPA

$$P_{\mathrm{Mu-g}}^{\mathrm{LO}} = P_{K-1}^{\mathrm{LO}} + P_K^{\mathrm{ex}} , \tag{14}$$

while the number of the overall allocated bits is

$$B_{\mathrm{Mu-g}} = \sum_{k=0}^{K-1} B_k^{\mathrm{Mu-g}} + B_K^{\mathrm{u}} . \tag{15}$$
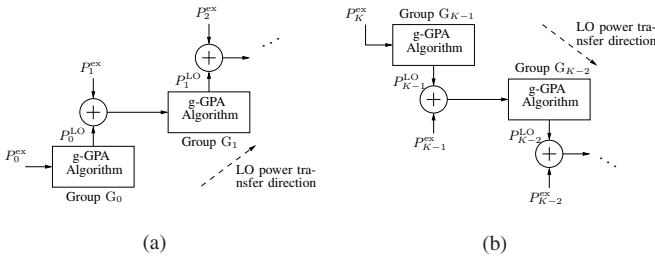

(a)   (b)

Figure 1. Algorithmic arrangements for (a) Mu-GPA and (b) Md-GPA with final LO powers given in (14) and (16), respectively.

C. Md-GPA Algorithm

A second algorithm is proposed to exploit the residual power P_k^{LO} of each QAM group of the g-GPA algorithm but in a reverse direction compared to the Mu-GPA algorithm. Starting from QAM group G_{K-1} downwards to QAM group G_0, these procedures are illustrated in Fig. 1(b) which show the direction of the LO

power flow. Proceeding downwards, at the kth stage the Md-GPA algorithm applies the g-GPA algorithm for the available power that comprises both the excess power missed by the UPA algorithm and the LO power of the previous QAM group (G_{k+1} in this case), i.e., $P_{k+1}^{\mathrm{ex}} + P_{k+1}^{\mathrm{LO}}$ (cf. Fig. 1(b)). This will finally result in a LO power of

$$P_{\mathrm{Md-g}}^{\mathrm{LO}} = P_0^{\mathrm{LO}} + P_0^{\mathrm{ex}} , \tag{16}$$

and an overall number of allocated bits of

$$B_{\mathrm{Md-g}} = \sum_{k=0}^{K-1} B_k^{\mathrm{Md-g}} + B_K^{\mathrm{u}} . \tag{17}$$

IV. COMPLEXITY EVALUATION

The key idea behind this work is to reduce the complexity associated with the implementation of the GPA algorithm. Instead of jointly applying GPA across all subcarriers which consequently requires high computational complexity specifically for large numbers of subcarriers, the g-GPA algorithm only addresses a subset of subcarriers within a specific QAM group at a time. With the aid of Fig. 2 it is obvious that the search step (4) of Table I, which represents the complexity bottleneck of the GPA algorithm, has to include all subcarriers N in every iteration regardless of initial subcarriers ordering. This is because it is possible to find subcarriers in lower QAM levels that require less power to upgrade than others in higher QAM levels (cf. Fig. 2). Beyond the division of the QAM grouping concept, a further reduction in complexity can be achieved if subcarriers are initially ordered in their gains CNR_i.
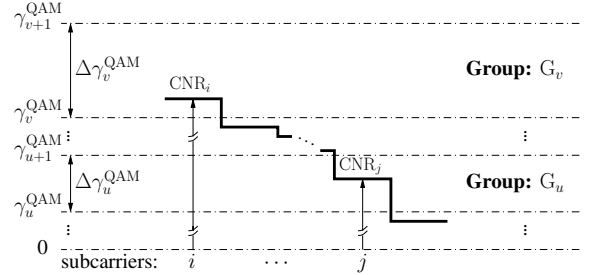

Figure 2. Illustration of the necessity to address all subcarriers N in every iteration in the case of the GPA algorithm as it is possible to find $P_j^{\mathrm{up}} = \Delta\gamma_u^{\mathrm{QAM}}/\mathrm{CNR}_j \leq P_i^{\mathrm{up}} = \Delta\gamma_v^{\mathrm{QAM}}/\mathrm{CNR}_i$. Contrarily, for the g-GPA algorithm since $\Delta\gamma_k^{\mathrm{QAM}}$ will be fixed for all subcarriers in group G_k, ordering $\mathrm{CNR}_i, i \in G_k$ will simplify the search step (3) of Table II with only an incremental indexing instead.

Referring to Table I and Table II, the computational complexities of both GPA and g-GPA algorithms are summarised in Table III, whereby the number of operations is computed for each algorithm. We consider the cases where subcarrier SNRs are either ordered prior to involving g-GPA, or where the ordering is left to any of the g-GPAs. Note that for the full GPA algorithm ordering subcarriers does not lead to any improvement in complexity as discussed above.

The quantities L_1 and $L_2 = \sum_{k=0}^{K-1} L_2^k$ in Table III denote the average number of iterations of the while loop for the GPA (Table I) and the g-GPA (Table II) algorithms, respectively. Obviously, N_k cannot be easily quantified as it depends on both CNR_i, which is a χ^2 random variable, and the operating SNR. Therefore the complexity of g-GPA is evaluated in a heuristic fashion. In the worst case and by assuming that subcarriers are uniformly distributed across all QAM groups, i.e. $N_k = \frac{N}{K}$, the complexity of the g-GPA algorithm can be approximated as given in Table III which is lower than its GPA counterpart.

Table III
COMPUTATIONAL ANALYSIS FOR BOTH GPA AND g-GPA ALGORITHMS

algorithm	no. of operations
GPA (order and no order)	$L_1(2N+7) + 4N + 1$
g-GPA (no order)	$\sum_{k=0}^{K-1} L_2^k(2N_k+4) + 2N_k + 2$ $\approx L_2(2\frac{N}{K} + 4) + 2\frac{N}{K} + 2$
g-GPA (order)	$\sum_{k=0}^{K-1} L_2^k(N_k+5) + 2N_k + 2$ $\approx L_2(\frac{N}{K} + 5) + 2\frac{N}{K} + 2$

V. SIMULATION RESULTS AND DISCUSSION

Computer simulations are conducted to investigate the performance of our proposed algorithms compared with both UPA and GPA algorithms. A 4x4 MIMO-OFDM system characterised by an ISI MIMO channel **H** of 6-tabs FIR is considered, where the entries of **H** are complex Gaussian random variables with zero-mean and unit-variance, i.e., $h_{ij} \in \mathcal{CN}(0,1)$. Results are averaged over 5,000 different channel realisations for target BER $\mathcal{P}_b^{\text{target}} = 10^{-4}$ and various levels of SNRs using square QAM modulation schemes: QPSK, 16-QAM, and 64-QAM, i.e., $M_k = 2^k, k = 2, 4, 6$.

System throughput is examined and shown in Fig. 3 for a 24-subcarrier system. It is evident that UPA represents an inefficient way of power allocation since the performance is approximately 2 to 8 dB below other algorithms, and provides approximately only half the throughput at 12.5 dB SNR. By considering power transfer between groups, both Mu-GPA and Md-GPA algorithms outperform the g-GPA. Interestingly, Mu-GPA performs better at low SNR, while Md-GPA performs better at higher SNR. This can be explained with the aid of (14) and (16), that is, for low-to-medium SNRs it is most likely that $P_K^{\text{ex}} < P_0^{\text{ex}}$ as transmit power cannot afford occupation of subcarriers in the highest QAM group, as a consequence Mu-GPA performs better in this SNR region. In contrary, for medium-to-high SNRs $P_K^{\text{ex}} > P_0^{\text{ex}}$ is expected to be high and then Md-GPA is likely to be advantageous as it exploits P_K^{ex} in its power re-distribution.

In order to highlight the simplification gained by our proposed scheme, Fig. 4 demonstrates the cumulative distribution function (CDF) of the computation time of the g-GPA algorithm compared to the standard GPA algorithm for a 1024-subcarrier system. Two different SNRs values of 15 and 35 dB are considered, and it is clear that the g-GPA algorithm has a higher computational efficiency in particular for large SNR values. The effect of subcarrier ordering is also evident as discussed in Sec. IV.
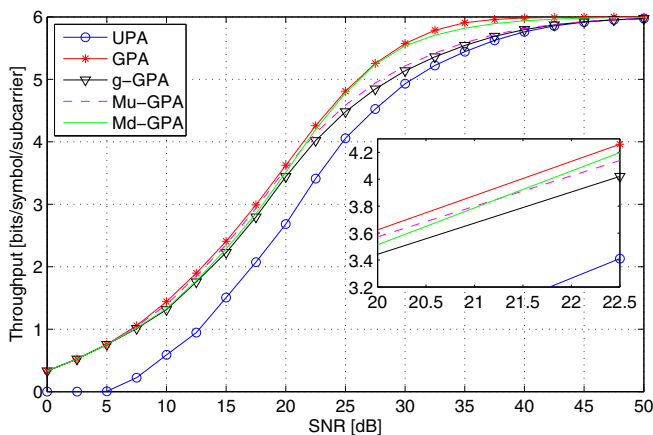

Figure 3. Overall throughput for a 24-subcarrier MIMO-OFDM system with $\mathcal{P}_b^{\text{target}} = 10^{-4}$.
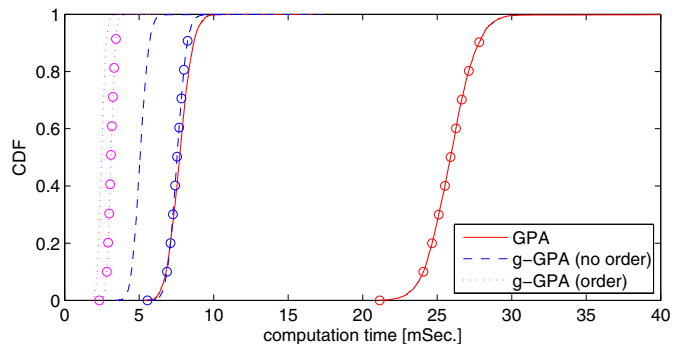

Figure 4. Cumulative distribution function of the computation time for a 1024-subcarrier system with $\mathcal{P}_b^{\text{target}} = 10^{-4}$ at SNR values of 15 dB (without circles) and 35 dB (with circles).

VI. CONCLUSIONS

Suboptimal discrete bit loading schemes have been proposed in this paper. Compared to optimum greedy power allocation (GPA) algorithm, these schemes perform GPA on groups of subcarriers. Two of these schemes have been suggested with a refined power allocation stage. Simulations show that performance very close to the GPA algorithm can be attained by the two algorithms — for the Mu-GPA at low SNR, for the Md-GPA at high SNR — at a much reduced computational complexity.

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