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EXTENDED ANALYTIC LINEAR MODEL OF HYDRAULIC CYLINDER WITH RESPECT DIFFERENT PISTON AREAS AND VOLUMES

ROZŠÍŘENÝ ANALYTICKÝ MODEL HYDRAULICKÉHO VÁLCE S RESPEKTOVÁNÍM ROZDÍLNÝCH ČINNÝCH PLOCH PÍSTNICE A OBJEMŮ

Abstract

Standard analytic linear model of hydraulic cylinder usually comes from assumptions of identical action piston areas on both sides of hydraulic cylinder (double piston rod) and suitable operation point, which is usually chosen in the middle of piston. By reason of that volumes inside of cylinder are than same. Moreover for control of that arrangement of hydraulic cylinder, usually controlled by 4/3 servovalve, the same mount of flows comes in and comes out to each of chambers of hydraulic cylinder. Presented paper deal with development of extended form of analytic linear model of single piston rod hydraulic cylinder which respects different action piston areas and volumes inside of chambers of hydraulic cylinder and also two different input flows of hydraulic cylinder. In extended model are also considered possibilities of different dead volumes in hoses and intake parts of hydraulic cylinder. Dead volume has impact on damping of hydraulic cylinder. Because the system of hydraulic cylinder is generally presented as a integrative system with inertia of second order: $G_{(s)} = K_M / s \cdot (T_M{}^2 s^2 + 2 \cdot T_M \xi_M s + 1)$, we can than obtain time constants and damping of hydraulic cylinder for each of analytic form model. The model has arisen for needs of model fractionation on two parts. Part of behaviour of chamber A and part of behaviour of chamber B of cylinder. It was created for the reason of analysis and synthesis of control parameters of regulation circuit of multivalve control concept of hydraulic drive with separately controlled chamber A and B which could be then used for.

Abstrakt

Standardní analytický lineární model hydraulického válce obvykle vychází z předpokladu shodných činných ploch na obou stranách pístnice (oboustranná pístnice) a vhodného pracovního bodu, který je obvykle volen uprostřed pístnice. Z toho důvodu jsou pak objemy uvnitř jednotlivých komor hydraulického válce stejné. Mimoto pro řízení takovéhoto uspořádávání hydraulického válce, obvykle řízeného čtyřcestným, třístavovým servoventilem 4/3 přitékají a odtékají stejná množství průtoků do každé z komor válce. Presentovaný příspěvek se zabývá vývojem rozšířené formy analytického lineárního modelu hydraulického válce s jednostrannou pístnicí, který respektuje rozdílné činné plochy na pístnici a rozdílné objemy uvnitř komor hydraulického válce a rovněž také umožňuje uvažovat oba dva vstupy průtoků do hydraulického válce. V rozšířeném modelu jsou rovněž uvažovány možnosti různých mrtvých objemů v hadicích a přívodních částech hydraulického válce. Mrtvý objem má velký vliv na tlumení hydromotoru. Protože systém hydraulického válce můžeme obecně považovat za integrační systém se setrvačností druhého řádu $G_{(s)} = K_M / s \cdot (T_M{}^2 s^2 + 2 \cdot T_M \xi_M s + 1)$, můžeme pro každý z analytických modelů obdržet časové konstanty a konstanty tlumení hydraulického válce. Model vznikl pro potřeby rozdělení modelu na dvě části, část chování komory A a část chování komory B z důvodu potřeby analýzy a dále syntézy řídicích parametrů regulačního obvodu víceventilové řídicí koncepce hydraulického pohonu se samostatně řízenou komorou A a komorou B hydromotoru, pro kterou může být následně využit.

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1 INTRODUCTION

For creation of standard analytical model of hydraulic cylinder we usually come out from idea of symmetrical cylinder with two side piston and from the same action areas on both sides of cylinder $S = S_a = S_b$, see figure 1. In that case the changes of leakage Q_{li}, flows Q_A a Q_B and volumes V_a a V_b inside of chambers of cylinder are the same. Than it is easy to obtain cylinder mathematical formulation in form of analytically model described in working area point, which is usually chosen in the middle of stroke of hydraulic cylinder. Described model generally present integration system with inertia of second order with one input value Q, which is flow, and with one output value x, which is piston position [Noskievič, P., 1999].

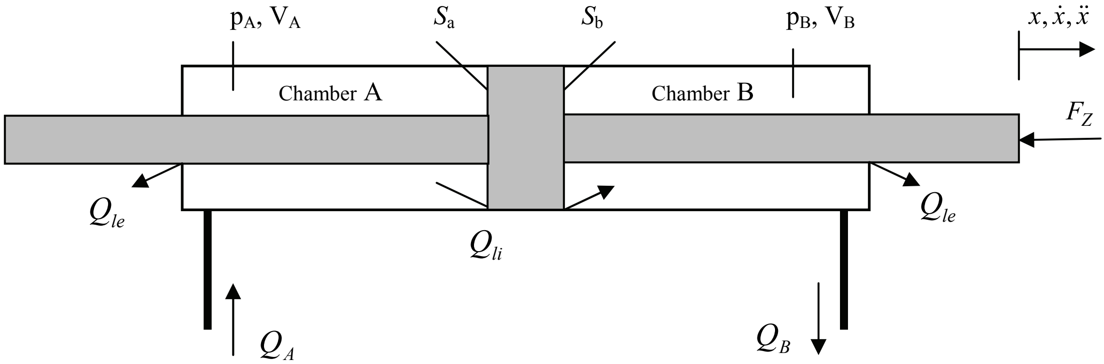

Fig. 1 Situation scheme of double rod cylinder

Mentioned integrating transfer function with inertia of second order can be taken in form

$$G_{(s)} = K_M \big/ s \cdot (T_M{}^2 \cdot s^2 + 2 \cdot T_M \cdot \xi_M \cdot s + 1)\,. \tag{1}$$

For needs of linear analysis of behaviour of hydraulic circuits is usually this model sufficient, but some differences between behaviour of single rod cylinder and double rod cylinder are not taken on account. Than it is obvious, coefficients of characteristic polynomial and calculation of cylinder damping and time constant can be different. Linear model, which can respect mentioned differences should be used to easy synthesis of control of hydraulic drive, multivalve control concept of hydraulic drive, to computing of variable constants of gains during piston stroking and operation parameters changing.

If we are thinking about single rod cylinder, another situation as with the previous double rod cylinder arises. Due to another construction design of cylinder and different action areas of piston S_a and S_b, state values are changing and also flows, leakage and volumes inside of cylinder chambers are different according as the piston position is changing. These differences and characteristics are necessary take into account and consider them in model.

2 ORIGINAL MATEMATICAL MODEL OF HYDRAULIC CYLINDER

Nonlinear mathematical model of cylinder and its movement equation (3) comes from balance of forces of pressures in chambers A and B on relevant action areas, from load force F_Z, eventually from friction, viscous and other forces.

Equations (4) and (5) are valid for pressure definitions. Equations (4) and (5) are possible to express pressure as an integration of relevant flows in each of chambers of cylinder multiplied with reciprocal value of hydraulic capacity C. Hydraulic capacity depends on relevant piston position, eventually on temperature changes and on the other characteristics, and it can be generally defined as

$$C = \frac{V}{K}\,, \tag{2}$$

like ratio between volume V [m^3] and liquid bulk modulus K [$N\ m^{-2}$].

$$m \cdot \ddot{x} + b \cdot \dot{x} = S_a \cdot p_A - S_b \cdot p_B - F_Z - F_T \cdot \mathrm{sgn}(\dot{x}), \tag{3}$$

$$\dot{p}_A = \frac{K}{V_A} \cdot \left[Q_A - S_a \cdot \dot{x} - G_{li} \cdot (p_A - p_B) - G_{le} \cdot p_A \right], \tag{4}$$

$$\dot{p}_B = \frac{K}{V_B} \cdot \left[S_b \cdot \dot{x} - Q_B + G_{li} \cdot (p_A - p_B) - G_{le} \cdot p_B \right], \tag{5}$$

where	m	=	Reduced mass of piston of cylinder
	b	=	Coefficient of damping of cylinder
	x	=	Piston position of hydromotor
	$X_{\max}$	=	Maximal piston stroke of cylinder
	F_z	=	Load force on piston of cylinder
	F_T	=	Friction force of piston of cylinder
	S_a, S_b	=	Action areas of piston of cylinder
	p_A, p_B	=	Pressure in chambers A and B of cylinder
	K	=	Oil bulk modulus
	V_A, V_B	=	Volumes in chambers A and B
	Q, Q_A, Q_B	=	Flow, flow to chamber A, flow to chamber B of cylinder
	G_{li}	=	Internal leakage between chambers A and B of cylinder
	G_{le}	=	External leakage.

3 EXTENDED MATHEMATICAL MODEL

If we neglect friction forces F_T and external leakage on cylinder G_{le} and if we will take initial conditions of state variables as absolute conditions $p_{A0}, p_{B0}, x_0, \dot{x}_0 = 0$, than from previous equations (3, 4, 5) we could obtain new equations by L-transformation with complex variable s. And furthermore with supplementation and expansion about parts of $V_A = V_{0A} + S_a \cdot X_{(s)}$ and $V_B = V_{0B} + V_{B\max} - S_b \cdot X_{(s)}$ can be written relations (6, 7, 8):

$$m \cdot X_{(s)} \cdot s^2 + b \cdot X_{(s)} \cdot s = S_a \cdot P_{A(s)} - S_b \cdot P_{B(s)} - F_{Z(s)} \tag{6}$$

$$P_{A(s)} \cdot s = \frac{K}{V_{0A} + S_a \cdot X_{(s)}} \cdot \left[Q_{A(s)} - S_a \cdot X_{(s)} \cdot s - G_{li} \cdot (P_{A(s)} - P_{B(s)}) \right] \tag{7}$$

$$P_{B(s)} \cdot s = \frac{K}{V_{0B} + V_{B\max} - S_b \cdot X_{(s)}} \cdot \left[S_b \cdot X_{(s)} \cdot s - Q_{B(s)} + G_{li} \cdot (P_{A(s)} - P_{B(s)}) \right], \tag{8}$$

where	V_{A0}, V_{B0}	=	Dead volumes in intake parts of pipelines to the chambers A and B of cylinder
	V, $V_{B\max}$	=	Volume, maximal possible volume in chamber B, $V_{B\max} = X_{\max} \cdot S_b$.

Than for forces difference in consequence of pressure drop on cylinder is possible to define:

$$\mathrm{Pa}(s) \cdot \mathrm{Sa} - \mathrm{Pb}(s) \cdot \mathrm{Sb} = \tag{9}$$

$$\frac{\left(V_{B0} + Sb \cdot Xmax\right) \cdot Sa \cdot Q_A(s) + \left(V_{A0}\right) \cdot Sb \cdot Q_B(s) + \overset{*}{Sb \cdot Sa \cdot \left(Q_B(s) - Q_A(s)\right) \cdot X(s)} + \left[\overset{**}{\left(Sa^2 \cdot Sb - Sa \cdot Sb^2\right) \cdot X(s)^2} - \left(V_{A0} \cdot Sb^2 + V_{B0} \cdot Sa^2 + Sa^2 \cdot Sb \cdot Xmax\right) \cdot X(s) \right] \cdot s}{\left(V_{A0} \cdot V_{B0} + V_{A0} \cdot Sb \cdot Xmax - V_{A0} \cdot Sb \cdot X(s) + V_{B0} \cdot Sa \cdot X(s) + Sb \cdot Xmax Sa \cdot X(s) - Sa \cdot Sb \cdot X(s)^2\right) \cdot \frac{s}{K} + G_{li} \cdot \left(V_{B0} + Sxx \cdot Xmax + V_{A0}\right)}$$

Area $S_{xx} \in \langle S_A; S_B \rangle$ is area, which can takes values between values of areas S_b a S_a or which can be equal and which is also depend on direction of leakage flow inside of cylinder. Its effect is negligible. Inception and introduction of area S_{xx} arisen due to simplification of relation in area of internal leakage influence, where assumption of identical piston areas, $S_a = S_b$ was established. Due to this it was possible to express equation (9) and mentioned area is marked as parameter S_{xx}, which is able to correct leakage than with.

If we consider knowledge of equation of continuity and constant density of medium, than in equation (9) is possible to consider

$$S_b \cdot S_a \cdot Q_{A(s)} \cdot X_{(s)} = S_b \cdot {S_a}^2 \cdot s \cdot {X_{(s)}}^2 \text{ and similarly } S_b \cdot S_a \cdot Q_{B(s)} \cdot X_{(s)} = {S_b}^2 \cdot S_a \cdot s \cdot {X_{(s)}}^2 . \qquad (10)$$

Expressions marked as * and ** we can consider as the identical with opposite sign, so we can suspend them out of equation (6) and than obtain complete equation of movement of cylinder piston (11). The equation is only function of input flows variables, force variable and function of piston position of cylinder.

$$\left[\frac{-m\cdot Sa\cdot Sb}{K}\cdot X(s)^2 + \frac{m\cdot Sa\cdot V_{B0} - m\cdot V_{A0}\cdot Sb + m\cdot Sa\cdot Sb\cdot Xmax}{K}\cdot X(s) + \left(\frac{m\cdot V_{A0}\cdot V_{B0}}{K} + \frac{m\cdot V_{A0}\cdot Sb\cdot Xmax}{K}\right)\right]\cdot s^3\cdot X(s) \; +$$

$$\left[\frac{-b\cdot Sa\cdot Sb}{K}\cdot X(s)^2 + \frac{b\cdot Sa\cdot V_{B0} - b\cdot V_{A0}\cdot Sb + b\cdot Sa\cdot Sb\cdot Xmax}{K}\cdot X(s) + \left[m\cdot G_{li}\cdot\left(V_{B0} + Sxx\cdot Xmax + V_{A0}\right) + \frac{b\cdot V_{A0}\cdot V_{B0}}{K} + \frac{b\cdot V_{A0}\cdot Sb\cdot Xmax}{K}\right]\right]\cdot s^2\cdot X(s) \; +$$

$$\left[b\cdot G_{li}\cdot\left(V_{B0} + Sxx\cdot Xmax + V_{A0}\right) + \left(V_{A0}\cdot Sb^2 + V_{B0}\cdot Sa^2 + Sa^2\cdot Sb\cdot Xmax\right)\right]\cdot s\cdot X = \qquad (11)$$

$$\left(V_{B0}\cdot Sa + Sb\cdot Sa\cdot Xmax\right)\cdot Q_A(s) + V_{A0}\cdot Sb\cdot Q_B(s) \; -$$

$$Fz(s)\cdot\left[\frac{s}{K}\cdot\left[-Sa\cdot Sb\cdot X(s)^2 + \left(Sa\cdot V_{B0} - V_{A0}\cdot Sb + Sa\cdot Sb\cdot Xmax\right)\cdot X(s) + \left(V_{A0}\cdot V_{B0} + V_{A0}\cdot Sb\cdot Xmax\right)\right] + G_{li}\cdot\left(V_{B0} + Sxx\cdot Xmax + V_{A0}\right)\right]$$

From equation (11) is possible and to solve fragmental coefficients of characteristic polynomial of wanted transfer function between output X(s) and fragmental inputs Q_A, Q_B and F_Z. Namely by charge with s.X(s) from left side of equation (11) and by solving of roots of square and linear sub-equations and by help of more adjustments was possible to express piston position of cylinder $X_{(s)}$, equation (12). During all mathematical adjustments physical dimension and mathematical units were watched, controlled. It is necessary to make remark, that due to consideration of continuity equation, variables that should be used as a tool to define different operation point are suspend out of equation. Whole equation of movement is now similar to original mathematical model, built only for one operation point. Differences between them are mainly in number of inputs variables in numerator of overall transfer function, where is now featured load force F_z on piston of cylinder and two flows Q_A and Q_B.

$$X = \frac{K\cdot Q_A\cdot\left(\frac{V_{B0}}{V_{A0}} + Sb\cdot\frac{Xmax}{V_{A0}}\right) + K\cdot Q_B\cdot\left(\frac{Sb}{Sa}\right) - Fz\cdot\left(a_1\cdot s + K\cdot G_{li}\left(\frac{1}{Sa} + \frac{V_{B0}}{Sa\cdot V_{A0}} + \frac{Sb\cdot Xmax}{Sa\cdot V_{A0}}\right)\right)}{s\cdot\left(a_1\cdot m\cdot s^2 + a_2\cdot b\cdot s + \frac{K}{Sa}\cdot a_3\right)}, \qquad (12)$$

where

$$a_1 = \frac{V_{B0} + Sb\cdot Xmax}{Sb} \qquad a_3 = \frac{b\cdot G_{li}}{V_{A0}}\cdot\left(V_{B0} + Sb\cdot Xmax + V_{A0}\right) + \left(Sb^2 + \frac{V_{B0}\cdot Sa^2}{V_{A0}} + \frac{Sa^2\cdot Sb\cdot Xmax}{V_{A0}}\right)$$

$$a_2 = \frac{Sa\cdot V_{B0} - Sb\cdot V_{A0} + Sa\cdot Sb\cdot Xmax}{2\cdot Sa\cdot Sb} + \sqrt{\frac{Sa^2\cdot V_{B0}^2 + Sb^2\cdot V_{A0}^2 + Sa^2\cdot Sb^2\cdot Xmax^2 + 2\cdot Sa\cdot Sb\cdot V_{A0}\cdot V_{B0} + 2\cdot Sa\cdot Sb^2\cdot V_{A0}\cdot Xmax + 2\cdot Sa^2\cdot Sb\cdot V_{B0}\cdot Xmax +}{2\cdot Sa\cdot Sb} + \frac{4\cdot G_{li}\cdot K\cdot Sa\cdot Sb^2\cdot Xmax\cdot m + 4\cdot G_{li}\cdot K\cdot Sa\cdot Sb\cdot V_{A0}\cdot m + 4\cdot G_{li}\cdot K\cdot Sa\cdot Sb\cdot V_{B0}\cdot m}{b\cdot 2\cdot Sa\cdot Sb}}$$

From equation (12) we can than express time constant and also coefficient of damping of hydraulic cylinder defined like:

$$T_M = \sqrt{\frac{m \cdot Sa}{K \cdot a_3 \cdot Sb} \cdot \left(V_{B0} + Sb \cdot Xmax\right)} \qquad \xi_M = \frac{b \cdot Sa \cdot a_2}{2 \cdot T_M \cdot K \cdot a_3} \tag{13}$$

Therefore experimental measurements and experience results shows, the cylinder damping is changing during cylinder piston movement. From previous it appears, that for computation of different course of cylinder damping is necessary not to use compensation which is coming from equation (10), but solve the given model together with rest of variables. $X_{(s)}$ can't be than quite suspend out of movement equation (12) and stay there as variable. In consequence of that variable is possible to define different operation point of cylinder and obtain courses of damping and time constants, both dependent on piston position changing of cylinder. These solutions are not analyzed here in the paper.

4 MODEL'S COMPARISON IN MATLAB/SIMULINK ENVIRONMENT

Extended mathematical model was modelled, simulated and compared in simulation program Matlab/Simulink together with original nonlinear and linear model of cylinder. Nonlinear model realization of cylinder could be found in [Noskievič, P., 1999]. Simulation scheme with all designed three model's realizations can be seen in figure 2. Simulation results of pistons positions from nonlinear, original linear and extended linear model are compared in figure 3. From figure 3 is obvious small improvement of piston position compare to original linear model. The piston position course of extended linear model is more similar to nonlinear one.

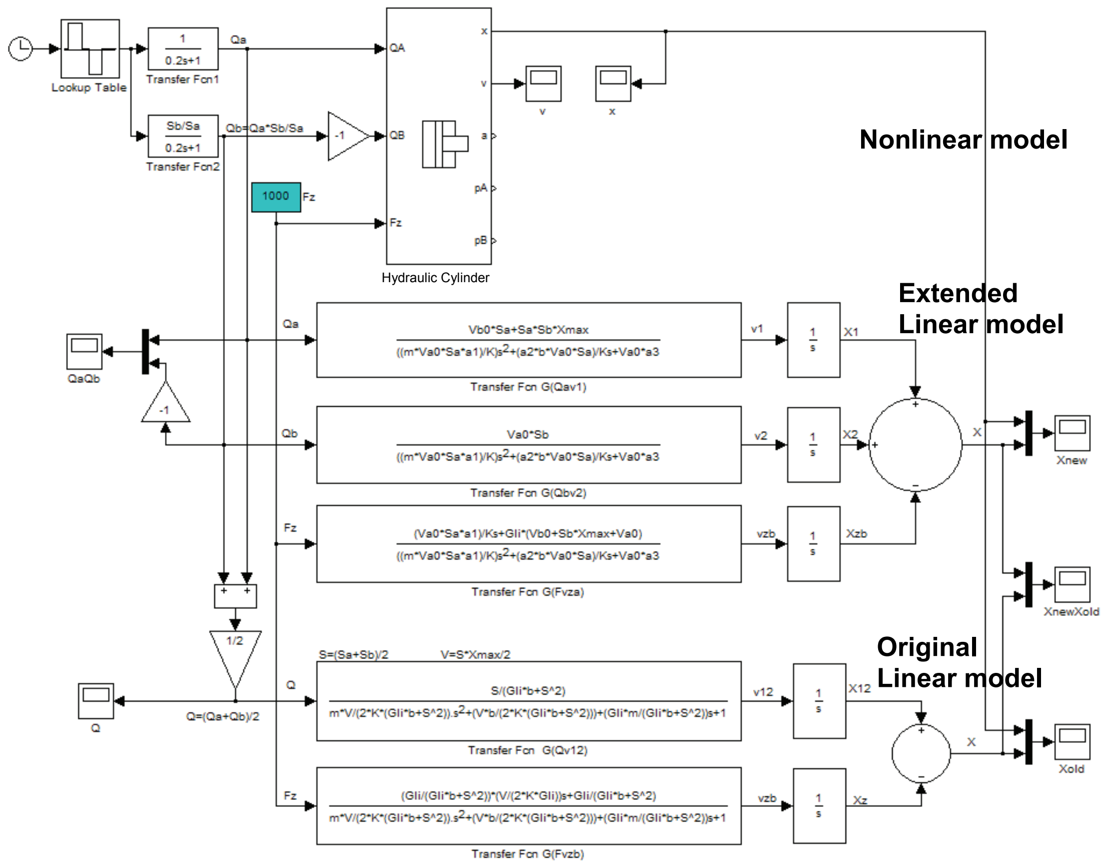

Fig. 2 Model's comparison in simulation program MATLAB/Simulink

In figure 2 we can see simulation scheme and its realization in open loop and we can also see there a simplified modelled course of input flow(s) to models of systems of hydraulic cylinders. Flow

Q_A comes into the chamber A and flow Q_B comes into the chamber B. Q_B is equal with flow Q_A, multiplied with ratio of different action areas on side of chambers A and B under the expression (14).

$$Q_B = Q_A \cdot \frac{S_b}{S_a} \tag{14}$$

Input to the original linear model, which is assuming only one input flow Q, is modelled like mean value of flows Q_A and Q_B. All models are loaded by constant load force F_z = 1000N on piston of hydraulic cylinder.

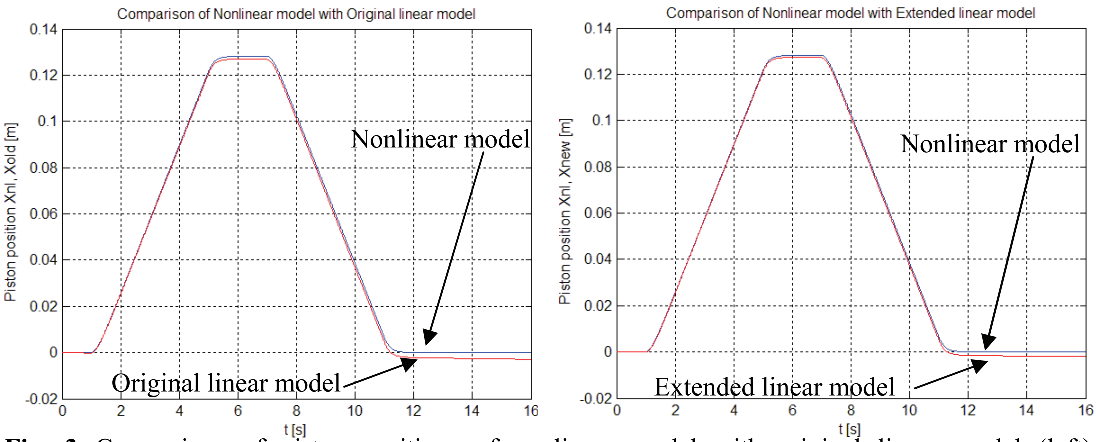

Fig. 3 Comparison of piston positions of nonlinear model with original linear model (left) and comparison of piston positions of nonlinear model with extended linear model (right)

5 CONCLUSION

In the paper extended analytical linear model of hydraulic single rod cylinder with different action areas on piton and with different working volumes in chambers of hydraulic cylinder was described. Extended model form presented in equation (12) is describing piston position of cylinder, which is generally expressed by three linear transfer functions. These transfer functions can be easy mathematically modelled and simulated by the help of linear block algebra. From type of system and from de-numerator of transfer function can be then calculated time constant a damping of hydraulic cylinder under the equation (13). Extended model was modelled, simulated and compared in simulation program Matlab/Simulink together with original nonlinear and linear model of cylinder. Results and piston position during ejection and retraction of cylinder rod are viewed and compared in figure 3. Extended model should be later used for analysis and synthesis of control parameters of regulation circuit of multivalve control concept of hydraulic drive, where separate control of each chamber A and B is required.

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