MATHEMATICAL MODELS OF MIMO CHANNEL

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Summary In this contribution, we try to resolve the problem with multipath MIMO channel in wireless environment. To understand the environment, where multipath signal is transported, theoretical models will be applied. We will take a look on theoretical models and results reached in mathematical programmable language MATLAB. Results will be shown on graphs at the end of this paper.

1. INTRODUCTION

Multipath channel is one of the most important points in wireless technology for the future. This is the way, how can be the bit rate increased and how we can transport a signal as best as possible through wireless communication channel. Technology, which describes how to transport the signal through multipath channel with high efficiency is named MIMO (Multiple Input Multiple Output) technology.

2. THEORY

To multipath channel we have to assign antenna systems, which are using more than one antenna element. These special systems are able to increase the capacity of transport wireless channel and bit rates. Complexity of these antenna systems is increasing with number of antenna elements.

Wireless systems can be divided into few groups:

- *SISO* (Single Input Single Output),
- *SIMO* (Single Input Multiple Output),
- *MISO* (Multiple input Single Output),
- *MIMO* (Multiple Input Multiple Output).

This paper is dealing with the last one, MIMO system. The basic point of MIMO exploitation is to have the right environment in which we would like to transport wireless signal. We must have sufficient quantity of scatterers, which will help to transport the signal through wireless environment by multipath propagation. The way how the signal is transported by clusters of scatterers will be shown on mathematical models.

2.1 Eliptical model

This model assumes that cluster of scatterers is distributed on ellipse. Let us assume that mobilestation (MS) and base station (BS) are situated in focal points of ellipse. The distance between two stations is 2f and MS is moving at direction, as shown on *figure 1*.

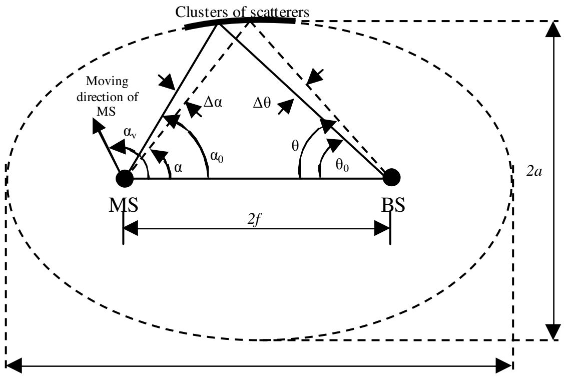

Fig. 1. Elliptical model

Signal transported from MS at angle α_0 is received by BS at angle θ_0. Signals can be transmitted or received in angle interval [-π, π). Departure or arrival angles can be estimated from equations in [1] and [2]. Each angle of departure or arrival depends on probability density function of clusters distribution. We can get the transfer matrix of each path, as it is shown on *Fig. 2* [2].

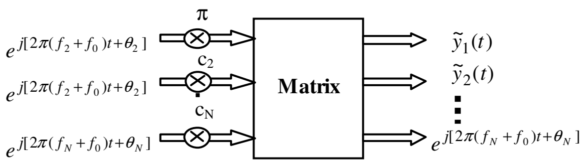

Fig. 2. Transfer Channel Matrix

Each path is represented by time delay, which have to be consider by each output function *y(t)* mentioned in [1] or [2], as it is shown on *Fig.3*. Each delay is represented by ellipses, which fenced MS and BS. This mean, that each transported signal have to be described by delays, on which the transport time depends.

From delays assigned to each group of scatterers clusters, the impulse response has to be estimated [2].

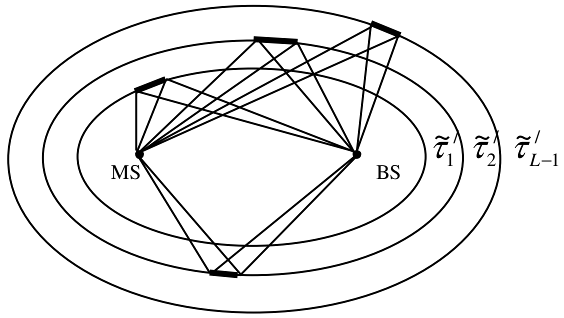

Fig. 3. Groups of clusters of scatterers with variable time delays

Impulse response can be estimated from equation shown in [2]

From impulse responses, the channel matrix can be estimated and size of this matrix depends on the number of channels established between MS and BS.

2.2 Two ring model

This model has the best performance for indoor environment. In this case, clusters of scatterers are situated on two rings, which surround MS and BS (Fig. 4). Let's have an antenna system with two antenna elements on both sides. All angles are pointed to main axis traversing MS and BS [2].

We can imagine the signal propagation as a wave, transmitted from antenna element A_t trough groups of clusters of scatterers S_T and S_R. Signal is coming to receiver at angle Φ_R.

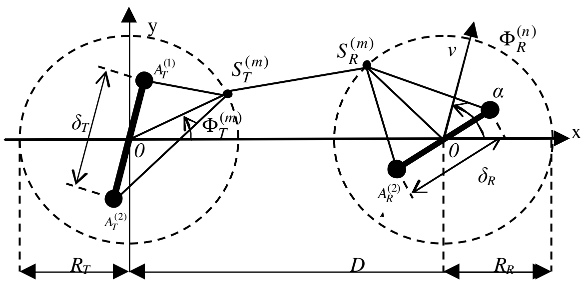

Fig. 4. Two ring model

MS is moving in direction along vector *v*. First of all we must estimate the transfer function [2] that will be used for transfer matrix

$$H(t) = \begin{pmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{pmatrix}, \quad (2)$$

This matrix is describing two ring mathematical model of MIMO channel with Rayleigh fading. Channel matrix will be used in capacity estimation [1], [2] by next equation:

$$C(t) := \log_2\left[\det\left(I_2 + \frac{P_T}{M_T N_0} H(t) H^H(t)\right)\right] \quad (3)$$

I represents the unit matrix, P_T is the whole transported power, N_0 is noise spectral power density.

2.3 3GPP model

3GPP model differs from other models by its look onto environment. It is more complicated, but on the other side, has higher precision.

This model divides environment into 3 groups [2], [3].

- ***Suburban macrocell*** (Approximately 3km distance between MS and BS).
- ***Urban macrocell*** (Approximately 1km distance between MS and BS).
- ***Urban Microcell*** (less than 1km distance between MS and BS).

The fig. 5 shows the 3GPP model in environment with multipath.

The signal is transported through cluster of scatterers between MS and BS. Signal strength and transport time depends on location of cluster.

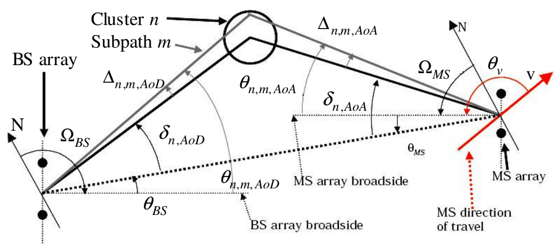

Fig. 5. BS and MS angle parameters of 3GPP model

Ω_{BS} - BS antenna array orientation, defined as the difference between the broadside of the BS array and the absolute North (N) reference direction.

θ_{BS} - BS antenna array orientation, defined as the difference between the broadside of the BS array and the absolute North (N) reference direction.

$\delta_{n,AoD}$ - Angle of Departure (AoD) for the *n*-th (n = 1 … N) path with respect to Line Of Sight (LOS) AoD θ_0.

$\Delta_{n,m,AoD}$ - Offset for *m*-th (m = 1 … M) subpath of *n*-th path with respect to $\delta_{n,AoD}$.

$\theta_{n,m,AoD}$ - Absolute AoD for the *m*-th (m = 1 … M) subpath of the *n*-th path at the BS with respect to the BS broadside.

Ω_{MS} - MS antenna array orientation, defined as the difference between the broadside of the MS array and the absolute North reference direction.

$\delta_{n,AoA}$ – Angle of arrival (AoA) for the *n*-th (n = 1 … N) path with respect to the LOS AoA $\theta_{0,MS}$.

$\Delta_{n,m,AoA}$ - Offset for the *m*-th (m = 1 … M) subpath of the *n*-th path assigned to $\delta_{n,AoA}$.

$\theta_{n,m,AoA}$ - Absolute AoA for the *m*-th (m = 1 … M) subpath of the *n*-th path at the MS with respect to the BS broadside.

v - MS velocity vector

θ_v - Angle of the velocity vector with respect to the MS broadside

Environments have impact on parameters used for estimation of channel capacity. We have to consider that there are not the same groups of cluster of scatterers, which have impact onto channel capacity. It was stated that we need the proper environment for multiple channel transport. As will be shown later, we demonstrated this impact onto simulation model.

The used parameters are shown in Table 5.1 Environment parameters in [3].

The first step is to find out user's parameters [2], [3]:

- We have to choose, in which environment 3GPP model will be used.
- To determine various distance and orientation parameters.
- To determine *Delay Spread (DS), Angle Spread (AS)* and *Spreading Factor (SF).*
- The 3GPP model is specific in that, it calculates 6 main paths which are established between MS and BS. Because of this fact, it has to be consider the delay of each path. So we have to consider 6 delays.
- In receiver we can find, that each path transports the signal with different power, so we have to assign different powers to each of 6 paths.

$$P(\theta,\sigma,\bar{\theta}) = N_0 \exp\left[\frac{-\sqrt{2}\left|\theta-\bar{\theta}\right|}{\sigma}\right] G(\theta) , \quad (4)$$

- Average powers are normalized and we get the equation:

$$P_n = \frac{P'_n}{\sum_{j=1}^{6} P'_j} , \quad (5)$$

- To determine angles of departure (*AoDs*) of *N* multipath components.
-

$$\delta_{n,AoA} \sim \eta(0,\sigma^2_{n,AoA}) , \quad (6)$$

where $\sigma_{n,AoA} = 104.12(1-\exp(-0.265\left|10\log_{10}(P_n)\right|))$, [3],

- To associate the multipath delays with *AoDs.*
- To determine powers, phases, and offsets *AoDs* of *M = 20 subpaths* for each of *N*-paths.

Our target is to estimate the impulse response *h* of each *n*-th channel, from which the channel matrix will be build. There after we can find the channel capacity of the whole multipath model. As it is shown in [2] and [3] equation for multipath response is:

$$h^{LOS+NLOS}_{s,u,n} = \sqrt{\frac{1}{1+K}} h_{s,u,1}(t) + \sigma_{SF}\sqrt{\frac{K}{K+1}}\begin{pmatrix} \sqrt{G_{BS}(\theta_{BS})}\exp(jkd_s \sin(\theta_{BS}))\times \\ \sqrt{G_{MS}(\theta_{MS})}\exp(jkd_u \sin(\theta_{MS})+\Phi_{LOS})\times \\ \exp(jk\|v\|\cos(\theta_{MS}-\theta_v)t) \end{pmatrix}, \quad (7)$$

This equation represents the elements of multipath channel matrix.

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \ldots & h_{1u} \\ h_{21} & h_{22} & \ldots & h_{2u} \\ \vdots & \vdots & \vdots & \vdots \\ h_{s1} & h_{s2} & \ldots & h_{su} \end{pmatrix}, \quad (8)$$

Result of theoretical model simulation is the multipath channel capacity based on equation:

$$C = \log_2\left| I + \frac{\rho}{n} HH^H \right| , \quad (9)$$

3. 3GPP MODEL SIMULATION

We have realized the 3GPP model in MATLAB environment. The basic idea of proposed simulation model is to find out, how will vary the capacity of multipath system, with changing of some parameters in communication system.

As it was said in theoretical part, the first step is the *environment selection.* Program will find out all necessary parameters assigned to selected environment.

After that, the program is divided into 4 functions.

In *function no.1* the user will be asked for some values, that it can't be set randomly, like *distance, number of used antenna elements* etc.

Program will apply these values into equations where first of all impulse responses (7) will be estimated. We have to calculate also with possibility, that not only Non Line Of Sight (NLOS) environment will be used. Because of this fact, the LOS component is applied. Its estimation can be found in [2] or [3].

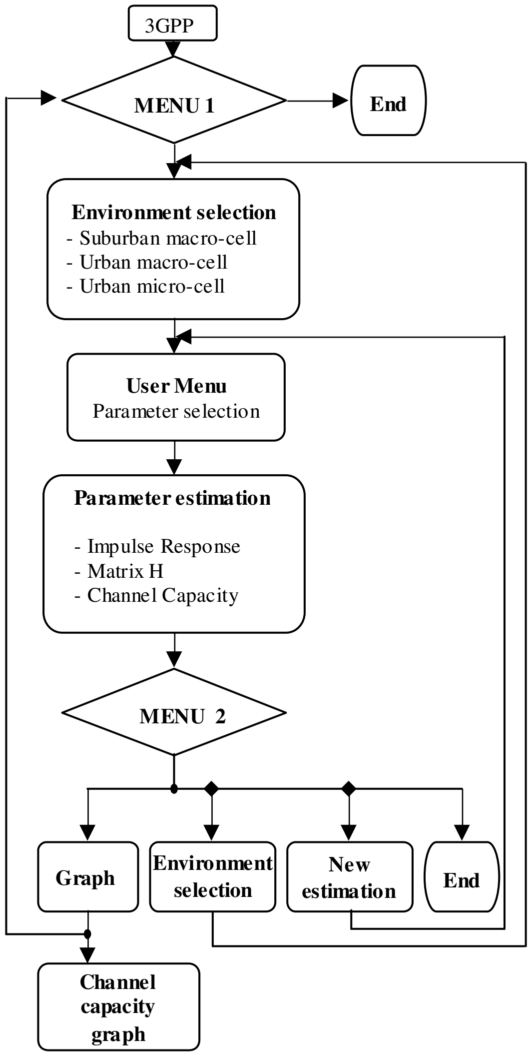

Fig. 6. Structure of simulation model

Function no.2 represents the channel matrix estimation, where impulse response will be applied to equation (8).

Function no.3 is devoted to capacity estimation, where channel matrix will be applied.

In *function no.4* we can choose to look onto a graph, estimate a new capacity or change environment.

The example of capacity estimation is at Fig.7. Red line represents the average capacity.

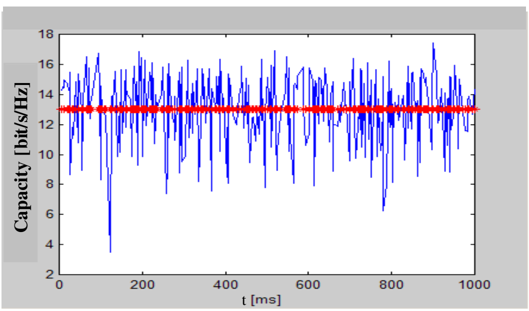

Fig. 7. Channel capacity in response to time

Channel capacity was estimated with the change of some factors, which may have impact onto capacity estimation [2]. Results are shown on next graphs (Fig. 8.1,...,8.4).

From results we have found, that only a few factors have impact onto channel capacity.

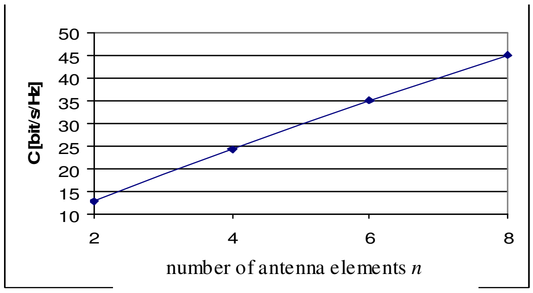

Fig. 8.1 Capacity dependence on number of antenna elements

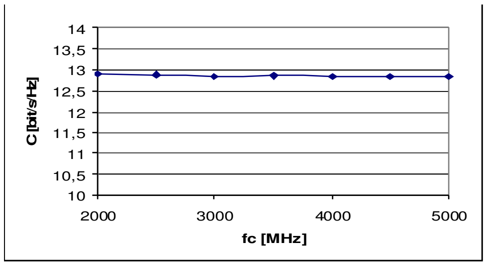

Fig. 8.2 Capacity dependence on frequency

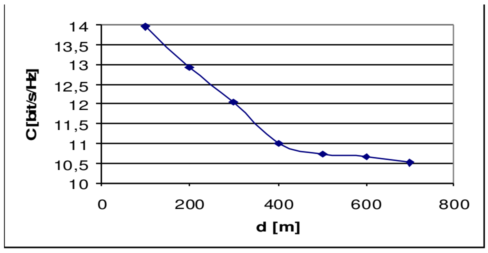

Fig. 8.3 Capacity dependence on distance between MS and BS

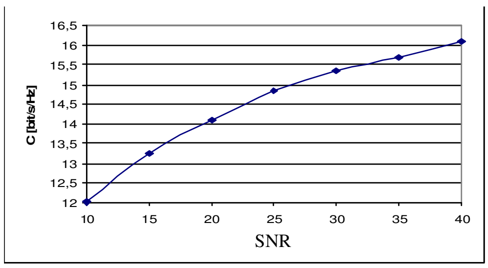

Fig. 8.4 Capacity dependence on Signal to Noice Ratio

Simulations show [2], that capacity varies only with changing of antenna elements number of, with the distance between MS and BS or signal to noise ratio (SNR). Variation of other parameters, like frequency, has no effective impact on the channel capacity.

Acknowledgement

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