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MODEL-FREE AUTOTUNING TESTING ON A MODEL OF A THREE-TANK CASCADE

TESTOVÁNÍ BEZMODELOVÉ SAMOSEŘIZOVACÍ METODY  
NA LABORATORNÍM MODELU KASKÁDY TŘÍ NÁDRŽÍ

**Abstract**

A newly developed model-free autotuning method based on frequency response analysis has been tested on a laboratory set-up that represents a physical model of a three-tank cascade. This laboratory model was chosen for the following reasons: a) the laboratory model was ready for computer control; b) simultaneously, computer simulation could be effectively utilized, because a mathematical description of the cascade based on quite exactly valid relations was available; c) the set-up provided the necessary degree of nonlinearity and changeable properties. The improvement of the laboratory set-up instrumentation presented here was necessary because the results obtained from the first experimental identification did not correspond to the results provided by the simulation. The data was evidently imprecise, because the available sensors and the conditions for process settling were inadequate.

**Abstrakt**

Pro testování nově vyvíjené bezmodelové samoseřizovací metody byl vybrán existující laboratorní model kaskády tří nádrží. Tento laboratorní model byl vybrán z následujících důvodů: a) laboratorní model je říditelný pomocí počítače, b) jednoduchý matematický popis, c) dostatečný stupeň nelinearity a proměnnosti v čase. Prezentované rozšíření vybavení laboratorního modelu bylo nevyhnutelné, neboť výsledky získané z prvních experimentů nepříliš dobře odpovídaly výsledkům získaných pomocí simulace.

**1 INTRODUCTION**

Many controller tuning methods have been developed. Most of them are designated for tuning PID controllers [1]. However, the most widely-used tuning method remains the Ziegler-Nichols method [11]. Nowadays, the relay method is becoming popular [10], but some disadvantages remain, like the necessity to disconnect the controller while tuning. However, it is a useful tool for identification [4]. Special tuning methods have been developed to tune controllers of two degrees of freedom [5]. For this reason, we decided to develop a new tuning method that fulfils the requirements of industrial practice – the method must be model-free, it should operate without interrupting the control process, and it must be possible to use the tuning method without deep knowledge of control theory.

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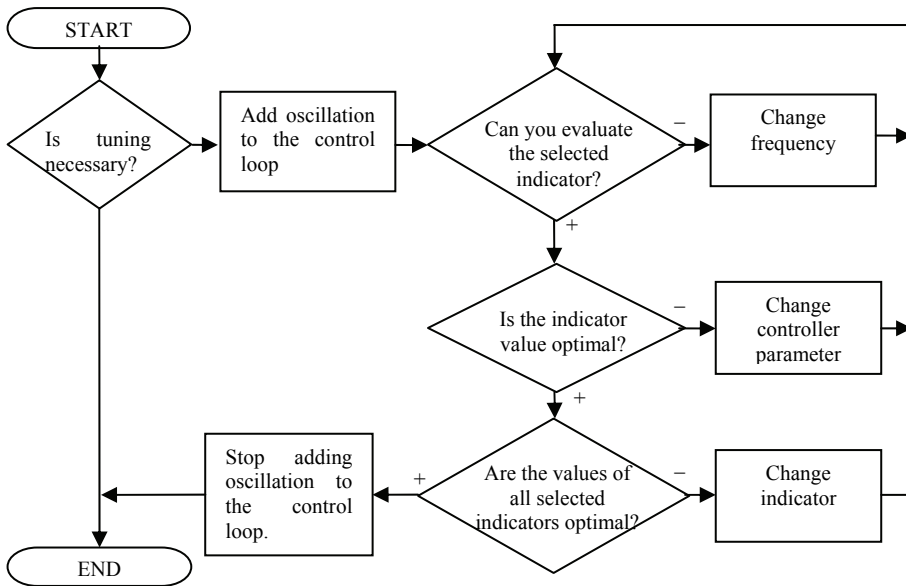
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When new control or setting algorithms are developed, it is a good principle to test them by means of simulation. In these tests, unsuitable simulation models are often used. Either they are too idealized, thus imprecise, or, on the other hand, they may be unrealistic, and mutually unlinked changes in the parameters of the model are artificially proposed in a way that the controller is not able to deal with. Pilot devices or laboratory replica models are an ideal solution for such controller testing. Changes in parameters are physically conditioned, the impacts of physical limits cannot be wrongly modelled in simulation (integral wind up), and if such a set up is well-chosen or well-designed, the mathematical description of important process phenomena is quite exact. We used a laboratory model of a cascade of three interconnected tanks to verify a new frequency-based controller autotuning method. This laboratory model was able to provide the necessary degree of nonlinearity and variability in time. A mathematical description of its dynamic behaviour is not difficult to derive. However, the laboratory set up has been developed for other purposes before we started to use it for these testing tasks, and as a result some instrument reconstruction was necessary.

## 2 MODEL-FREE AUTOTUNING

The new frequency-based autotuning method utilizes added harmonic oscillation in a closed control loop, which allows us to measure frequency-based control quality indicators as if they were measured in an open loop [6]. The advantage is that no model of controlled plant is used and the controller tuning is fully operable while it is being tuned. This tuning method can be added to the control loop purely by means of software. No additional instrumentation or structures are necessary when the controller is based on a PLC or on some other programmable device. These advantages will make this method easily applicable in industrial practice, when long-term adaptation is required, sometimes only at the request of the operator.



**Fig. 1** Flowchart of model-free frequency-based autotuning

Figure 1 shows the basic steps of model-free frequency-based autotuning in the form of a flowchart. Further information about the tuning method was published in [7]. Detailed description of the autotuning mechanism was published in [8].

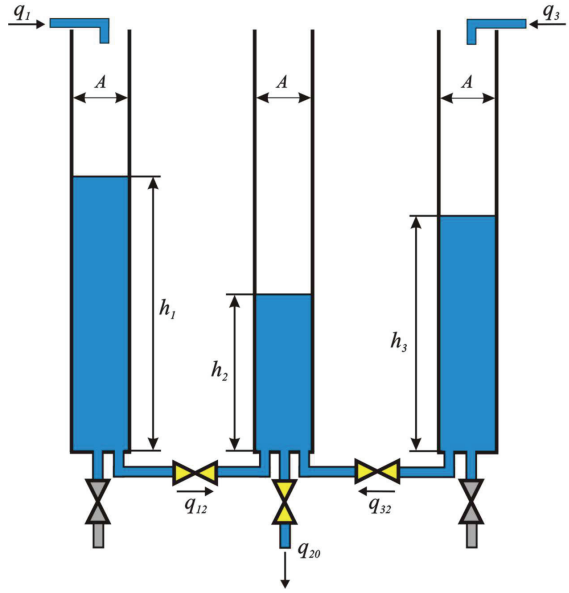
## 3 THREE-TANK CASCADE LABORATORY MODEL

The laboratory model consists of three interconnected tanks (Figure 2). Water is supplied into Tank One and Tank Three. Each tank is equipped with a pressure sensor that is used for measuring

the water level. In addition to the mutual interconnection, each of the tanks in the cascade has its own outlet valve, enabling various operation modes with different dynamics to be simulated. All valves in the laboratory model are adjustable only manually.

Originally, electronics of own design were used to control the cascade [2]. We replaced this by the WinCon W-8741-G programmable automation controller [9], while the other equipment remained unchanged. WinCon ran the water pumps and evaluated the data from the pressure sensors. It also converted the measured data from the sensors into the heights of the water levels in all three tanks.

We used the cascade in the configuration as depicted in Figure 2. This is not the only possible way of mutually interconnecting the Tanks, but if we take height  $h_2$  of the level in the second tank as the controlled variable, this option has a quite reasonable technical interpretation – in the second tank two solutes are mixed in a fixed volume. The inlet flow to the first tank  $q_1$  is manipulated by a controller, while the inlet flow to the third tank  $q_3$  represents the disturbance.



**Fig. 2** The three tank cascade model arrangement for control algorithm testing

#### 4 MATHEMATICAL MODEL DERIVATION AND PARAMETER IDENTIFICATION

Stated above, the simulation model is a good tool for autotuning function verification. For this purpose the derivation of a mathematical model of the set-up based on measured data was needed. The standard procedure for deductive identification was used.

From the volumic flow-rate balances we get equations of a nonlinear dynamic model of the cascade

$$\frac{d}{dt} h_1(t) = \frac{1}{A} q_1(t) - \frac{Kv_{12} \sqrt{h_1(t) - h_2(t)}}{A}, \quad (1)$$

$$\frac{d}{dt} h_2(t) = \frac{Kv_{12} \sqrt{h_1(t) - h_2(t)}}{A} + \frac{Kv_{12} \sqrt{h_3(t) - h_2(t)}}{A} - \frac{Kv_{20} \sqrt{h_2(t)}}{A}, \quad (2)$$

$$\frac{d}{dt} h_3(t) = \frac{1}{A} q_3(t) - \frac{Kv_{32} \sqrt{h_3(t) - h_2(t)}}{A}, \quad (3)$$

where:

$A$  – cross section (the same for all three tanks) [dm<sup>2</sup>],

$Kv_{xy}$  – flow rate coefficients used in the equations quantifying the (volumic) flow rate through the valve from tank  $x$  to tank  $y$  [dm<sup>2.5</sup>min<sup>-1</sup>],

$h_x$  – water level in the tank  $x$  [dm],

$q_x$  – supply flow rate, or mutual flow rates between the tanks ( $x, y = 1, 2, 3$ ) or to the atmosphere ( $y = 0$ ) [ $\text{dm}^3\text{min}^{-1}$ ].

In the next step, we had to identify the  $Kv_{xy}$  coefficients. First, it was necessary to measure the values of the flow rates  $q_1$  and  $q_3$ . The easiest way to do this was to close the output valve of the selected tank and to measure the time in which the defined level is reached. When the flow rate had been measured, the  $Kv_{xy}$  values were obtained for each steady state according to the formulas

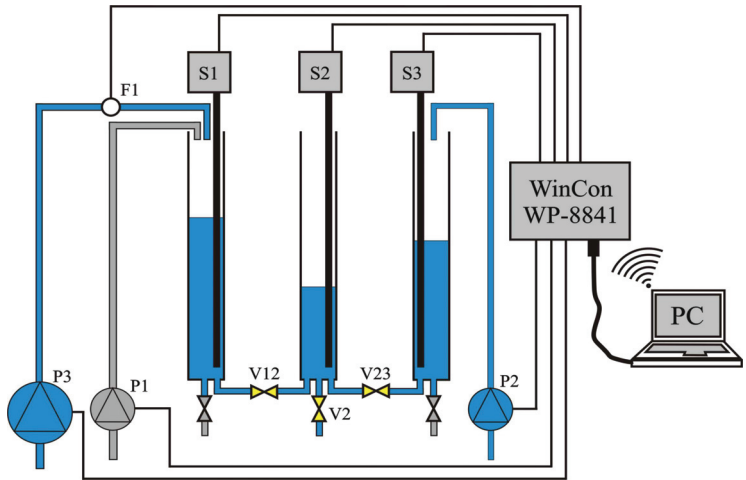
$$Kv_{12} = \frac{q_{10}(t)}{\sqrt{h_{10}(t) - h_{20}(t)}}, \quad Kv_{20} = \frac{q_{10}(t) + q_{30}(t)}{\sqrt{h_{20}(t)}}, \quad Kv_{32} = \frac{q_{30}(t)}{\sqrt{h_{30}(t) - h_{20}(t)}} \quad (4)$$

During the experimental measurements we found that the  $Kv_{20}$  and  $Kv_{32}$  coefficients in various steady states did not differ so significantly. However, the value of  $Kv_{12}$  coefficient did change its value significantly. Since no changes were made in the valve opening, the values of  $Kv_{12}$  should remain the same. One of the obvious explanations for this variability was that the  $Kv_{xy}$  values were obtained using a primitive and imprecise way of measuring the flow rate in situ (a stop watch and a graduated vessel), which led to imprecise measured flow rates values. Another reason was that the water pumps were operating below their nominal performance range. While we wanted to attain a steady state with a constant flow-rate, the flow rate in fact varied due to fluctuation of the water pump revolutions. The speed fluctuations differed for each pump; they were evidently greater in the case of the water pump supplying Tank One.

## 5 IMPROVEMENTS TO THE LABORATORY MODEL

To avoid water pump fluctuations, we equipped the set-up with a third water pump of a different type, equipped with a flow meter. This water pump replaced the previous pump supplying Tank One. The flow meter even enables us to control the water inlet flow rate by a control circuit.

Figure 3 shows a scheme of the topical three-tank cascade equipment. Pumps P1 and P2 are original submersible pumps [2]. Pump P1 is not used in our experiments. Honeywell 142PC01G pressure sensors remained as water level sensors (sensors S1 to S3). The WinCon W-8741-G programmable automation controller was replaced by a newer WinPAC WP-8841. The main parts of the additional equipment were the KELLER 5,5 l/min 0,4 bar water pump (pump P3) and the VISION 2006 2F66 flow meter (flow meter F1) produced by Badger meter. We chose these devices and their electronics on the basis of a personal recommendation by Dr. Hlava and the description in [3]. Two rotameters (not depicted in Figure 3) were added temporarily in order to measure the flow rates).



**Fig. 3** Scheme of a the three-tank cascade after the changes in instrument equipment

The WinCon W-8741-G programmable automation controller was replaced by a newer WinPAC WP-8841. The main parts of the additional equipment were the KELLER 5,5 l/min 0,4 bar water pump (pump P3) and the VISION 2006 2F66 flow meter (flow meter F1) produced by Badger meter. We chose these devices and their electronics on the basis of a personal recommendation by Dr. Hlava and the description in [3]. Two rotameters (not depicted in Figure 3) were added temporarily in order to measure the flow rates).

Figure 4 shows the steady state values of the levels in the tanks for various values of the inlet flow rate  $q_1$  (manipulated variable), while the flow rate  $q_3$  (disturbance variable) was fixed. The characteristics when  $q_3 = 1,00 \text{ l}\cdot\text{min}^{-1}$  and  $q_3 = 1,11 \text{ l}\cdot\text{min}^{-1}$  cannot be in the same range of  $q_1$  as when  $q_3 = 0,91 \text{ l}\cdot\text{min}^{-1}$ , because the tank height is 5 dm.

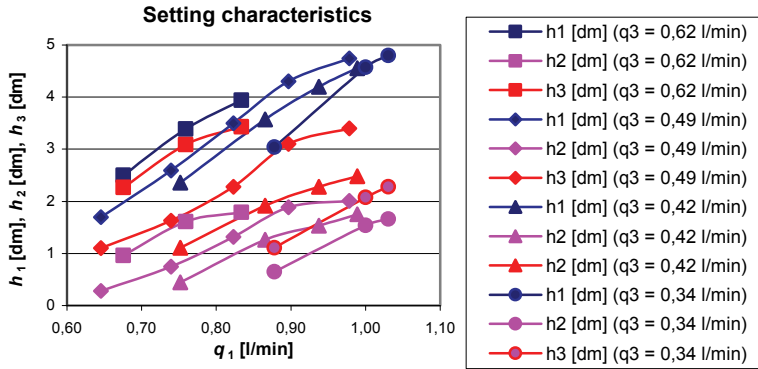


Fig. 4 Setting characteristics of the improved three-tank cascade laboratory model

## 6 SIMULATION RESULTS

We used the average of the  $K_{V_{12}}$ ,  $K_{V_{20}}$  and  $K_{V_{32}}$  values ( $K_{V_{12}} = 0,56 \text{ dm}^{2,5} \cdot \text{min}^{-1}$ ,  $K_{V_{20}} = 1,12 \text{ dm}^{2,5} \cdot \text{min}^{-1}$ ,  $K_{V_{32}} = 0,51 \text{ dm}^{2,5} \cdot \text{min}^{-1}$ ) in the simulations. The dynamics of the water pump was simulated by means of blocks representing an intuitively estimated transfer function  $G_{\text{pump}} = (0,1s + 1)^{-1}$ .

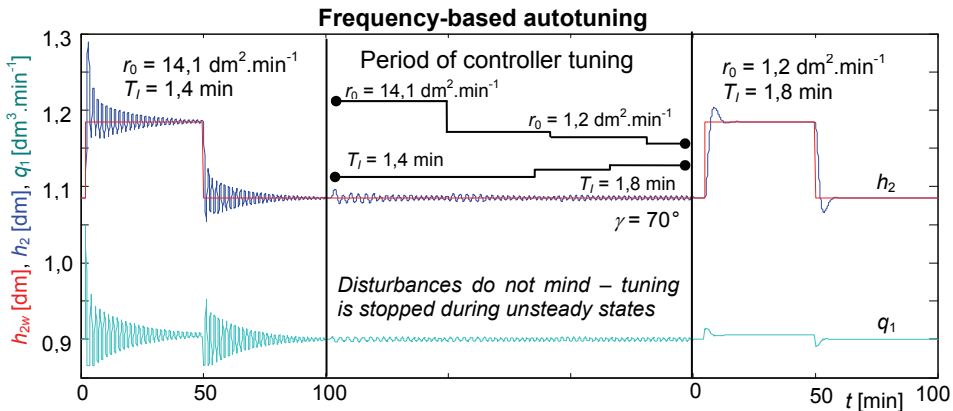


Fig. 5 Frequency-based autotuning experiment (manipulated variable  $q_1$  depicted after dividing the values by 10)

The real water pump dynamics is complicated to model, because no information is available about it. The autotuning features are simulated in the following sequence: first, using changes in the controller setpoint, the water level in Tank Two is increased by  $\Delta h_2 = 0,1 \text{ dm}$ , then it is decreased back to the initial water level. Then, during the second phase, the controller is tuned, and, finally, the same experiment as in step one is repeated, but with a new controller setting after autotuning.

## 7 CONCLUSIONS

The simulation result shown in Figure 5 confirmed good autotuning capability as far as finding the optimal controller setting is concerned. The tuning procedure is performed without disconnecting the controller at all.

The additional equipment of the three-tank cascade laboratory model helped in obtaining more precise results, especially in achieving a better fit between simulation and measurement. The main goal of the improvements was fulfilled. Simulated experiments, which can be easier to perform, enable conclusions on the controller function in real conditions to be drawn with greater confidence.

Our main interest is in evaluating the comparative usability of a fixed-set PI controller and a PI controller equipped with model-free autotuning features.

When all instrument improvements are finished, we expect to be able to use the set-up to make comparative tests of control algorithms in a distance mode. We hope that the autotuning capability of the model-free tuning algorithm will be sufficiently proved in these comparisons.

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