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Parameter estimation of nonlinear econometric models using particle swarm optimization

Mark P. WACHOWIAK, Nipissing University, Canada[i]
Renata WACHOWIAK-SMOLÍKOVÁ, Nipissing University, Canada[i]
Dušan SMOLÍK, VŠB-TU Ostrava [ii]

Abstract

Global optimization is an essential component of econometric modeling. Optimization in econometrics is often difficult due to irregular cost functions characterized by multiple local optima. The goal of this paper is to apply a relatively new stochastic global technique, particle swarm optimization, to the well-known but difficult disequilibrium problem. Because of its co-operative nature and balance of local and global search, particle swarm is successful in optimizing the disequilibrium maximum likelihood function, providing better values than those reported in the literature obtained using other stochastic techniques. These encouraging results suggest that particle swarm optimization may be successfully applied to difficult econometrics problems, possibly in conjunction with existing methods.

Keywords

Disequilibrium, econometric modeling, econometrics, optimization, parameter estimation, particle swarm optimization

JEL Classification: C02, C13, C51, C61

[i] Department of Computer Science and Mathematics, Nipissing University, North Bay, ON, P1B 8L7 Canada.
[ii] Department of Regional and Environmental Economics, Faculty of Economics, VŠB-Technical University of Ostrava, Sokolská 33, 701 21 Ostrava, Czech Republic.
markw@nipissingu.ca (corresponding author)

1. Introduction

Global optimization is the determination of the globally best solution of models – which, in econometrics, are typically nonlinear – in the presence of many local minima. Optimization is a key component in estimation and modeling problems that are intractable by using standard numerical methods (see Gilli and Winker, 2009). Because these problems are ubiquitous in econometrics, there is a growing need for robust and efficient global optimization techniques. Typical problems include long-term financial planning (see Maranas et al., 1997), portfolio management (see Dallagnol et al., 2009), time series forecasting models (see Behnamian and Fatemi Ghomi, 2010), estimating GARCH models (see Jerrell and Campione, 2001), and numerous applications in the emerging field of spatial econometrics (see LeSage, 2005), to name just a few.

Because models of substantial explanatory and predictive power (models that are not overly-simplified and that can be used for prediction and extrapolation) are frequently quite complex systems, global optimization has been required in parameter estimation problems, where data observations are frequently incomplete or noisy. Parameter estimation techniques range from ordinary least squares (OLS) for relatively simple autoregressive models to advanced global optimization methods (see Jerrell and Campione, 2001). The latter approaches are frequently needed when a model unit or observation reacts instantaneously to other units, without a time delay.

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For these situations, techniques such as OLS fail, and other methods must be applied.

In this paper, a global optimization approach based upon computational intelligence is proposed to specify a complex econometric model, and to motivate its use for more advanced applications. Specifically, the efficacy of particle swarm optimization (PSO) to estimate parameters of an important class of econometrics problems, the disequilibrium model, is demonstrated. The disequilibrium problem was chosen primarily because of its complexity, its analytical cost function, and because of its utility in comparing various approaches (see Jerrell and Campione (2001), Fair and Jaffe (1972)).

The remainder of the paper is organized as follows: Section 2 describes the maximum likelihood estimator (MLE) cost function, which is to be optimized. Section 3 introduces particle swarm optimization. Section 4 describes the specific econometrics problem studied in this paper – the disequilibrium problem. In Section 5, the MLE cost function for the disequilibrium probem is given. Improvements to and adaptations of the particle swarm approach to the disequilibrium problem are also described. Experimental results on realistic simulated data are provided in Section 6, and comparisons are made to results using other optimization methods found in the literature, while Section 7 discusses these results in the context of other, more complex econometrics problems, and offers some suggestions for future improvements and directions.

2. Fitness Functions in Econometrics

Assume that an economic model can be represented as $y_t = f(\mathbf{x}_t, \boldsymbol{\beta}) + \varepsilon_t$, where y_t is a scalar-valued observation at time t, $\mathbf{x}_t$ is a vector of explanatory (predictor, or independent) variables, $\boldsymbol{\beta}$ is a vector of parameters, and ε_t represents the sampling error for observation t. The goal is to obtain estimates of $\boldsymbol{\beta}$ so that the model best predicts the behavior of the modeled entity. The maximum likelihood estimator (MLE) finds the parameter estimates that give the highest probability of generating the observed sample. Given T samples, the goal is to find estimates of $\boldsymbol{\beta}$ and σ^2 that maximize the likelihood L (see Jerrell and Campione, 2001):

$$L(\hat{\boldsymbol{\beta}}, \hat{\sigma}^2) = \frac{1}{(2\pi\hat{\sigma}^2)^{T/2}} \cdot \\ \cdot \exp\left\{ - \frac{[\mathbf{y} - \mathbf{f}(\mathbf{X}, \hat{\boldsymbol{\beta}})]^{\mathrm{T}} [\mathbf{y} - \mathbf{f}(\mathbf{X}, \hat{\boldsymbol{\beta}})]}{2\hat{\sigma}^2} \right\}. \tag{1}$$

In the case of linear models, the problem results in a system of linear equations that can be solved using standard linear algebra techniques. However, in large or nonlinear models, maximizing likelihood may pose numerical problems. For instance, even simple model specifications can produce near-flat likelihood functions or functions with numerous local optima. As a result, standard numerical approximation algorithms may fail (see Gilli and Winker, 2009).

Derivative-based optimization methods are preferred if derivatives are available or can be accurately estimated. However, for many MLE functions, derivatives can be complex and difficult to compute, and, in some cases, the functions may not even be differentiable. Numerical derivative calculations may also fail because of truncation or rounding errors. Therefore, deterministic, derivative-based optimization methods are often not effective, and may fail completely.

Stochastic methods – many of which are inspired by natural phenomena – rely on some degree of randomness. They address the shortcomings of deterministic optimization, and are robust in the presence of difficult, non-smooth functions characterized by many local optima. They have also been shown to provide remarkably robust solutions to difficult optimization problems, where traditional techniques have failed because of entrapment in local optima (see Gilli and Winker, 2009), (see Riders on a Swarm (2010) for a non-technical account of computational intelligence). However, these methods generally require greater computation time, and multiple runs of the same technique on the same problem may produce different results. Several of these stochastic techniques have been applied to the disequilibrium problem, including genetic algorithms (GA) and evolutionary strategies (ES), both of which are population-based competitive algorithms, simulated annealing, tabu search, and hybrid methods (see Gilli and Winker (2009), Jerrell and Campione (2001)). The current paper proposes the application of a different nature based method: particle swarm optimization.

3. Particle Swarm Optimization

PSO is a relatively new stochastic global optimization algorithm (see Kennedy and Eberhart (1995), Kennedy et al. (2001), Parsopoulos and Vrahatis (2002)). Like GA and ES, PSO is an iterative, stochastic population-based technique. However, in contrast to GAs and ESs, which exploit the competitive characteristics of biological evolution, PSO simulates cooperative and social behavior, such as fish schooling, birds flocking, or insects swarming. A diffuse population of P individuals, now termed as particles, explores the search space, gradually forming smaller swarms in minima regions. PSO has been successful

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in many problems, including those in econometrics and manufacturing (see Dallagnol et al. (2009), Behnamian and Fatemi Ghomi (2010), Liu et al. (2006)), discrete scheduling problems (see Liao et al., 2007), and in multi-objective problems (see Yang et al., 2008). Investigation into the theoretical and convergence properties of PSO is an active, on-going research pursuit (see Parsopoulos and Vrahatis (2002), Niknam and Amiri (2010), Pedersen and Chipperfield (2010), Zhan et al. (2009), Mendes et al. (2004)).

At each iteration, a *d*-dimensional particle $\mathbf{p}_i$, $i = 1, \ldots, P$, representing a point in the search space, evaluates $f(\mathbf{p}_i)$, where *f* denotes the cost function. The particle then moves through the search space by the addition of a velocity vector, which is a function of the best position found by that particle – its personal best $(\mathbf{p}_i^{\mathbf{best}})$ – and of the best position – the global best $(\mathbf{g}^{\mathbf{best}})$ – found so far among all particles. In this way, particles tend to *swarm* around the best position, and to converge about this point. During this process, particles may move on a trajectory in which a better response is encountered, in which case the other particles adjust their movements to gather around the new point. However, there is considerable freedom in the particle's movement, as it is influenced not only by the best global position, but also by its personal best position and by random effects. Details of the algorithm are found in Kennedy et al. (2001). PSO offers several benefits: it requires only simple mathematical operators; it is computationally inexpensive, in terms of both memory requirements and speed; its population-based aspects make PSO resistant to the problem of local minima; very few parameters are needed, often resulting in less fine-tuning; PSO exhibits diverse response in that the search does not occur along excessively narrow channels in the search space (see Parsopoulos and Vrahatis, 2002); and, very importantly, the algorithm is inherently parallel, and can make use of new, inexpensive parallel computer hardware. In addition, PSO provides good solutions to global optimization problems in noisy or continuously changing environments (see Parsopoulos and Vrahatis, 2002).

Of the parameters that PSO does require, the inertial weight *w* indicates the relative influence of the velocity in the previous iteration and determines the degree to which the particle should move in the same direction as in the last iteration. It is usually a monotonically decreasing function of the iteration *t*, but can be adjusted adaptively depending upon the progress of the search. This parameter is critical, as it regulates the balance between global (over the entire search space) and local (nearby) searches. Large values of *w* facilitate global exploration, while small values tend to improve solutions in small neighborhoods (see Parsopoulos and Vrahatis, 2002). Two other parameters, C_1 and C_2, are not critical for the operation of PSO, but tuning these parameters may result in better convergence and avoidance of local optima. C_1 is known as a cognitive (self or personal) parameter, while C_2 is the social (swarm) parameter. Usually, $C_1 = C_2$, but some investigators have suggested that the cognitive parameter be set higher than the social parameter (see Parsopoulos and Vrahatis, 2002).

With these parameters, the basic velocity update equation for particle *i* is given as:

$$\mathbf{v}_i(t+1) = w(t)\mathbf{v}_i(t) + C_1\varphi_1\left(\mathbf{p}_i^{\text{best}} - \mathbf{x}_i(t)\right) + C_2\varphi_2\left(\mathbf{g}^{\text{best}}(t) - \mathbf{x}_i(t)\right), \tag{2}$$

where $\mathbf{v}_i(t)$ is the velocity of a particle *i* at time *t*, $\mathbf{v}_i(t+1)$ is the velocity in the next time period $t+1$, $\mathbf{p}_i^{\mathbf{best}}$ denotes the best position of $\mathbf{p}_i$ thus far in the search, $\mathbf{x}_i(t)$ denotes the particle's current position at time *t*, and random numbers φ_1, $\varphi_2 = U(0,1)$ add stochasticity to the update and diversity to the swarm population.

After the new velocity is computed, the position of the *i*-th particle is updated as:

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t+1). \tag{3}$$

Convergence criteria for PSO include reaching a pre-specified maximum number of iterations, reaching a pre-specified number of iterations without improvement in the cost function, and attaining a variance of the responses from all particles falling below a threshold. A flowchart of the basic PSO algorithm is shown in Figure 1.

Numerous adaptations and enhancements have been made to PSO, many of which were designed to solve specific optimization problems, or to address shortcomings to which PSO is susceptible (see Parsopoulos and Vrahatis, 2002).

4. The Disequilibrium Problem

Markets, especially those for commodities, are considered to be in equilibrium if the supply for that market is equal to the demand. However, whether markets are in equilibrium cannot be accurately determined due to the long acquisition time of economic data. The difficulty lies in estimating supply and demand schedules for disequilibrium markets. That is, in such markets, it cannot be determined whether a particular quantity traded was a supply or

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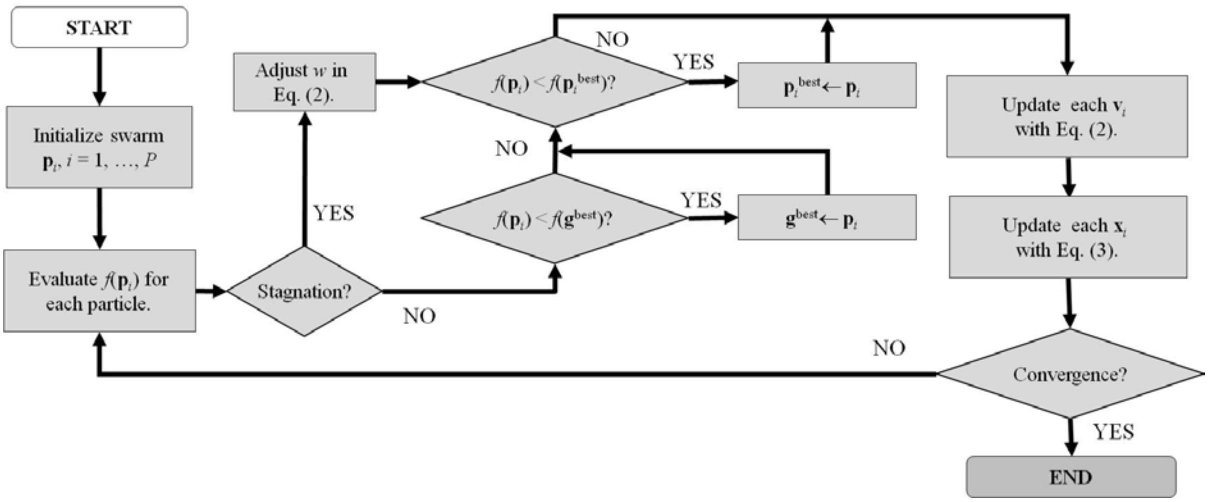

Figure 1 The PSO algorithm.

demand quantity. Because consumers cannot buy more than what is offered, and do not in general purchase more than they demand, the observed quantity purchased (or traded) at a particular time period *t* is the lesser of the quantity supplied (S_t) and the quantity demanded (D_t) (see Jerrell and Campione, 2001). Specifically,

$$D_t = \mathbf{X}_{1t}'\boldsymbol{\beta}_1 + u_t, \quad S_t = \mathbf{X}_{2t}'\boldsymbol{\beta}_2 + v_t, \quad Q_t = \min(D_t, S_t). \tag{4}$$

Here, D_t and S_t respectively denote the quantities demanded and supplied during time period *t*, Q_t denotes the quantity transacted, $\mathbf{X}_{1t}$ and $\mathbf{X}_{2t}$ are *k*-dimensional vectors of explanatory variables, and u_t and v_t denote independently distributed error terms (see Sapra, 1986).

5. Methods

In the disequilibrium model discussed above, it is assumed that the error terms are distributed such that $u_t = N(0, \sigma_1)$ and $v_t = N(0, \sigma_2)$. $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are the parameters to be estimated. The number of $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ parameters are the same as the number of the explanatory variables $\mathbf{X}_{1t}$ and $\mathbf{X}_{2t}$, respectively. The cost function is the MLE for the disequilibrium model, which is then given as follows (see Maddala and Nelson, 1974):

$$h_{1t} = \frac{Q_t - \mathbf{X}_{1t}'\boldsymbol{\beta}_1}{\sigma_1}, \; h_{2t} = \frac{Q_t - \mathbf{X}_{2t}'\boldsymbol{\beta}_2}{\sigma_2},$$

$$f_{1t} = \frac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-h_{1t}^2/2\right), \quad f_{2t} = \frac{1}{\sqrt{2\pi}\sigma_2}\exp\left(-h_{2t}^2/2\right),$$

$$F_{1t} = \int_{h_{1t}}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left(-u^2/2\right)du = \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{h_{1t}}{\sqrt{2}}\right)\right),$$

$$F_{2t} = \int_{h_{2t}}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left(-u^2/2\right)du = \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{h_{2t}}{\sqrt{2}}\right)\right), \tag{5}$$

and the log-likelihood is given as:

$$L = \sum_{t=1}^{T}\log\left(f_{1t}F_{2t} + f_{2t}F_{1t}\right), \tag{6}$$

where *T* indicates the number of samples.

A disequilibrium problem was used to test the efficacy of PSO for this particular model. Eight (8) parameters are to be estimated from simulated data for the following model:

$$\begin{aligned} D_t &= \alpha_0 + \alpha_1 X_{1t} + \alpha_2 P_{t-1} + u_t, \\ S_t &= \beta_0 + \beta_1 X_{2t} + \beta_2 P_{t-1} + v_t, \\ Q_t &= \min(D_t, S_t). \end{aligned} \tag{7}$$

Here, D_t and S_t respectively denote the quantities demanded and supplied during time period *t*, P_t is the price in period *t*, and X_{1t} and X_{2t} are explanatory variables for time period *t* (see Sapra, 1986). During time period *t*, Q_t and P_t are the only observed quantities. In this model,

$$\begin{aligned} &X_{1t} = U(0,1), \; X_{2t} = U(0,1), \\ &P_t = \frac{1}{2}P_{t-1}\frac{3D - S}{D}, \end{aligned} \tag{8}$$

and

$$u_t = N(0, \sigma_1^2), \; v_t = N(0, \sigma_2^2).$$

The goal is to estimate α_i, β_i, $i = 0,1,2$, as well as σ_1 and σ_2.

Simulated data of $T = 100$ samples are generated with the following parameters (see Maddala and Nelson, 1974): $P_0 = 1.0$, $\sigma_u^2 = 0.0625$, $\sigma_v^2 = 0.01$, $\alpha_0 = 2.0$, $\alpha_1 = 1.0$, $\alpha_2 = -0.5$, $\beta_0 = 1.4$, $\beta_1 = 1.0$ and

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$\beta_2 = 0.1$. The search space was constrained to $-30 < \alpha_i$, $\beta_i < 30$ and $10^{-6} < \sigma_u^2$, $\sigma_v^2 < 50$. An example of 100 observations of Q_t is shown in Figure 2.

One of the shortcomings of PSO (and many other global optimization methods) is stagnation, where there is no improvement in the cost function for several iterations. In the current work, the inertial parameter *w* was adaptively adjusted during the search to avoid stagnation. If no improvement is made in the global best solution after 50 iterations, the parameter is initially increased by 0.01, decreasing the effect of the globally best particle, and resulting in a more global search. After reaching a maximum of 1.03, the parameter is decreased in increments of 0.01, to a minimum of 0.97, to facilitate a more intense local search around the current global best. It was empirically noted that for this problem, *w* < 0.97 results in pre-mature convergence. This method of adaptive adjustment was found to provide faster convergence and high quality solutions.

Twenty-five (25) experiments were performed using PSO with an adaptive inertial parameter *w*, as described above. The population consisted of 1000 particles. The experiments are terminated when an expansion of *w* to 1.03 followed by a contraction to 0.97 does not improve the result after 50 iterations.

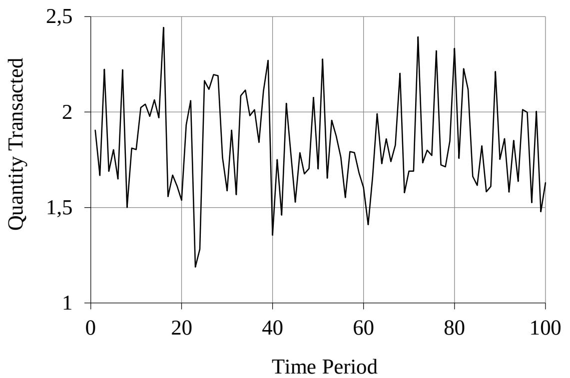

Figure 2 Quantity transacted (Q_t) for the disequilibrium problem over 100 periods.

6. Results

The experimental results obtained by using PSO estimation were compared with those obtained by Jerrell and Campione from stochastic evolutionary strategies (ES), genetic algorithms (GA), and simulated annealing (SA) (see Jerrell and Campione, 2001). The true parameter values, as given above, that were used to generate the simulated data resulted in an MLE of 63.0330. Using the data obtained in the current experiment, the values reported by Jerrel and Campione (2001) resulted in a log-likelihood of 57.8038. The success rates were: 100% for ES, 90% for GA, and 100% for SA (see Table 5 in Jerrell and Campione, 2001). The Euclidean distance between the obtained parameters (see Table 4 in Jerrell and Campione, 2001) and the true values was 0.2724.

For the PSO method, of the 25 trials, one trial resulted in a failure in which the final optimum was unacceptably far from the true values, with a low log-likelihood. The remaining 24 successful trials (a 96% success rate) yielded an average log-likelihood value of 66.0185, with an average Euclidean distance between the obtained and true parameters of 0.2390. PSO therefore had a better MLE (66.0185 > 57.8038) and a smaller Euclidean distance between the estimated and true values (0.2390 < 0.2724).

The average number of iterations for the successful trials was 8 207.42. The sorted results are shown in Figure 3. For the 1000 particles, this results in an average of about 8.2 million function evaluations. The total times for the methods tested by Jerrel and Campione are given in Table 5 in Jerell and Campione, (2001), but timing results are highly dependent upon the computer hardware platform (processor, memory, etc.), as well as the software running the experiments.

As mentioned above, one of the advantages of PSO is its ability to escape local optima. An example of this characteristic behavior is shown in Figure 4, where the swarm is represented along three of the eight coordinate axes of the disequilibrium cost function (α_1, α_2 and β_1).

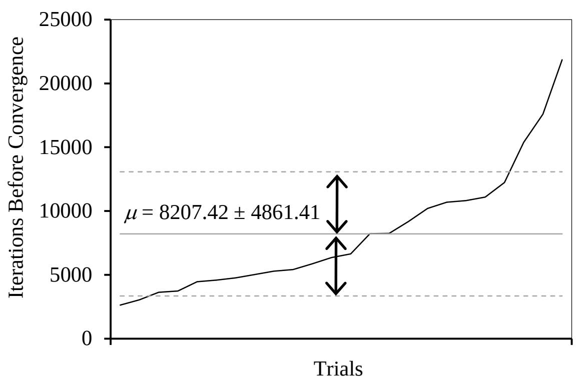

Figure 3 Sorted number of iterations until convergence for successful trials.

As is common among stochastic methods, a drawback of PSO is the high number of function evaluations required for convergence. However, with the advent of multicore CPUs and the offloading of computational tasks to relatively inexpensive and massively parallel graphics processing units (GPUs), the inherently parallel nature of PSO can be exploited to reduce overall computation time. Consequently, the number of function evaluations is not as great a concern as it has been in the past. Another concern

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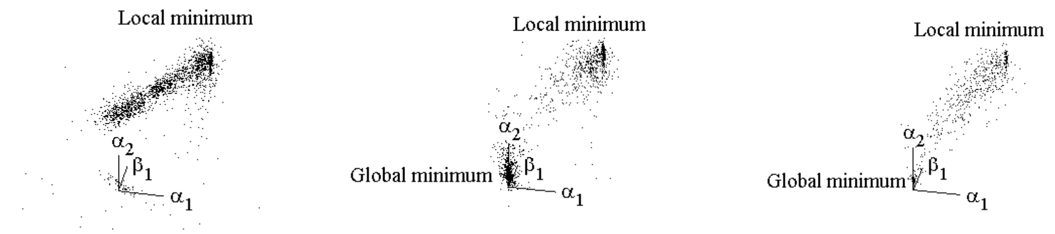

(a) Swarm is concentrated about local minimum.

(b) Swarm begins to escape local minimum.

(c) Swarm is now more concentrated about global minimum.

Figure 4 Progress of PSO as the swarm escapes from a local minimum.

with PSO, shared with other stochastic global optimization techniques, is the variation in the convergence time. This is the direct result of stochasticity, which provides robustness in probing the search space and in finding good solutions, but sometimes at the expense of computation time. The results presented in this paper exhibit a large standard deviation (> 4 000 iterations) for the 24 successful experiments. One possible solution is to apply a local optimization when local optima are suspected, or when the search begins to stagnate. Since local techniques generally require fewer iterations, excessive computation time would not be expended exploring these local optima (see Wachowiak et al. (2004), Tsoulos and Stavrakoudis (2010)).

7. Conclusion

The present paper focused on one instance of a disequilibrium model. However, this model is an excellent test problem, as it was devised to study model misspecification and difficulites in parameter estimation (see Jerrell and Campione (2001), Maddala and Nelson (1974)). The MLE of other models of real data can also be determined, and various estimation approaches can be compared on these functions. Although a comparative study of these methods on real data is a topic of future research, one may expect that the success rate, quality of solutions, and convergence time will be analogous to the results presented here, primarily because of the robustness of PSO in optimizing objective functions that are noisy or imprecisely defined (see Parsopoulos and Vrahatis, 2002).

Econometric models are becoming increasingly complex and higher in dimensionality. Accurately determining the parameters of these models will require more robust global optimization techniques. Although widely accepted in many other fields, these methods still find only limited use in econometric modeling and estimation (see Gilli and Winker, 2009). However, as shown in this paper and in others in the literature, stochastic methods, such as PSO, have demonstrated efficacy in a variety of econometrics applications, including the disequilibrium problem presented here. The high quality of the solutions obtained after convergence, the relative simplicity of parameter tuning in the algorithm, and its heuristic capability to be tailored to specialized applications make PSO an attractive choice for global optimization of econometrics functions. However, a drawback of this technique is the high number of iterations required for convergence, which affects overall efficiency. Although the focus in the current work is the quality of the solutions obtained (which is of primary performance, as it is not helpful to converge quickly to a bad solution), there exist several efficiency enhancements that can be easily incorporated (see Tsoulos and Stavrakoudis, 2010). Furthermore, as indicated earlier, PSO is an inherently parallel population-based technique, and can greatly benefit from low-cost multicore and GPU hardware. The success of PSO in solving a model disequilibrium problem, along with other numerous economics applications cited in this paper, encourage its use in other, more complex models. Although derivatives can be analytically computed in the current test problem, there are many useful problems in econometrics where derivatives are not available, and, furthermore, are characterized by multiple strong local optima in high dimensions. For this reason, it is necessary to continue to develop derivative-free, stochastic global methods to address some of these important problems. Because stochastic global optimization methods can solve problems more complex than those that can be solved by linear least squares, they are expected to become more common in econometrics. However, issues such as efficiency, the random nature of these methods, and the development of a common standard for presentation and evaluation of results are necessary for more widespread acceptance (see Gilli and Winker, 2009).

Future work will concentrate on: (1) Improving the efficiency of PSO and mapping function evaluations to GPUs, (2) Hybridizing PSO with local (and possibly other global) search techniques (which is especially important in higher-dimensional problems), (3) Rigorous comparison of PSO and other stochastic and deterministic optimization methods, and (4) Applying PSO and its variants to instantaneous spatial lag problems in spatial econometrics. Because of the high

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complexity of cost functions typically found in spatial econometrics, the inherent parallelism of PSO and its implementation on multicore and GPU hardware are particularly promising areas to exploit.

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