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# Wall Following to Escape Local Minima for Swarms of Agents Using Internal States and Emergent Behaviour

Mabrouk M. H. and McInnes C. R.

*Abstract* — Natural examples of emergent behaviour, in groups due to interactions among the group's individuals, are numerous. Our aim, in this paper, is to use complex emergent behaviour among agents that interact via pair-wise attractive and repulsive potentials, to solve the local minima problem in the artificial potential based navigation method. We present a modified potential field based path planning algorithm, which uses agent internal states and swarm emergent behaviour to enhance group performance. The algorithm is used successfully to solve a reactive path-planning problem that cannot be solved using conventional static potential fields due to local minima formation. Simulation results demonstrate the ability of a swarm of agents to perform problem solving using the dynamic internal states of the agents along with emergent behaviour of the entire group.

*Index Terms*— Agent Internal States, Local Minima Escape, Swarm Emergent Behaviour, Wall Following.

## I. INTRODUCTION

Common emergent patterns in natural systems such as coherent flock, single-mill states, and double-mill pattern have been observed and reported for various species whose members have high rates of information exchange [1]–[3]. As the researchers become more concerned in investigating such phenomenon, terms like complexity, emergence, and stigmergy have been defined [4] and models of natural or artificial individuals, which interact through pair-wise long-range attraction and short-range repulsion within a swarm, have been introduced [5]–[8]. Such behaviour may offer new approaches to many classes of information processing problems, which currently prove infeasible, and to design systems that can accomplish their tasks more reliably, faster and cheaper than could be achieved by conventional systems [9].

## II. SWARM MODEL

Investigating the motion of swarms using artificial potential fields shows that swarms of interacting particles can relax into vortex-like states [8]. The model consists of  $N_p$  agents with mass  $m_i$ , position  $\mathbf{r}_i$ , velocity  $\mathbf{v}_i$  and relative distance  $\mathbf{r}_{ij}$  between the  $i^{\text{th}}$  and  $j^{\text{th}}$  agents. The agents interact by means of a cohesive two-body generalized Morse potential  $V_{interaction}(\mathbf{r}_i)$  with weak long range attraction and strong short range repulsion.

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M. H. Mabrouk is with the Department of Mechanical Engineering, University of Strathclyde, Glasgow, UK. (phone:+44 (0) 141 548 4851; fax: +44 (0) 141 552 5105; e-mail: mohamed.mabrouk@strath.ac.uk).  
C. R. McInnes is with the Department of Mechanical Engineering, University of Strathclyde, Glasgow, UK.

For simplicity, we will consider identical agents of unit mass. To control the speed of the  $i^{\text{th}}$  agent a linear dissipative term with a positive coefficient  $\beta_i$  is added [10]. The total potential field, which affects the  $i^{\text{th}}$  agent, is then characterized by other agent's attractive and repulsive potential fields of strength  $C_a$  and  $C_r$  with ranges  $l_a$  and  $l_r$  respectively along with obstacle potentials  $V_{obstacles}(\mathbf{r}_i)$  of strength  $C_{io}$  with range  $l_{io}$  ( $C_a, C_r, l_a, l_r, C_{io}, l_{io} \geq 0$ ). For the goal we use a hyperbolic attractive well of strength  $w_g$  to ensure convergence of the agents to the goal [11].

To make the swarm of agents dissipate energy while the total angular momentum is conserved as the swarm relaxes, the agents encounter orientation forces  $\mathbf{F}_{orientation}(\mathbf{r}_i, \mathbf{v}_i)$ , which act directly on the agents' velocities to orient the individuals' velocities with respect to one another [8]. The constant  $C_A$  is the magnitude of the orientation force and  $L_A$  is the range over which the orientation interaction occurs ( $C_A, L_A \geq 0$ ). In general, the equations of motion for  $N_p$  agents moving in a workspace that contains  $N_o$  point obstacles at locations  $\mathbf{r}_o$  and one goal  $G$  at position  $\mathbf{r}_g$  are then defined by:

$$\mathbf{v}_i = \dot{\mathbf{r}}_i \quad (1)$$

$$m_i \dot{\mathbf{v}}_i = \mathbf{F}_{total}(\mathbf{r}_i, \mathbf{v}_i) \quad (2)$$

where,  $\mathbf{F}_{total}(\mathbf{r}_i, \mathbf{v}_i)$  is the sum of all forces exerted on the  $i^{\text{th}}$  agent. To calculate the force in Eq. (2), the global potential is now defined as:

$$V_{total} = V_{interaction} + V_{obstacles} + V_{goal} \quad (3)$$

where, the interaction potential  $V_{interaction}(\mathbf{r}_i)$  is defined as the sum of the repulsion potential and the attraction potential among the agents. We use generalized Morse potential, of the exponentially decaying nature, to obtain interactions that are close to real biological systems as follows:

$$V_{interaction}(\mathbf{r}_i) = \sum_{j \neq i}^{N_p} C_{r_j} e^{-|\mathbf{r}_i - \mathbf{r}_j|/l_{r_j}} - \sum_{j \neq i}^{N_p} C_{a_j} e^{-|\mathbf{r}_i - \mathbf{r}_j|/l_{a_j}} \quad (4)$$

The obstacles and goal potentials are defined as:

$$V_{obstacles}(\mathbf{r}_i) = \sum_{z=1}^{N_o} C_{o_z} e^{-|\mathbf{r}_i - \mathbf{r}_{o_z}|/l_{o_z}} \quad (5)$$

$$V_{goal}(\mathbf{r}_i) = w_g \left( \sqrt{1 + |\mathbf{r}_i - \mathbf{r}_g|^2} - 1 \right) \quad (6)$$

$\mathbf{F}_{total}(\mathbf{r}_i, \mathbf{v}_i)$  in Eq. (2) consists of the following:

$$\mathbf{F}_{total}(\mathbf{r}_i, \mathbf{v}_i) = \mathbf{F}_{dissipation}(\mathbf{v}_i) + \mathbf{F}_{interaction}(\mathbf{r}_i) + \mathbf{F}_{orientation}(\mathbf{r}_i, \mathbf{v}_i) + \mathbf{F}_{goal}(\mathbf{r}_i) + \mathbf{F}_{obstacles}(\mathbf{r}_i) \quad (7)$$

where,

$$\mathbf{F}_{dissipation}(\mathbf{v}_i) = -\beta_i \mathbf{v}_i \quad (8)$$

$$\mathbf{F}_{interaction}(\mathbf{r}_i) = -\nabla V_{interaction}(\mathbf{r}_i) = \sum_{j \neq i}^{N_p} \left[ \frac{C_{a_j}}{l_{a_j}} e^{-|\mathbf{r}_i - \mathbf{r}_j|/l_{a_j}} - \frac{C_{r_j}}{l_{r_j}} e^{-|\mathbf{r}_i - \mathbf{r}_j|/l_{r_j}} \right] \hat{\mathbf{r}}_{ij} \quad (9)$$

$$\mathbf{F}_{goal}(\mathbf{r}_i) = -\nabla V_{goal}(\mathbf{r}_i) = -\frac{w_g(\mathbf{r}_i - \mathbf{r}_G)}{\sqrt{1 + |\mathbf{r}_i - \mathbf{r}_G|^2}} \quad (10)$$

$$\mathbf{F}_{obstacles}(\mathbf{r}_i) = -\nabla V_{obstacles}(\mathbf{r}_i) = \sum_{z=1}^{N_o} \frac{C_{io_z}}{l_{io_z}} e^{-|\mathbf{r}_i - \mathbf{r}_{io_z}|/l_{io_z}} \hat{\mathbf{r}}_{io_z} \quad (11)$$

The orientation force is defined in (McInnes,2007) as:

$$\mathbf{F}_{orientation}(\mathbf{r}_i, \mathbf{v}_i) = -\sum_{j \neq i}^{N_p} C_A(\mathbf{v}_j \cdot \hat{\mathbf{r}}_{ij}) e^{-|\mathbf{r}_j|/l_A} \hat{\mathbf{r}}_{ij} \quad (12)$$

Substituting from Eq. (3), Eq. (7-12) in Eq. (2), it can be seen that:

$$m_i \dot{\mathbf{v}}_i = \mathbf{F}_{dissipation}(\mathbf{v}_i) + \mathbf{F}_{orientation}(\mathbf{r}_i, \mathbf{v}_i) - \nabla_i V_{total}(\mathbf{r}_i) \quad (13)$$

For a complex potential field such as that represented by Eq. (3-6), the potential can possess multiple local minima. A key issue is to identify how the agents will realize that they are trapped in a local minimum so that they can then attempt to escape. To solve the problem in this case, the agents must discount their immediate sensory information (attraction of the goal) by endowing them with higher-level perception concerning the environment.

### III. PROBLEM DEFINITION

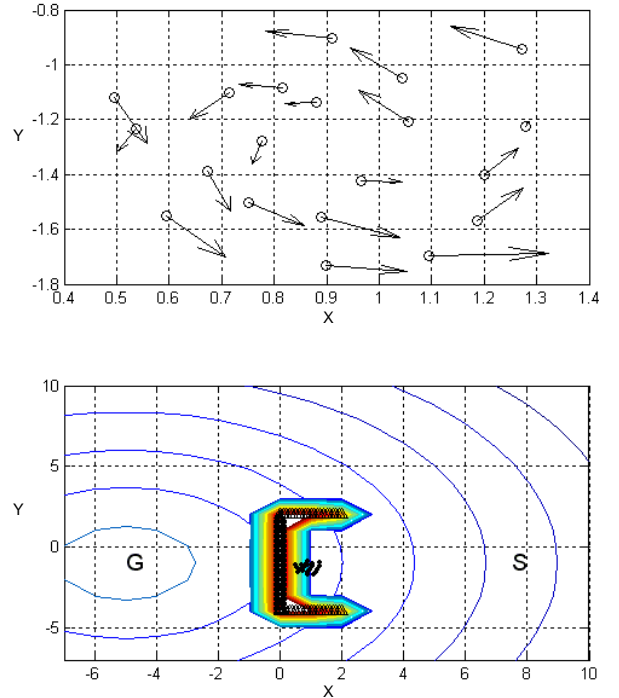
The local minima problem has been an issue of concern for potential field methods [12]. Several attempts, which can be categorized into local minima avoidance (LMA) techniques and local minima escape (LME) techniques, have been made to overcome it [13]. In our previous work we introduced the approach of using dynamic internal states for a swarm of robots to escape local minima by manipulating the global potential of the environment. The performance of the swarm was enhanced by using some aspects of swarming behaviours such that swarm leader concept [14], and the collective behaviour [10], which can be found in real biological systems.

In this paper we introduce a solution to the problem performed by a swarm of robots, which encounter mutual interaction. By choosing the proper interaction parameters, a vortex-like pattern will emerge [8]. We will

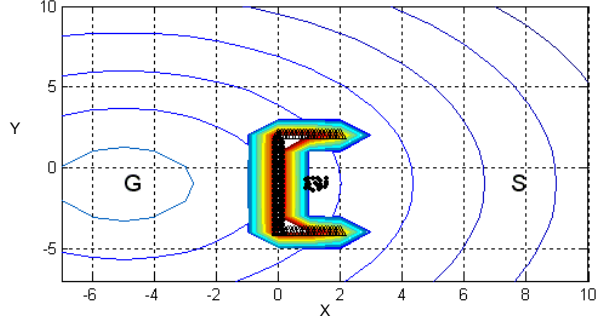
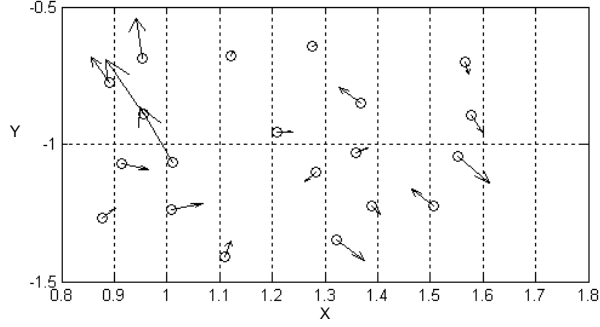
use the agents' internal states to employ this emergent behaviour to make the swarm of agents escape local minima by following the boundaries of obstacles.

The reactive problem of a swarm of agents attracted to a goal point at position  $G$  is shown in Fig. 1, where we can see the group moves towards the goal as a flock with individuals' velocities increasing until reaching the goal. Then they are trapped in the local minimum, which is a barrier that consists of a number of identical obstacle points located in the path of the swarm to the goal such that the goal is visible from the swarm individuals' initial positions but they cannot pass through the barrier. Considering this case, the whole swarm will be trapped at the barrier because the agents trapped inside the barrier will suffer two opposite forces; the first force is the repulsion from the barrier while the other one will be the attraction to the goal.

Fig. 2 shows the nearly sinusoidal change in the group angular momentum for the swarm in Fig. 1 indicating the frequent attempts of the group to go to the goal through the obstacles and that the angular momentum of the swarm decreases as the swarm is repulsed. Then, the swarm group angular momentum almost decays with time. In that case, the swarm rotates around its center with a decaying angular momentum, which indicates that the swarm will never escape the local minimum.

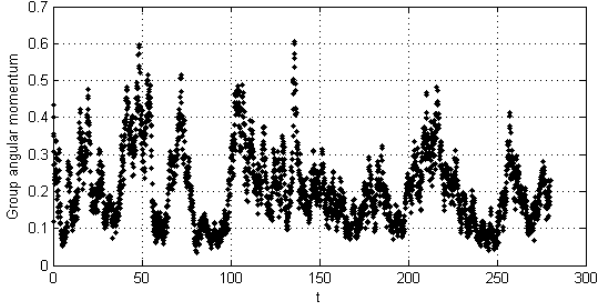


**Fig.1.a.** The swarm starts from position  $S$  and then becomes trapped in the local minimum,  $t = 95$



**Fig.1.b.** The swarm fails to escape the local minimum,  $t = 270$

**Fig.1.** Behaviour of a swarm of agents that use fixed internal states.



**Fig. 2.** Group angular momentum with time for the swarm in Fig.1.

#### IV. INTERNAL STATE MODEL

We use one of the most interesting aspects in swarming behaviours, which is the emergence of vortex pattern among agents that interact via pair-wise attractive and repulsive potentials [15], as a new technique to escape the local minimum position. The solution depends mainly on increasing the group perception about the swarm state by linking the goal gradient potential in the equation of motion to one of the swarming parameters, the swarm center velocity  $\mathbf{v}_c$ , in a way that when the velocity of the swarm decreases the goal effect diminishes. This helps in eliminating the local minimum from the global potential, which in turn enables the formed vortex pattern amongst the group to solve the problem. The attraction strength  $w_g$  in Eq. (13) is now defined as:

$$w_g = (1 - \lambda_c e^{-\lambda_c |\mathbf{v}_c|})k \quad (14)$$

where  $k$  is a positive coefficient. This effect will not solve the problem by itself because the vortex pattern emerges and the local minimum disappears but the agents may rotate around their center behind the wall, as shown in Fig. 1, which indicates that the swarm will not follow the obstacle boundaries. At this point comes the role of manipulating one of the agents' internal states, the dissipation coefficient  $\beta$ , to achieve a pure rolling in a way that makes the group follow obstacle boundaries. The dissipation coefficient will be defined as:

$$\beta_i = \beta_o + \frac{e^{-R_i}}{e^{\lambda_\beta |\mathbf{v}_c|}} \quad (15)$$

and,

$$R_i = \text{Min}_{z=1}^{N_p} (|\mathbf{r}_{i o_z}|) \quad (16)$$

where  $\beta_o$  is the minimum dissipation coefficient necessary to prevent the agents from escaping the group [10],  $R_i$  is the minimum distance between  $i^{\text{th}}$  agent and the obstacles,  $\lambda_c$  is a positive coefficient that controls the effect of the swarm center velocity on the goal attraction potential strength in Eq. (14),  $\lambda_g$  is a positive coefficient that guarantees that  $w_g$  is always positive and  $\lambda_\beta$  is a positive coefficient that controls the effect of the swarm center velocity on the dissipation coefficient in Eq. (15). The effect of manipulating the dissipation coefficient  $\beta$  guarantees that when the swarm is trapped, the swarm's individuals closer to the boundary of the obstacles will gain higher values of dissipation coefficient (i.e. lower velocities). Meanwhile, the individuals who are far from the obstacles will gain lower values of dissipation coefficient and consequently higher velocities to form a pure rolling action.

#### V. STABILITY ANALYSIS

We will follow Mogilner's approach [6] to discuss the stability of the system. From Eq. (13) it can be deduced that:

$$m_i \dot{\mathbf{v}}_i = -\beta_i \mathbf{v}_i - \sum_{j \neq i}^{N_p} C_A(\mathbf{v}_{ij} \cdot \hat{\mathbf{r}}_{ij}) e^{-|\mathbf{r}_{ij}|/L_A} \hat{\mathbf{r}}_{ij} - \nabla_i V_{total}(\mathbf{r}_i) \quad (17)$$

where  $V_{total}(\mathbf{r}_i)$  is from Eq. (3). Now, the total energy of the system is defined as:

$$\phi = \sum_{i=1}^{N_p} \frac{1}{2} m_i \mathbf{v}_i^2 + V_{total}(\mathbf{r}_i) \quad (18)$$

so that

$$\dot{\phi} = \sum_{i=1}^{N_p} \mathbf{v}_i \cdot (m_i \dot{\mathbf{v}}_i + \nabla_i V_{total}(\mathbf{r}_i)) \quad (19)$$

Substituting from Eq. (8), Eq. (10) and Eq. (17) in Eq. (19):

$$\dot{\phi} = -\sum_{i=1}^{N_p} \beta_i \mathbf{v}_i^2 - \sum_{i=1}^{N_p} \mathbf{v}_i \cdot \sum_{j \neq i}^{N_p} C_A(\mathbf{v}_{ij} \cdot \hat{\mathbf{r}}_{ij}) e^{-|\mathbf{r}_{ij}|/L_A} \hat{\mathbf{r}}_{ij} \quad (20)$$

Then, it can be concluded that:

$$\dot{\phi} = -\sum_{i=1}^{N_p} \beta_i \mathbf{v}_i^2 - \frac{1}{2} \sum_{i=1}^{N_p} \sum_{j \neq i}^{N_p} C_A (\mathbf{v}_{ij} \cdot \hat{\mathbf{r}}_{ij})^2 e^{-|\mathbf{r}_{ij}|/l_A} \quad (21)$$

Knowing that  $\beta > 0$ ,  $w_g \geq 0$ ,  $\lambda_g \geq 0$ ,  $C_A \geq 0$ ,  $k > 0$ , then  $\dot{\phi} < 0$ , therefore the system is Lyapunov stable, so that the group will slowly leak energy and relax to a minimum-energy state.

## VI. NUMERICAL RESULTS

### A. Problem Solving

Simulation results, demonstrated in Fig. 3, show a swarm of agents escaping local minimum while Fig. 4 shows the group angular momentum during the problem-solving phase. We can see in Fig. 3.a) that the swarm moves towards the goal as almost aligned flock until it enters the local minimum. This is shown regime I of Fig. 4, where the group angular momentum is low and of almost constant value.

The effect of the term in that increases the perception of the group about the environment is obvious in Fig. 3.b) to Fig. 3.e), which show that when the swarm is stuck the goal effect on the group is decreased. This makes the local minimum disappears, as shown in Fig. 3.b) to Fig. 3.e), and the vortex pattern emerge with higher angular momentum amongst the group. Fig. 3.b) to Fig. 3.e) also show the effect of using Eq. (15) and Eq. (16) which manipulate the values of the dissipation coefficient making the individuals closer to the obstacle to gain higher dissipation coefficient that makes them of lower velocity than those who are located far from the obstacle walls. This guarantees pure rolling motion, making the group to follow the wall boundary even if there is no direct contact to the obstacle wall. The pure rolling-wall following action is very clear in Fig. 3.b) to Fig. 3.e). Fig. 4 shows that the swarm maintains almost constant group angular momentum to follow boundaries of the obstacles. Zones II, III, IV and V of Fig. 4 respectively show the group angular momentum corresponding to the boundaries following for the lower horizontal inner wall, vertical inner wall, the higher horizontal inner wall, and the outer boundaries of the higher horizontal wall until the swarm escapes from the local minimum. Again the effect of the term that increases the perception of the group about the environment is obvious in Fig. 3.f) and Fig. 3.g) in a way that as the agents escape from the local minimum, the swarm center velocity increases and consequently the goal effect on the group increases which makes the group moves towards the goal with relatively higher velocity as an almost aligned flock. This is shown in region VI of Fig. 4 where the swarm group angular momentum decreases as the swarm moves toward the goal. Using the model ensures sinusoidal change in the agent's dissipation coefficient, especially the peripheral ones, with time as shown in Fig. 5. This effect guarantees that

the swarm will follow the obstacle wall in a pure rolling motion even in the absence of the goal effect and consequently the absence of the direct touch of the swarm to the wall.

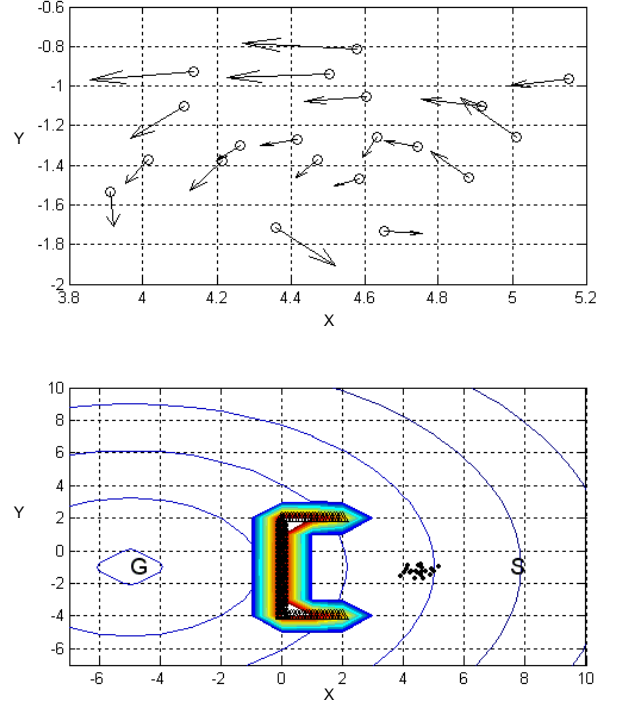


Fig. 3.a. The swarm at the initial position,  $t=17$

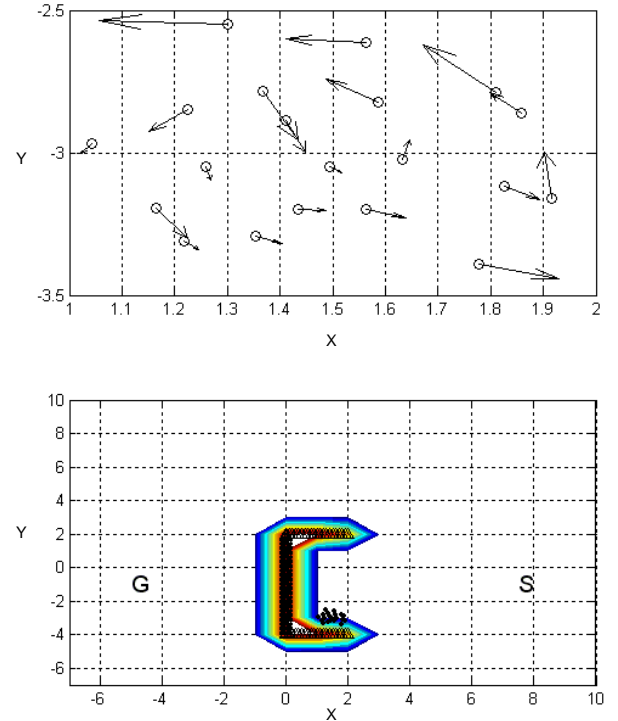


Fig. 3.b.  $t=39$

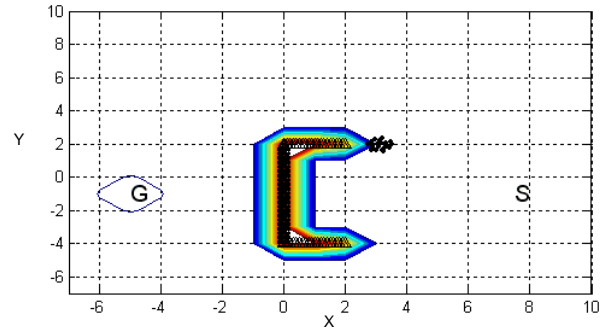
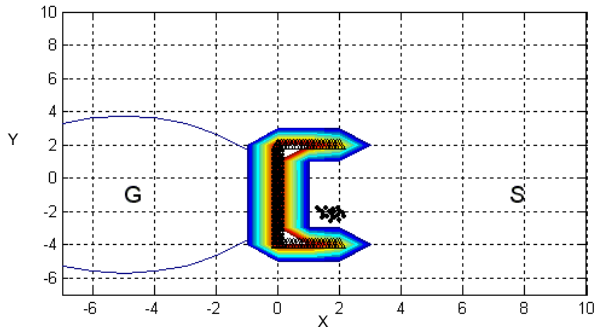
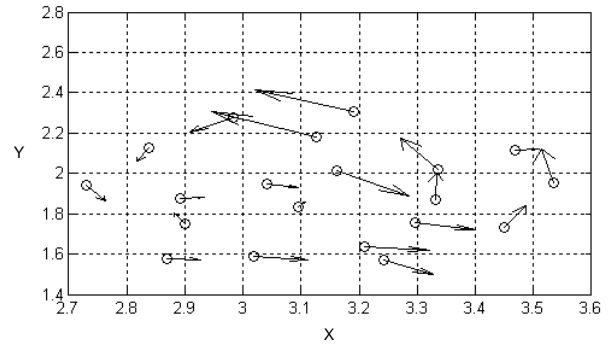
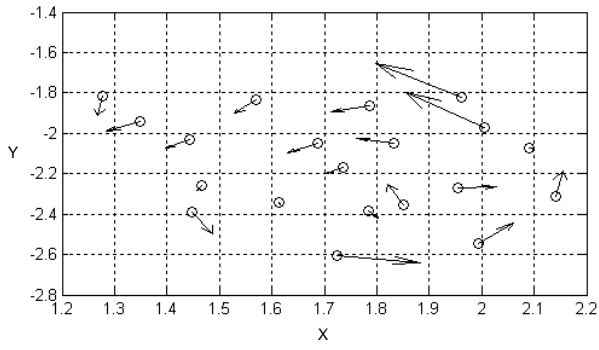


Fig. 3.c.  $t=55$

Fig. 3.e.  $t=220$

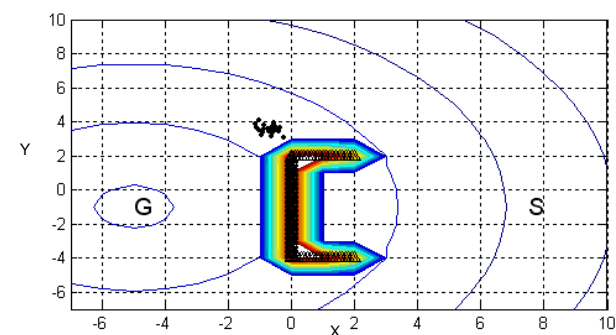
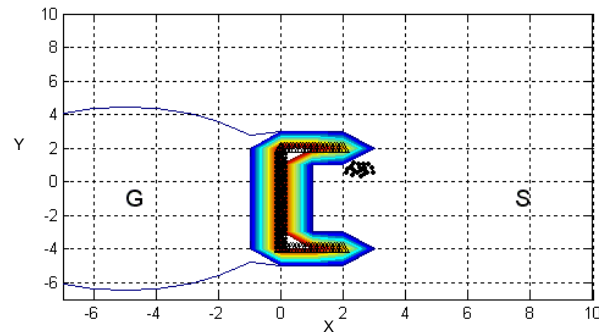
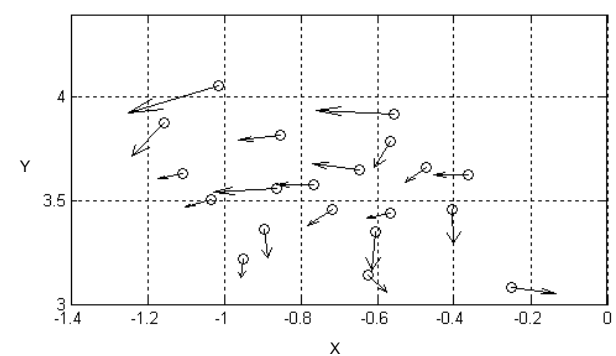
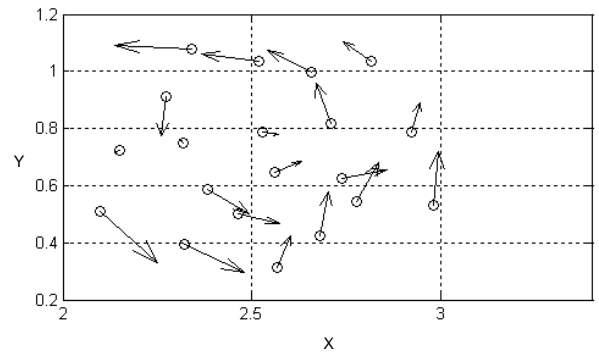


Fig. 3.d.  $t=200$

Fig. 3.f.  $t=260$

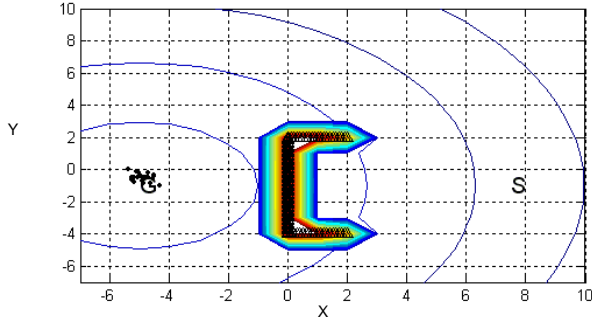
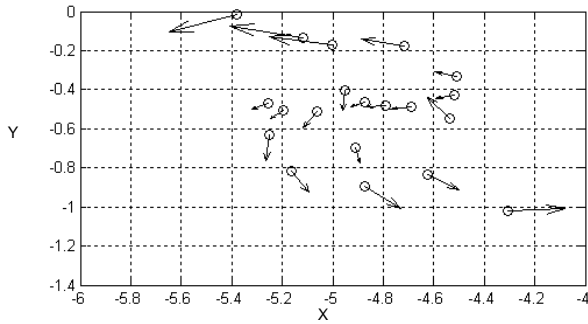


Fig. 3.g. The swarm escape the local minimum,  $t=270$

Fig.3. Behaviour of a swarm using the internal state model with agents' above zoom window.

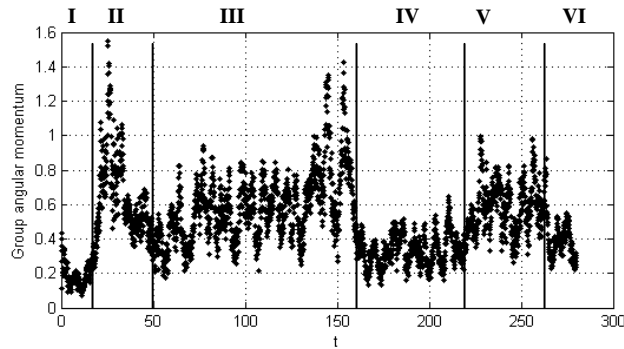


Fig.4. Group angular momentum with time for swarm in Fig.3.

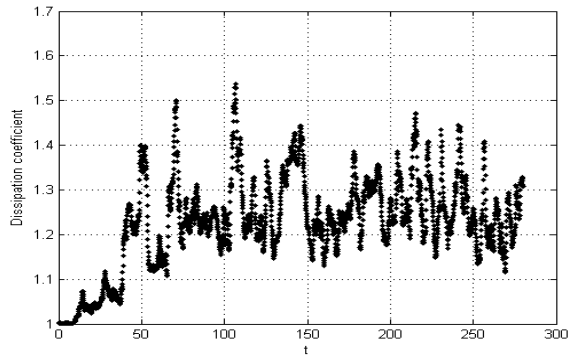


Fig.5. A peripheral agent's dissipation coefficient with time

### B. Solving a Maze Application

We now consider two groups of agents attempting to reach a single goal in a maze whose potential field has multiple local minima. The groups navigate from a starting point  $S$  and attempt to reach a goal position  $G$  through a 4-level maze. One of these two groups, swarm  $A$ , is using the internal state model supported by the wall following technique to solve the maze while the other group, swarm  $B$ , is using a conventional static potential field. The simulation results, shown in Fig. 6, demonstrate the capability of the swarm using the internal state model to solve the problem and reach the goal, while the other conventional swarm is trapped in the first level of the maze. Fig. 7 shows the path of the center-of-mass of swarm  $A$  through the maze to the goal.

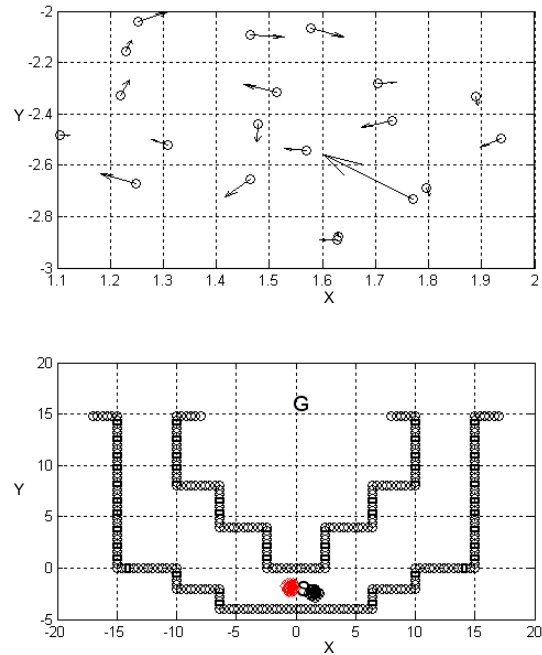


Fig.6.a. The two swarms as the starting position,  $t=0$



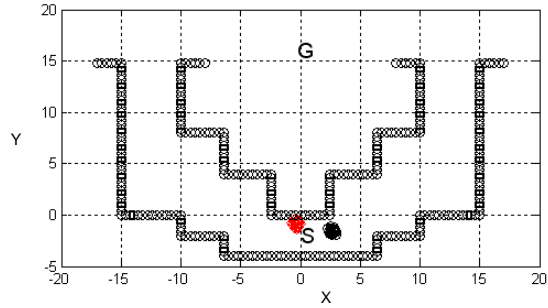
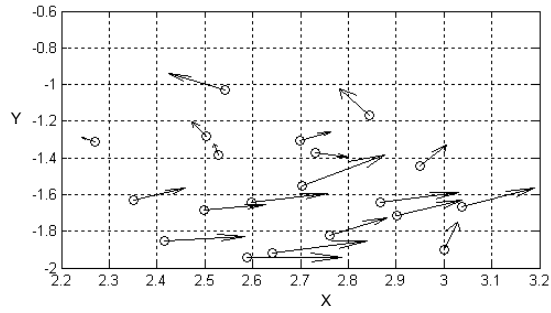


Fig. 6.b.  $t=71$

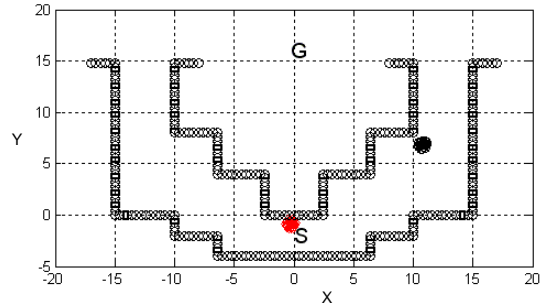
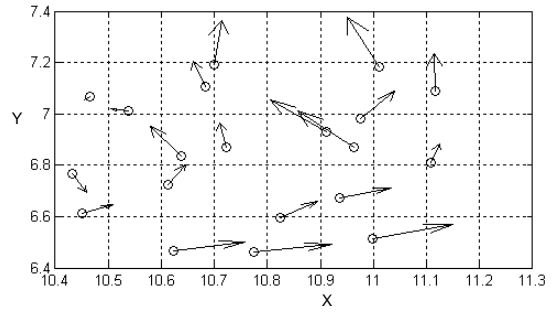


Fig. 6.d.  $t=346$

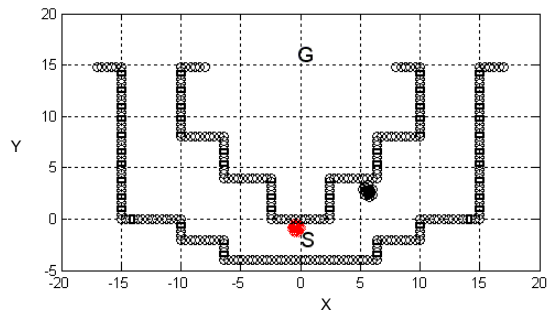
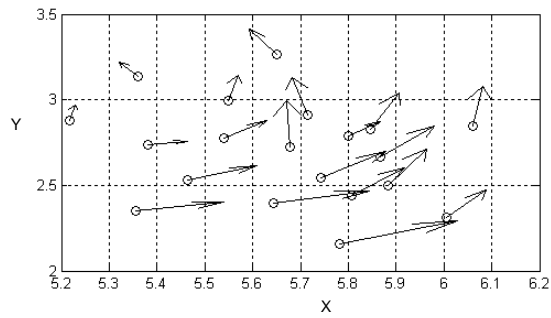


Fig. 6.c.  $t=125$

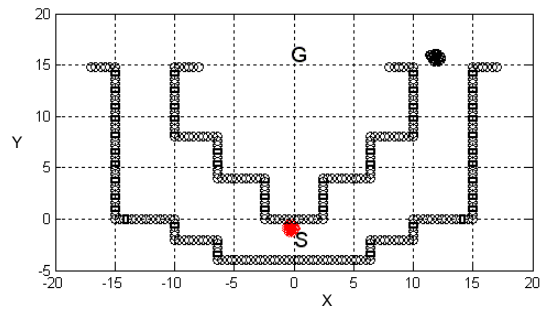
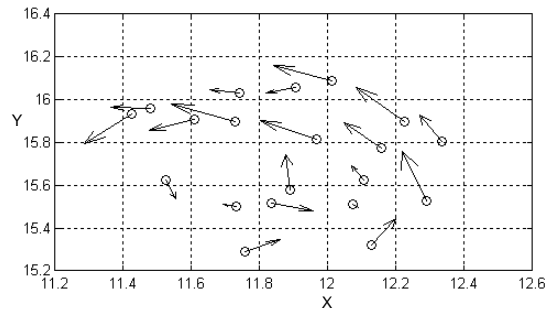
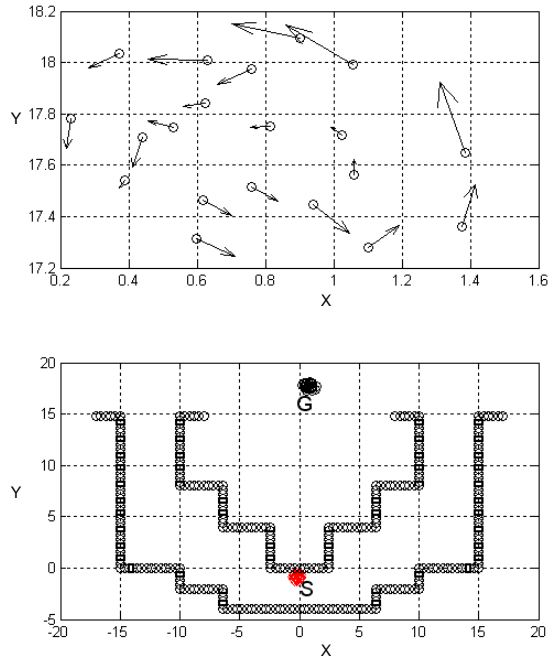
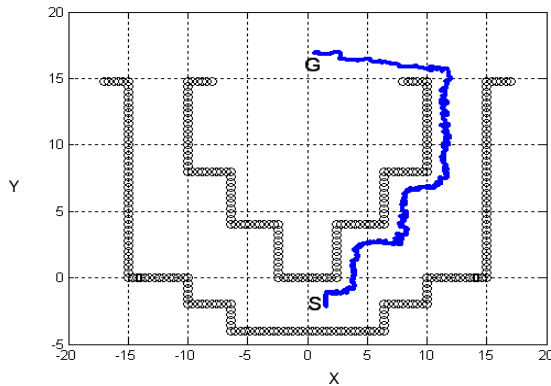


Fig. 6.e.  $t=550$



**Fig. 6.f.** The swarm that use internal state model solves the maze while the swarm with fixed internal states fails.  $t=810$

**Fig.6.** Two swarms in a maze application



**Fig.7.** The path of the swarm center inside the maze

## VII. CONCLUSION

This paper presents a development of our work in overcoming the local minima problem by using the agent's Internal states along with the emergent behaviour of the agents. The model uses the swarm center velocity to solve the problem in two ways. The first way links the goal attraction potential strength in the equation of motion to the swarm center velocity in a way that as the swarm center decreases, the goal effect decreases and the local minimum disappears. At the same time when the goal effect decreases, a swarm vortex pattern emerges. This activates the pure rolling motion of the swarm through which the agents

near to the wall of obstacles acquire higher dissipation coefficient, consequently having lower velocities than those individuals that are far from the wall. This will enable the swarm of agents to achieve pure rolling motion in which the swarm follows the wall boundaries even in case of indirect contact. The simulation results show that, rather than moving in a static potential field, the agents are able to manipulate the potential according to their estimation of whether they are moving towards the goal or stuck in a local minimum and the method allows a swarm of agents to escape from and to manoeuvre around a local minimum in the potential field to reach a goal. This new methodology successfully solves reactive path planning problem, such as a complex maze with multiple local minima, which cannot be solved using conventional static potential fields.

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