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A CM BASED EQUALIZER FOR SPACE-TIME SPREADING OVER CHANNELS WITH INTER-SYMBOL INTERFERENCE

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ABSTRACT

Space-time block coding (STBC) and a number of derivative techniques have been developed to maximize the diversity gain of a multi-input multi-output (MIMO) channel. This was later generalized for multi-user DS-CDMA systems through space-time spreading (STS), [3] and [4], which assumed flat fading as well as the availability of full channel state information (CSI) at the receiver. This paper focuses on the development of a blind chip-rate method for the equalization of STS over channels with inter-symbol interference (ISI). Simulation results are presented to demonstrate the convergence and noise resilience of the derived algorithm.

Keywords: Space-time spreading, multi-user, constant modulus algorithm, blind equalization, frequency selective channels.

1. INTRODUCTION

Multi-input multi-output (MIMO) channels are known to increase the capacity of a transmission link. This can be exploited to increase either the multiplexing gain or the diversity gain, which leads to a higher data throughput or a better resilience of the link to fading, respectively.

In the pioneering work by Alamouti [2], a transmitter diversity scheme, named Space-Time Block Coding (STBC), was derived which maximizes the level of diversity obtained in flat fading channels. Inspired by STBC, a transmit diversity scheme referred to as space-time spreading (STS) was derived in [4] for the downlink of DS-CDMA systems. Unlike most other algorithms, STS only requires one orthogonal code per user, which makes it an attractive technique. However, in [4] it is assumed that the MIMO channel matrix is non-dispersive and perfectly known at the receiver. For dispersive channels, a few schemes have been proposed in the literature. These are mainly block based and introduce redundancy as well as assuming block stationarity of the channel. Knowledge of the channel can be obtained through the injection of training sequences but the channel is assumed staionary during the transmission of data. This assumption is not always valid, thus a blind equalizer appears advantageous.

The most widely used blind equalizer is the constant modulus algorithm (CMA), first introduced in [6], which adapts the weight vectors based on the assumption that the transmitted constellation has a constant modulus. In a multi-user scenario, this property is not preserved. In [1], a Constant Modulus (CM) algorithm was derived for the blind equalization of DS-CDMA systems, named FIRMER-CMA. The algorithm operates at the chip rate and forces the various user signals onto a constant modulus without any additional requirements such as minimizing the mutual cross-correlation between the multiple users.

In this paper, a similar CM algorithm for the blind equalization of STS systems over frequency selective channels is derived. The new algorithm forces the despread signals to have a constant modulus.

In the following, we introduce the data model for STS and the proposed equalisation algorithm. A derrivation of the algorithm is detailed in section 3. In order to assess the performance of the novel algorithm, the zero-forcing solution has been chosen as a benchmark. The inverse of the polynomial channel matrix is calculated in frequency domain, as in [5]. Simulation results are presented in section 5 to demonstrate the convergence and error performance of the STS-CM algorithm.

2. DATA MODEL

Inspired by STBC coding, STS was introduced in [4] as a transmit diversity scheme for wideband-CDMA systems. Similar to STBC, the source data is split into odd and even symbol sequences, $b_1[n]$ and $b_2[n]$, respectively. The index n is referred to as symbol time, whereas the transmission will be at a chip rate K times higher than the symbol rate, with chip index m. The chip rate signals transmitted from the two antennas, $s_1[m]$ and $s_2[m]$, are defined as

$$\begin{aligned} s_1[m] &= \frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty}\left(c_1[m-nK]b_1[n]+c_2[m-nK]b_2^*[n]\right) \\ s_2[m] &= \frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty}\left(c_1[m-nK]b_2[n]-c_2[m-nK]b_1^*[n]\right), \end{aligned}$$

where $(\cdot)^*$ denotes complex conjugation and $c_1[m]$ and $c_2[m]$ are orthonormal codes of length K, whose coefficients in the following are arranged in vectors $\mathbf{c}_i \in \mathbb{R}^K$, $i = \{1,2\}$. The code length defines the CDMA spreading gain as $K/2$. If only one code, $\mathbf{c}$, is assigned to every user, then the two codes can be defined as

$$\mathbf{c}_1 = \begin{bmatrix} \mathbf{c} \\ \mathbf{c} \end{bmatrix} \quad \text{and} \quad \mathbf{c}_2 = \begin{bmatrix} \mathbf{c} \\ -\mathbf{c} \end{bmatrix}. \qquad (1)$$

Note that the orthonormality of the user codes is preserved, i.e. $\mathbf{c}_1^H\mathbf{c}_2 = 0$, where superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose operations, respectively.

The received signals over a window of L samples is given

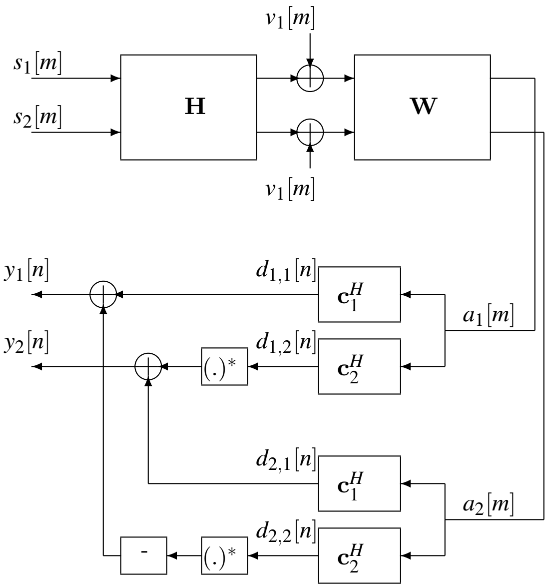

Figure 1: Space-Time equalization for STS.

by

$$\mathbf{R}_m = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \end{bmatrix} = \sum_{\nu=0}^{N-1} \mathbf{H}_\nu \mathbf{S}_{m-\nu} + \mathbf{V}_m. \quad (2)$$

where $\mathbf{S}_m$ is the matrix of transmitted data

$$\mathbf{S}_m = \begin{bmatrix} s_1[m] & \cdots & s_1[m-L+1] \\ s_2[m] & \cdots & s_2[m-L+1] \end{bmatrix}. \quad (3)$$

Matrix $\mathbf{H}_\nu$ is the ν^{th} time slice of the channel transfer function and is defined as

$$\mathbf{H}_\nu = \begin{bmatrix} h_{1,1}[\nu] & h_{1,2}[\nu] \\ h_{2,1}[\nu] & h_{2,2}[\nu] \end{bmatrix}, \quad (4)$$

where $\mathbf{h}_{i,j}$ is the frequency selective channel from the j^{th} transmit antenna to the i^{th} receive antenna. The channels have a maximum length of N. The transfer function of the dispersive MIMO channel can be written as

$$\mathbf{H}(z) = \sum_{\nu=0}^{N-1} \mathbf{H}_\nu z^{-\nu}. \quad (5)$$

Additive white Gaussian noise (AWGN), $\mathbf{V}_m$, with zero mean and $\mathscr{E}\{\mathbf{V}\mathbf{V}^H\} = L\sigma_r \mathbf{I}_{2\times 2}$ is assumed to corrupt the signals at the receiver.

As shown in Fig. 1, a MIMO equalizer is used, whereby each of the four subfilters has length L. It was proven in [7] that perfect Zero-Forcing (ZF) equalization of the p-by-u MIMO channel can be achieved if the length of the subequalizers, L satisfies

$$L \geq \underline{L} = \left\lceil \frac{N(p-1)}{u-p} \right\rceil, \quad (6)$$

where $\lceil \cdot \rceil$ denotes rounding operation to the next larger integer.

3. THE STS-CMA ALGORITHM

Due to its simplicity and low requirements, the constant modulus algorithm (CMA) is the most commonly used blind equalizer. The algorithm, first introduced in [6], assumes a constant modulus constellation set at the transmitter and works by forcing the received signal to have the same modulus. A MIMO-CMA algorithm was derived in [8] which in addition explicitly forces the cross-correlation between the outputs to zero in order to avoid multiple extractions of the same source.

As shown in Fig. 1, the equalizer outputs $a_1[m]$ and $a_2[m]$ will ideally identify the transmitted sequences $s_1[m]$ and $s_2[m]$, respectively. Due to the fact that the transmitted signals are not constant modulus and the fact that multiple users are present in the system, the conventional MIMO-CMA cannot be used. In [1], a CM algorithm was derived for the equalization of a single antenna DS-CDMA system based on the orthogonality of the match filtered outputs. To exploit the STS scenario similar to [1], we consider the cost function

$$\xi = \mathscr{E}\left\{ \sum_{i=1}^{2} \sum_{l=1}^{2} (|d_{i,l}[n]|^2 - \gamma^2)^2 \right\}, \quad (7)$$

where γ is the modulus of the source constellation and $d_{i,l}[n]$ are the despread signals as shown in Fig. 1. Due to the explicit orthogonality of the despread outputs, the proposed cost function is simply the CM criterion calculated over the four signals. The despread outputs are defined as

$$d_{i,l}[n] = \mathbf{c}_l^H \cdot \begin{bmatrix} a_i[nK] \\ \vdots \\ a_i[nK-K+1] \end{bmatrix}, \quad (8)$$

where the output of the i^{th} space-time equalizer is given by

$$a_i[nK] = \mathbf{w}_i^H \cdot \begin{bmatrix} r_1[nK] \\ \vdots \\ r_1[nK-L+1] \\ r_2[nK] \\ \vdots \\ r_2[nK-L+1] \end{bmatrix} \text{ with } \mathbf{w}_i = \begin{bmatrix} \mathbf{w}_{i,1} \\ \mathbf{w}_{i,2} \end{bmatrix}. \quad (9)$$

Expanding (8) yields:

$$d_{i,l}[n] = \mathbf{c}_l^H \cdot \mathbf{W}_i \cdot \begin{bmatrix} r_1[nK] \\ \vdots \\ r_1[nK-K-L+2] \\ r_2[nK] \\ \vdots \\ r_2[nK-K-L+2] \end{bmatrix}, \quad (10)$$

where $\mathbf{W}_i$ is the convolutional matrix for the i^{th} equalizer defined by

$$\mathbf{W}_i = \left[\begin{array}{ccc|ccc} \mathbf{w}_{i,1}^H & & \mathbf{0} & \mathbf{w}_{i,2}^H & & \mathbf{0} \\ & \ddots & & & \ddots & \\ \mathbf{0} & & \mathbf{w}_{i,1}^H & \mathbf{0} & & \mathbf{w}_{i,2}^H \end{array}\right]. \quad (11)$$

Table 1: Summary of the STS-CMA equalization algorithm.

1:	Update $\mathbf{x}_l[n] = \mathbf{C}_l \cdot \mathbf{r}_{nK}$, for $l = 1,2$.
2:	$d_{i,l}[n] = \mathbf{w}_i^H[n] \cdot \mathbf{x}_l[n]$, for $i,l = 1,2$.
3:	$e_{i,l}[n] = d_{i,l}[n] \cdot \left(\lvert d_{i,l}[n]\rvert^2 - \gamma^2\right)$, $i,l = 1,2$.
4:	$\mathbf{w}_i[n+1] = \mathbf{w}_i[n] - \mu \cdot \sum_{l=1}^{2} e_{i,l}^*[n]\mathbf{x}_l[n]$, $i = 1,2$.

The terms of Equation (10) can be rearranged to produce:

$$d_{i,l}[n] = \mathbf{w}_i^H \cdot \mathbf{C}_l \cdot \mathbf{r}_{nk}, \tag{12}$$

where

$$\mathbf{C}_l = \left[\begin{array}{ccc|ccc} \mathbf{c}_l^H & & \mathbf{0} & & & \\ & \ddots & & & \mathbf{0} & \\ \mathbf{0} & & \mathbf{c}_l^H & & & \\ \hline & & & \mathbf{c}_l^H & & \mathbf{0} \\ & \mathbf{0} & & & \ddots & \\ & & & \mathbf{0} & & \mathbf{c}_l^H \end{array}\right], \tag{13}$$

and

$$\mathbf{r}_{nK} = \begin{bmatrix} r_1[nK] \\ \vdots \\ r_1[nK-K-L+2] \\ r_2[nK] \\ \vdots \\ r_2[nK-K-L+2] \end{bmatrix}. \tag{14}$$

Estimating the instantaneous gradient of ξ through omitting the expectation operator in (7), the stochastic gradient descent for the new algorithm is defined as

$$\mathbf{w}_i[n+1] = \mathbf{w}_i[n] - \mu \cdot \bigtriangledown_{\mathbf{w}_i^*} \hat{\xi}_1, \text{for } i = 1,2. \tag{15}$$

The instantaneous gradient of ξ for the i^{th} space-time equalizer is calculated similar to the conventional CMA as follows:

$$\bigtriangledown_{\mathbf{w}_i^*} \hat{\xi}_1 = \sum_{l=1}^{2} \left(e_{i,l}^*[n]\mathbf{C}_l \mathbf{r}_{nK}\right), \tag{16}$$

where

$$e_{i,l}[n] = d_{i,l}[n] \cdot \left(\lvert d_{i,l}[n]\rvert^2 - \gamma^2\right), \quad \text{for } i,l = 1,2. \tag{17}$$

A summary of the iterative STS-CM algorithm is detailed in Tab. 1.

4. ZERO-FORCING EQUALIZATION

The Zero-Forcing (ZF) equalizer has been chosen as a benchmark for the derived STS-CMA algorithm. The ZF receiver pre-multiplies the received data matrix with the inverse of the channel matrix. The transfer function of the equalizer is given by

$$\mathbf{W}(z) = \begin{bmatrix} \mathbf{w}_{1,1}(z) & \mathbf{w}_{1,2}(z) \\ \mathbf{w}_{2,1}(z) & \mathbf{w}_{2,2}(z) \end{bmatrix} \tag{18}$$

Table 2: Frequency domain inversion of the polynomial matrix $\mathbf{C}$.

1:	$\mathbf{D}$ is a $2 \times 2 \times N$ matrix with $D_\nu = I$ for all ν.
2:	$\mathbf{G}(e^{j\Omega_i}) = DFT(\mathbf{G}_i)$,
	where $\Omega_i = \frac{2\pi i}{L}$ for $i = 0, \cdots, N_{DFT} - 1$
3:	$\mathbf{D}(e^{j\Omega_i}) = DFT(\mathbf{D}_i)$
4:	for $i = 1,2,\ldots$
	$\mathbf{G}^{-1}(e^{j\Omega_i}) = D(e^{j\Omega_i}) * \mathbf{G}^\dagger(e^{j\Omega_i})$
5:	$\mathbf{G}^{-1} = IDFT(\mathbf{W}(e^{j\Omega_i}))$

With a 2-transmit and p receive antenna configuration, the polynomial channel matrix $\mathbf{H}(z)$ may be rectangular, which makes it non-invertible. In order to overcome this problem, the pseudo inverse is used as follows:

$$\begin{aligned} \mathbf{W}(z) &= \mathbf{H}^\dagger(z) \\ &= \left(\tilde{\mathbf{H}}(z)\mathbf{H}(z)\right)^{-1} \tilde{\mathbf{H}}(z) \\ &= \mathbf{G}^{-1}\tilde{\mathbf{H}}(z), \end{aligned} \tag{19}$$

where $\tilde{(\cdot)}$ denotes the para-Hermitian operator.

In [5], a frequency domain approach was proposed for the inversion of a polynomial matrix. After taking the Discrete Fourier Transform (DFT) of the channel matrix, the algorithm performs a bin-wise pseudo-inverse operation. A regularization matrix $\mathbf{D}$ is used to ensure that the optimal filters decay away quickly enough to minimize the circular convolution effect. A summary of the inversion algorithm is given in Tab. 2.

5. SIMULATION RESULTS

Computer simulation results are presented in this section to provide insight into the propose scheme and to demonstrate the convergence of the derived STS-FIRMER-CMA algorithm. A 2×2 MIMO model as indicated in Figure 1 is used for simulations with $k = 4$ users. QPSK modulation is used at the transmitter with a modulus equal to $\sqrt{2}$. At the receiver, signals are corrupted by AWGN at a Signal-to-Noise Ratio (SNR) of 20 dB. The length of the subequalizers is set to $L = 7$. The step size is initialized to $\mu = 3 \cdot 10^{-3}$, and the coefficient vectors for the four subequalizers are set to all zeros except the first entries of $\mathbf{w}_{11}$ and $\mathbf{w}_{21}$, which are set to unity. Figure 2 shows the pole-zero diagrams of the dispersive channels used in the simulations.

Fig. 3 shows the symbol values at the output of the adapted MIMO equalizer, indicating that the equalizer has correctly extracted the transmitted sequences of the user of interest. A residual rotation is present at the output due to the CMA's phase invariance.

To demonstrate the robustness in noise, the channel model shown in Fig. 2 has been randomized by using the

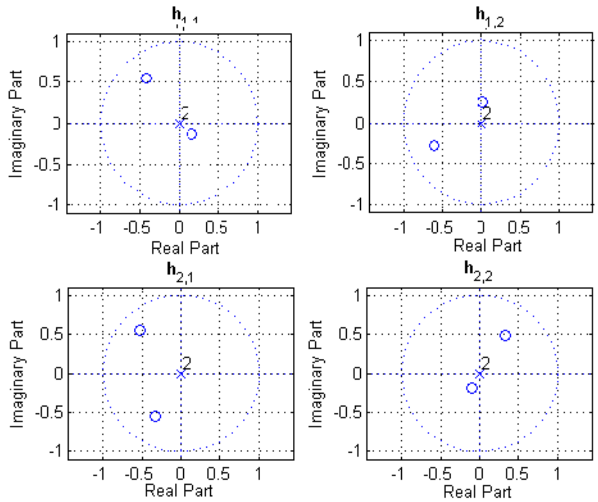

Figure 2: Pole-zero diagrams for the dispersive channels.

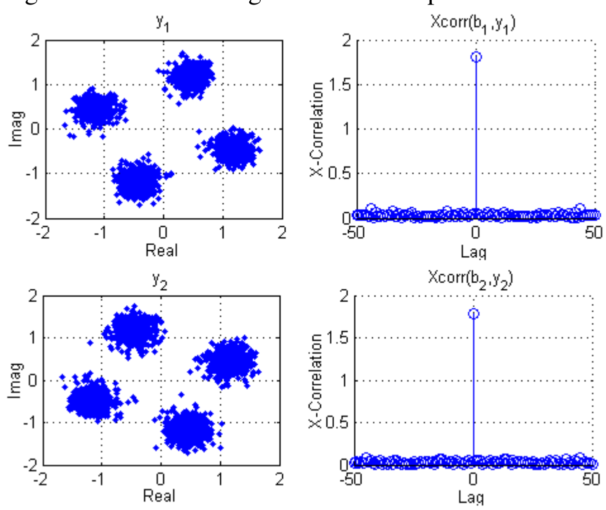

Figure 3: Equalizer outputs and their cross-correlation with the source signals, SNR = 20dB.

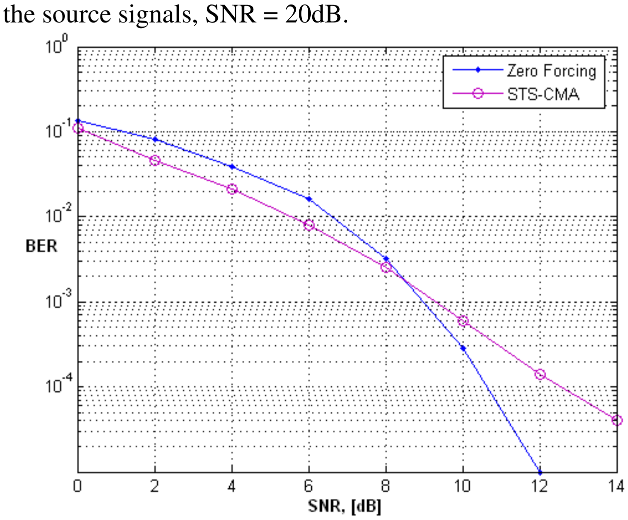

Figure 4: BER curve for the derived STS-CM Algorithm.

subchannel impulse responses to define a power profile, from which 50 Rayleigh distributed ensemble probes are drawn. The BER performance of the proposed algorithm and the Zero-Forcing receiver averaged over the 50 runs is shown in Fig. 4 over a range of SNR values. It can be clearly observed that the new algorithm performs better over low SNR values. This is due to the noise amplification inherent in the matrix inversion operation of the Zero-Forcing algorithm. When the SNR increases beyond 8dB, the performance of the derived algorithm drops in comparison to that of the Zero-Forcing receiver, which can be expected since the former assumes no knowledge of the channel.

6. CONCLUSION

In this paper, a novel algorithm based on the Constant Modulus (CM) criterion has been derived for the blind equalization of STS systems over frequency selective channels. Similar to FIRMER-CMA derived in [1], the new algorithm forces the constant modulus property of the matched filtered signals of the user of interest rather than the immediate equalizer outputs, and therefore implicitly achieves orthogonality of the extracted signals.

Extensive computer simulations have been performed to highlight the stability of the algorithm, whereby some representative results have been presented here. The noise performance of the derived algorithm has been compared to that of the Zero-Forcing equalizer. Over low SNRs, the derived algorithm achieves better performance without the need for any Channel State Information (CSI).

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