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Greedy Power Allocation for Multicarrier Systems with Reduced Complexity

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Abstract

In this paper we consider a reduced complexity discrete bit loading for Multicarrier systems based on the greedy power allocation (GPA) under the constraints of transmit power budget, target BER, and maximum permissible QAM modulation order. Compared to the standard GPA, which is optimal in terms of maximising the data throughput, three suboptimal schemes are proposed, which perform GPA on subsets of subcarriers only. These subsets are created by considering the minimum SNR boundaries of QAM levels for a given BER. We demonstrate how these schemes can reduce complexity. Two of the proposed algorithms can achieve near optimal performance by including a transfer of residual power between groups at the expense of a very small extra cost. It is shown that the two near optimal schemes, while greatly reducing complexity, perform best in two separate and distinct SNR regions.

1 Introduction

In OFDM, or general transmultiplexing techniques a number of independent subcarriers arise for transmission, which differ in SNR. Maximising the channel capacity or data throughput under the constraint of limited transmit power leads to the well-known and simple waterfilling algorithm [1]. Waterfilling is generally followed by bit loading, where b_i bits are allocated to the QAM symbols transmitted over the *i*th subcarrier. To achieve an identical target bit error ratio (BER) across all subcarriers leads to $b_i \in \mathbb{R}$, which needs to be rounded off to the nearest integer $b_i^{(r)} = \lfloor b_i \rfloor$, thus lowering the overall throughput. Furthermore, unbounded modulation orders $b_i^{(r)} \to \infty$ in the case of infinite SNR are required to efficiently utilise the transmit power but are practically unfeasible.

Pure waterfilling-based solutions have been reported in [2, 3, 4], leading to some of the above stated problems. Reallocation of the excess power when realising the target BER given $b_i^{(r)} \in \mathbb{Z}$ and the SNR in the *i*th subcarrier has lead to a rate-optimal algorithm known as the greedy algorithm [5, 6], of which a number of difference variation have emerged constraining the average BER [7] or the total power [8]. For a good review of greedy algorithms, please refer to [9].

While achieving rate optimality, the family of greedy algorithms is also known to be greedy in terms of computing requirements. Therefore, reduced complexity schemes are either waterfilling-based only [2] or aim at simplifications [10]. Different from our previous work of MIMO sum-rate maximisation presented in [11, 12], the interest of this paper is focusing on OFDM Multicarrier systems to more elaborate on complexity reduction especially for higher numbers of subcarriers. A novel suboptimal greedy algorithm is proposed, whereby the power re-allocation is performed in subsets of the subcarriers. We show that some simple overall redistribution can be included at very low cost, whereby two different methods on terms of approximate overall optimisation are discussed. These suboptimal schemes, while greatly reducing complexity, are hardly sacrifice any performance compared to the full greedy algorithm, provided that the correct algorithmic version is applied for specific SNR regions.

The paper is outlined as follows. The greedy approach is first reviewed in Sec. 2. Thereafter, our proposed reducedcomplexity schemes are outlined in Sec. 3, where computational complexity are evaluated in Sec. 4. These Schemes are evaluated by a number of simulations results, which are reported and discussed in Sec. 5. Finally, conclusions are drawn in Sec. 6.

2 Greedy Approach Review

2.1 Problem Statement

We consider the problem of maximising the transmission rate over an OFDM Multicarrier system. A single-input single-output (SISO) OFDM system, whereby the ISI channel characterised by a FIR vector $\mathbf{h} = [h_0 \cdots h_L] \in \mathbb{C}^{L+1}$ of order *L* is converted into an *N*-subcarrier system with different gains g_i , $i = 1 \cdots N$. The extension to the MIMO OFDM case is straightforward. The *i*th subcarrier experiencing the gain g_i will be used to transmit b_i bits per symbol. Maximising the sum-rate

$$\max\sum_{i=1}^{N} b_i,\tag{1}$$

with total power budget, target bit error ratio (BER), and maximum permissible QAM modulation order constraints can be formulated as

$$\sum_{i=1}^{N} P_{i} \leq P_{\text{budget}}, \quad \mathscr{P}_{b,i} = \mathscr{P}_{b}^{\text{target}} \quad \text{and} \quad b_{i} \leq b^{\max}, \forall i$$
(2)

where P_i is the amount of power allocated to the *i*th subcarrier to achieve a BER $\mathscr{P}_{b,i}$, and b^{max} is the maximum number of permissible allocated bits per subcarrier. Note that BERs are assumed equal, i.e. $\mathscr{P}_{b,i} = \mathscr{P}_b^{\text{target}}$ in (2) for all subcarriers $i = 1 \cdots N$ and therefore the subscript *i* will be dropped from the BER notation.

The carrier-to-noise ratio of the *i*th subcarrier can be defined as

$$CNR_i = \frac{g_i^2}{\mathcal{N}_0},\tag{3}$$

where \mathcal{N}_0 is the total noise power at the receiver, whereas the SNR of this subcarrier is

$$\gamma_i = P_i \times \text{CNR}_i \,. \tag{4}$$

For BPSK modulation, BER can be related to the subcarrier SNR γ_i as $\mathscr{P}_b = Q(\sqrt{2\gamma_i})$, while for rectangular M-QAM modulation of order M_k , BER is given by [13]

$$\mathscr{P}_{b} = \frac{1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{M_{k}}}\right)Q\left(\sqrt{\frac{3\gamma_{i}}{M_{k} - 1}}\right)\right]^{2}}{\log_{2}M_{k}}.$$
(5)

Therefore, symbols of b_k -bits, $b_k = \log_2 M_k$ can be loaded to a subcarrier with minimum required SNR obtained from (5) as

$$\gamma_{k}^{\text{QAM}} = \frac{M_{k} - 1}{3} \left[Q^{-1} \left(\frac{1 - \sqrt{1 - \mathscr{P}_{b} \log_{2} M_{k}}}{2 \left(1 - 1/\sqrt{M_{k}} \right)} \right) \right]^{2}, \tag{6}$$

where Q^{-1} is the inverse of the well-known Q function, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du$.

The problem is solved in two steps, (i) a uniform power allocation (UPA) initialisation step and (ii) the Greedy algorithm, both described below.

2.2 UPA Algorithm and Initialisation Setup

The uniform power allocation is performed by the following steps:

- 1. Calculate γ_k^{QAM} for all M_k , $1 \le k \le K$ and $\mathscr{P}_b = \mathscr{P}_b^{\text{target}}$ using (6), where M_K is the maximum QAM constellation that is potentially permissible by the transmission system, i.e., $M_K = 2^{b^{\text{max}}}$.
- 2. Equally allocate P_{budget} among all subcarriers $1 \le i \le N$

$$\gamma_i = P_i \times \text{CNR}_i = \frac{P_{budget}}{N} \times \text{CNR}_i.$$
(7)

3. Reside subcarriers according to their SNR γ_i into QAM groups $G_k, 0 \le k \le K$ bounded by QAM levels γ_k^{QAM} and $\gamma_{k+1}^{\text{QAM}}$ with $\gamma_0^{\text{QAM}} = 0$ and $\gamma_{K+1}^{\text{QAM}} = +\infty$ (cf. Fig. 1) such that

$$\gamma_i \ge \gamma_k^{\text{QAM}} \quad \text{and} \quad \gamma_i < \gamma_{k+1}^{\text{QAM}}.$$
 (8)

4. For each group G_k , load subcarriers within this group with QAM constellation M_k and compute the group's total allocated bits

$$B_k^{\mathbf{u}} = \sum_{i \in G_k} b_{i,k}^{\mathbf{u}} = \sum_{i \in G_k} \log_2 M_k \tag{9}$$

with $B_0^u = 0$. It is clear at this point and from step (3) that subcarriers are resided into QAM groups of SNR levels that is less than their original SNRs; $\gamma_k^{\text{QAM}} \leq \gamma_i$ and therefore leaving some unused (excess) power of

$$P_k^{\text{ex}} = \sum_{i \in G_k} \frac{\gamma_i - \gamma_k^{\text{QAM}}}{\text{CNR}_i} = \sum_{i \in G_k} P_i - \frac{\gamma_k^{\text{QAM}}}{\text{CNR}_i}.$$
 (10)

5. The overall system allocated bits and used power for the uniform power allocation scheme are therefore,

$$B_u = \sum_{k=1}^K B_k^u \tag{11a}$$

$$P_{\rm u}^{\rm used} = P_{\rm budget} - \sum_{k=0}^{K} P_k^{\rm ex} = P_{\rm budget} - P^{\rm ex}, \tag{11b}$$

where P^{ex} is the overall excess power missed by the UPA scheme. Note that the summation in (11a) starts from group G₁ since none of the subcarriers in G₀ will be loaded in this initialisation.

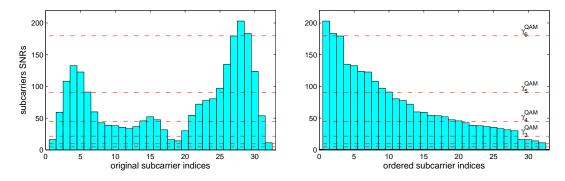


Figure 1: An illustrative example for grouping subcarriers of a 32-subcarrier system of SNR = 25 dB and $\mathcal{P}_{b}^{\text{target}} = 10^{-3}$ with $b^{\text{max}} = 6$ bits.

It is clear from (11b) that P_k^{ex} has an obvious impact on the performance of the UPA scheme. The worst cases are P_0^{ex} and P_K^{ex} which reveal inefficient power allocations in situations of low-to-medium and medium-to-high SNRs, respectively, as will be discussed in Sec. 5.

2.3 Full Greedy Power Allocation (GPA) Algorithm

Based on the initialisation step described in the previous section, the full GPA algorithm [8] performs an iterative redistribution of the unallocated power of the UPA algorithm P^{ex} by applying the algorithmic steps detailed in Table 1. At each iteration, this algorithm tries to increase bit loading by upgrading (to the next higher QAM level) the subcarrier of the least power requirements through an exhaustive search, step (4) in Table 1 for all subcarriers N. When either of the following events occurred: i) the remaining power cannot afford any further upgrades or ii) all subcarriers appear in the highest QAM level K, the algorithm stops resulting in an overall system allocated bits

$$B_{\rm gpa} = \sum_{i=1}^{N} b_i^{\rm gpa}.$$
 (12)

Table 1: Full GPA algorithm applied to the initialisation step of the UPA algorithm

	Initialisation:				
1.	Set power available for GPA to $P_a^{\text{gpa}} = P^{\text{ex}}$ in (11b)				
	For each subcarrier <i>i</i> do the following:				
2.	Set $b_i^{\text{gpa}} = b_i^{\text{u}}$ in (9) and index $k_i = k$ in (8)				
3.	Cal. the min required upgrade power: $P_i^{\text{up}} = \frac{\gamma_{k_i+1}^{QAM} - \gamma_{k_i}^{QAM}}{CNR_i}$				
	Recursion:				
	while $P_a^{\text{gpa}} \ge \min(P_i^{\text{up}})$ and $\min(k_i) < K, 1 \le i \le N$				
4.	$j = \operatorname{argmin}(P_i^{\operatorname{up}})$				
5.	$k_i = k_i + 1, P_a^{\text{gpa}} = P_a^{\text{gpa}} - P_i^{\text{up}}$				
5.	$k_j = k_j + 1, r_a = r_a r_j$ if $k_i = 1$				
	J				
6.	$b_j^{ ext{gpa}} = \log_2 M_1, P_j^{ ext{up}} = rac{\gamma_{k_j+1}^{ ext{QAM}} - \gamma_{k_j}^{ ext{QAM}}}{ ext{CNR}_j}$				
	elseif $k_j < K$				
7.	$b_j^{ ext{gpa}} = b_j^{ ext{gpa}} + \log_2\left(rac{M_{k_j}}{M_{k_j-1}} ight), P_j^{ ext{up}} = rac{\gamma_{k_j+1}^{ ext{QAM}} - \gamma_{k_j}^{ ext{QAM}}}{ ext{CNR}_j}$				
	else				
8.	$b_j^{ ext{gpa}} = b_j^{ ext{gpa}} + \log_2{\left(rac{M_{k_j}}{M_{k_j-1}} ight)}, P_j^{ ext{up}} = +\infty$				
	end				
	end				

3 Proposed Low-Cost GPA

Given B_k^u as defined in (9) and P_k^{ex} in (10), three low-cost greedy algorithms are proposed to efficiently utilise the total excess power of the uniform power allocation in (11b) using the QAM grouping concept. More precisely, GPA is separately accomplished for each QAM group G_k aiming to increase the total bit allocation to this group and therefore the overall system allocated bits. Based on the way of making use of P_k^{ex} , we propose three different algorithms, which below are referred to as (i) grouped GPA (g-GPA), (ii) power Moving-up GPA (Mu-GPA) and (iii) power Moving-down GPA (Md-GPA).

3.1 g-GPA Algorithm

As discussed in Sec. 2, optimum discrete bit loading with total power and maximum permissible QAM order constraints can be performed by the GPA approach. However, the direct application of GPA is computationally very costly due to the fact that at each simulation iteration an exhaustive sorting of all subcarriers is required as evident from Table 1.

A simplification of GPA can be achieved if subcarriers are firstly divided into QAM groups G_k , $0 \le k \le K$ according to their SNRs as shown in Fig. 1, where we assume a Multicarrier systems with subcarriers not ordered with respect to their SNR yet. GPA is therefore independently applied to each group G_k , trying to allocate as much of the excess power P_k^{ex} within this QAM group. This excess power is iteratively allocated to subcarriers within this group according to the greedy concept with the aim of upgrading as many subcarriers as possible to the next QAM level.

The pseudo code for the *k*th QAM group G_k of the g-GPA algorithm is given in Table 2. Note that different from the standard GPA, this algorithm permits upgrades to the next QAM level only for a given QAM group $(P_j^{up}$ is set to $+\infty$ in steps (5) and (6) in Table 2) and therefore may leave some left-over (LO) power P_k^{LO} for each QAM group G_k , resulting in a total left-over power of

$$P_{\rm g}^{\rm LO} = \sum_{k=0}^{K-1} P_k^{\rm LO} + P_K^{\rm ex} \,. \tag{13}$$

Intuitively, for the overall performance of the g-GPA algorithm, the algorithm in Table 2 has to be executed *K* times, one for each QAM group, from G_0 to G_{K-1} resulting in an overall system that allocated bits given by

$$B_{\rm g} = \sum_{k=0}^{K-1} B_k^{\rm g} + B_K^{\rm u}.$$
 (14)

Input: $b_{i,k}^{u}$, P_{k}^{ex} , $\gamma_{d,k}^{QAM} = \gamma_{k+1}^{QAM} - \gamma_{k}^{QAM}$, CNR_{i} Output: B_{k}^{g} , P_{k}^{LO}				
1.	$\forall i \in G_k$, cal. the min required upgrade power: $P_i^{\text{up}} = \frac{\gamma_{d,k}^{\text{pAM}}}{\text{CNR}_i}$			
2.	Initiate $b_{i,k}^{g} = b_{i,k}^{u}$ and $P_{k}^{LO} = P_{k}^{ex}$			
	while $P_k^{\text{LO}} \ge \min(P_i^{\text{up}})^{n}$			
3.	$j = \operatorname{argmin}(P_i^{\operatorname{up}})$			
4.	$P_k^{\mathrm{LO}} = P_k^{\mathrm{LO}} - P_j^{\mathrm{up}}$			
	$\mathbf{if} \ k = 0$			
5.	$b_{j,k}^{\mathrm{g}} = \log_2 M_1, P_j^{\mathrm{up}} = +\infty$			
	else			
6.	$b^{\mathrm{g}}_{j,k}=b^{\mathrm{g}}_{j,k}+\mathrm{log}_2rac{M_{k+1}}{M_k}, P^{\mathrm{up}}_j=+\infty$			
	end			
	end			
7.	$B_k^{ m g} = \sum\limits_{i \in { m G}_k} b_{i,k}^{ m g}$			

Table 2: g-GPA algorithm for subcarriers in the *k*th QAM group G_k

3.2 Mu-GPA Algorithm

The g-GPA algorithm results in unused P_k^{LO} for each QAM group. This residual power can be exploited by a second stage, whereby it is proposed to move power upwards starting from the lowest QAM group, as outlined in Fig. 2(a). This modifies the g-GPA algorithm by considering the left-over power P_0^{LO} of the QAM group G_0 after running the g-GPA algorithm on that group , and assign this power for redistribution to group G_1 . Any left-over power after running g-GPA on G_1 is then passed further upwards to G_2 , and so forth. At the *k*th algorithmic iteration, the Mu-GPA algorithm is working with G_k and tries to allocate the sum of the excess power missed by the UPA algorithm of that group as well as the left-over power of the application of the g-GPA algorithm to the previous group G_{k-1} , i.e., $P_k^{\text{ex}} + P_{k-1}^{\text{LO}}$ (cf. Fig. 2(a)). Finally, the left-over power resulting from the QAM group G_{K-1} is added to the excess power of the K^{th} QAM group P_K^{ex} to end up with a final left-over power

$$P_{\rm Mu-g}^{\rm LO} = P_{K-1}^{\rm LO} + P_K^{\rm ex}$$
(15)

and overall system allocated bits

$$B_{\rm Mu-g} = \sum_{k=0}^{K-1} B_k^{\rm Mu-g} + B_K^{\rm u} \,. \tag{16}$$

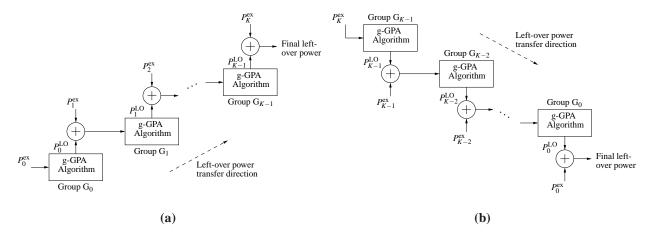


Figure 2: Algorithmic arrangements for (a) Mu-GPA and (b) Md-GPA with final left-over power in (15) and (17), respectively.

3.3 Md-GPA Algorithm

A second algorithm is proposed to exploit the residual power P_k^{LO} of each QAM group but in a reverse direction compared to the Mu-GPA algorithm, starting from the highest-indexed QAM group G_{K-1} downwards to the leastindex QAM group G_0 . This procedures is illustrated in Fig. 2(b) which show the direction of the left-over power flow. Proceeding downwards, at the *k*th stage this algorithm applies the g-GPA algorithm for the available power that comprises both the excess power missed by the UPA algorithm of the previous QAM group (G_{k+1} in this case) and the left-over power of the previous stage, i.e., $P_{k+1}^{\text{ex}} + P_{k+1}^{\text{LO}}$, as also characterised in Fig. 2(b). Therefore, the excess power of the QAM group under consideration is not utilised within this group but is transferred to the next working group along with the left-over power of the former QAM group. This will finally results in a left-over power of

$$P_{\rm Md-g}^{\rm LO} = P_0^{\rm LO} + P_0^{\rm ex},$$
(17)

and overall system allocated bits

$$B_{\rm Md-g} = \sum_{k=0}^{K-1} B_k^{\rm Md-g} + B_K^{\rm u}.$$
 (18)

4 Complexity Evaluation

Instead of jointly applying GPA algorithm across all subcarriers which consequently requires high system complexity especially for large numbers of subcarriers, the g-GPA algorithm only addresses a subset of subcarriers within a specific QAM group at a time. Beyond the division of the QAM grouping concept, a further reduction in complexity can be achieved if subcarriers are initially ordered in their gains CNR_i (see Fig. 1). In this case the search step (3) in Table 2 can be replaced by a simple incremental indexing.

Referring to Table 1 and Table 2 the computational complexity of both GPA and g-GPA algorithms is summarised in Table 3, whereby the no. of operations is computed for each algorithm. Both subcarriers "no order" and "order"

Table 3: Computational analysis for both GPA and g-GPA algorithms

algorithm	GPA (order and no order)	g-GPA (no order)	g-GPA (order)
no. of operations	$L_1(2N+7) + 4N + 1$	$\alpha \left[L_2(2\beta + 4) + 2\beta + 2 \right]$	$\alpha \left[L_2(\beta+5) + 2\beta + 2 \right]$
		$\approx K \left[L_2 \left(\frac{2N}{K} + 4 \right) + \frac{2N}{K} + 2 \right]$	$\approx K \left[L_2 \left(\frac{N}{K} + 5 \right) + \frac{2N}{K} + 2 \right]$

cases are considered. Note that for the GPA algorithm ordering subcarriers does not lead to any improvement in complexity as the search step (4) in the **while** loop has to include all subcarriers. This is due to the fact — which represents the core idea behind this work — that by relaxing the grouping concept it is possible to find subcarriers in lower QAM levels that need less power to upgrade than others in higher QAM levels. The quantities L_1 and L_2 in Table 3 denote the no. of iterations of the **while** loops for the GPA (Table 1) and the g-GPA (Table 2) algorithms, respectively. For the g-GPA algorithm α and β stand, respectively, for the average no. of QAM groups occupied by all subcarriers *N* and the average no. of subcarriers per QAM group. Obviously, α and β cannot be easily quantified as they both depend on CNR_i, which is a χ^2 random variable, and the operating SNR, therefore the complexity of the g-GPA algorithm is assessed in a heuristic fashion. In the worst case and by assuming that subcarriers are uniformly distributed across all QAM groups, the g-GPA computational complexity is approximately given by the second line formula of Table 3 which is still less than its GPA counterpart.

5 Simulation Results and Discussion

Secs. 3.2 and 3.3 have shown that both Mu-GPA and Md-GPA algorithms work very similarly in utilising the left-over power P_k^{LO} for all groups $k, 0 \le k \le K - 1$ that remained unused by the g-GPA algorithm. The two algorithms differ in the direction in which P_k^{LO} is transferred. Below we compare the two algorithms with the UPA, GPA, and the g-GPA approaches.

Simulations are conducted for a SISO system of 6-tabs FIR, where the entries of the SISO channel **h** are drawn from complex Gaussian processes with zero-mean and unit-variance, i.e., $h_l \in \mathscr{CN}(0,1)$. Results presented below refer to ensemble averages across 10^3 channel realisations for target BER $\mathscr{P}_b^{\text{target}} = 10^{-3}$ and various levels of SNRs

using QAM modulation schemes $M_k = 2^k, k = 1 \cdots K$ with K = 6 being the maximum permissible QAM level of constellation size $M_K = 64$ which is equivalent to encoding 6 bits per data symbol.

The total system throughput is examined and shown in Fig. 3 for all proposed algorithms in addition to both UPA and standard GPA algorithms. It is evident that UPA represents an inefficient way of bit loading since the performance is approximately 2 to 8 dB below other algorithms, and provide approximately half the throughput at 15 dB SNR.

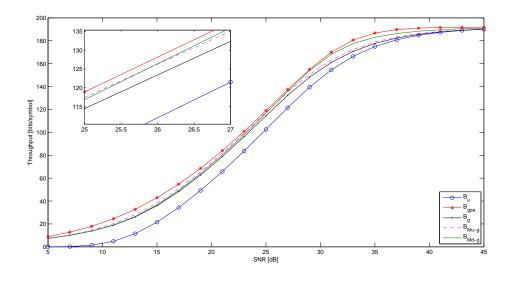


Figure 3: Overall throughput for 32-subcarrier system with $\mathscr{P}_{h}^{\text{target}} = 10^{-3}$.

Of the proposed reduced complexity greedy algorithms, both Mu-GPA and Md-GPA algorithms outperform the g-GPA without the refinement stage to allocate the residual power across QAM groups. Interestingly, Mu-GPA performs better at low SNR, while Md-GPA performs better at higher SNRs. This can be attributed to the fact that for low-to-medium SNRs P_K^{ex} (which is missed by the Mu-GPA) in this case will be relatively low and can be allocated without violating the constraint on maximum QAM levels. While P_0^{ex} (which is missed by the Mg-GPA) is most likely to be high, please see (10) and Fig. 1. For medium-to-high SNRs $P_K^{\text{ex}} > P_0^{\text{ex}}$ can be expected to be high, and then Md-GPA is likely to be advantageous in its bit allocation as P_K^{ex} is fully utilised by the Md-GPA algorithm.

Finally, for very high SNRs most subcarriers will appear in the highest QAM group G_K as their SNRs, γ_i in (7), exceed the highest QAM level γ_K^{QAM} in (6). As a result, the overall system throughput of all different algorithms reaches its expected maximum.

In order to assess the computational complexity of the proposed scheme compared to the standard GPA algorithm, Fig. 4 shows the computation time against the no. of subcarriers N for the g-GPA algorithm with both "no order" and "order" cases compared to the GPA algorithm. Two different SNRs values of 15 and 35 dB that is suitable for the application of mobile and fixed wireless communication, respectively, are considered. It is clear that the g-GPA algorithm is much computationally efficient in particular for large values of N and SNRs, the effect of subcarriers ordering is also evident as discussed in Sec. 4.

6 Conclusions

The optimum solution of discrete bit loading is provided by the Greedy algorithm, which operates across all subcarriers but is computationally very expensive. Therefore, in this paper suboptimal reduced complexity alternatives have been explored. The common theme amongst the proposed algorithms is to restrict the Greedy algorithm to subsets of subcarriers, which are grouped according to the QAM levels assigned to them in the uniform power allocation stage. Two different schemes have been suggested, of which one moves the left-over power upwards from the lowest to the highest subgroup, where in the high SNR case a limitation by the maximum defined QAM level can restrict the performance. A second scheme moves the power from the highest towards the lower subgroups, whereby at low SNR the channel quality in the lowest subgroup may not be such that it can be lifted across the lowest QAM level, and

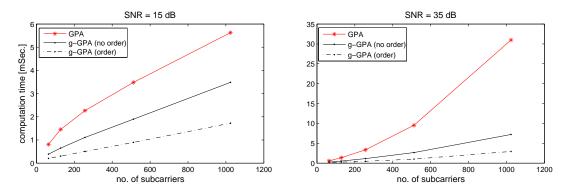


Figure 4: Average computation time comparison of g-GPA and GPA algorithms.

hence no bits may be loaded with the excess power. However, in general both algorithms perform very close to the GPA in their respective domains of preferred operation, thus permitting to allocate power close to the performance of the GPA at a much reduced cost.

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