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MATHEMATICAL MODEL OF FILM-WISE CONDENSATION FOR EVALUATION OF HEAT TRANSFER COEFFICIENT

MATEMATICKÝ MODEL BLÁNOVÉ KONDENZACE PRO STANOVENÍ SOUČINITELE PŘESTUPU TEPLA

Abstract

The paper presents a methodology to the prediction of local heat transfer coefficients of a condensing vapor in the presence of a noncondensable gas. The methodology is based on the theory of diffusion layer model from which sufficiently accurate results can be obtained for most practical purposes. The theoretical model has been applied on film-wise condensation mode of steam with carbon dioxide forming so-called steam-gas mixture used as a working fluid in CES Rankine cycle concept. The theoretical solution has been validated using experimental data from an investigation performed with steam-CO2 mixture flowing downward in vertical tube condenser.

Abstrakt

Příspěvek představuje metodiku výpočtu lokálního součinitele přestupu tepla při kondenzaci páry v přítomnosti nekondenzujícího plynu. Metodika je založena na teorii difúzní vrstvy, která poskytuje dostatečně přesné výsledky ve většině praktických aplikací. Teoretický model byl zpracován pro režim blánové kondenzace směsi vodní páry a oxidu uhličitého označované jako paroplynová směs a využívané jako pracovní látka v RC cyklu s CES generátorem. Teoretické výpočty byly ověřeny srovnáním s daty z experimentálního šetření kondenzace paroplynové směsi proudící vertikálním trubkovým kondenzátorem.

1 INTRODUCTION

The film-wise condensation is only one of several regimes of condensation process, nevertheless it is this mode of phase transition which engineers must keep in mind when designing almost all types of applications and equipment. This situation stems from the fact that the most common working fluid in technical practice is water and construction materials usually have hydrophilic surface. In such a case, a saturated vapor in contact with a cooled surface undergoes condensation and a continuous liquid film is formed on the heat transfer surface. The liquid film provides a major fraction of the overall resistance to heat transfer. The latent heat released at the phase interface must be removed which is done by conduction through the layer of condensate. Fourier's law shows that the intensity of heat transfer depends solely on the liquid film thickness whose estimation is based on the well-known Nusselt analysis. The analysis in many cases of laminar condensate provides a sufficiently accurate data despite the fact that only viscous forces opposing motion and gravity are taken into account. At higher velocities of the steam, a correction including vapor shear effect on the interface profile must be introduced.

In many industry applications, not only the process ones, we may encounter cases when the steam is fraction of a working fluid or is polluted with gases. An example of this could be steam-gas mixture consisting of water vapor and carbon dioxide resulting from oxy-fuel combustion of natural gas in the CES generator [1]. After expansion in a turbine, the mixture constituents must be separated

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and processed individually in order to prevent carbon dioxide entering the atmosphere and maintain a power plant in zero-emission mode. The best possible solution to this situation is the condensation of water vapor, while carbon dioxide is removed in a gaseous phase. The presence of CO_2 has very profound impact on the condensation process, thus a design of condensation equipment must be based on knowledge of specific differences between a pure steam and mixture with inert gas.

As the condensation proceeds, the noncondensable mass fraction increases which leads to reduction of vapor partial pressure and corresponding saturation temperature. The net result is that the temperature gradient between the bulk mixture and the coolant is also reduced which significantly affects convective heat transfer. The vapor mass fraction profile develops not only in a flow direction but in a lateral direction as well. The molar flux of vapor sweeps carbon dioxide toward the liquid-gas interface, where the gas builds up to higher concentration and makes boundary layer adjacent to the liquid film inhibiting vapor diffusion. Thus Nusselt analysis must be extended by mass transfer phenomena.

2 THEORETICAL MODEL

The solution of the film-wise condensation problem lies in the equation describing a heat balance at the liquid-gas interface. The total heat flux from the gaseous core comprising of the sensible heat flux due to temperature gradient between the bulk mixture and the interface and of the latent heat flux due to the condensing vapor must be equal to the heat flux from the condensate film to the wall. The computational procedure should reflect the fact that thermodynamic properties and the heat transfer coefficient (HTC) vary greatly as the condensation process proceeds. To provide maximum computational accuracy, dividing the condenser into small intervals is required [2]. In each interval the interface temperature affecting the heat and mass transfer is estimated iteratively. The developed procedure is based on the following basic relations.

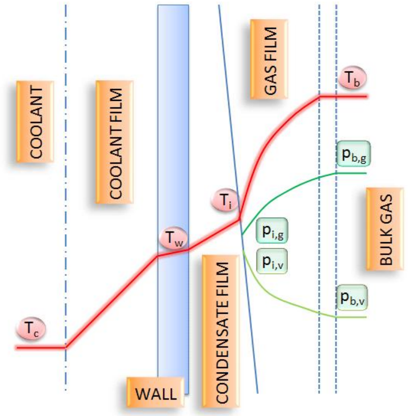

Fig. 1 Diffusion layer model of condensation

The heat balance at the interface can be written in terms of energy per unit surface

$$q = m_{cond} \cdot h_{fg} + a'_g \cdot (T_b - T_i) = a_f \cdot (T_i - T_w) \tag{1}$$

where q is the heat flux, m_{cond} represents the condensate mass flux, h_{fg} is the latent heat of condensation, a'_g and a_f are the sensible HTC and film HTC, respectively and T_b. T_w and T_i denote the bulk, wall and interface temperature, respectively.

The condensation heat transfer coefficient can be defined as

$$a_{cond} \cdot (T_b - T_i) = m_{cond} \cdot h_{fg} \tag{2}$$

so Eq. 1 can be manipulated into the form

$$\left(a_{cond} + a'_{g}\right)\cdot\left(T_b - T_i\right) = a_f \cdot \left(T_i - T_w\right) \tag{3}$$

which means that the liquid and gas side resistance act in series and the total heat transfer coefficient can be written as follows

$$a_{tot} = \left[\frac{1}{a_f} + \frac{1}{a_{cond} + a'_{g}}\right]^{-1} \tag{4}$$

2.1 Heat transfer coefficients in the gas phase

The sensible HTC can be expressed as

$$a_g = Nu \cdot \frac{l_g}{d_h} \tag{5}$$

where λ_g represents the thermal conductivity of gas phase and d_h is hydraulic diameter.

Estimation of Nusselt number is based on Dittus-Boelter correlation which predicts turbulent convective heat transfer very well for fluids in the Prandtl number range of 0.7-120.

$$Nu = 0{,}023 \cdot \mathrm{Re}^{0{,}8}\,\mathrm{Pr}^{0{,}3} \tag{6}$$

Ackermann correction should be used in order to take into account influence of mass transfer on heat transfer.

$$a'_g = a_g \frac{f_v}{e^{f_v} - 1} \tag{7}$$

where Φ_v is given by

$$f_v = \frac{n_v \cdot M_v \cdot c_{pv}}{a_g} \tag{8}$$

The condensate mass flux m_{cond} is given by diffusive and convective contributions to molar flux of water vapor at the interface n_v described by Fick’s law in Eq. 9.

$$n_v = -D_{AB} \cdot \frac{dC_v}{dy} + n_v \cdot \frac{C_v}{C_T} \tag{9}$$

which can be rearranged into form of a function of the pressure driving force

$$n_v = \frac{b \cdot C_T}{p_m}\left(p_{v,i} - p_{v,b}\right) \tag{10}$$

where C_T denotes the total molar concentration of a mixture, p_m represents the logarithmic mean partial pressure of an inert gas, $p_{v,i}$ and $p_{v,b}$ are the partial pressure of vapor at interface and in bulk, respectively.

The condensation heat transfer coefficient formulation relies on the application of the heat and mass transfer analogy. Thus Sherwood number can be correlated in the same form as the Nusselt number.

$$Sh = 0{,}023 \cdot \mathrm{Re}^{0{,}8}\,Sc^{0{,}3} \tag{11}$$

The mass transfer coefficient b then can be calculated from following expression.

$$b = Sh \cdot \frac{D_{AB}}{d_h} \tag{12}$$

Various methods have been proposed for determination of binary diffusivity coefficients D_{AB}. One of the most reliable and widely used formulas is that of Fuller [3].

$$D_{AB} = \frac{10^{-7}\cdot T^{1,75}}{p_t\cdot\left(v_A^{1/3}+v_B^{1/3}\right)^2}\cdot\sqrt{\frac{1}{M_A}+\frac{1}{M_B}} \tag{13}$$

where T is the absolute temperature in K, p_t is the total pressure in bar, M_i and v_i are molecular weight and diffusion volume of each component, respectively.

2.2 Heat transfer coefficient in the liquid phase

The heat transfer coefficient of the liquid film was evaluated from the local film Nusselt number defined as the ratio of condensate conductivity and film thickness. A transition region between laminar and turbulent behavior suggested by Blangetti [4] is incorporated in the expression

$$Nu_f = \frac{a_f\cdot L}{l_l} = \left(Nu_{f,la}^4 + Nu_{f,tu}^4\right)^{1/4} \tag{14}$$

The laminar part of Nu comes from hydrodynamic analysis for a falling liquid film

$$Nu_{f,la} = \frac{1}{d^*} \tag{15}$$

where δ^* is dimensionless thickness obtained by solving the equation

$$\frac{\mathrm{Re}_f}{1-r_m/r_l} = \frac{d^{*3}}{3}+\frac{t^*\cdot d^{*2}}{2} \tag{16}$$

The dimensionless parameter τ^* introduces the interfacial shear stress considering the suction effect.

The turbulent part is given by correlation

$$Nu_{f,tu} = a\cdot\mathrm{Re}_f^b\,\mathrm{Pr}^c\cdot\left(1+e\cdot t^*\right) \tag{17}$$

All the factors and exponents related to smooth film are tabulated for different ranges of the vapor shear [4]. Since turbulence in the liquid film enhances energy transfer rates due to rippling effect which alters the boundary layer characteristics, the roughness correction must be considered. Norris [5] proposed following expression

$$Nu_{f,tu}^r = Nu_{f,tu}\cdot\left(\frac{f_r}{f_s}\right)^n \tag{18}$$

where n is an empirical constant representing $0{,}68\mathrm{Pr}^{0,215}$ and smooth friction factor f_s is $0{,}079\mathrm{Re}^{-0,25}$.

For estimation of f_r Wallis model for interfacial friction in the vertical annular flow has been used [6].

$$f_r = f_s\cdot\left(1+300\frac{d}{d}\right) \tag{19}$$

2.3 Solution procedure

Fig. 2 depicts the flowchart of the algorithm for the solution of condensation process of a vapor in the presence of a non-condensing gas. The procedure is based on Colburn and Hougen iterative method employing preceding equations for the energy balance calculation. The interface temperature is the only unknown parameter whose value must be assumed. By using the trial value of the interface temperature, the local heat flux is evaluated. If recalculated temperature differs from the trial value more than 0,1 K, guessed value is improved and steps are repeated.

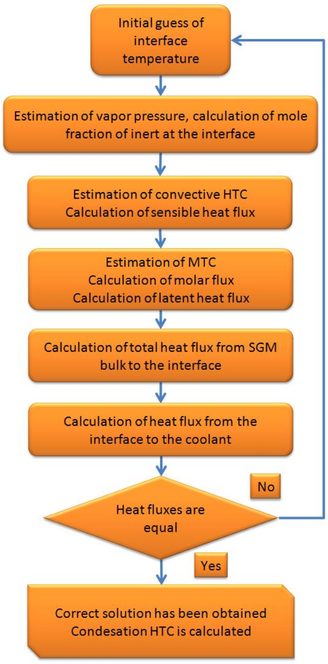

Fig. 2 Step-by-step solution

3 EXPERIMENTAL INVESTIGATION

3.1 Test facility

The theoretical model for the prediction of the overall heat transfer coefficient presented in this paper has been validated using results of experimental investigation of condensation of steam from steam-CO_2 mixture. The experimental apparatus schematically shown in Fig. 3 consists of

- steam generator,
- gas supply line,
- mixing chamber,
- test section,
- coolant water supply,
- data acquisition system.

The central element of the facility is a vertical condenser – 3m long double concentric pipes forming cylindrical annulus where the mixture flows in downward direction. The coolant water flows in the core tube counter-current to the primary fluid. The mixture bulk temperature measurement is ensured by set of thermocouples mounted on the condenser wall. The condensate flow is calculated from the rate of water level rise in a sight glass on the condensate tank. Outputs from all the sensors were continuously recorded and visualized in the data acquisition software.

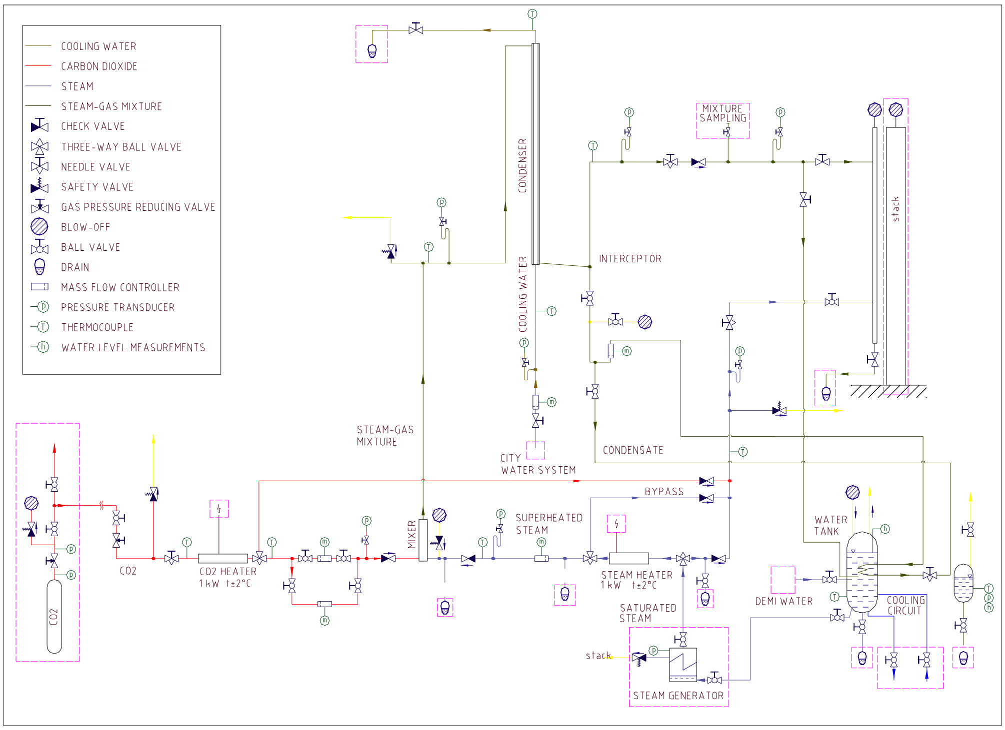

Fig. 3 Test facility layout

3.1 Test conditions

The experimental work included 45 runs with steam-CO_2 mixture at test conditions listed in Tab. 1, which were determined by expected parameters of the mixture expanded in HP (IP) turbine of CES cycle. The range of inlet CO_2 mass fraction was chosen to cover possible composition of the mixture generated in the CES combustor. To obtain reliable data, the steam mass flow rate, CO_2 flow rate, inlet pressure, coolant inlet temperature and coolant flow rate were kept constant during each test. Data were recorded when the system had been at a steady state for at least 10 minutes.

Tab. 1 Test conditions

Parameter	Range
Primary side pressure [MPa]	0,4-0,6
Steam inlet flow rate [g/s]	8,6-17
Mixture inlet velocity [m/s]	4-10
Inlet Reynolds number of mixture	14000-30000
NC inlet mass fraction [%]	5-15
Coolant flow rate [g/s]	170
Coolant inlet temperature [°C]	10

At these conditions, the complete condensation never occurred and outlet CO_2 concentration was specified by following methods:

1. mass balance approach,
2. temperature measurement of the mixture at the condenser exit,
3. infrared spectroscopy .

4 RESULTS AND DISCUSSION

Fig. 3 depicts the heat gained by the cooling water plotted against the heat load by the condensing side. The heat gained by the water can be calculated directly from the measured flow rate and the temperature difference between inlet and outlet, whereas the energy transferred from the mixture can be obtained from the amount of condensate and the latent heat of condensation corrected by sensible heat of the mixture and the condensate. The energy balance deviation is within 8 % for all runs which indicates good quality of the experimental data.

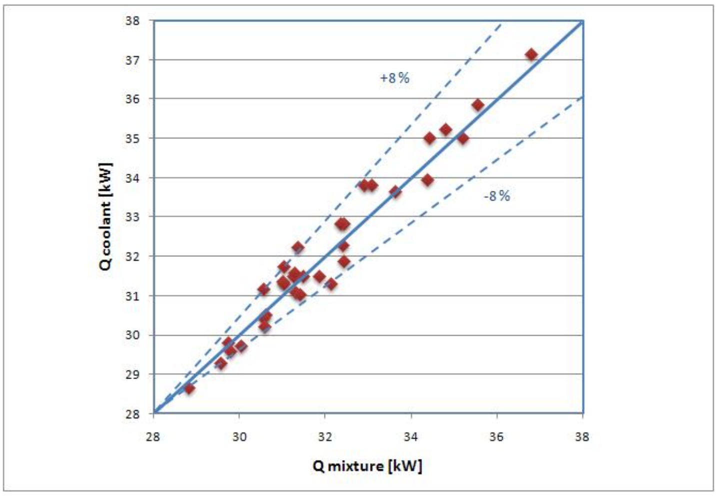

Fig. 3 Energy balance of the condenser

The bulk mixture temperature profile (Fig. 4) illustrates typical behavior of a condensing mixture in the presence of a noncondensing gas. Since the pressure drop is less than 1 kPa at all the test conditions, the change in the temperature is contributed to the accumulation of the gas which reduces the steam partial pressure and corresponding saturation temperature. This behavior reveals that an extra resistance is added to the condensing side in comparison with pure steam.

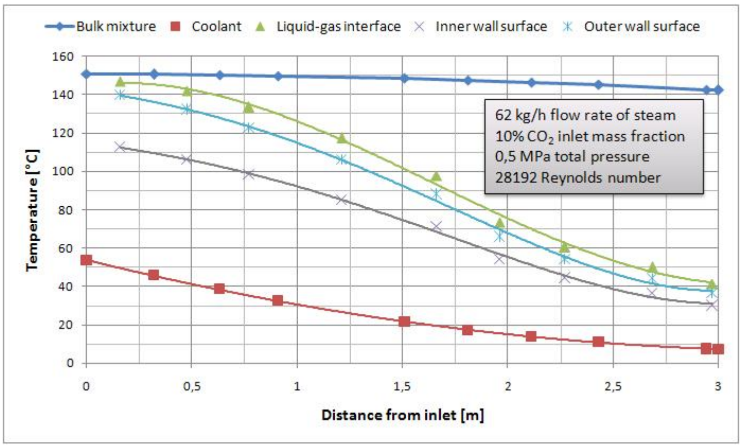

Fig. 4 Temperature profile along the condenser

The increasing mass fraction of carbon dioxide prevents the steam from reaching the condensing surface, which results in deterioration of mass transfer and impetuous reduction of the condensation heat transfer coefficient along the test tube (Fig. 5). The figure also shows that the condensate heat transfer coefficient is much above the condensing one, therefore represents minor thermal resistance to the condensation process. This can be explained by high velocity of the mixture which induces shear forces leading to thinning and disruptions of the condensate boundary layer. The overall heat transfer coefficient refers to the ability of the series of conductive and convective barriers to transfer heat from the mixture side to the coolant side.

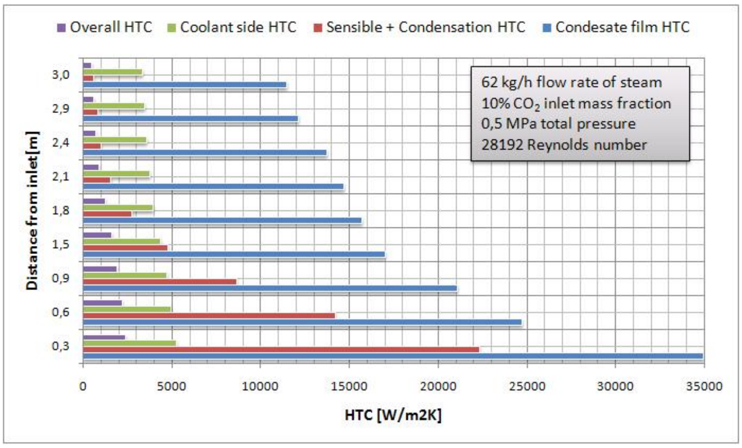

Fig. 5 Heat transfer coefficients in the condenser

The experimentally obtained heat transfer coefficient on the gas side and the theoretical one shows following figure. The heat transfer coefficient presented here combines the condensate film, the sensible and the condensation HTC. The experimental values come from the measured heat flux and temperature gradient between the bulk mixture and structure surface. The theoretical data were obtained from the mathematical solution using proved semi-empirical correlations with the heat and mass transfer analogy. The proposed model takes account of the effect of transpiration which affects the temperature and concentration profiles and alters structure of the boundary layer considerably.

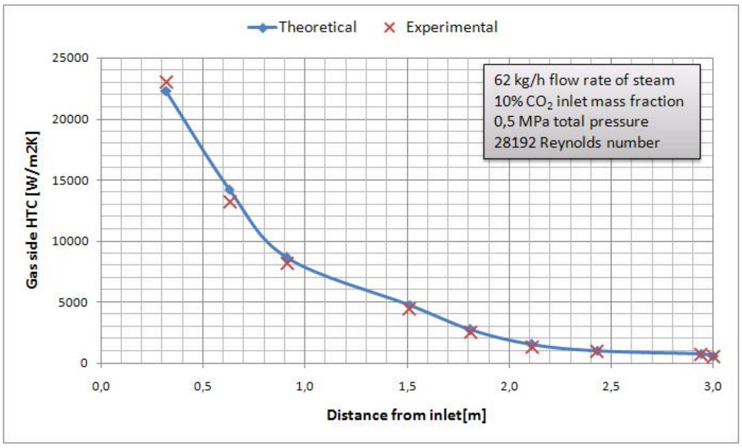

Fig. 6 Experimental and theoretical heat transfer coefficient

From figure 7 can be seen the impact of decreasing trend of HTC which adversely determinate performance of the condenser and its overall efficiency. At the condenser exit, carbon dioxide mass fraction varied from 25 to 55% (according to the total pressure and the inlet composition) despite the fact that 77% and more of steam inlet mass flow rate has been condensed. The complete condensation must be achieved in order to separate compounds from each other. Such effort puts extreme demands on the heat transfer area and economical possibilities must be considered.

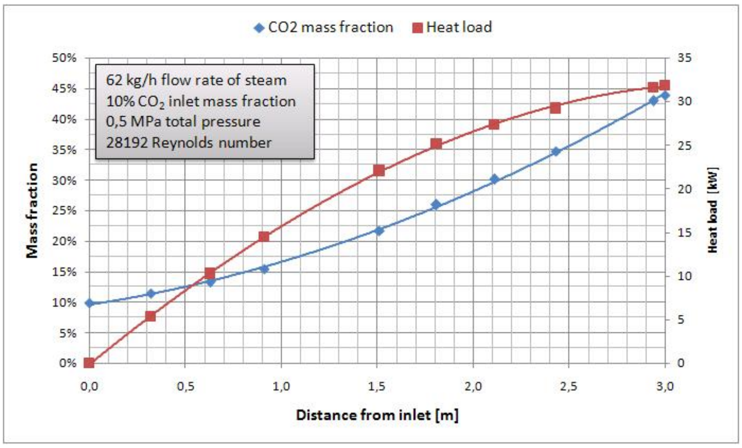

Fig. 7 Concentration profile of CO_2 in the condenser

5 CONCLUSIONS

Series of experiments with a condensing steam-gas mixture has been carried out in order to investigate the influence of carbon dioxide on a film-wise condensation in a vertical tube condenser. The experimental measurements clearly demonstrated significant deterioration of the local heat transfer coefficient as the condensation process proceeds and distinctive decrease in performance of terminal parts of the condenser. The experimentally obtained heat transfer coefficients were compared with theoretical values received by solution of a mathematical model arising from a theory of diffusion boundary layer. Correlations taking into account effects of a flow regime and shape of the condensate film have been integrated into Colburn and Hougen iterative procedure. Outputs of the mathematical model show very good agreement with the experimentally observed parameters. The relative deviation does not exceed 15%.

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