Evaluating Plastic Loads in Torispherical Heads Using a New Criterion of Collapse

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Abstract

In ASME Design by Analysis, the plastic load of pressure vessels is established using the Twice Elastic Slope criterion of plastic collapse. This is based on a characteristic load-deformation plot obtained by inelastic analysis. This study investigates an alternative plastic criteria based on plastic work dissipation where the ratio of plastic to total work is monitored. Two sample analyses of medium thickness torispherical pressure vessels are presented. Elastic-perfectly plastic and strain hardening material models are considered in both small and large deformation analyses. The calculated plastic loads are assessed in comparison with experimental results from the literature.

1. Introduction

Design procedures for pressure vessels are divided into two methods: Design-By-Formula, based on strength of materials type analysis and experimental tests, and Design-By-Analysis (DBA), based on detailed stress analysis of the configuration. While design-by-formula is well established, in the last decade Design-by-analysis methods has become more viable and easier to apply. Furthermore, DBA methods offer the possibility to analyze more complex structures. Guidelines for DBA are given in the ASME B&PV Code Section VIII Division 2 Appendix 4 [1]. These are based on elastic and inelastic analyses. In elastic

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analysis the material model is linear elasticity and the calculated elastic stress is categorized into three classes; primary, secondary and peak stress. The allowable load is determined by limiting the categorized (and collected) stress to specified allowable values (for example the general primary stresses should not exceed the design stress S_m). The main problem in this approach is identifying the three stress categories. This problem does not arise in DBA based on inelastic stress analysis, in which the allowable load is determined directly from the elastic-plastic response of the vessel.

Two inelastic analysis methods may be used to calculate the allowable static load of a vessel with reference to the gross plastic deformation failure mechanism: limit analysis and plastic analysis. Limit analysis assumes an elastic-perfectly plastic material model and small deformation theory. The *limit load* is the highest load satisfying equilibrium between external and internal forces. In DBA terminology, a plastic analysis is an inelastic analysis assuming material strain hardening and/or large deformation effects. In practice, strain hardening enhances the static load carrying capacity of a vessel compared to the idealized elastic-perfectly plastic material model. Including large deformation effects may lead to geometric strengthening or geometric weakening, depending on the structural configuration. In plastic analysis the critical load, referred to as the plastic load, is defined as the load at which the plastic deformation becomes excessive; i.e. the load at the onset of gross plastic deformation.

2. Plastic Criteria

Several criteria of collapse load have been proposed in literature to identify the plastic load. A review of the most common criteria, is given by Gerdeen [3]. The most commonly used criterion is the ASME Twice Elastic Slope, TES, technique. In the TES criteria the structural response of a pressure vessel is defined by plotting a load parameter versus a deformation parameter. The plastic load is defined as the load corresponding to the intersection of the load-deformation curve and a straight line emanating from the origin and having twice the slope of the respective elastic response (with respect to the load axis) in the load-deformation curve, as shown in Figure 1a. The latter line is known as the collapse limit line. A number of limitations associated with the TES criterion are identified in the literature [4, 5, 6, 7, 8]. The TES criterion is empirical and intended to yield a conservative plastic load on the basis of experience in experimental stress analysis (tests). It does not specify the nature or form of the gross plastic deformation mechanism in a quantitative manner. In practical terms, the criterion requires an appropriate local deformation parameter has to be selected that will represent the global ductile failure of the vessel. Gerdeen, recommends that the characterizing load and deformation parameters should be chosen such that their product represents work; e.g. moment and rotation. However, this choice is not always appropriate, as has been shown to be the case for torispherical heads. Internally pressurized torispherical heads are subject to complex ductile failure mechanisms, exhibiting (three point) bending mechanisms in the knuckle region and membrane deformation in the cylinder and/or domed end (possibly leading to local thinning and tensile plastic instability). The chosen deformation parameter must represent the limiting ductile failure mechanism experienced by a specific configuration.

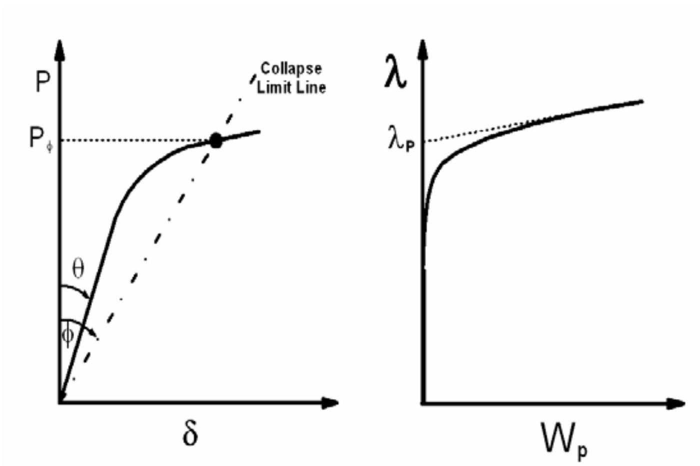


Figure 1: Twice elastic slope and plastic work *criteria*

The plastic work (PW) criterion proposed by Muscat et. al. [9] sets out to define a collapse criterion on a more global basis. In this instance the total plastic work dissipated in the structure is considered, thus representing the global inelastic response of the vessel. Muscat's Plastic Work criterion is characterized by a plot of a load parameter against plastic work dissipation in the vessel. The plastic load is determined by drawing a straight-line tangent from the load-plastic work curve to the load axis as shown in Figure 1b. However the location of the tangent is subjective and the choice of location affects the calculated plastic load. In an attempt to define a concise plastic load the plastic work criterion was subsequently developed by Li and Mackenzie [10], who proposed the Plastic Work Curvature, PWC, criterion. The PWC identifies the plastic load by considering the curvature of the load-plastic work curve, as shown in Figure 2. The curvature characterizes the rate of change of plastic deformation. In the elastic region the curvature is zero while during post yielding and stress redistribution the curvature starts to increase. The maximum curvature corresponds to the maximum stress redistribution, after which the curvature decrease until it settles to minimum constant, indicating gross-plastic deformation.

The PWC criterion was applied to simple bar, beam and cylinder structures in [10] and to a cylindrical nozzle intersection under combined loading in [11]. In these examples, the plastic load was specified as the load at which the PWC returns to zero or near zero following a local maxima in the curve. However, specifying "near zero" PWC is subjective and a more specific definition of plastic load is required. The PWC was applied to torispherical dished ends in [8]. Due to the complex failure mechanisms in the torispherical heads, several regions of stress redistributions with corresponding maxima in the PWC plot were found to occur. In many cases the maxima associated with initial gross plastic deformation was not clearly identified due to scaling effects and the relative magnitude of subsequent maxima. In this paper a new criterion is proposed which relates plastic load and PWC but leads to a distinct value of plastic load, suitable for design applications.

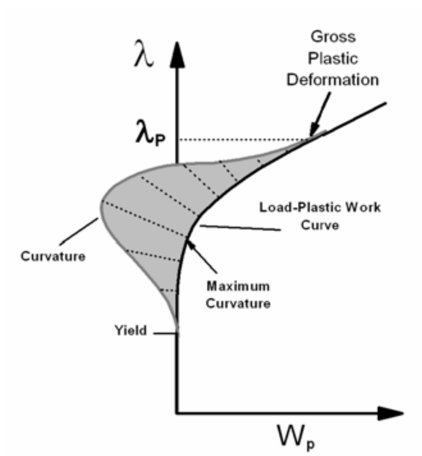


Figure 2: Plastic work curvature criteria

3. A New Plastic Criterion

The new criterion is developed from Gerdeen's proposal that the plastic load can be defined as the load when the plastic work W_p is equal to the elastic work W_e factored by an undefined scalar parameter α [3].

Figure 3a shows a typical element stress–strain response undergoing plastic deformation. Considering the areas under the stress–strain curve, the elastic and plastic work are given by:

$$W_e = \sum_{i=1}^{n} W_{e(i)} = \tfrac{1}{2}\sum_{i=1}^{n} Vol_i \sigma_{eqv(i)} \varepsilon_{eqv(elastic)(i)} \tag{1}$$

$$W_p = \sum_{i=1}^{n} W_{p(i)} = \tfrac{1}{2}\sum_{i=1}^{n} Vol_i \left(\sigma_{eqv(i)} + \sigma_{eqv(Yield)}\right) \varepsilon_{eqv(plastic)(i)} \tag{2}$$

When DBA is based on Finite Element Analysis, the plastic work and elastic work (strain energy) are standard results calculated by the program.

Li and Mackenzie have showed that the value of α in Gerdeen's criterion is problem dependent and a general value cannot be defined. In the criterion proposed here, the development of the gross plastic deformation mechanism is characterized by plotting the ratio of plastic work W_p to the total work (W_p+W_e) against the applied load. The curvature of the work ratio-load curve is then obtained from the inverse of the circumradius of three consecutive points, as shown in Figure 3b and equation 3

$$R = \frac{abc}{4\sqrt{s(a+b-s)(a+c-s)(b+c-s)}} \tag{3}$$

where s is the semiperimeter given by:

$$s = \frac{a+b+c}{2} \tag{4}$$

The curvature of the work ratio-load plot is thus obtained as shown in Figure 4. The plot is divided into an elastic and a plastic dominant phase. The latter stage occurs when the plastic work is equal or higher than the elastic work. In the plastic phase, the curvature describes the rate of change from plastic deformation to gross plastic deformation, followed by a 'near zero' curvature that will lead to global collapse. In this criterion, referred to as Ratio Plastic Work Curvature (RPWC) criterion, the onset of gross plastic deformation (plastic load) is identified at the load equivalent to the maximum curvature.

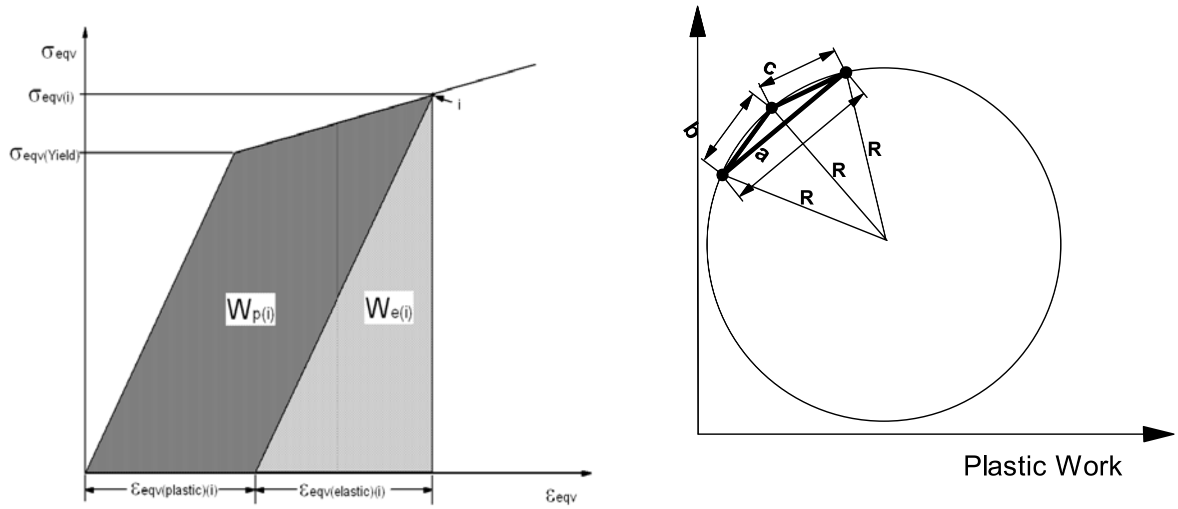


Figure 3: a) Elastic and plastic work b) Circumradius of three points

4. Sample Problems

The criteria of collapse were applied to two cylindrical vessels previously investigated experimentally by Kalnins and Rana [12]. The vessels have a welded torispherical head at one end and a 2:1 elliptical head at the other. Experimental analysis showed that gross plastic deformation occurred in the torispherical ends. Therefore, in the finite element analyses, only half of the vessels (the torispherical half) were modelled, with symmetric boundary conditions applied to the end of the cylindrical section of the vessel. The geometry and dimensions of the models are defined in Figure 5. The measured mean dimensional values of the vessels were used in the analyses. The heads were modelled using ANSYS 8 noded axisymmetric Plane82 elements [13]. The mesh consisted of a total of 2688 elements having 6 elements through thickness and refined at the knuckle and crown region.

Elastic perfectly plastic and strain hardening material models were used. In the strain hardening model, the experimentally determined ultimate tensile stress and strain together with the yield stress / strain [12], were used to establish a linear plastic modulus. Thereafter, the material is assumed to exhibit perfect plasticity, as shown in Figure 6. As opposed to [12], the strain-hardening assumes a linear relationship between the stress and strain. Table 1 gives the material properties for Vessels 1 and 2. These values were used for small and large deformation analyses, where engineering and true stress-strain models are used respectively. For direct comparison with the experimental results, different material properties were applied to the cylinder and head. The ANSYS inelastic routines used in the analyses are based on the von Mises model of yielding and associated flow rule.

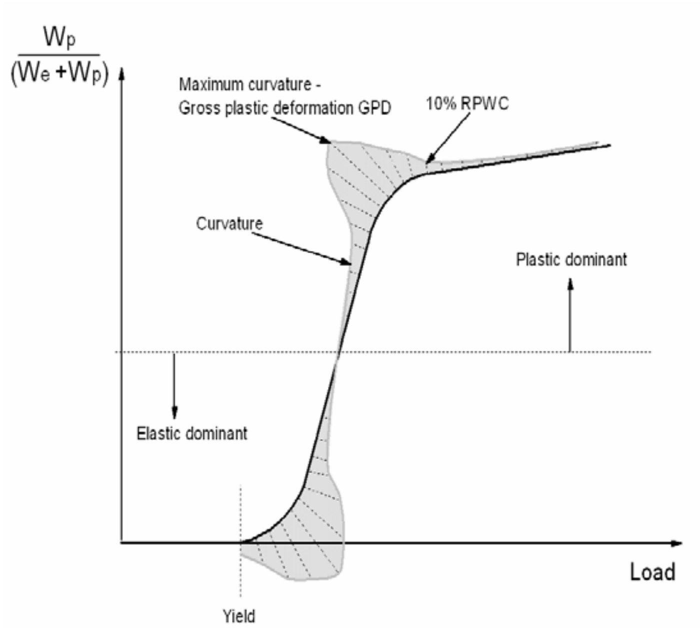


Figure 4: Ratio plastic work curvature criteria

	Vessel 1		**Vessel 2**	
	Cylinder	Head	Cylinder	Head
Young's modulus (GPa)	205.5	205.5	210.5	208.9
Yield strength (MPa)	365.7	424.9	387.3	375.3
Engineering ultimate tensile strength (MPa)	533.2	572.3	579.9	550.9
Engineering ultimate tensile strain	0.191	0.161	0.178	0.200

Table 1: Material properties of pressure vessels

5. Results

The design load for each vessel was calculated using the TES criterion (load-deformation plots) and RPWC criterion (load - ratio plastic/total work curvature plots). The load at which the numerical solution failed to converge (due to violation of equilibrium, excessive deformation or excessive plastic strain increment) was also recorded. This is referred to as the numerical instability load. This corresponds to the limit load when small deformation theory and elastic-perfectly material are assumed and to the *tensile plastic instability* load [14] when large deformation theory and strain hardening material are assumed in the analysis. The deformation parameters used in the TES criterion were the radial displacement of the middle of the knuckle, the radial displacement of the cylinder and the axial displacement of the crown.

In interpreting the results of the curvature plots, the PWC and RPWC are normalised with respect to the maximum value of the PWC and RPWC respectively obtained in the analyses. Here, the pressure corresponding to the maximum RPWC is identified and specified as the plastic load. The pressures corresponding to local maxima identified in the PWC are reported for comparison and discussion purposes

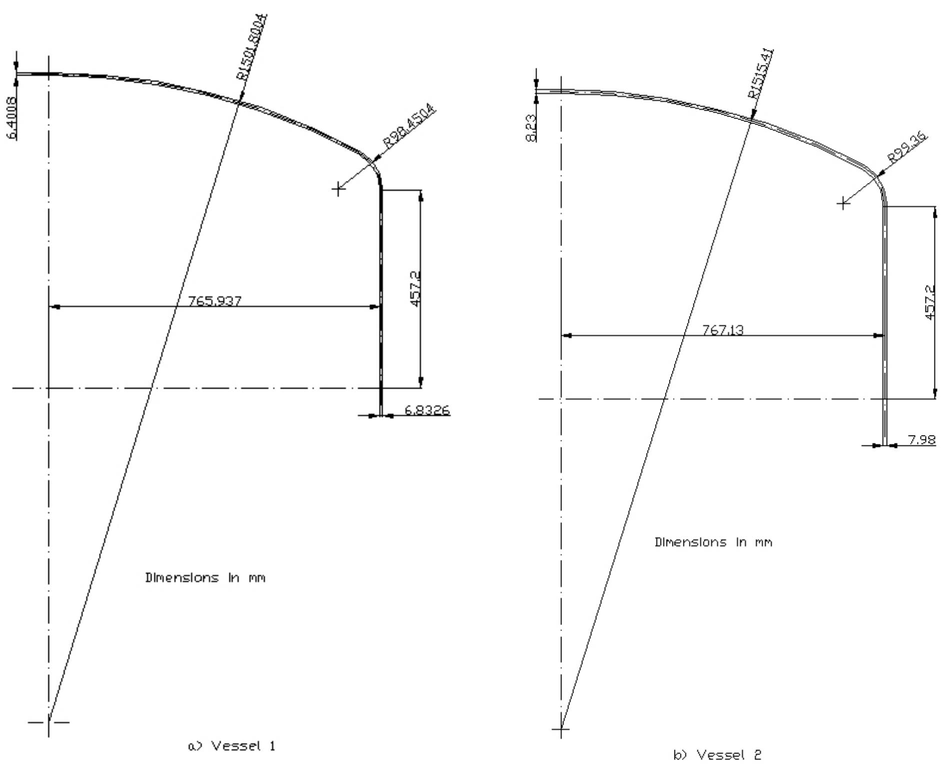


Figure 5: Schematic diagram of pressure vessels

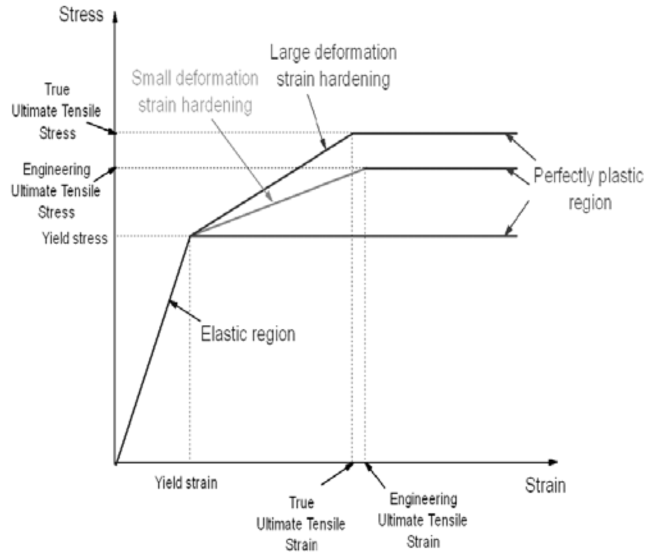
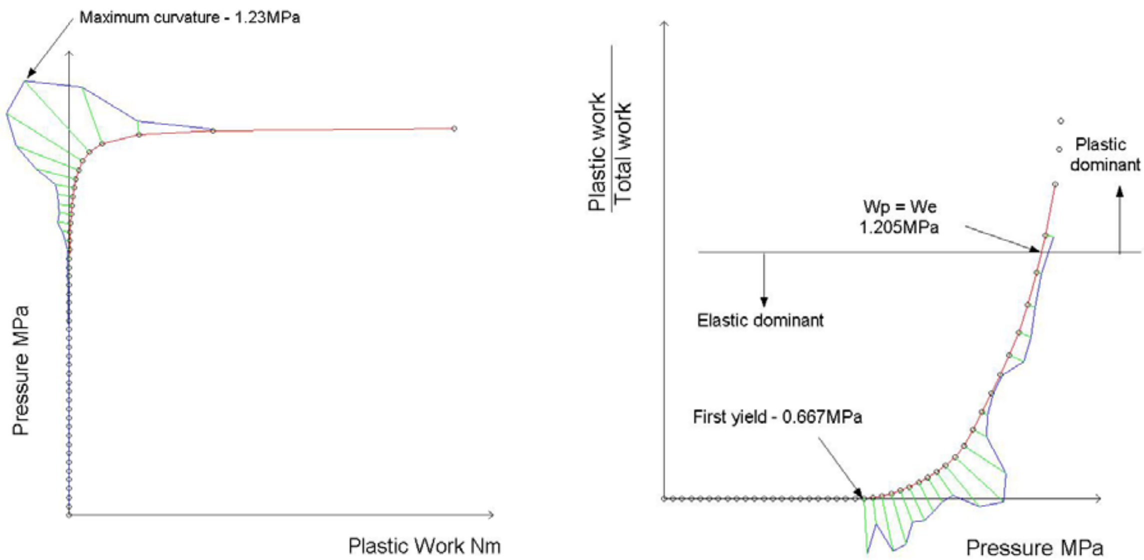


Figure 6: Elastic perfectly plastic and strain hardening material models

5.1 Vessel 1

Initial yielding occurs at the inside surface of the knuckle region at a pressure of 0.667MPa for all analyses. In limit analysis, a three point bending hinge mechanism forms at the knuckle region and develops through the thickness until the analysis fails to converge at a pressure of 1.310MPa. As shown in Figure 7, a distinct peak is observed in the load-PWC plot but in the RPWC-load plot the analysis terminates before the point at which gross plastic deformation is identified (due to the perfect plasticity and small deformation assumptions). In this instance the RPWC fails to give a curvature plot for the plastic dominant phase. A conservative approach to calculating the design load for limit load analysis and using the RPWC is to assume the design load to be the point when the structure becomes plastically dominant or when $W_p = W_e$. In this instance the corresponding load is 1.205MPa.

A three point bending hinge mechanism is also observed when strain hardening material properties and small deformation theory are assumed for Vessel 1. However, in the load–PWC two distinct peaks are observed in Figure 8. The first peak at 1.320MPa is due to the change from elastic to plastic behaviour and the second peak at 1.650MPa reflects the change from a strain hardening model to a perfectly plastic model. However in the RPWC–load plot one distinct peak is observed at 1.410MPa and this load is specified as the point when gross plastic deformation occurs. This value is close to the minimum value of PWC occurring between the two peaks in the curve (which occurs at 1.440MPa).



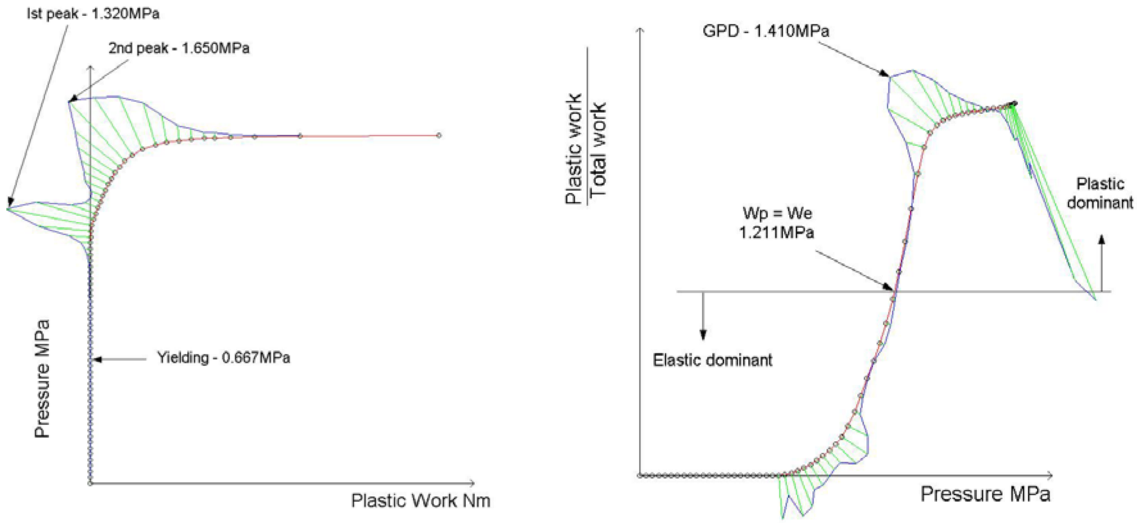
a) PWC criterion

b) RPWC criterion

Figure 7: Limit load analysis – Vessel 1

When large deformation theory is applied to Vessel 1 (for both material models) the vessel is demonstrates two types of failure mechanisms; a three point bending hinge mechanism at the knuckle region and membrane deformation in first the cylindrical and later the crown regions. The two mechanisms are identified with the two distinct peaks in the curvature plot. The first peak corresponds to the bending mechanism and the second peak to the membrane mechanism. As seen in Figure 9, the large change in stress redistribution associated with the membrane response, the second peak, dominates the curvature plot to the extent that the first peak is not clearly visible. This makes interpretation of plastic pressure by the PWC as applied in reference [8] potentially difficult in a design environment. The RPWC is simpler to apply as the gross plastic deformation is assumed to occur at the load corresponding to the clearly identified maximum RPWC.

Table 2 shows the plastic loads when using the RPWC and TES criteria. The pressures corresponding to the first and second maxima in the PWC plot are included for comparison, as are the experimental failure pressure and calculated burst pressure from [12]. In the experiment reported in [12], the vessel failed by brittle fracture at a flaw in the nozzle to shell wall at a pressure of 4.826MPa prior to the occurrence of significant plastic deformation; the predicted ductile burst load was 8.515MPa [12].



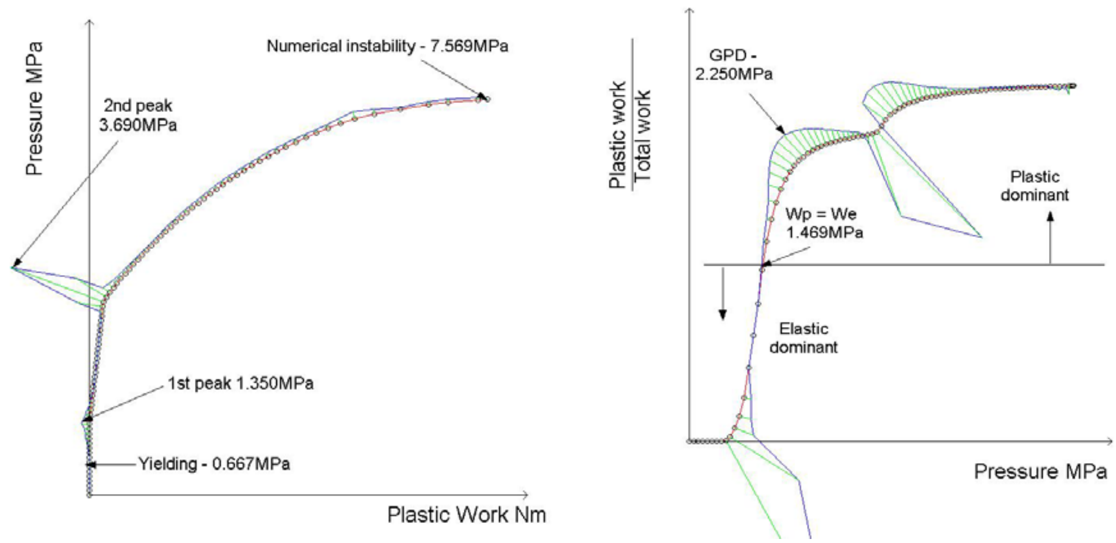
a) PWC criterion

b) RPWC criterion

Figure 8: Small deformation strain hardening model – Vessel 1

As was found in [12], the limit load is much smaller than the experimental failure load and predicted plastic instability load. The numerical instability load for large deformation theory and strain hardening material is greater than the experimental burst pressure, close to the predicted burst pressure (or tensile plastic instability load).

The plastic loads given by the various plastic criteria are all significantly lower than the plastic instability load. These criteria are intended to establish the pressure at which a gross plastic deformation mechanism occurs rather than local thinning and tensile instability.



a) PWC criterion b) RPWC criterion

Figure 9: Large deformation strain hardening model – Vessel 1

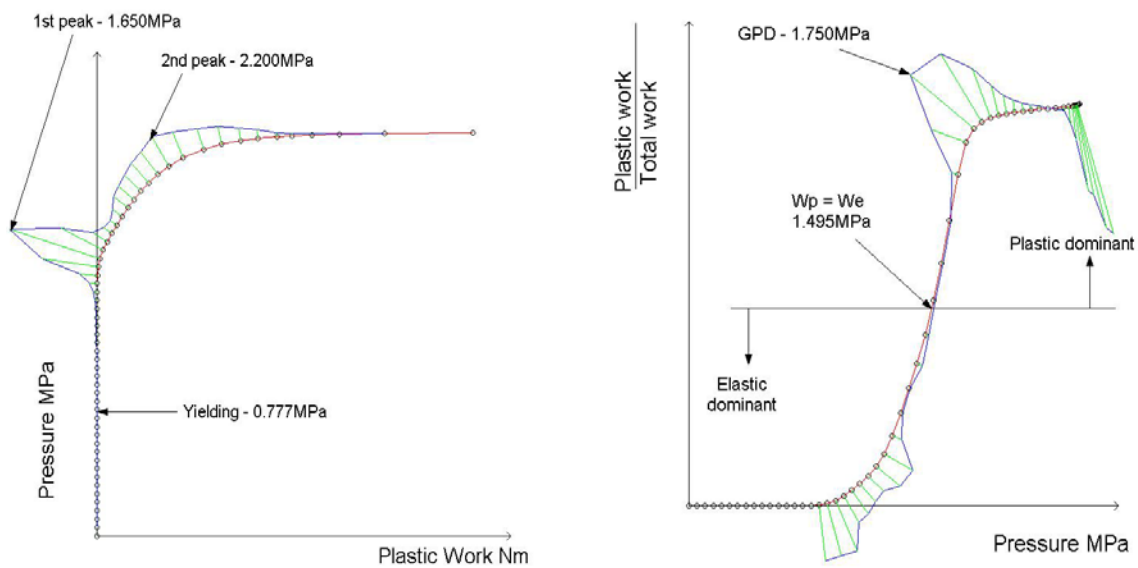
The TES criterion gives lower values of plastic load than the RPWC criterion, with very little strength enhancement due to material strain hardening. The RPWC criterion does not give a result for the limit load analysis: in design the limit load would be taken as that corresponding to the last convergent solution. In the strain hardening and large deformation analyses, the RPWC identifies a distinct point at which plastic deformation becomes dominant. The corresponding pressure lies between the two peaks in the PWC at the minimum value of curvature.

Pressure load in MPa		Small deformation analysis: Elastic – perfectly plastic	Small deformation analysis: Strain hardening model	Large deformation analysis: Elastic – perfectly plastic	Large deformation analysis: Strain hardening model
Numerical instability load		1.310	1.781	4.848	7.569
PWC criterion	First peak	1.230	1.320	1.350	1.350
	Second peak	n/a	1.650	3.700	3.690
RPWC criterion	Wp=We	1.205	1.211	1.450	1.469
	GPD-Plastic Load	n/a	1.410	2.000	2.250
TES criterion	Knuckle	1.241	1.219	1.536	1.546
	Crown	1.234	1.243	1.684	1.730
	Cylinder	1.238	1.250	1.595	1.620
Experimental and predicted results: Reference [12]:		Experimental TES crown deformation: n/a Experimental failure load: 4.826MPa Predicted burst pressure: 8.515MPa			

Table 2: Vessel 1 – plastic pressures

5.2 Vessel 2

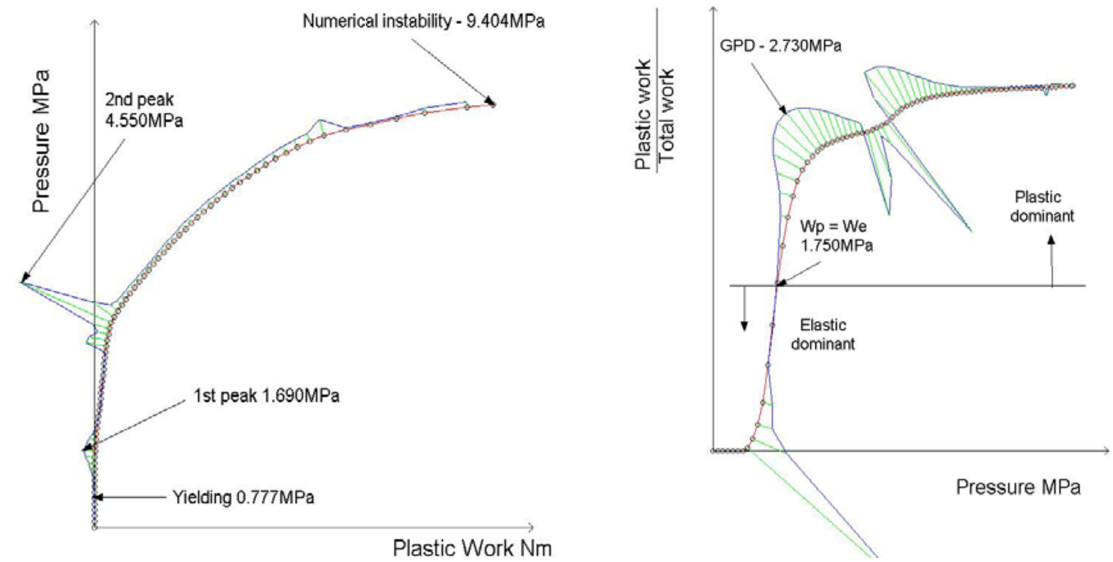
Similar stress redistributions and failure mechanisms are observed for pressure Vessel 2. Initial yielding also occurs at the inside surface of the knuckle region but at a higher pressure of 0.777MPa. This is independent of the material model and deformation theory used. In limit load analysis failure due to a three point bending hinge mechanism occurs at a pressure of 1.630MPa. A single peak is also observed in the load-PWC, while the RPWC criterion fails to give a maximum curvature RPWC for the plastic dominant phase. The change from elastic to plastic dominant deformation occurs at a pressure load of 1.487MPa. When a strain hardening small deformation model is applied to vessel 2 two distinct peaks at 1.650MPa and 2.200MPa are observed in the load –PWC (see Figure 10). As in pressure vessel one this is due to the material model assumed (elastic-strain hardening – perfect plasticity). Such changes are not seen in the RPWC – load plot and gross plastic deformation occurs after a pressure of 1.750MPa.



a) PWC criterion b) RPWC criterion

Figure 10: Small deformation strain hardening model – Vessel 2

In large deformation theory the vessel fails due to three point bending hinge mechanism at the knuckle region and due to membrane deformation at the crown and cylinder region. As in Vessel one, the two plastic deformation mechanisms are identified with two peaks in the PWC. Again, the first peak in the load-PWC plot is dwarfed by the second peak, however the RPWC-load plot clearly identifies the two stress redistributions (refer to figure 11).



a) PWC criterion b) RPWC criterion

Figure 11: Large deformation strain hardening model – Vessel 2

A comparison between the RPWC and TES criteria plastic pressures for Vessel 2 is given in Table 3, with the pressures corresponding to the first and second maxima in the PWC plot included for comparison. The experimental failure pressure and calculated burst pressure from [12] are also given. In the experiment reported in [12], the vessel failed at the longitudinal weld of the shell at a pressure of 7.446MPa before the predicted ductile burst load of 11.31MPa was achieved [12].

Pressure load in MPa		Small deformation analysis		Large deformation analysis	
		Elastic –perfectly plastic	Strain hardening model	Elastic –perfectly plastic	Strain hardening model
Instability -load		1.630	2.397	5.987	9.404
PWC criterion	First peak	1.500	1.650	1.700	1.690
	Second peak	n/a	2.200	4.500	4.550
RPWC criterion	Wp=We	1.487	1.495	1.728	1.750
	GPD-Plastic load	n/a	1.750	2.400	2.730
TES criterion	Knuckle	1.493	1.504	1.817	1.820
	Crown	1.526	1.535	1.960	1.977
	Cylinder	1.530	1.538	1.871	1.901
Experimental and predicted results: Reference [12]:		Experimental TES crown deformation – 2.551MPa Experimental failure load – 7.446 MPa Predicted burst pressure – 11.31MPa			

Table 3: Vessel 2 – plastic pressures

The TES criterion gives lower values of plastic load than the RPWC criterion, with no significant strength enhancement observed when strain-hardening effects are included. The maximum curvature of the RPWC criterion, gives a plastic load of 0.29 times the instability load.

6. Conclusions

A new criterion of plastic collapse, the Ratio Plastic Work Curvature criterion, has been proposed and applied to the complex problem of elastic-plastic behaviour of torispherical pressure vessels. The RPWC criterion is simple to apply and can be wholly automated for design purposes through user-routines and macros for commercial finite element programs or external coding. In the investigation, plastic loads evaluated using the RPWC criterion were compared with experimental results, Twice Elastic Slope criterion plastic pressures and the Plastic Work Curvature. The RPWC criterion was shown to clearly identify the load at which plastic deformation becomes dominant and gross plastic deformation occurs. The plastic loads calculated using the RPWC indicate the strength enhancing effect of material strain hardening and geometric non-linearity. However, the plastic loads are conservative with respect to the tensile plastic instability load and the reported experimental failure loads of the vessels (in which failure occurred by other mechanisms prior to the onset of tensile plastic instability). It is intended to investigate the performance of the proposed criterion for other pressure vessel configurations in future work.

Acknowledgments

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