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Sum-Rate Maximisation Comparison using Incremental Approaches with Different Constraints

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***Abstract*—In this work, the problem of rate maximisation of multichannel systems is considered. Two greedy allocation approaches using power (GPA) and bit (GBA) loading schemes with a slight difference in design constraints that aiming to maximise the overall system throughput are compared. Both algorithms use incremental bit loading whereby, the GPA is designed with main interest of efficient power utilisation. Whereas, the GBA sacrifices power utilisation to another design issue of achieving an average bit error ratio (BER) less than the target BER. Simulation results shows that with GPA algorithm better throughput is gained over the GBA algorithm while the latter guaranteed less BER.**

***Index Terms*—Incremental bit-loading, power allocation, waterfilling algorithms, constrained optimisation, greedy algorithms.**

I. INTRODUCTION

Adaptation of transmission resources to channel conditions in multichannel systems has been proved to significantly enhance the overall system performance provided that channel state information (CSI) is known to the transmitter [1], [2]. This includes the achievement of either higher data rates or lower power requirements under one or more practical/design constraints known respectively in the literature as *rate maximisation* [3], [4], or *margin maximisation* [5], [6]. Commonly, these multichannels arise for multicarrier systems (e.g., OFDM) by converting the frequency-selective channel into a number of narrowband subchannels and for MIMO systems by using singular value decomposition (SVD), the later is the interest of this paper. In both cases we result in a number of subchannels with different gains over which a reliable communication is aimed. The parameters to be considered in such loading problems are: data rate, bit error ratio (BER) and total transmit power. The sum-rate of a multichannel system with different subchannel gains is of particular interest from the system design point of view which can be optimised using power and/or bit loading schemes.

Research for power and bit allocation problems is usually considering closed form expressions for either channel capacity [3], [7] or probability of bit error [8], [9]. Alternatively, incremental (greedy) approaches optimising sum-rate using power [10] and bit [11] loading schemes achieve higher rate at the expense of computational complexity.

In this paper, the sum-rate maximisation is considered using both power and bit loading schemes. Two different greedy approaches are examined and compared, both are trying to maximise the overall system rate with the same set of constraints. However, one of these algorithms considers greedy power allocation (GPA) that achieves only the desirable target BER and therefore would save some unused power from the available transmit power budget. The other algorithm uses the greedy approach but with bit loading (power is uniform distributed among all subchannels) and with the main concern of achieving an average BER not to exceed the target BER, we call this algorithm: greedy bit allocation (GBA).

The rest of this paper is organised as follows. In Sec. II the sum-rate problem with different design constraints is described. While the greedy approach solutions to this problem is given in

Sec. III. Simulation results to these solutions are evaluated and discussed in Sec. IV, whereby a number of conclusions are drawn in Sec. V.

II. Problem Formalisation

We consider the problem of maximising the sum-rate of a narrowband MIMO system characterised by a $N_R \times N_T$ channel matrix $\mathbf{H}$ under the constraints of

1) a fixed total transmit power budget P_{budget},
2) a specified target BER $\mathcal{P}_b^{\text{target}}$, and a square-QAM modulation scheme

$$M_k = \begin{cases} 2^{b_k}, & 1 \le k \le K \\ 0, & k = 0 \end{cases}, \quad (1)$$

where the maximum constellation size $M_K = 2^{b^{\max}}$, with $b_k \in \{0, 2, 4, \cdots, b^{\max}\}$, is limited.

By means of a SVD, the channel matrix $\mathbf{H}$ can be decoupled into N independent subchannels with gains of descending order $\sigma_i^2, 1 \le i \le N$, where $N = \operatorname{rank}(\mathbf{H}) \le \min(N_R, N_T)$ and σ_i are the singular values of $\mathbf{H}$. This maximisation can be defined by the optimisation problem

$$\max \sum_{i=1}^{N} b_i, \quad (2)$$

subjected to the constraints

$$\sum_{i=1}^{N} P_i \le P_{\text{budget}} \quad \text{and} \quad \overline{\mathcal{P}}_b = \mathcal{P}_b^{\text{target}} \quad (2a)$$

or

$$\sum_{i=1}^{N} P_i = P_{\text{budget}} \quad \text{and} \quad \overline{\mathcal{P}}_b \le \mathcal{P}_b^{\text{target}}, \quad (2b)$$

where b_i and P_i are, respectively, the number of bits and amount of power allocated to the ith subchannel. The average system BER is defined as

$$\overline{\mathcal{P}}_b = \frac{\sum_{i=1}^{N} b_i \mathcal{P}_{b,i}}{\sum_{i=1}^{N} b_i} \quad (3)$$

with $\mathcal{P}_{b,i}$ being the BER of the ith subchannel. The aim of this paper is to explore the effect of these two different constraints on the overall data rate by using greedy algorithms that perform power or bit allocation , respectively.

The channel-to-noise ratio of the ith subchannel is given by

$$\text{CNR}_i = \frac{\sigma_i^2}{\mathcal{N}_0}, \quad (4)$$

while its signal-to-noise is

$$\gamma_i = P_i \times \text{CNR}_i \,. \quad (5)$$

Closed form expressions and solutions of the sum-rate in (2) are extensively considered in the literature — see for example [12], [6] for a review — based on the concept of the SNR-gap [13] as

$$b_i = \log_2 \left(1 + \frac{P_i \times \text{CNR}_i}{\Gamma}\right), \quad (6)$$

where Γ denotes the SNR-gap that signifies the loss in SNR of a particular transmission scheme when compared to the theoretical channel capacity. For QAM modulation schemes, this SNR-gap is given by

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{\mathcal{P}_{s,i}}{4}\right)\right]^2, \quad (7)$$

where Q^{-1} is the inverse of the well-known Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du$, and $\mathcal{P}_{s,i}$ is the symbol error rate (SER) of the ith subchannel. It is clear from (7) that Γ is not fixed for all subchannel but depends on the subchannel SER, which in turn depends on b_i and γ_i of (6). This dependence has to be taken into account whenever the rate or the gain in (6) is changed. Nevertheless, the system's operation is in fixed at very low BER — typically 10^{-6} — and higher QAM levels which is not usually the case for realistic applications [4].

Moreover, direct optimisation of (6) with the constraints in (2a) or (2b) under considerations leads to the well-know waterfilling solution [14]. However, the resultant bit allocation obtained by the waterfilling is real valued and requires rounding off to the nearest integer value. This quantisation leads to an overall loss in performance. Alternatively and more accurately, greedy approaches [5], [15], [16] have been proven optimal in this sense [17].

For a certain BER

$$\mathcal{P}_{b,i} \approx \mathcal{P}_{s,i} / \log_2 M_k, \quad (8a)$$

where

$$\mathcal{P}_{s,i} = 1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{M_k}}\right) Q\left(\sqrt{\frac{3\gamma_i}{M_k - 1}}\right)\right]^2 \tag{8b}$$

is the SER for the rectangular QAM modulation [18], the ith subchannel can carry b_i bits per symbols. The word length $b_i = \log_2 M_k$ is drawn from the QAM constellation of size $M_k, 1 \leq k \leq K$ with the minimum required SNR γ_i obtained from (8b) and (8a) as [19]

$$\gamma_k^{\mathrm{QAM}} = \frac{M_k - 1}{3}\left[Q^{-1}\left(\frac{1 - \sqrt{1 - \mathcal{P}_b \log_2 M_k}}{2\left(1 - 1/\sqrt{M_k}\right)}\right)\right]^2 . \tag{9}$$

III. Bit Loading with Different Constraints

Expression (9) is of particular interest in implementing the GPA algorithm, as it returns the minimum required allocated power ($\frac{\gamma_k^{\mathrm{QAM}}}{\mathrm{CNR}_i}$) for a certain subchannel with CNR_i to be loaded with a square QAM modulation scheme of constellation size M_k to achieve a target BER $\mathcal{P}_b$. Therefore optimality is guaranteed in terms of saving power [10]. In the case of GBA algorithm, (8a) and (8b) represent the core issue for this algorithm. Note that different from GPA algorithm, GBA algorithm proceeds with the optimal strategy of avoiding the worst bit-loading that violates the condition of $\mathcal{P}_b^{\mathrm{target}}$, similar to the algorithm of [11]. As (9), (8a), and (8b) are deduced from each other, a fair comparison between GPA and GBA algorithms would be expected. The difference between the algorithms lies in the strategy of optimality considered by each of them as highlighted in this paragraph.

In turn, we will provide details of the algorithmic steps for both GPA and GBA algorithms.

A. Greedy Power Allocation (GPA) Algorithm:

In this algorithm, the sum-rate of a MIMO system with a target BER $\overline{\mathcal{P}}_b = \mathcal{P}_b^{\mathrm{target}}$ is maximised subjected to the constraints in (2a). The GPA algorithm achieves optimality by finding, at each iteration, the subchannel of minimum required power to upgrade to the next QAM level. The initialisation procedures is done using the uniform power allocation (UPA) arrangements (the initialisation part of Table I), whereby subchannels are resided into QAM levels according to their SNRs $\gamma_i, 1 \leq i \leq N$ and a uniform power allocation across all subchannels

$$\gamma_i = \frac{P_{\mathrm{budget}}}{N} \times \mathrm{CNR}_i\,. \tag{10}$$

The procedures of the GPA algorithm with the UPA initialisation is illustrated by Fig. 1 and given completely in Table I. The algorithm starts with loading subchannels with bits to the nearest QAM level that is just less in power than the subchannel SNRs. Then extra power difference from P_{budget}, P_d^{upa} is collected and iteratively allocated to subchannels that do not yet reach their maximum allowable QAM level $M_K = 2^{b^{\max}}$. The sum-rate of this algorithm B_{gpa} and its final power difference from P_{budget}, P_d^{gpa}, are evaluated. The usage power of both UPA and GPA algorithms are therefore

$$P_{used}^{\mathrm{upa}} = P_{\mathrm{budget}} - P_d^{\mathrm{upa}}, \tag{11a}$$

$$\text{and} \quad P_{used}^{\mathrm{gpa}} = P_{\mathrm{budget}} - P_d^{\mathrm{gpa}}, \tag{11b}$$

this is a useful measure of how efficient, in terms of power utilisation, both algorithms are. Note that this quantity is not defined for the GBA algorithm as it uses, by definition, the total power budget.

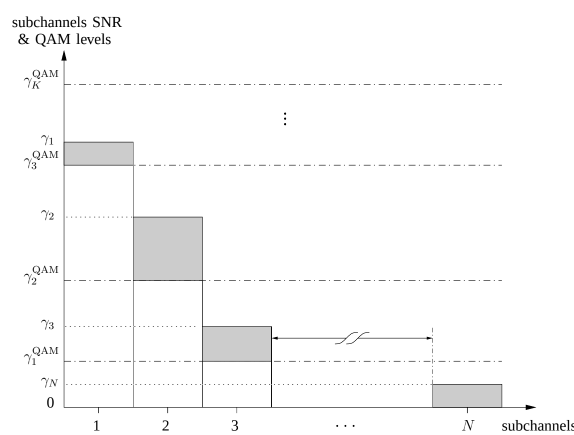

Figure 1: Subchannels residing into QAM levels according to their SNRs and UPA

B. Greedy Bit Allocation (GBA) Algorithm:

In this algorithm another greedy-like approach is proposed, similar to the algorithm in [11],

Table I: Bit Loading using UPA and GPA - Constraint (2a)

Initialisation:
Calculate γ_k^{QAM} for all M_k and $\mathcal{P}_b = \mathcal{P}_b^{\text{target}}$ using (9)
Equally allocate P_{budget} among subchannels using (10)
for $i = 1$ to N
  Find k_i that satisfy: $\gamma_i \geq \gamma_{k_i}^{\text{QAM}}$ and $\gamma_i < \gamma_{k_i+1}^{\text{QAM}}$
  if $k_i = 0$
    $b_i^{\text{upa}} = 0$, $P_i^{\text{up}} = \frac{\gamma_1^{\text{QAM}}}{\text{CNR}_i}$
  elseif $k_i < K$
    $b_i^{\text{upa}} = \log_2 M_{k_i}$, $P_i^{\text{up}} = \frac{\gamma_{k_i+1}^{\text{QAM}} - \gamma_{k_i}^{\text{QAM}}}{\text{CNR}_i}$
  else
    $b_i^{\text{upa}} = \log_2 M_{k_i}$, $P_i^{\text{up}} = +\infty$
  end
end
$B_{\text{upa}} = \sum_{i=1}^{N} b_i^{\text{upa}}$
Collect power difference from total budget: $P_d^{\text{upa}} = \sum_{i=1}^{N} \frac{\gamma_i - \gamma_{k_i}^{\text{QAM}}}{\text{CNR}_i}$
Initiate greedy bit allocation to $b_i^{\text{gpa}} = b_i^{\text{upa}} \, \forall i$ and $P_d^{\text{gpa}} = P_d^{\text{upa}}$
Recursion:
while $P_d^{\text{gpa}} \geq \min(P_i^{\text{up}})$ and $\min(k_i) < K$, $1 \leq i \leq N$
  $j = \underset{1 \leq i \leq N}{\text{argmin}}(P_i^{\text{up}})$
  $k_j = k_j + 1$, $P_d^{\text{gpa}} = P_d^{\text{gpa}} - P_j^{\text{up}}$
  if $k_j = 1$
    $b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 M_1$, $P_j^{\text{up}} = \frac{\gamma_{k_j+1}^{\text{QAM}} - \gamma_{k_j}^{\text{QAM}}}{\text{CNR}_j}$
  elseif $k_j < K$
    $b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 \left(\frac{M_{k_j}}{M_{k_j - 1}} \right)$, $P_j^{\text{up}} = \frac{\gamma_{k_j+1}^{\text{QAM}} - \gamma_{k_j}^{\text{QAM}}}{\text{CNR}_j}$
  else
    $b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 \left(\frac{M_{k_j}}{M_{k_j - 1}} \right)$, $P_j^{\text{up}} = +\infty$
  end
end
$B_{\text{gpa}} = \sum_{i=1}^{N} b_i^{\text{gpa}}$

to maximise the sum-rate of a MIMO system with fully utilisation of the total transmit power budget $\sum_{i=1}^{N} P_i = P_{\text{budget}}$ and achieving an average BER not to exceed the target BER $\overline{\mathcal{P}}_b \leq \mathcal{P}_b^{\text{target}}$ as the constraints in (2b). This algorithm starts with loading all subchannels to bits of the maximum allowable QAM level, i.e. $b^{\max}$, then computes the average BER for all subchannels using (3). Thereafter, the algorithm proceeds with the bit removal approach [15] tied up by keeping the average BER less than or equal to the target BER.

The approach of this algorithm is different from that of the GPA algorithm in the sense that the overall available transmit power budget is completely utilised and the sum-rate maximisation is therefore being sought, at each algorithmic iteration, by removing the bits with the highest degradation impact on the the average BER $\overline{\mathcal{P}}_b$. In section IV the performance of the overall bit rate of this algorithm B_{gba} is compared with that of GPA and UPA algorithms and some useful comments and discussion of both GPA and GBA algorithms are highlighted which clarify system design requirements issues.

Table II: Bit Loading using GBA - Constraint (2b)

Initialisation:
Calculate γ_i for all subchannels with upa ($P_i = \frac{P_{budget}}{N}$) using (5)
Load all subchannels with M_K, i.e., $k_i = K$ and $b_i^{\text{gba}} = \log_2 M_K \quad \forall i$
Calculate $\mathcal{P}_{b,i}$ for all subchannels $1 \leq i \leq N$ using (8a) and (8b)
$\overline{\mathcal{P}}_b = \frac{\sum_{i=1}^{N} \mathcal{P}_{b,i}}{N}$
if $\overline{\mathcal{P}}_b \leq \mathcal{P}_b^{\text{target}}$
  Maintain current loading
else
  Recursion:
  while $\overline{\mathcal{P}}_b > \mathcal{P}_b^{\text{target}}$
    $j = \underset{1 \leq i \leq N}{\text{argmax}}(\mathcal{P}_{b,i})$
    $k_j = k_j - 1$
    if $k_j \geq 1$
      $b_j^{\text{gba}} = b_j^{\text{gba}} - \log_2 \left(\frac{M_{k_j+1}}{M_{k_j}} \right)$
      Re-calculate $\mathcal{P}_{b,j}$ with γ_j and M_{k_j} using (8a) and (8b)
      Update $\overline{\mathcal{P}}_b$ using (3)
    else
      $b_j^{\text{gba}} = 0$, $\mathcal{P}_{b,j} = 0$
      Update $\overline{\mathcal{P}}_b$ using (3)
      if $\sum_j b_j^{\text{gba}} = 0$
        $\overline{\mathcal{P}}_b = 0$
      end
    end
  end
end
$B_{\text{gba}} = \sum_{i=1}^{N} b_i^{\text{gba}}$

IV. Simulation Results

Computer simulations are conducted to investigate the performance of both GPA and GBA algorithms. A 4x4 MIMO system of frequency-flat channel $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ with entries $h_{ij} \in \mathcal{CN}(0,1)$ and $b^{\max}$ is set to 8 bits. It is shown from the sum-rate results for a $P_b^{\text{target}} = 10^{-3}$ in Fig. 2 that GPA algorithm performs better than both GBA and UPA algorithms. An explanation to this is as follows: since the power allocation of the GBA algorithm is done using the UPA, which is inefficient power allocation scheme, therefore, wasting power for unnecessary improvement of the average BER $\overline{\mathcal{P}}_b < \mathcal{P}_b^{\text{target}}$. On the other hand, the GPA algorithm is efficiently utilise the total power budget P_{budget} (power is allocated according to the greedy approach) to maximise the overall throughput while achieving BER to its maximum requirements, $\mathcal{P}_b^{\text{target}}$. This means better investment of the total power towards the rate maximisation problem.

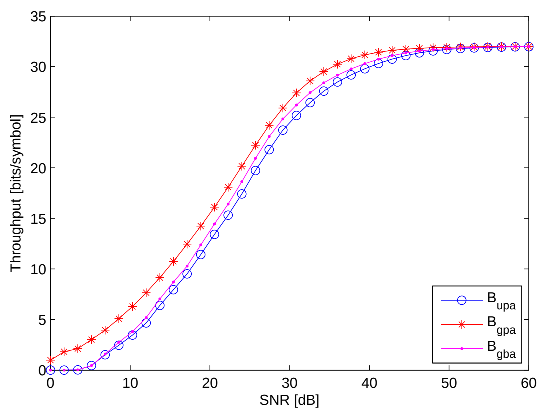

Figure 2: Sum-rate results for a 4x4 MIMO system with $\mathcal{P}_{\mathbf{b}}^{\text{target}} = 10^{-3}$ and varying SNR

Fig. 3 shows the throughput versus target BER at SNR=30 dB, intuitively, throughput is increasing with the increase of target BER. The GPA algorithm outperforms its GBA counterpart by more than 2 bits for a $\mathcal{P}_b^{\text{target}}$ of 10^{-7} and thereafter the improvement is decreasing with further increasing of $\mathcal{P}_b^{\text{target}}$. In Fig. 4, the power utilisation of UPA and GPA algorithms is compared, which shows better performance of GPA over UPA algorithm. Note that GBA algorithm (shown as the P_{budget} curve) cannot be compared here as it spends the total power budget getting improvement in the achieved average BER as shown in Fig. 5. Once the throughput reaches its expected maximum of $4(\text{subchannel})\times 8(\text{bits}) = 32(\text{bits})$, extra power is no longer required. Therefore, the effective used power for both UPA and GPA algorithms in (11a) and (11b), respectively, starts to saturate to the minimum power that is required to achieve the maximum bit-loading $b^{\max}$ for

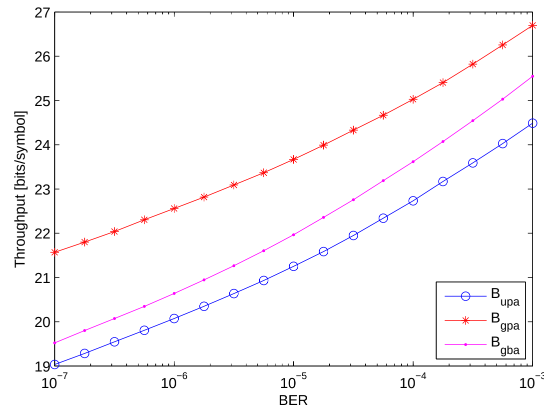

Figure 3: Sum-rate results for a 4x4 MIMO system at SNR = 30 dB and varying $\mathcal{P}_{\mathbf{b}}^{\text{target}}$

all subchannels, i.e., $\sum_i \frac{\gamma_K^{\text{QAM}}}{\text{CNR}_i}$ which is found to be $\approx 45.94\,\text{dB}$ (shown in Fig. 4 by the dashed line). Fig. 5 shows the actual achieved BER of the GBA algorithm, which is less than the target BER. Again both the UPA and GPA algorithms cannot be compared in this results as they both achieve only the target BER (shown as $\mathcal{P}_b^{\text{target}} = 10^{-3}$).

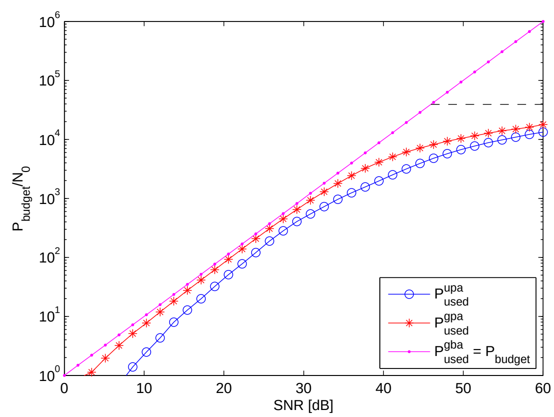

Figure 4: Power utilisation of UPA and GPA algorithms - constraint (2a)

Note that both approaches of GPA and GBA algorithms can be viewed as a conversion between the throughput and BER performance such that if the overall system throughput is of prime interest, then it is better to consider the GPA algorithm. On the other hand, if the average BER performance $\overline{\mathcal{P}}_b$ is the major design issue, then

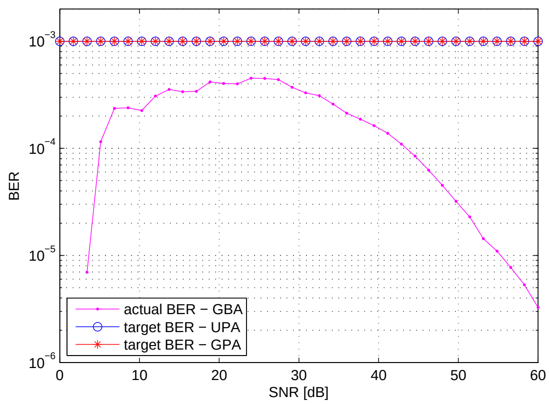

Figure 5: Actual BER of GBA algorithm - constraints (2b)

GBA would be the right algorithm as it always achieves $\overline{\mathcal{P}}_b < \mathcal{P}_b^{\text{target}}$ as shown in Fig. 5.

V. Conclusions

Rate maximisation using greedy power GPA and bit GBA allocation schemes with different constraints is considered. Both algorithms share the main target of optimising the overall system throughput. GPA algorithm tackles this from the efficient power utilisation point of view keeping the target BER to its maximum requirements. While GBA algorithm guarantees less average BER than target BER. This can be thought, respectively, as a conversion between achieving higher system throughput with target BER or attaining higher quality of service in terms of BER with some degradation in throughput, which is useful in design selection of systems with particular interests.

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