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A New Low-Cost Discrete Bit Loading using Greedy Power Allocation

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***Abstract*—In this paper we consider a low cost bit loading based on the greedy power allocation (GPA). Compared to the standard GPA, which is optimal in terms of maximising the data throughput, three suboptimal schemes are suggested, which perform GPA on subsets of subchannels only. We demonstrate how these schemes can reduce complexity. Two of the proposed algorithms can achieve near optimal performance by including a transfer of residual power between subsets at the expense of a very small extra cost. By simulations, we show that the two near optimal schemes perform best in two separate and distinct SNR regions.**

***Index Terms*—Adaptive loading, discrete bit-loading, power allocation, water-filling algorithms, constrained optimisation, greedy algorithms.**

I. Introduction

In OFDM, multiplexing over MIMO channels, or general transmultiplexing techniques a number of independent subcarrier or subchannel arise for transmission, which differ in SNR. Maximising the channel capacity or data throughput under the constraint of limited transmit power leads to the well-known and simple waterfilling algorithm [1]. Waterfilling is generally followed by bit loading, where b_i bits are allocated to the QAM symbols transmitted over the ith subchannel. To achieve an identical target bit error ratio (BER) across all subchannels leads to $b_i \in \mathbb{R}$, which needs to be rounded off to the nearest integer $b_i^{(\mathrm{r})} = \lfloor b_i \rfloor$, thus lowering the overall throughput. Furthermore, unbounded modulation orders $b_i^{(\mathrm{r})} \to \infty$ in the case of infinite SNR are required to efficiently utilise the transmit power but are practically unfeasible.

In order to optimise capacity and throughput, a wide range of methods has been suggested in the literature. Pure waterfilling-based solutions have been reported in [2], [3], [4], leading to some of the above stated problems. Reallocation of the excess power when realising the target BER given $b_i^{(\mathrm{r})} \in \mathbb{Z}$ and the SNR in the ith subchannel has lead to a rate-optimal algorithm known as the greedy algorithm [5], [6], of which a number of difference variation have emerged constraining the average BER [7] or the total power [8]. For a good review of greedy algorithms, please refer to [9].

While achieving rate optimality, the family of greedy algorithms is also known to be greedy in terms of computing requirements. Therefore, reduced complexity schemes are either waterfilling-based only [2] or aim at simplifications [10]. In this paper we propose a novel suboptimal greedy algorithm, whereby the power re-allocation is performed in subsets of the subchannels. We show that some simple overall redistribution can be included at very low cost, whereby two different methods on terms of approximate overall optimisation are discussed. We show that these suboptimal schemes, while greatly reducing complexity, hardly sacrifice any performance compared to the full greedy algorithm, provided that the correct algorithmic version is applied for specific SNR regions.

The paper is outlined as follows. The greedy approach is first reviewed in Sec. II. Thereafter, our proposed low-cost schemes are outlined in Sec. III, and are evaluated by a number of simulations results, which are reported and discussed in Sec. IV. Finally, conclusions are drawn in Sec. V.

II. GREEDY APPROACH REVIEW

A. *Problem Statement*

We consider the problem of maximising the transmission rate over an $N_R \times N_T$ narrowband MIMO system, whereby the channel is characterised by a matrix $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ of complex coefficients which describe the complex gains between pairs of the N_T transmit and the N_R receive antennas. The singular value decomposition (SVD) can be used to decouple the system $\mathbf{H}$ into $N = \mathrm{rank}\{\mathbf{H}\} \leq \min\{N_T, N_R\}$ subchannels whose gains are represented by the singular values $\sigma_i, \quad i = 1 \ldots N$ and are ordered such that $\sigma_i \geq \sigma_{i+1}$. The ith subchannel experiencing the gain σ_i will be used to transmit b_i bits per symbol. We here consider maximising the sum rate

$$\max \sum_{i=1}^{N} b_i, \tag{1}$$

with total power budget and target bit error ratio (BER) constraints. This set of constraints can be formulated as

$$\sum_{i=1}^{N} P_i \leq P_{\mathrm{budget}},\, \mathcal{P}_{b,i} = \mathcal{P}_b^{\mathrm{target}} \,\mathrm{and}\, b_i \leq b^{\max}, \forall i \tag{2}$$

where P_i is the amount of power allocated to the ith subchannel to achieve a BER $\mathcal{P}_{b,i}$, and $b^{\max}$ is the maximum number of permissible allocated bits per subchannel. Note that BERs are assumed equal, i.e. $\mathcal{P}_{b,i} = \mathcal{P}_b^{\mathrm{target}}$ in (2) for all subchannels $i = 1 \cdots N$ and therefore the subscript i will be dropped from the BER notation.

In order to further elaborate on the constraints, the channel-to-noise ratio of the ith subchannel can be defined as

$$\mathrm{CNR}_i = \frac{\sigma_i^2}{\mathcal{N}_0}, \tag{3}$$

where $\mathcal{N}_0$ is the total noise power at the receiver. The signal-to-noise ratio of this subchannel is

$$\gamma_i = P_i \times \mathrm{CNR}_i \,. \tag{4}$$

BER can be related to the symbol error rate (SER) $\mathcal{P}_s$ as

$$\mathcal{P}_b \approx \mathcal{P}_s / \log_2 M_k, \tag{5}$$

where

$$\mathcal{P}_s = 1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{M_k}}\right) Q\left(\sqrt{\frac{3\gamma_i}{M_k - 1}}\right)\right]^2 \tag{6}$$

is the SER for a square QAM modulation of order M_k for a subchannel SNR γ_i [11]. The ith subchannel can carry symbols of b_k-bits, $b_k = \log_2 M_k$ with the minimum required SNR obtained from (6) and (5) as

$$\gamma_k^{\mathrm{QAM}} = \frac{M_k - 1}{3}\left[Q^{-1}\left(\frac{1 - \sqrt{1 - \mathcal{P}_b \log_2 M_k}}{2\left(1 - 1/\sqrt{M_k}\right)}\right)\right]^2, \tag{7}$$

where Q^{-1} is the inverse of the well-known Q function (the tail probability of the normalised Gaussian distribution)

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du \,. \tag{8}$$

The problem is solved in two steps, (i) a uniform power allocation (UPA) initialisation step and (ii) the Greedy algorithm, both described below.

B. *UPA Algorithm and Initialisation Setup*

The initial step of uniform power allocation is performed by the following steps:

1) Calculate γ_k^{QAM} for all $M_k, 1 \leq k \leq K$ and $\mathcal{P}_b = \mathcal{P}_b^{\mathrm{target}}$ using (7), where M_K is the maximum QAM constellation that is potentially permissible by the transmission system, i.e., $M_K = 2^{b^{\max}}$.
2) Equally allocate P_{budget} among all subchannels $1 \leq i \leq N$:

$$\gamma_i = P_i \times \mathrm{CNR}_i = \frac{P_{budget}}{N} \times \mathrm{CNR}_i \,. \tag{9}$$

3) Reside subchannels according to their SNR γ_i into QAM groups $\mathrm{G}_k, 0 \leq k \leq K$ bounded by QAM levels γ_k^{QAM} and $\gamma_{k+1}^{\mathrm{QAM}}$ with $\gamma_0^{\mathrm{QAM}} = 0$ and $\gamma_{K+1}^{\mathrm{QAM}} = +\infty$ (cf. Fig. 1 and Fig. 2) such as:

$$\gamma_i \geq \gamma_k^{\mathrm{QAM}} \,\mathrm{and}\, \gamma_i < \gamma_{k+1}^{\mathrm{QAM}} \,. \tag{10}$$

4) For each group G_k, load subchannels within this group with QAM constellation

M_k and compute the group's total allocated bits

$$B_k^{\mathrm{u}} = \sum_{i \in \mathrm{G}_k} b_{i,k}^{\mathrm{u}} = \sum_{i \in \mathrm{G}_k} \log_2 M_k \qquad (11)$$

with $B_0^u = 0$, and the total excess (unused) power

$$\begin{aligned} P_k^{\mathrm{ex}} &= \sum_{i \in \mathrm{G}_k} \frac{\left(\gamma_i - \gamma_k^{\mathrm{QAM}}\right)}{\mathrm{CNR}_i} \\ &= \sum_{i \in \mathrm{G}_k} P_i - \frac{\gamma_k^{\mathrm{QAM}}}{\mathrm{CNR}_i} . \end{aligned} \qquad (12)$$

5) The overall system allocated bits and used power for the uniform power allocation scheme are therefore,

$$B_u = \sum_{k=1}^{K} B_k^{\mathrm{u}} \qquad (13a)$$

$$P_{\mathrm{u}}^{\mathrm{used}} = P_{\mathrm{budget}} - P^{\mathrm{ex}}, \qquad (13b)$$

where P^{ex} is the overall excess (unused) power of the UPA scheme given by,

$$P^{\mathrm{ex}} = \sum_{k=0}^{K} P_k^{\mathrm{ex}} . \qquad (14)$$

Note that the summation in (13a) starts from group G_1 since none of the subchannels in G_0 will be loaded in this initialisation.

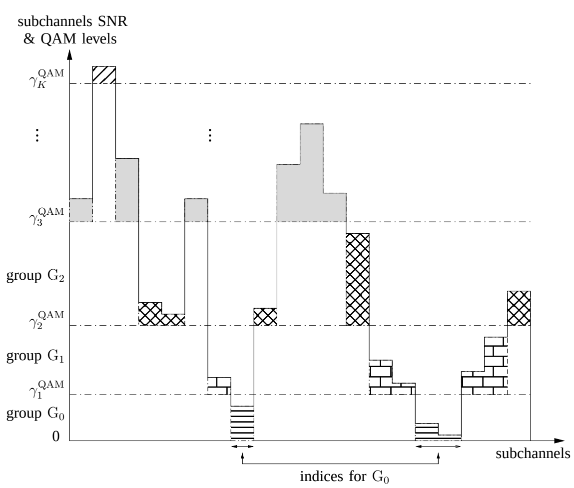

Figure 1: Grouping subchannels of multicarrier systems into QAM groups according to their SNRs in (9) and step (3) Sec. II-B

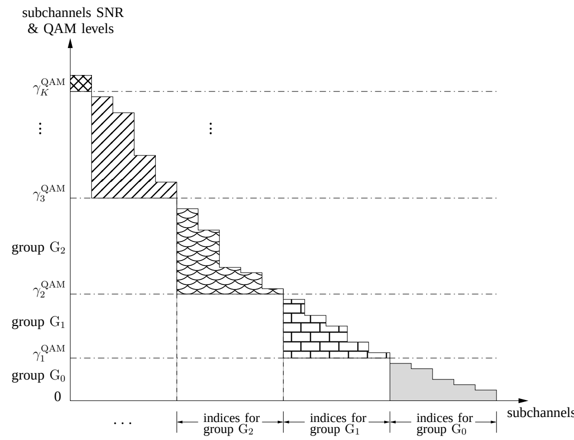

Figure 2: Grouping ordered subchannels of MIMO systems into QAM groups according to their SNRs in (9) and step (3) in Sec. II-B

The difference between the power budget and the overall power $P_{\mathrm{u}}^{\mathrm{used}}$ allocated by the UPA scheme can be improved by a number of algorithms, this represents a useful measure to indicate how well a bit loading scheme utilises the total system transmit power P_{budget}. The closer the $P_{\mathrm{u}}^{\mathrm{used}}$ is to P_{budget}, the better is the utilisation of power achieved by a specific power loading scheme. Therefore, it is clear from (13b) that the amount of excess power P_k^{ex} that is left unused has an obvious impact on the performance of the uniform power allocation scheme. The worst cases are P_0^{ex} and P_K^{ex} which reveal inefficient power allocations in situations of low-to-medium and medium-to-high SNRs, respectively, as will be discussed in Sec. IV.

C. Greedy Power Allocation (GPA) Algorithm

Based on the initialisation step described in the previous section, the full GPA algorithm [5], [6], [9] performs an iterative re-distribution of the unallocated power of the UPA algorithm given in (14) applying the algorithmic steps detailed in Table I. At each iteration, this algorithm tries to increase bit loading by upgrading the subchannel of the least power requirements to the next higher QAM level through an exhaustive search, step (5) in Table I. When either of the following events occurred: i) the remaining power cannot afford any further upgrades or ii) all subchannels appear in the highest QAM level K, the algorithm stops resulting in an overall system allocated bits and used power given, respectively, by

$$B_{\text{gpa}} = \sum_{i=1}^{N} b_i^{\text{gpa}} \tag{15a}$$

$$P_{\text{gpa}}^{\text{used}} = P_{\text{budget}} - P_d^{\text{gpa}} \, . \tag{15b}$$

Table I: Full GPA algorithm applied to the initialisation step of the UPA algorithm

	Initialisation:
1.	Set power difference from total budget of GPA $P_d^{\text{gpa}} = P^{\text{ex}}$ in (14)
	For each subchannel i do the following:
2.	Set $b_i^{\text{gpa}} = b_i^{\text{upa}}$ in (11)
3.	Initiate index $k_i = k$ in (10)
4.	Cal. the min required upgrade power: $P_i^{\text{up}} = \frac{\gamma_{k_i+1}^{\text{QAM}} - \gamma_{k_i}^{\text{QAM}}}{\text{CNR}_i}$
	Recursion:
	while $P_d^{\text{gpa}} \geq \min(P_i^{\text{up}})$ and $\min(k_i) < K$, $1 \leq i \leq N$
5.	$j = \underset{1 \leq i \leq N}{\text{argmin}}(P_i^{\text{up}})$
6.	$k_j = k_j + 1$, $P_d^{\text{gpa}} = P_d^{\text{gpa}} - P_j^{\text{up}}$
	if $k_j = 1$
7.	$b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 M_1$, $P_j^{\text{up}} = \frac{\gamma_{k_j+1}^{\text{QAM}} - \gamma_{k_j}^{\text{QAM}}}{\text{CNR}_j}$
	elseif $k_j < K$
8.	$b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 \left(\frac{M_{k_j}}{M_{k_j-1}} \right)$, $P_j^{\text{up}} = \frac{\gamma_{k_j+1}^{\text{QAM}} - \gamma_{k_j}^{\text{QAM}}}{\text{CNR}_j}$
	else
9.	$b_j^{\text{gpa}} = b_j^{\text{gpa}} + \log_2 \left(\frac{M_{k_j}}{M_{k_j-1}} \right)$, $P_j^{\text{up}} = +\infty$
	end
	end

III. Proposed Low-Cost GPA

With B_k^{u} as defined in (11) and P_k^{ex} in (12), three low-cost greedy algorithms are proposed to efficiently utilise the total excess power of the uniform power allocation $P^{\text{ex}} = \sum_{k=0}^{K} P_k^{\text{ex}}$ and hence P_{budget}. More precisely, GPA is separately accomplished for each QAM group G_k aiming to increase the total bit allocation to this group and therefore the overall system allocated bits. Based on the way of making use of P_k^{ex}, we propose three different algorithms, which below are referred to as (i) QAM-Level Greedy Power Allocation (QAM-L-GPA), (ii) Power Moving-up Greedy Power Allocation (Mu-GPA) and (iii) Power Moving-down Greedy Power Allocation (Md-GPA).

A. QAM-L-GPA Algorithm

As discussed in Sec. II, optimum discrete bit loading with total power and maximum QAM level constraints can be performed by the greedy power allocation (GPA) approach. However, the direct application of GPA is computationally very costly due to the fact that at each simulation iteration an exhaustive sorting of all subchannels is required as evident from Table I.

*1) **Model Description**:* A simplification of GPA can be achieved if subchannels are firstly divided into QAM groups $\text{G}_k, 0 \leq k \leq K$ according to their SNRs as shown in Fig. 1, where we assume a multicarrier systems with subchannel not ordered with respect to their SNR yet. After ordering or due to implicit ordering of the singular values in case of SVD-based decoupling of MIMO systems, the grouping as shown in Fig. 2 arises. GPA is therefore independently applied to each group G_k, trying to allocate as much of the excess power P_k^{ex} that is remaining after application of the UPA algorithm within a QAM group. This excess power is iteratively allocated to subchannels within this group according to the greedy concept with the aim to upgrade as many subchannels as possible to the next QAM level.

Table II: QAM-L-GPA algorithm for subchannels in the kth QAM group G_k

	In: $b_{i,k}^{\text{u}}$, P_k^{ex}, $\gamma_{d,k}^{\text{QAM}} = \gamma_{k+1}^{\text{QAM}} - \gamma_k^{\text{QAM}}$, CNR_i **Out:** B_k^{g}, P_k^{LO}
1.	$\forall i \in \text{G}_k$, cal. the min required upgrade power: $P_i^{\text{up}} = \frac{\gamma_{d,k}^{\text{QAM}}}{\text{CNR}_i}$
2.	Initiate $b_{i,k}^{\text{g}} = b_{i,k}^{\text{u}}$ and $P_k^{\text{LO}} = P_k^{\text{ex}}$
	while $P_k^{\text{LO}} \geq \min(P_i^{\text{up}})$
3.	$j = \underset{i \in \text{G}_k}{\text{argmin}}(P_i^{\text{up}})$
4.	$P_k^{\text{LO}} = P_k^{\text{LO}} - P_j^{\text{up}}$
	if $k = 0$
5.	$b_{j,k}^{\text{g}} = \log_2 M_1$, $P_j^{\text{up}} = +\infty$
	else
6.	$b_{j,k}^{\text{g}} = b_{j,k}^{\text{g}} + \log_2 \frac{M_{k+1}}{M_k}$, $P_j^{\text{up}} = +\infty$
	end
	end
7.	$B_k^{\text{g}} = \sum_{i \in \text{G}_k} b_{i,k}^{\text{g}}$

The pseudo code for the above allocation within the kth QAM group G_k of the QAM-L-GPA algorithm is given in Table II. Note that different from the standard GPA, this algorithm permits upgrades to the next QAM level only for a given QAM group (P_j^{up} is set to $+\infty$ in steps (5) and (6) in Table II) and therefore may leave some left-over (LO) power P_k^{LO} for each QAM group G_k, resulting in a total left-over power of

$$P_{\mathrm{g}}^{\mathrm{LO}} = \sum_{k=0}^{K-1} P_k^{\mathrm{LO}} + P_K^{\mathrm{ex}} \,. \tag{16}$$

Intuitively, for the overall performance of the QAM-L-GPA algorithm, the algorithm in Table II has to be executed K times, once for each QAM group, from G_0 to G_{K-1} resulting in an overall system that allocates bits and uses power according to

$$B_{\mathrm{g}} = \sum_{k=0}^{K-1} B_k^{\mathrm{g}} + B_K^{\mathrm{u}} \tag{17a}$$

and

$$P_{\mathrm{g}}^{\mathrm{used}} = P_{\mathrm{budget}} - P_{\mathrm{g}}^{\mathrm{LO}} \,. \tag{17b}$$

2) ***Complexity Assessment****:* The QAM-L-GPA algorithm can be viewed as a GPA applied to individual QAM groups. Instead of jointly applying GPA algorithm across all subchannels which consequently requires high system complexity especially for large numbers of subchannels, the GPA algorithm only addresses a subset of subchannels within a specific QAM group at a time. Beyond the division of the QAM grouping concept, a further reduction in complexity can be achieved if subchannels are ordered in their gains CNR_i, as the case of SVD-based decoupling of subchannels for MIMO systems. In this case the search step (3) in Table II can be replaced by a simple incremental indexing.

Referring to Table I and Table II the computational complexity of both GPA and QAM-L-GPA algorithms is summarised in Table III, whereby the no. of operations is assessed for each algorithm. Both subchannels "no order" and "order" cases are considered. Note that for the GPA algorithm ordering subchannels does not lead to any improvement in complexity as the search step (5) in the **while** loop has to include all subchannels. This is due to the fact that by relaxing the grouping concept it is possible to find subchannels in lower QAM levels that need less power to upgrade than others in higher QAM levels. The quantities L_1, L_2 in Table III denote the no. of iterations of the **while** loops for GPA (Table I) and QAM-L-GPA (Table II), respectively. Note that it is expected that $L_1 \geq L_2$ as the P^{ex} in (14) collected from all subchannels has to be re-distributed by the GPA algorithm, while P_k^{ex} in (12) collected from only subchannels $i \in \mathrm{G}_k$ is considered by the QAM-L-GPA algorithm. For the QAM-L-GPA algorithm α and β stand, respectively, for the no. of QAM groups occupied by all subchannels N and the no. of subchannels per QAM group. Obviously, α and β are not easily quantified as they both depend on CNR_i which is a χ^2 random variable, therefore the complexity of QAM-L-GPA is assessed in a heuristic fashion. In the worst case and by assuming that subchannels are uniformly distributed across all QAM groups the complexity of QAM-L-GPA is approximately given by the second line formula (cf. Table III) which is still less than its GPA counterpart.

Table III: Computational analysis for both GPA and QAM-L-GPA algorithms

algorithm	no. of operations
GPA (no order)	$L_1(2N+7)+4N+1$
GPA (order)	same as (no order)
QAM-L-GPA (no order)	$\alpha\left[L_2(2\beta+4)+2\beta+2\right] \approx$ $K\left[L_2(\frac{2N}{K}+4)+\frac{2N}{K}+2\right]$
QAM-L-GPA (order)	$\alpha\left[L_2(\beta+5)+2\beta+2\right] \approx$ $K\left[L_2(\frac{N}{K}+5)+\frac{2N}{K}+2\right]$

B. Mu-GPA Algorithm

The QAM-L-GPA algorithm results in unused P_k^{LO} for each QAM group. This residual power can be exploited by a second stage, whereby it is proposed to move power upwards starting from the lowest QAM group, as outlined in Fig. 3 and by the flowchart in Fig. 4. This modifies the QAM-L-GPA algorithm by considering the left-over power P_0^{LO} of the QAM group G_0 after running the QAM-L-GPA algorithm on that group , and assign this power for redistribution to group G_1. Any left-over power after running QAM-L-GPA on G_1 is then passed further upwards to G_2, and so forth. At the kth algorithmic iteration, the Mu-GPA algorithm is working with G_k and tries to allocate the sum of the excess power missed by the UPA algorithm of that group as well as the

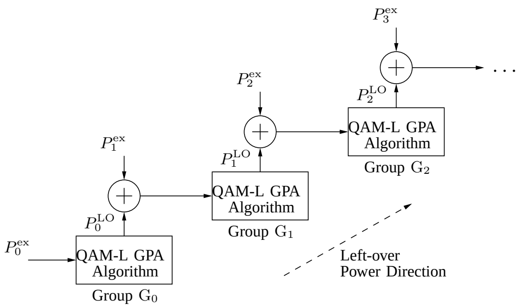

Figure 3: Mu-GPA algorithm arrangements with final left-over power in (18)

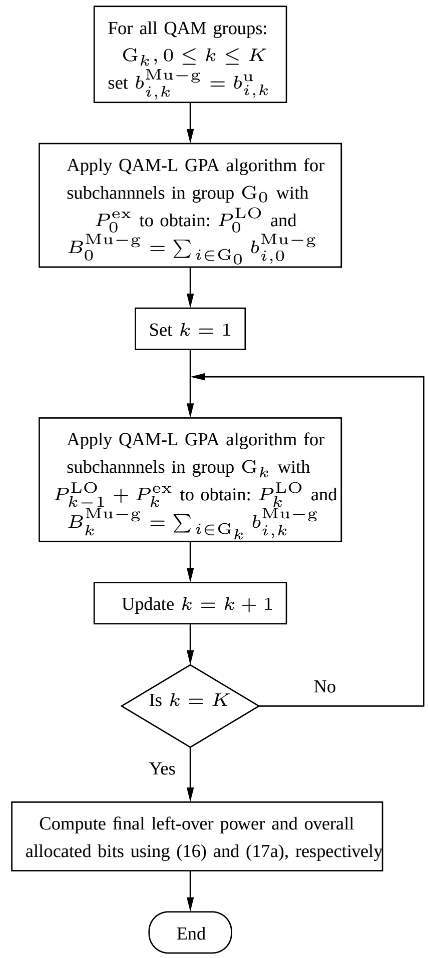

Figure 4: Flowchart of the Mu-GPA algorithm

left-over power of the application of the QAM-L-GPA algorithm to the previous group G_{k-1}, i.e., $P_k^{ex} + P_{k-1}^{LO}$ (cf. Fig. 3). Finally, the left-over power resulting from the QAM group G_{K-1} is added to the excess power of the K^{th} QAM group P_K^{ex} to end up with a final left-over power

$$P_{Mu-g}^{LO} = P_{K-1}^{LO} + P_K^{ex} \tag{18}$$

of this algorithm. The overall system allocated bits and used power for this algorithm are, respectively,

$$B_{Mu-g} = \sum_{k=0}^{K-1} B_k^{Mu-g} + B_K^{u} \tag{19a}$$

$$P_{Mu-g}^{used} = P_{budget} - P_{Mu-g}^{LO} \tag{19b}$$

C. *Md-GPA Algorithm*

A second algorithm is proposed to exploit the residual power P_k^{LO} of each QAM group but in a reverse direction compared to the Mu-GPA algorithm, starting from the highest-indexed QAM group G_{K-1} downwards to the least-index QAM group G_0. This procedures is illustrated in Fig. 5 which show the direction of the left-over power flow. Proceeding downwards, at the kth stage this algorithm applies the QAM-L-GPA algorithm for the available power, comprising of the excess power missed by the UPA algorithm of the previous QAM group (G_{k+1} in this case) as well as the left-over power of the previous stage, i.e., $P_{k+1}^{ex} + P_{k+1}^{LO}$, as also characterised in Fig. 5. Therefore, the excess power of the QAM group under consideration is not utilised within this group but is transferred to the next working group along with the left-over power of the former QAM group. This will finally results in a left-over power of

$$P_{Md-g}^{LO} = P_0^{LO} + P_0^{ex}\,. \tag{20}$$

The flowchart of this algorithm is analogous to the Mu-GPA algorithm. The overall system allocated bits and used power are, respectively,

$$B_{Md-g} = \sum_{k=0}^{K-1} B_k^{Md-g} + B_K^{u} \tag{21a}$$

$$P_{Md-g}^{used} = P_{budget} - P_{Md-g}^{LO}\,. \tag{21b}$$

IV. Simulation Results and Discussion

Secs. III-B and III-C have shown that both Mu-GPA and Md-GPA algorithms work very

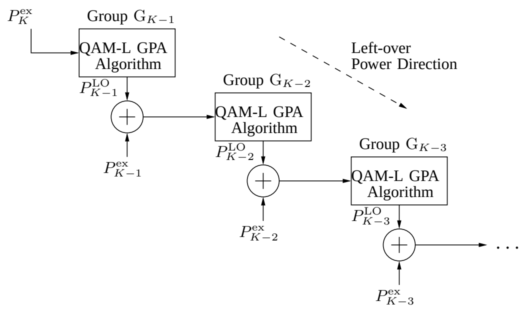

Figure 5: Md-GPA algorithm arrangements with final left-over power in (20)

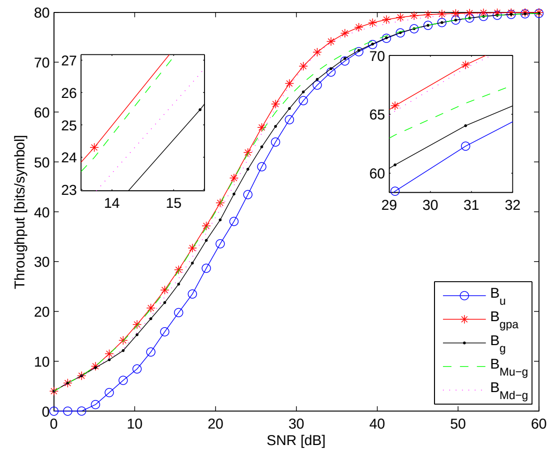

Figure 6: Overall throughput for a 10x10 MIMO system with $\mathcal{P}_{\mathbf{b}}^{\text{target}} = \mathbf{10^{-3}}$

similarly in utilising the power P_k^{LO} for all groups $k, 0 \leq k \leq K-1$ that remained unused by the QAM-L-GPA algorithm. The two algorithms differ in the direction in which left-over power is transferred. Below we compare the two algorithms with the UPA, GPA, and the QAM-L-GPA approaches.

Simulations are conducted over 10^4 instances of a 10x10 MIMO system, where the entries of the MIMO channel $\mathbf{H}$ are drawn from a complex Gaussian distribution with zero-mean and unit-variance, i.e., $h_{ij} \in \mathcal{CN}(0, 1)$. Results presented below refer to ensemble averages across the 10^4 channel realisations for target BER $\mathcal{P}_b^{\text{target}} = 10^{-3}$ and various levels of SNRs using square QAM modulation schemes $M_k = 4^k, k = 1 \cdots K$ with $K = 4$ being the maximum permissible QAM level of constellation size, i.e., $M_K = 256$ which is equivalent to encoding 8 bits per data symbol.

The total system throughput is examined and shown in Fig. 6 for all proposed algorithms in addition to both UPA and standard GPA algorithms. It is evident that UPA represents an inefficient way of bit loading since the performance is approximately 5 to 10 dB below other algorithms, and provide approximately half the throughput at 10 dB SNR.

Of the proposed low-cost greedy algorithms, both Mu-GPA and Md-GPA algorithms outperform the QAM-L-GPA without the refinement stage to allocate residual power across QAM groups. Interestingly, Mu-GPA performs better at low SNR, while Md-GPA performs better at higher SNRs. This can be attributed to the fact that for low-to-medium SNRs P_K^{ex} (which is missed by the Mu-GPA) in this case will be relatively low and can be allocated without violating the constraint on maximum QAM levels. While P_0^{ex} (which is missed by the Mg-GPA) is most likely to be high — please see (12) and Fig. 2 — such that the remaining power in the lowest QAM group is insufficient to lift subchannels across the QPSK boundary. For medium-to-high SNRs $P_K^{\text{ex}} > P_0^{\text{ex}}$ can be expected to be high, and then Md-GPA is likely to be advantageous in its bit allocation, as the maximum QAM level constraint is beginning to be felt.

The data throughput performance of the various algorithms can also be confirmed when considering the power utilisation. Fig. 7 shows the total power available for allocation, and the levels of power allocation that is reached by the different algorithms. For Md-GPA and Mu-GPA, it can be noted that within their respective superiority regions, both are very close to the performance of the standard GPA which demonstrate the good utilisation of the left-over power missed by the QAM-L-GPA algorithm. For high SNR, both QAM-L-GPA and Mu-GPA algorithms behaves like the UPA algorithm due to the increase of P_K^{ex} which is missed by both of them and therefore deteriorates their performances.

Finally, for very high SNRs most subchannels will appear in the highest QAM group G_K as their SNRs, γ_i in (9), exceed the highest QAM level γ_K^{QAM} in (7). As a result, the overall system throughput of all different algorithms reaches its expected maximum.

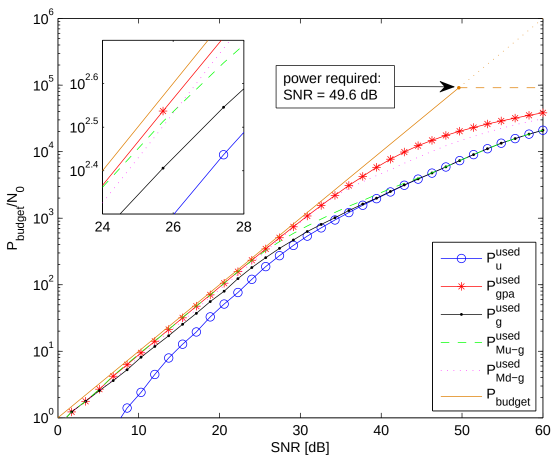

Figure 7: Transmit power used to achieve $\mathcal{P}_{\mathbf{b}}^{\text{target}} = 10^{-3}$ with a min transmit power required by (7) of approximately 49.6 dB

V. Conclusions

Power allocation to achieve maximum data throughput under constraints on the transmit power and the maximum QAM level has been discussed. The optimum solution is provided by the Greedy Algorithm, which operates across all subchannels but is computationally very expensive. Therefore, in this paper sub-optimal low-cost alternatives have been explored. The common theme amongst the proposed algorithms is to restrict the Greedy Algorithm to subsets of subchannels, which are grouped according to the QAM level assigned to them in the uniform power allocation stage. In order to exploit excess (unused) power in each subset, two algorithms were created which carry left-over power forward into the next subset that is optimised by a local greedy algorithms. Two different schemes have been suggested, of which one moves the left-over power upwards from the lowest to the highest subgroup, where in the high SNR case a limitation by the maximum defined QAM level can restrict the performance. A second scheme moves the power from the highest towards the lower subgroups, whereby at low SNR the channel quality in the lowest subgroups may not be such that it can be lifted across the lowest QAM level, and hence no bits may be loaded with the excess power. However, in general both algorithms perform very close to the GPA in their respective domains of preferred operation, thus permitting to allocate power close to the performance of the GPA at a much reduced cost.

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