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DETERMINATION OF MEASUREMENT UNCERTAINTIES OF STRAIN HARDENING EXPONENTS BY STATISTICAL METHODS

STANOVENÍ NEJISTOT MĚŘENÍ EXPONENTŮ DEFORMAČNÍHO ZPEVNĚNÍ MATERIÁLU STATISTICKÝMI METODAMI

Abstract

Paper deals with a determination of confidence bands of instantaneous actual stress-instantaneous actual logarithmic strain relationship and with a determination of measurement uncertainties of strain hardening exponent at the tensile tests with the use of three test bars in orientations 0°, 45° and 90° towards sheet-metal rolling direction. The standard for tensile tests ČSN EN ISO 6892-1 does not include determination of measurement uncertainties of mechanical properties. The standard for determination of strain hardening exponent ČSN ISO 10275 does not include determination of measurement uncertainties, only determines the rounding method of strain hardening exponent values. In this paper the confidence bands of mean instantaneous actual stress-instantaneous actual logarithmic strain relationship, scattering analysis and resulting uncertainties of strain hardening exponents of steel strip DD11 (11 320.0) are calculated with the use of statistical analysis of scattering by ANOVA method.

For calculations the programmes MS Excel and Statgraphics have been used. Based on the calculated values of medium strain hardening exponent and degree of plain anisotropy of strain hardening exponent, the suitability of tested steel for drawing operations, at which predominate tensile mechanical diagrams of stress, was evaluated.

Abstrakt

Příspěvek se týká stanovení pásem spolehlivosti závislosti napětí na deformaci a stanovení nejistot měření exponentů deformačního zpevnění u zkoušek tahem při použití tří tyčí ve směrech 0°, 45° a 90° vůči směru válcování plechu. Norma pro zkoušky tahem ČSN EN ISO 6892-1 neobsahuje stanovení nejistot měření mechanických vlastností. Norma pro stanovení exponentu deformačního zpevnění ČSN ISO 10275 neobsahuje stanovení nejistot měření, pouze stanovuje způsob zaokrouhlení hodnot exponentu deformačního zpevnění. V příspěvku jsou pomocí statistické analýzy rozptylu metodou ANOVA vypočteny intervaly spolehlivosti závislosti střední hodnoty okamžitého skutečného napětí na deformaci, analýza rozptylu a výsledná nejistota exponentů deformačního zpevnění pásové oceli DD11 (11 320.0).

K výpočtům byly použity programy MS Excel a Statgraphics. Na základě vypočtených hodnot průměrného exponentu deformačního zpevnění a stupně plošné anizotropie exponentu deformačního zpevnění byla zhodnocena vhodnost zkoušené oceli pro tažné operace, u kterých převládají tahová mechanická schémata napjatosti.

INTRODUCTION

Before drawing process starts, in practice it is very important to assess the formability of sheet-metal because of spoilage in production and increasing production efficiency. To assess the formability of sheet-metal, the unconventional assessment criteria, that include plastic strain ratio and strain hardening exponent, are used. These criteria allow better classification of sheet-metal than mechanical properties evaluated by tensile test.

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For finding of strain hardening exponent in individual orientations *x* towards sheet-metal rolling direction n_x, the results of the tensile tests according to ČSN EN ISO 6892-1 of test bars taken from sheet-metal in directions 0°, 45° and 90° towards rolling direction are used. The values of strain hardening exponent are calculated according to ČSN ISO 10275. Unlike the previous standard ČSN ISO 10275 (42 0436) from November 1995 an elastic deformation component was not taken into account in the calculation of the actual deformation from total strain in the case that it was below 10 % of actual strain. In the standard ČSN ISO 10275 (42 0436) from January 2008 the actual strain is always taken away from the total strain in calculation of elastic component. Values obtained in such way already contain an uncertainty measurement. The standard for specimens modifies cases, where tensile strain hardening exponent *n* and plastic strain ratio *r* are established simultaneously – the specimens must comply conditions in the standard for determination of plastic anisotropy ratios ČSN ISO 10113.

The standard for tensile tests ČSN EN ISO 6892-1 solves measurement uncertainties only by informative manner with meaning of determination of input values parameters, tested specimens shape and determination of measurement equipment uncertainties, it do not include uncertainties of measured mechanical properties values. The standard for determination of strain hardening exponent by tensile tests ČSN ISO 10275 does not deal with results reliability, i. e. determining of the leakage dependence of instantaneous actual stress-instantaneous actual logarithmic strain relationship by selected regression function, confidence intervals for regression parameters of strain hardening exponent *n* and strength coefficient *C,* confidence bands of approximate instantaneous actual stress-instantaneous actual logarithmic strain relationship.

Estimation of uncertainty measurement for assessing the reliability of measurement results of strain hardening exponent can be determined by analysis of variance – ANOVA method. The estimation allows validating the selected regression model and determination of confidence interval of strain hardening exponent. Total variance of the whole set of measurements is given as a combination of variances between groups and within groups.

Processing of data by means of statistical methods is adapted by standards ČSN ISO 3534–1:1994, ČSN ISO 3534–2:1994 and ČSN ISO 3534–3:2001.

In this paper the determination of uncertainties of strain hardeneing exponent and the degree of planar anisotropy of strain hardening exponent of steel strip DD11 (11 320.0) is made, which was selected as an example of drawing steel with using of three test bars, which is the minimum number prescribed by standard, in directions 0°, 45° and 90° towards sheet-metal rolling direction used at the tensile tests. According to analysis of variance (ANOVA method) the values for regression function were determined, the estimates of regression parameters and the coefficients of determination were calculated, the confidence intervals for regression parameters were calculated and the confidence intervals for mean values of the instantaneous actual stress were calculated as well. An analysis of dispersal values of strain hardening exponent was made, directional strain hardening exponent was determined and resulting uncertainty of strain hardening exponent from the average values of interval of strain hardening exponent for directions 0°, 45° and 90° towards sheet-metal rolling direction was determined. Based on the calculated values of average strain hardening exponent of the tested steel strip was concluded, that the tested steel is suitable for drawing operations at which the pressure-tension mechanical diagrams of stress predominate.

1 CHARACTERISTIC OF TESTED MATERIAL

For tests the strip from steel DD11 (11 320.0) with dimensions according to ČSN 42 5355: thickness of strip (2,6 ± 0,1) mm, strip width with natural edges (171 ± 1,35) mm, was chosen. Due to ČSN 41 1320 this steel corresponds to steel St 22 according to DIN 1614 ÷ 74 and St 12 according to DIN 1623 ÷ 72. The steel is hot rolled, recrystallizationally annealed, not re-rolled with mill surface. According to ČSN 41 1320 the steel is suitable for drawing and cold forming operations and has a certain welding characteristic according to ČSN 41 1320. The demands on chemical composition of steel DD11 (11 320.0) according to ČSN 41 1320 are in Tab. 1, desirable values of mechanical properties are in Tab. 2.

Tab. 1 Demands on chemical composition of steel DD11 (11 320.0) according to ČSN 41 1320

C (wt %)	P (wt %)	S (wt %)
max. 0,11	max. 0,045	max. 0,045

Tab. 2 Demands on chemical properties of steel DD11 (11 320.0) according to ČSN 41 1320

Yield strength R_e crosswise (MPa)	Tensile strengh R_m crosswise (MPa)	Ductility A_{10} along (%)	Test brittleness for D = 0 mm
max. 0,8 · R_m	270 ÷ 370	min. 30	a = 180°

2 REGRESSION FUNCTION OF INSTANTANEOUS ACTUAL STRESS-INSTANTANEOUS ACTUAL LOGARITHMIC STRAIN RELATIONSHIP

Strain hardening exponent characterizes the intensity of sheet-metal strain hardening during uniaxial tension plastic deformation. The exponent is numerically equal to the exponent in the equation (1), mathematically it express approximated parabolic dependence of the instantaneous actual stress on the instantaneous actual logarithmic strain. Strain hardening exponent is a material constant – for deep drawing steel sheets it is always less than 1, it can be determined from the relation:

$$s_i = C_x \cdot j_i^{\,n_x} \,, \tag{1}$$

where n_x – strain hardening exponent in orientation x towards sheet-metal rolling direction,
C_x – coefficient of strength in orientation x towards sheet-metal rolling direction (MPa). Strength factor is numerically equal to the extrapolated value of instantaneous actual stress at the instantaneous actual logarithmic strain $\varphi = 1$.
i – point of the measured interval.

Strain hardening exponent is an important indicator to evaluate formability of sheet-metal especially in those cases where tensile strain predominate.

Directive rating of formability:

$n_m < 0,215$ – low formability,
$n_m = 0,215$ up to 0,250 – good formability,
$n_m > 0,250$ – very good formability.

From the above results it is obvious that relatively small deviations in mean coefficient of strain hardening exponent n_m have resulted in substantial changes in the assessment of material. The high demands on accuracy of the test and its evaluation are ruled by the standard ČSN ISO 10275.

By expression of the equation (1) in logarithmic form the line equation is obtained:

$$\begin{aligned} \ln y &= \ln a + b \cdot \ln x \\ \ln s_i &= \ln C_x + n_x \cdot \ln j_i \end{aligned} \tag{2}$$

Normal equation:

$$\begin{aligned} \sum \ln s_i &= N \cdot \ln C_x + \ln j_i \\ \sum \ln j_i \cdot \ln s_i &= \ln C_x \cdot \sum \ln j_i + n_x \cdot \sum (\ln j_i)^2 \end{aligned} \tag{3}$$

Strain hardening exponent in different orientations x towards sheet-metal rolling direction is determined by the tensile tests according to ČSN EN ISO 6892–1 and ČSN ISO 10275.

2.1 Utilization of variance analysis to determine the regression function values

Analysis of variance allows the comparison of several mean values of independent random selections. In its parametrical form, it assumes normality of distribution and identical variances that determine: mean of samples, samples standard deviation and samples standard deviation of sample averages.

Analysis of variance (ANOVA method) is a standard statistical method and can be used in the analysis of measurement errors and other sources of data variance. Analysis of variance is based on the idea that the variability (volatility, distraction, dispersion), with which fluctuates the values of observed random variable around its mean value distribution, is a result of various factors, each of which contributes to the overall variability with particular share. ANOVA method is a technique that allows assessment of the individual sources of variability of the data, because the measurements in general are not prone to deviations. These random deviations can cause difficulties in detecting significant difference between groups of replicates (parallel measurements).

The analysis follows the partial contribution of one or more selected factors in addition to the remaining effects that are not monitored in the measurement. Selected factor is changed resulting in increased dispersion of measured values of the so-called residual variance (residual), which always occurs as a result of random fluctuations of the remaining factors, that are not measured directly during the measurement of random changes and thus inducing just residual variance. By mutual comparison of variances calculated from the measured results (a comparison using F-tests) can be determined which of the considered factors are for the random variables variability significant and also the incremental variance value can be determined.

To determine the regression function of specimens instantaneous actual stress s on instantaneous actual logarithmic strain j relationship of material DD11 (11 320.0) the test bars removed from sheet-metal in direction 45° towards sheet-metal rolling direction were used. At this direction an explanation how to determine the uncertainty of measurement for determining of the strain hardening exponent of sheet-metals and strips will be given. By the same procedure it was determined in directions 0° and 90° towards sheet-metal rolling direction. The values measured by the test bars No. 1, 2 and 3 are in Tab. 3.

The values used as example of calculation of the regression power functions are shown in Tab 4. To accommodate the instantaneous actual stress and the corresponding instantaneous actual strain the six values at every test bar were determined according to ČSN ISO 10275. Each measurement point corresponds to the value ***i*** that are given in Tab. 3.

Tab. 3 Values of instantaneous actual stress and instantaneous actual evaluated by test bars No. 1, 2 and 3 in direction 45° towards sheet-metal rolling direction

i	Bar No. 1		Bar No. 2		Bar No. 3	
	ln φ (-)	ln σ (-)	ln φ (-)	ln σ (-)	ln φ (-)	ln σ (-)
1	-3,21888	5,75479	-3,50656	5,74236	-3,91202	5,60801
2	-2,81341	5,86817	-2,81341	5,87099	-2,81341	5,78321
3	-2,40795	5,94254	-2,40795	5,94175	-2,40795	5,86562
4	-2,12026	5,99096	-2,12026	5,99096	-2,12026	5,92024
5	-1,96611	6,02828	-1,96611	6,02514	-1,89712	5,96023
6	-1,77196	6,05444	-1,77196	6,05256	-1,71480	5,99471

Example of calculation procedure is performed for test bar No. 1 in direction 45° towards sheet-metal rolling direction. In Tab. 4 the continuously evaluated values to determine the strain hardening exponent are recorded, where ***i*** indicates the selected point of measurement.

Tab. 4 Values for calculation of the regression power function for test bar No. 1 in direction 45° towards sheet-metal rolling direction

i	j_i (-)	s_i (MPa)	ln j_i (-)	ln s_i (-)	ln j_i · ln s_i (-)	(ln j_i)^2 (-)	(ln s_i)^2 (-)
1	0,04	315,7	-3,21888	5,75479	-18,52396	10,36116	33,11764
2	0,06	353,6	-2,81341	5,86817	-16,50956	7,91528	34,43538
3	0,09	380,9	-2,40795	5,94254	-14,30931	5,79820	35,31374
4	0,12	399,8	-2,12026	5,99096	-12,70242	4,49552	35,89165
5	0,14	415,0	-1,96611	6,02828	-11,85228	3,86560	36,34014
6	0,17	426,0	-1,77196	6,05444	-10,72821	3,13983	36,65624
Σ	–	–	-14,29857	35,63918	-84,62573	35,57559	211,75479

2.2 Estimates of regression parameters and the coefficient of determination

The method of measurement according to ČSN ISO 10275 uses not-graduated test bars. Strain hardening exponent is calculated from equation (4) using least-squares method regression in which like the regression function the line is selected. Strain hardening exponent in direction 45° towards sheet-metal rolling direction can then be calculated according to formula:

$$n_x = \frac{N \cdot \sum \ln j_i \cdot \ln s_i - \sum \ln j_i \cdot \sum \ln s_i}{N \cdot \sum (\ln j_i)^2 - (\sum \ln j_i)^2} = 0{,}20375 \qquad (4)$$

$$\ln C_x = \frac{\sum \ln s_i}{N} - n_x \cdot \frac{\sum \ln j_i}{N} = 6{,}42543 \qquad (5)$$

Transformed model for the regression line:

$$\ln s_i = 6{,}42543 + 0{,}20375 \cdot \ln j_i \qquad (6)$$

Inverse transformation:

$$s_i = 617{,}3 \cdot j_i^{\,0{,}20375} \qquad (7)$$

The total sum of squares:

$$S_{\ln s} = \sum (\ln s_i)^2 - \frac{(\sum \ln s_i)^2}{N} = 0{,}06296 \qquad (8)$$

Theoretical sum of squares explained by regression model:

$$S_T = \ln C_x \cdot \sum \ln s_i + n_x \cdot \sum \ln j_i \cdot \ln s_i - \frac{(\sum \ln s_i)^2}{N} = 0{,}06231 \qquad (9)$$

where N is the number of measurements.

The coefficient of determination (confidence value) evaluates the suitability of the chosen regression model.

$$R^2 = \frac{S_T}{S_{\ln s}} = 0{,}98967 \qquad (10)$$

2.3 Confidence interval for regression parameters

The confidence interval for the regression parameters of strain hardening exponent n and strength coefficient C can be determined by analysis of variance which will verify the uncertainty regards of the measured values. Although the model is adequate to experimental values listed in Tab. 5 it certainly does not mean that they discrepancies could not be found. Analysis of the residues follows a set of deviations between experimental and calculated values, i. e. $\overline{y_i} - y_i$. Residues to a certain widen, represent a rough approximation of random effects. Residue determines the error between the actual and the model value.

Tab. 5 Regression statistics for the test bars No. 1, 2 and 3 in direction 45° towards sheet-metal rolling direction

Regression statistics	Bar No. 1	Bar No. 2	Bar No. 3
Number of observations	6	6	6
Sample multiple correlation coefficient	0,999683	0,999683	0,998842
Coefficient of determination (confidence value) R^2	0,999368	0,999368	0,997681
Adjusted value of reliability	0,996130	0,996130	0,990022
Mean value error (standard error) s_R	0,003238	0,003238	0,007658

Residual sum of squares:

$$S_R = S_{\ln s} - S_T = 0{,}062955 - 0{,}032305 = 0{,}000650 \tag{11}$$

Standard error (mean value error):

$$s_R = \sqrt{\frac{S_R}{N-2}} = 0{,}012752 \tag{12}$$

$$S_{(\ln C)} = s_R \cdot \sqrt{\frac{\sum(\ln j_i)^2}{N \cdot \sum(\ln j_i)^2 - (\sum \ln j_i)^2}} = 0{,}025351 \tag{13}$$

$$S_{(n)} = s_R \cdot \sqrt{\frac{N}{N \cdot \sum(\ln j_i)^2 - (\sum \ln j_i)^2}} = 0{,}010412 \tag{14}$$

The confidence interval for the strength coefficient C_x:

$$\ln C_x \pm t_{0,975(N-2)} \cdot S_{(\ln C)} \tag{15}$$

The confidence interval for the strain hardening exponent n:

$$n_x \pm t_{0,975(N-2)} \cdot S_{(n)} \tag{16}$$

where $t_{0,975\,(N-2)}$ is 95 % quantile of t-distribution with (N-2) degrees of freedom. Values are given in Tab. 6 to Tab. 8 and describe the results of data analysis by ANOVA method that consisting of 6 columns:

- column of *Source variability* is the cause of variability,
- column of *SS* indicates variability or Sum of Squares and indicates the mean deviation of resources,
- column of *Degrees of freedom* is connected with the resource,
- column of *MS* indicates the corresponding type of variance or the mean square divided by degree of freedom,
- column of ***F-ratio*** calculated to determine statistical significance of the value of resources,
- column ***P-value*** column indicates the minimum significance level at which you can still reject the null hypothesis, i. e. the probability of test criteria will be greater than the calculated value *F-ratio*. Great value of the *F-ratio* will result in small values of *P-value* which leads into rejection of the null hypothesis. ***F-ratio*** will be high when the internal variability constitutes a negligible portion of total variation and when interclass variability is an important part of total variance.

Tab. 6 Values and regression parameters of test bar No. 1 in direction 45° towards sheet-metal rolling direction determined by ANOVA method

Source variability	*SS*	*Degrees of freedom*	*MS*	*F-ratio*	*P-value*
Total	0,066301	5	–	6310,09524	0,0042
Regression (interclass)	0,066256	1	0,066256		
Residual Sum of Squares (inner)	0,000042	4	0,000011		
Regression parameters	Coefficients		Confidence interval	Error mean	t stat
ln $C_{45\text{-}1}$	6,42543		<6,35502;6,49582>	0,02535	253,49311
$n_{45\text{-}1}$	0,204		<1,191;1,262>	0,010	19,5735

Tab. 7 Values and regression parameters of test bar No. 2 in direction 45° towards sheet-metal rolling direction determined by ANOVA method

Source variability	*SS*	*Degrees of freedom*	*MS*	*F-ratio*	*P-value*
Total	0,066301	5	–	6310,09524	0,0053
Regression (interclass)	0,066256	1	0,066256		
Residual Sum of Squares (inner)	0,000042	4	0,000011		
Regression parameters	Coefficients		Confidence interval	Error mean	t stat
ln $C_{45\text{-}2}$	6,37428		<6,35861;6,38993>	0,00565	1127,56120
$n_{45\text{-}2}$	0,180		<1,189;1,205>	0,002	79,5025

Tab. 8 Values and regression parameters of test bar No. 3 in direction 45° towards sheet-metal rolling direction determined by ANOVA method

Source variability	*SS*	*Degrees of freedom*	*MS*	*F-ratio*	*P-value*
Total	0,101120	5	–	1717,20851	0,0038
Regression (interclass)	0,100886	1	0,100886		
Residual Sum of Squares (inner)	0,000235	4	0,000059		
Regression parameters	Coefficients		Confidence interval	Error mean	t stat
ln $C_{45\text{-}3}$	6,29377		<6,2632;6,3244>	0,01102	570,95510
$n_{45\text{-}3}$	0,177		<0,165;0,189>	0,004	41,4766

Values of ***P-value*** listed in Tab. 6 to Tab. 8 are less than 0,01 which implies 99 % reliability of instantaneous actual stress-instantaneous actual logarithmic strain relationship of the given test bar in the selected range of instantaneous actual stress values s.

2.4 Confidence intervals for the instantaneous actual stress mean values

For values of instantaneous actual stress s the confidence belts intervals were calculated as defined in ISO/IEC Guide 98-3 for the uncertainty in 95 % confidence level. Tab. 9 shows the values of the confidence interval for the mean instantaneous actual stress s for test bar No. 1 in direction 45° towards sheet-metal rolling direction. From comparison of measured values in Tab. 9 with Tab. 3 is seen that the value of the instantaneous actual logarithmic strain $j_2 = 0{,}06$ is closer to the actual instantaneous instantaneous actual stress s close to the upper limit of the confidence interval and for instantaneous actual logarithmic strain $j_6 = 0{,}17$ the value of the actual instantaneous instantaneous actual stress s approaches the lower limit of the interval.

Tab. 9 Calculated values of the confidence interval for the mean values of the instantaneous actual stress s_i evaluated by test bar No. 1 in direction 45° towards sheet-metal rolling direction

i	j_i (-)	$\ln s_i$ (-)	$S_{(\ln\sigma i)}$ (-)	Confidence interval for $\ln s_i$ (-)	Confidence interval for s_i (MPa)
1	0,04	5,76958	0,01014	<5,74139;5,79788>	<311,5;329,6>
2	0,06	5,85218	0,00687	<5,83305;5,87127>	<341,4;354,7>
3	0,09	5,93480	0,00521	<5,92024;5,94934>	<372,5;383,5>
4	0,12	5,99342	0,00588	<5,97711;5,99342>	<394,3;407,4>
5	0,14	6,02482	0,00678	<6,00611;6,04168>	<405,9;420,6>
6	0,17	6,06438	0,00822	<6,04358;6,08723>	<421,4;440,2>

Determination of the narrowest confidence belt in 95 % confidence level and the most accurate expected final value of confidence belt, which happened to minimize the value of $S_{(\ln s)}$, is established according to equations (17) and (18). The width of the strip could be, to some extent, influenced by the appropriate choice of points j_1 to shows the parabolic course of the instantaneous actual stress s depending on the instantaneous actual logarithmic strain j.

$$\ln s \pm t_{0,975(N-2)} \cdot S_{(\ln\sigma)} \tag{17}$$

$$S_{(\ln s)} = S_{\mathrm{R}} \cdot \sqrt{\frac{1}{N} + \frac{\left(\ln j_i - \frac{\sum \ln j_i}{N}\right)^2}{\sum (\ln j_i)^2 - \frac{\left(\sum \ln j_i\right)^2}{N}}} \tag{18}$$

The example of calculation is given for the value of the instantaneous actual logarithmic strain $j_{45\text{-}1} = 0{,}04$ immediately after loading the test bar:

$$\ln s_1 = C_{45-1} + n_{45-1} \cdot \ln j_1 \tag{19}$$

$$\ln s_1 = 6{,}42543 + 0{,}20375 \cdot (-3{,}21888) = 5{,}76958 \tag{20}$$

$$S_{(\ln\sigma)} = 0{,}04022 \tag{21}$$

Determination of confidence interval:

- lower confidence interval

$$\ln s_{1\mathrm{L}} = \ln s_1 - t_{0,975(N-2)} \cdot S_{(\ln\sigma)} = 5{,}65793 \tag{22}$$

- upper confidence interval

$$\ln s_{1H} = \ln s_1 + t_{0,975(N-2)} \cdot S_{(\ln\sigma)} = 5{,}88125 \tag{23}$$

After the transformation it is valid for:

$$s_1 = (320{,}5 \pm 9{,}0)\ \text{MPa} \tag{24}$$

These interval limits for the continuously changing values of the instantaneous actual stress s form the belt of the reliability round the regression line. By transformation of the regression line the confidence belt for values of instantaneous actual stress s is got.

After the transformation the calculated confidence interval for the mean value falls with 95 % probability among values, i. e. for s_1 <311,5;329,6> MPa to s_6 <421,4;440.2> MPa.

Graph of specimen No. 1

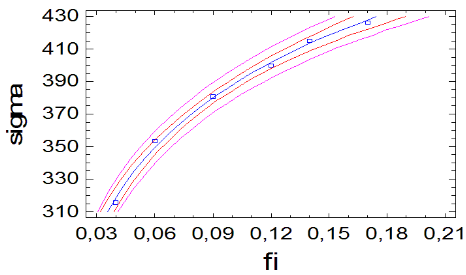

Fig. 1 Relationship of the instantaneous actual stress s on instantaneous actual logarithmic strain j with confidence intervals around the curve evaluated by test bar No. 1 in direction 45° towards sheet-metal rolling direction (the output of the statistical software Statgraphics)

The standard for tensile tests ČSN EN ISO 6892-1 provides the determination of measurement uncertainties for the 95 % confidence interval in regards with the recommendations contained in the GUM (Guide to Uncertainty in Measurement) and can be compared with the calculated values of uncertainty. Confidence band does not inform about the inaccuracy of the regression line as a whole but only about the individual intervals, always for a single value of independent variable. These interval estimates form the confidence band around the regression line. It is a band which width for a fixed value x indicates the reliability security of the calculated value from selected model of 95 % confidence interval for the mean instantaneous actual stress at a given level σ, i. e. 95 % of the σ mean values lie in this interval. The second zone corresponds to 90 % confidence interval for instantaneous actual stress σ and it is attached only from informational point of view to exclude test bar from the measurement in case the value of instantaneous actual stress σ is here.

Graph of specimen No. 2

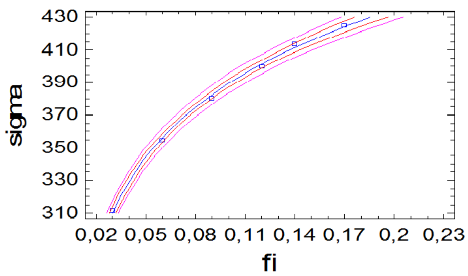

Fig. 2 Relationship of the instantaneous actual stress s on instantaneous actual logarithmic strain j with confidence intervals around the curve evaluated by test bar No. 2 in direction 45° towards sheet-metal rolling direction (the output of the statistical software Statgraphics)

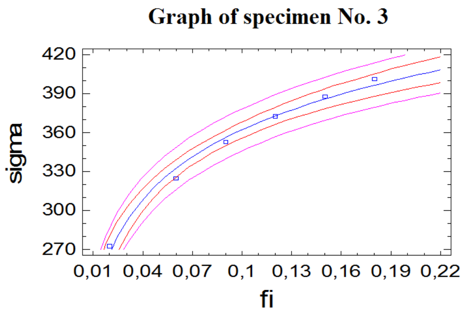

Fig. 3 Relationship of the instantaneous actual stress *s* on instantaneous actual logarithmic strain *j* with confidence intervals around the curve evaluated by test bar No. 3 in direction 45° towards sheet-metal rolling direction (the output of the statistical software Statgraphics)

If the measured points are not present in the band of 95 % confidence interval (see Fig. 3 for $j_2 = 0{,}06$) the actual value of the instantaneous actual stress *s* is outside the confidence interval which implies that the input uncertainty is being present. In case that more than two measured values of the instantaneous actual stress *s* do not occur in this zone, the current tested bar must be excluded from the measurement because standard ČSN ISO 10275 provides for the measurement of at least 5 points spaced in a geometric sequence according to equation (2) using least-squares method.

3 ANALYSIS OF VARIANCE OF STRAIN HARDENING EXPONENT MEAN VALUES

Analysis of variance was performed on three bars in each direction 0°, 45° and 90° towards sheet-metal rolling direction. This analysis can determine the uncertainty of measurement of mean directional values of strain hardening exponent n_x, the overall measurement uncertainty and its deviation from the average sample listed in Tab. 10. The column in the table presented as $y_{i,j}$ gives the values of strain hardening exponent in the direction towards sheet-metal rolling direction for every of three test bars. The value N_i in Tab. 10 indicates the total number of tested bars.

Tab. 10 Directional values of strain hardening exponent for analysis of variance

i	y_{ij}			N_i	$\sum_{j=1}^{N_i} y_{ij}$	$\bar{y}_i$	$\sum_{j=1}^{N_i} y_{ij}^2$	$N_i \bar{y}_i^2$
Direction 0°	0,15	0,16	0,20	3	0,51	0,170	0,0881	0,0867
Direction 45°	0,20	0,19	0,19	3	0,58	0,193	0,1122	0,1121
Direction 90°	0,22	0,15	0,20	3	0,57	0,190	0,1109	0,1083
Total	–	–	–	9	1,66	0,184	0,3112	0,3071

According to the table of analysis of variance of strain hardening exponent the separate sums of squares were calculated.

Overall (total) sum of squares:

$$S_y = \sum_{i=1}^{k}\sum_{j=1}^{n} y_{ij}^2 - \frac{1}{n}\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}\right)^2 = 0{,}00502 \tag{25}$$

The sum of squares between groups (interclass):

$$S_{ym} = \sum_{i=1}^{k} n_i \bar{y}_i^2 - \frac{1}{n}\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}\right)^2 = 0{,}00096 \tag{26}$$

The sum of squares within groups (internal):

$$S_{yv} = S_y - S_{ym} = 0{,}00407 \tag{27}$$

Tightness of dependence is expressed by the determination ratio:

$$p^2 = \frac{S_{ym}}{S_y} = 0{,}19027 \tag{28}$$

The value of 0 corresponds to the equality of all group averages, the value of 1 has a null group variability.

Correlation ratio:

$$P = \sqrt{p^2} = 0{,}43619 \tag{29}$$

The value of test criterion:

$$F = \frac{\frac{S_{ym}}{k-1}}{\frac{S_{yv}}{n-k}} = 0{,}705 \tag{30}$$

$$F_{0,95}(2{,}13) = 3{,}806 \tag{31}$$

$$F < F_{0,95}(2{,}13) \tag{32}$$

It does not belong to critical area defined by quantile $F_{0,95}(2{,}13) = 3{,}806$, so the hypothesis of independence of the strain hardening exponent ***n*** at inclination *a* (°) is denied. On the significance level *a* = 0,05 the difference between the strain hardening exponent among directions 0°, 45° and 90° towards sheet-metal rolling direction did not been demonstrated. By the above mentioned method the accuracy of measurement can be verified and if the test bars in directions are in the confidence interval. Results of analysis of variance are presented in the following Tab. 11.

Tab. 11 Results of the analysis of values of strain hardening exponent ***n*** using ANOVA method

Source variability	SS	Degree of freedom	MS	F-ratio	P-value
Total	0,005022	8	–	0,705	0,0053
Regression (interclass)	0,000961	2	0,000478		
Residual Sum of Squares (inner)	0,004072	6	0,000678		

The same conclusion may also occur at the minimum level of significance ***P-value*** ANOVA.

The value of ***P-value*** listed in Tab. 11 is less than 0,01 implying 99 % reliability of measured values of strain hardening exponent ***n***.

3.1 Determination of directional strain hardening exponent

The resulting value of strain hardening exponent in direction 45° towards sheet-metal rolling direction is determined by:

$$n_x = \frac{\sum n_{xi}}{N_V} \tag{33}$$

where N_V is total number of values in sheet-metal rolling direction *x*.

The standard deviation of direction of strain hardening exponent values:

$$s_{(n)} = \frac{1}{N_V} \cdot \sqrt{\sum s_{(ni)}}^2 \tag{34}$$

As an example of calculation of directional strain hardening exponent from three values evaluated by three tests bars the values in direction 45° towards sheet-metal rolling direction are used (see Tab. 6 to Tab. 8). Also the standard deviation of mean value of directional strain hardening exponent was determined:

$$n_{45} = \frac{0{,}2037 + 0{,}1795 + 0{,}1770}{3} = 0{,}187 \ ,$$

$$s_{(n)} = \frac{1}{3}\sqrt{0{,}01041^2 + 0{,}00226^2 + 0{,}00427^2} = 0{,}0038 \, . \tag{35}$$

Directional strain hardening exponent has the measurement uncertainty:

$$n_{45} \pm t_{0{,}975\,(N-2)} \cdot s_{(n)} = 0{,}187 \pm 2{,}12 \cdot 0{,}004 = 0{,}187 \pm 0{,}066 \tag{36}$$

4 UNCERTAINTY OF STRAIN HARDENING EXPONENT MEAN VALUE AND OF THE PLANAR ANISOTROPY DEGREE OF STRAIN HARDENING EXPONENT

The mean strain hardening exponent can be determined from formula:

$$n_m = \frac{1}{4} \cdot \left(n_0 + 2n_{45} + n_{90}\right) \tag{37}$$

where n_0, n_{45} and n_{90} are the mean values of strain hardening exponent interval in directions 0°, 45° and 90° towards sheet-metal rolling direction.

Practically, the standard ČSN ISO 10275 sets rounding of value of strain hardening exponent n_m to the nearest value with precision of 0,01, but in terms of clearer classification of the suitability of sheet-metal formability the earlier standard has been preferred.

The resulting value of strain hardening exponent for values of the confidence interval bands:

$$n_m = 0{,}183 \pm 0{,}008 \tag{38}$$

The planar anisotropy degree of strain hardening exponent is calculated from formula:

$$\Delta n = \frac{1}{2} \cdot \left(n_0 - 2 \cdot n_{45} + n_{90}\right) \tag{39}$$

$$Dn = -0{,}011 \pm 0{,}008 \tag{40}$$

According to the number of decimal places of input values, which were measured with occurrence of two decimal places, the measurement uncertainty is not significant.

5 CONCLUSIONS

From the experimental results of tensile tests according to ČSN ISO 6892-1 of the steel strip DD11 (11 320.0) follows that the steel strip has in all directions towards rolling direction yield strength R_e at the beginning, in the center and at the end of the coil, which indicates low suitability of steel for deep drawing.

The values of strain hardening exponent in directions 0°, 45° and 90° towards sheet-metal rolling direction, the mean value of strain hardening exponent and the planar anisotropy degree of strain hardening exponent *Dn* (see 4) were calculated in accordance with ČSN ISO 10275.

At all three tested bars in direction 45° towards sheet-metal rolling direction, which were selected for the calculation example of determining of confidence bands around the regression line of the instantaneous actual stress-instantaneous actual logarithmic strain relationship, the values of instantaneous actual stress varied within the 95 % confidence level, which is determined for tensile tests in standard ČSN EN ISO 6891-1. In the same way the measurement of the confidence interval band was proceeded for every from three test bars in directions 0° and 90° towards sheet-metal rolling direction, where acquired values of instantaneous actual stress matched given confidence intervals. Analysis of variance of strain hardening exponent directional values (see Tab. 10) and the results of analysis performed by ANOVA method listed in Tab. 11 verified the accuracy of measurements in directions 0°, 45° and 90° towards sheet-metal rolling direction.

Analysis of variance by ANOVA method enabled to separate the different sources of variance and to compare the partial variances to each other in order to determine whether the differences between them are statistically significant. Main advantages of this method compared to methods for average and spread lie in the fact that they are able to cope with the various experimental uncertainties, the method can provide more accurate estimation of the variance and allows obtaining of more information from experimental data. Compared with t-test for independent mean value, the ANOVA method is preferable for a smaller number of computations for achievement of the required accuracy range.

To determine the strain hardening exponent, the authors used 6 points spaced in a geometric sequence according to equation (2) to calculate by the method of least squares, which meet the requirement of the standard ČSN ISO 10275 that prescribes a minimum of 5 points. In the case where at least two of the values for the instantaneous actual stress relationship on instantaneous actual logarithmic strain are outside the zone of fixed confidence interval, it would be necessary to exclude the bar from a set of measured values, but this phenomenon did not occur in cases described in this paper. However for test bar No. 3 in direction 45° towards sheet-metal rolling direction (see Fig. 3) and for the value of the instantaneous actual logarithmic strain $j_2 = 0{,}06$ the actual value of the instantaneous actual stress s is outside the confidence band. In this case the initial uncertainty of the measured value was showed. At tested steel strip the evaluated uncertainty of mean strain hardening exponent calculated from the values in directions 0°, 45° and 90° towards sheet-metal rolling direction did not affect the result of evaluation of formability of steel strip since the calculated value is not even close to any of the limit of the intervals that are used to determine the suitability of the sheet-metal for forming.

From point of view of an unconventional sheet-metal formability criterion ($n_m < 0{,}215$) it can be concluded, that the material DD11 (11 320.0) has low suitability for cases of sheet-metal forming where the pressure-pull mechanical diagrams of stress predominate.

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