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Estimating Operational Validity Under Incidental Range Restriction: Some Important but Neglected Issues

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Operational validities are important to personnel selection research because they estimate how well a predictor in practical use correlates with a criterion construct, if the criterion measure were purged of measurement error variance. Because range restriction on a predictor or predictor composite creates incidental range restriction on the criterion, existing methodologies offer limited information and guidance for estimating operational validities. Although these effects of range restriction and criterion unreliability could be corrected with existing equations in a sequential fashion, proper use of sequential correction equations is not always as straightforward as it appears. This research reviews the existing equations for correcting validities, outlines the appropriate method for correcting validity coefficients via sequential equations, and proposes a new equation that performs a combined correction for the effects of incidental range restriction and criterion unreliability.

In the personnel selection literature, the *SIOP Principles for the Validation and Use of Personnel Selection Procedures* (SIOP, 2003) is a seminal reference. The *Principles* states that sample correlations in personnel selection typically are affected by range restriction and criterion unreliability and thus correlations should be psychometrically adjusted in order to "…obtain as unbiased an estimate as possible of the validity of the predictor" (p. 19). The present study focuses on psychometric equations employed in making these adjustments for estimating operational validity.

In personnel selection settings, *operational validity* refers to an estimate of the relationship between a predictor used in the practical context of selection and the theoretical construct that a criterion intends to measure (Binning & Barrett, 1989). However, top-down selection on the predictor (as might be typical in selection) obviously restricts predictor scores, and consequently, it also incidentally restricts scores on the criterion. This not only leads to a range-restricted validity coefficient; it also restricts the estimate of reliability for criterion scores (Sackett, Laczo, & Arvey, 2002). In addition to considering and correcting for range restriction, operational validities are corrected for measurement error in the criterion measure but not in the predictor measure. Taking the range restriction and measurement error variance phenomena together, the psychometric estimation of operational validity is more complicated than it appears, and thus we propose methods for appropriately performing corrections for the combined effects of range restriction and criterion unreliability.

More specifically, the current research (a) reviews the existing equations for correcting validities, (b) outlines the appropriate method for correcting validity via sequential equations, and (c) proposes a new equation that performs a combined correction for the effects of incidental range restriction and criterion unreliability. The goal is to ensure that researchers and practitioners are applying appropriate correction formulas to correlations typically found in personnel selection, as has been outlined in other frameworks for such corrections (e.g., Sackett & Yang, 2000). It is worth noting that the corrected correlation, although less biased, has a larger standard error; thus, one does not get something for nothing (Bobko & Reick, 1980; Oswald, Ercan, McAbee, Ock, & Shaw, 2015).

A Review of Existing Correlation Adjustment Equations

Before we present methods for correcting for multiple artifacts, a review of existing correction procedures is helpful. We will keep with notation conventions set by Schmidt, Hunter, and Urry (1976), where lower case letters will be used to indicate attenuated values (e.g., r_{xy} is the observed validity; s_x^2 is the predictor variance in the selected sample), and capital letters will be used to represent unrestricted or unattenuated values (e.g., R_{xy} is the operational validity; S_x^2 is the predictor variance in the applicant sample). Note that the terms *attenuated/unattenuated* refer to measurement error variance, and the terms *restricted/unrestricted* refer to range restriction.

Psychometric equations that adjust for range restriction and reliability attenuation must distinguish between the unrestricted criterion reliability and the restricted criterion reliability. Restricted criterion reliability occurs as a function of top-down selection on a correlated predictor, which incidentally restricts the range of criterion scores (and their underlying true scores). Thus, it is important to estimate unrestricted and restricted criterion reliability. We provide a series of correction formulas below that takes this into consideration; then we provide an approach to integrating these correction formulas into a single one.

Correcting criterion reliability for direct range restriction on the predictor. Schmidt et al. (1976; as corrected in Hunter, Schmidt, & Le, 2006, p. 598) provided an equation to correct an estimate of the criterion reliability for the effects of direct (top-down) range restriction on the predictor:

$$R_{yy} = 1 - \left[\frac{1 - r_{yy}}{1 + r_{xy}{}^2(U_X^2 - 1)}\right], \qquad (1)$$

where R_{yy} is the unrestricted criterion reliability, r_{yy} is the restricted criterion reliability, U_X^2 is the ratio of the unrestricted predictor variance S_X^2 to the restricted predictor variance s_x^2, and r_{xy} is the restricted validity coefficient. To offer an example of how this correction works, if the restricted criterion reliability (r_{yy}) is .60, the restricted validity coefficient (r_{xy}) is .30, the unrestricted predictor variance (S_X^2) is 16, and the restricted predictor variance (s_X^2) is 4 (i.e., the standard deviation is halved), then the unrestricted reliability coefficient is .69. In this case, note that the observed criterion reliability might be considered too low, yet when one understands and corrects for range-restriction effects, the estimated criterion reliability increases to more acceptable levels.

Correcting the validity coefficient for criterion unreliability. Likewise, the correction of r_{xy} for criterion unreliability is computed with another familiar equation that dates to Spearman (1904, p. 90):

$$R_{xy} = \frac{r_{xy}}{\sqrt{R_{yy}}}, \qquad (2)$$

where R_{xy} is the unattenuated validity coefficient; all other terms are defined as before. Use of Equation 2 without an accompanying range restriction correction implicitly assumes that there is no range restriction involved. It is possible to apply range restriction corrections after the reliability corrections; we approach this point later.

Correcting the validity coefficient for direct range restriction on the predictor. Thorndike's (1949, p. 173) correction of r_{xy} for the effects of direct range restriction on the validity coefficient is derived from the work of Pearson (1903):

$$R_{xy} = \frac{r_{xy}U_X}{\sqrt{1 + r_{xy}{}^2(U_X^2 - 1)}}, \qquad (3)$$

where R_{xy} is the unrestricted criterion-related validity coefficient, U_X is the ratio of the unrestricted predictor standard deviation to the restricted predictor standard deviation ($U_X = S_X / s_x$), with all other terms defined as before. As an example, if the restricted validity coefficient (r_{xy}) is .30, the unrestricted predictor variance (S_X^2) is 16, and the restricted predictor variance (s_X^2) is 4 (i.e., the standard deviation is halved), then in applying this formula, the unrestricted validity coefficient is .53.

Correcting the validity coefficient for incidental predictor range restriction. In addition to a correction for direct range restriction, Thorndike (1949, p. 174; derived from Pearson, 1903) offered a correction for incidental (or indirect) range restriction. With incidental range restriction, the extent of the restriction on r_{xy} is a function of the degree of restriction of scores on the operational predictor (*Z*, the predictor used to make selection decisions in the validity study), the relation between the experimental predictor (*X*, the focus of the validity study) and the operational predictor (r_{xz}), and the relation between the criterion (*Y*) and the operational predictor (r_{yz}). Thus, a correction for incidental range restriction is more complex than a correction for direct range restriction. Thorndike's equation to correct for the effects of incidental range restriction is as follows:

$$R_{xy} = \frac{r_{xy} + r_{xz}r_{yz}(U_Z^2 - 1)}{\sqrt{[1 + r_{xz}{}^2(U_Z^2 - 1)][1 + r_{yz}{}^2(U_Z^2 - 1)]}}, \qquad (4)$$

where r_{yz} is the restricted correlation between the criterion and the operational predictor, r_{xz} is the restricted correlation between the experimental and operational predictors, U_Z^2 is the ratio of the unrestricted variance of the operational predictor S_Z^2 to the restricted variance of the operational predictor s_Z^2, and all other terms are defined as before. As with the correction for direct range restriction, all correlations must be computed from the restricted sample. Of special concern is the correlation between variables *X* and *Z*, which will be available in its unrestricted form if all job applicants complete both measures. Using the unrestricted correlation between the operational (*Z*) and experimental (*X*) predictors in the above equation will result in an overcorrection of r_{xy}. As an example of the incidental range restriction correction, if the restricted correlation between the experimental predictor and the criterion (r_{xy}) is .30, the restricted correlation between the operational predictor and the criterion (r_{yz}) is .30, the restricted correlation between the two predictors (r_{xz}) is .50, the unrestricted operational predictor variance (S_Z^2) is 16, and the restricted operational predictor variance (s_Z^2) is 4, then the unrestricted validity coefficient is .50.

Simultaneous corrections for direct range restriction on the predictor and criterion unreliability. If r_{xy} is affected by both criterion unreliability and direct range restriction on the predictor, then operational validity can be estimated by adjusting for the effects of both psychometric artifacts. It is possible to perform the two corrections by using Equations 2 and 3 in a sequential fashion. Although the order of the corrections can vary, the sequence determines whether the reliability estimate used in Equation 2 is restricted or unrestricted. Bobko (1983, p. 585) offered a single correction equation in which the reliability correction for attenuation (Equation 2) is integrated into the correction for direct range restriction (Equation 3). That is, each r_{xy} within the range restriction equation is first corrected for reliability attenuation using a restricted estimate of criterion reliability. Bobko's multi-artifact correction equation is as follows:

$$R_{xy} = \frac{\frac{r_{xy}}{\sqrt{r_{yy}}}(U_X)}{\sqrt{1 + \frac{r_{xy}{}^2}{r_{yy}}(U_X^2 - 1)}}, \qquad (5)$$

where all terms are defined as before. To reiterate, criterion reliability should be computed from the restricted sample; use of an estimate of the unrestricted criterion reliability results in an undercorrection of r_{xy}. If separate, sequential corrections are used instead of Equation 5, and the correction for range restriction is made before the correction for unreliability (i.e., the opposite order of Equation 5), the unrestricted reliability should be used; use of the restricted reliability will lead to an overcorrection in such cases. As a demonstration of this combined correction equation, consider our earlier example for the direct range restriction equation (r_{xy} = .30, $S_X^2 = 16$, $s_X^2 = 4$) with the added element of criterion reliability. If the restricted criterion reliability (r_{xy}) is .60, then the unrestricted, unattenuated correlation is .64.

Proposed Correction

Research on correlation adjustments has not addressed the topic of adjustments for the combined effects of criterion unreliability and incidental range restriction, whether with sequential equations or with a single equation analogous to Bobko's (1983) equation. The research on direct range restriction and criterion unreliability is instructive; the nature of the reliability estimate depends on the order of the corrections. If the correction for unreliability (Equation 2) is performed before the correction for incidental range restriction (Equation 4), then the reliability estimate should be restricted. In a manner analogous to Bobko, the two equations can be integrated into one equation:

$$R_{xy} = \frac{\frac{r_{xy}}{\sqrt{r_{yy}}} + r_{xz}\left(\frac{r_{yz}}{\sqrt{r_{yy}}}\right)(U_Z^2 - 1)}{\sqrt{\left[1 + r_{xz}{}^2(U_Z^2 - 1)\right]\left[1 + \frac{r_{yz}{}^2}{r_{yy}}(U_Z^2 - 1)\right]}}, \qquad (6)$$

where all terms are defined as before. An essential element for an accurate correction for the combined effects of criterion unreliability and incidental range restriction is that, in order to be consistent with the goal of estimating operational validity (i.e., correlation with the criterion that is purged of measurement error variance), all correlations with the criterion variable must be corrected for unreliability. The necessity for these multiple unreliability corrections should be clear: failing to correct for criterion unreliability results in an undercorrection of all correlations involving variables that are correlated with the incidental selection variable, including r_{xy}. Finally, the same cautions that apply to Equations 4 and 5 regarding the effects of using unrestricted coefficients also apply to Equation 6. As an example of this combined correction for criterion unreliability and incidental range restriction, consider our previous example of incidental range restriction (r_{xy} = .30, r_{yz} = .30, r_{xz} = .50, S_Z^2 = 16, s_Z^2 = 4) with the added element of criterion reliability. If the restricted criterion reliability (r_{yy}) is .60, then the unrestricted, unattenuated correlation is .61.

Issues Related to Multi-Artifact Correction Equations

Stauffer and Mendoza (2001) identified an error in a procedure intended to correct for unreliability and range restriction that resulted in an overcorrection. This errant procedure was similar to Equation 5 except that it adjusts for direct range restriction and *predictor* (not criterion) unreliability. The issues raised by Stauffer and Mendoza likely also apply to criterion unreliability-based multi-artifact correction formulas. Stauffer and Mendoza stated the problem as follows. The extent of range restriction on the criterion variable is a direct function of the predictor range restriction and the observed restricted correlation. Thus, correcting for predictor unreliability prior to correcting for direct range restriction causes the range restriction adjustment to be performed on an inflated correlation, leading to an overcorrection. Stauffer and Mendoza proposed a new procedure to address this problem.

The correction procedure recommended by Stauffer and Mendoza (2001), adapted for criterion reliability, begins with a correction for either direct range restriction (Equation 3) or incidental range restriction (Equation 4), and is followed by a correction for criterion unreliability (Equation 2) based on the estimate of unrestricted criterion reliability (obtained via Equation 1). This procedure differs from Equations 5 and 6 regarding the order of the corrections and the nature of the reliability estimate. However, somewhat remarkably, both procedures yield the same results. An algebraic proof of the equivalence of Equation 6 to the Stauffer and Mendoza procedure is offered in Appendix A. Additional proofs regarding the equivalence of Equation 5 to the Stauffer and Mendoza procedure as well as the equivalence of Equation 6 to Raju, Edwards, and LoVerde's (1985) work are available from the authors upon request. Thus, the problem identified by Stauffer and Mendoza is not present in equations used in traditional personnel practice.

In summary, Stauffer and Mendoza's (2001) analysis is correct when one seeks to correct for predictor unreliability and range restriction, but this particular correction is seldom desired in the personnel selection context. It is much more common in personnel selection to estimate operational validity by correcting for criterion unreliability (but not predictor unreliability) and direct range restriction on the predictor. This paper demonstrates that the two correction methods are equivalent: A correction for range restriction followed by a correction for criterion unreliability, using an estimate of unrestricted criterion reliability, is equivalent to a correction for criterion unreliability, using the restricted reliability, followed by a range restriction correction. Given that in practice, the researcher is likely to obtain the restricted criterion reliability, Equations 5 and 6 are more efficient than Stauffer and Mendoza's multi-step procedure. As such, Equations 5 and 6 should be the preferred formulas for the personnel specialist who seeks to adjust validities for the effects of both criterion unreliability and range restriction.

Implications for Meta-Analysis

Meta-analyses of validity coefficients frequently employ corrections for predictor unreliability, criterion unreliability, and range restriction (direct or incidental). The issues raised in this study as well as in the Stauffer and Mendoza (2001) study apply to meta-analyses in which individual validity coefficients are corrected for these artifacts (no claims are made to meta-analyses employing assumed artifact distributions). The lessons of this study apply in these situations as well: the order of the corrections determines the type of criterion reliability estimate to be used. Furthermore, because predictor reliability corrections are also being performed, the concerns raised by Stauffer and Mendoza are relevant; the adjustment for predictor unreliability must be based on the unrestricted estimate and must be performed after the range restriction adjustment. The simplest procedure for a correction for these three artifacts begins with Equation 5 or 6 (using the restricted criterion reliability estimate) and follows with Equation 2 where the denominator is the unrestricted predictor reliability estimate.

Conclusions

There are many different psychometric formulas, designed for different purposes, available for adjusting sample correlation coefficients for range restriction and/or criterion unreliability. Careful consideration must be given to the design of the equation and the type of artifact estimate used in the equation (e.g., the restricted versus unrestricted values) when making appropriate adjustments. This research proposed a new equation that allows the researcher to correct for the combined effects of criterion unreliability and incidental range restriction. The issues raised by Stauffer and Mendoza (2001) were considered in light of this new equation as well as the existing correction for direct range restriction and criterion unreliability, and neither equation was found to be in error. In summary, researchers and practitioners who desire to adjust correlations for the effects of both range restriction and criterion unreliability are advised to use Equation 5 for direct range restriction and Equation 6 for incidental range restriction.

References

Binning, J. F., & Barrett, G. F., (1989). Validity of personnel decisions: A conceptual analysis of the inferential and evidential bases. *Journal of Applied Psychology*, 74, 478-494.

Bobko, P. (1983). An analysis of correlations corrected for attenuation and range restriction. *Journal of Applied Psychology*, 68, 584-589.

Bobko, P., & Reick, A. (1980). Large sample estimates for standard errors of functions of correlation coefficients. *Applied Psychological Measurement*, 4, 385-398.

Hunter, J. E., Schmidt, F. L., & Le, H. (2006). Implications of direct and indirect range restriction for meta-analysis methods and findings. *Journal of Applied Psychology*, 91, 594-612.

Oswald, F. L., Ercan, S. McAbee, S. T., Ock, J., & Shaw, A. (2015). Imperfect corrections or correct imperfections?: Psychometric corrections in meta-analysis. *Industrial and Organizational Psychology: Perspectives on Science and Practice*, 8, 1-4.

Pearson, K. (1903). Mathematical contributions to the theory of evolution: XI. On the influence of natural selection on the variability and correlation of organs. *Philosophical Transactions of the Royal Society of London.* Series A, 200, 1-66.

Raju, N. S., Edwards, J. E., & LoVerde, M. A. (1985). Corrected formulas for computing sample sizes under indirect range restriction. *Journal of Applied Psychology*, 70, 565-566.

Sackett, P. R., Laczo, R. M., & Arvey, R. D. (2002). The effects of range restriction on estimates of criterion interrater reliability: Implications for validation research. *Personnel Psychology*, 55, 807-825.

Sackett, P. R., & Yang, H. (2000). Correction for range restriction: An expanded typology. *Journal of Applied Psychology*, 85, 112-118.

Schmidt, F. L., Hunter, J. E., & Urry, V. W. (1976). Statistical power in criterion-related validation studies. *Journal of Applied Psychology*, 61, 473-485.

Society for Industrial and Organizational Psychology, Inc. (2003). Principles for the validation and use of personnel selection procedures (4th ed.). Author.

Spearman, C. (1904). The proof and measurement of association between two things. *American Journal of Psychology*, 15, 72-101.

Stauffer, J. M., & Mendoza, J. L. (2001). The proper sequence for correcting correlation coefficients for range restriction and unreliability. *Psychometrika*, 66, 63-68.

Thorndike, R. L. (1949). Personnel selection. New York: Wiley.

Appendix A

Equation 6 Proof

Proof to demonstrate that Stauffer and Mendoza's (2001) procedure to correct for direct range restriction and predictor unreliability:

$$R_{xy} = \frac{r_{xy}(U_X)}{\sqrt{R_{xx}}\sqrt{1 + r_{xy}^2(U_X^2 - 1)}}$$

Equals our Equation 6 (correction for indirect range restriction and criterion unreliability):

$$R_{xy} = \frac{\frac{r_{xy}}{\sqrt{r_{yy}}} + r_{xz}\left(\frac{r_{yz}}{\sqrt{r_{yy}}}\right)(U_Z^2 - 1)}{\sqrt{\left[1 + r_{xz}^2(U_Z^2 - 1)\right]\left[1 + \frac{r_{yz}^2}{r_{yy}}(U_Z^2 - 1)\right]}}$$

When the unrestricted predictor reliability in Stauffer and Mendoza's procedure is adapted for indirect range restriction and criterion unreliability.

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1. Stauffer and Mendoza's Equation 5:

$$R_{xy} = \frac{r_{xy}(U_X)}{\sqrt{R_{xx}}\sqrt{1 + r_{xy}^2(U_X^2 - 1)}}$$

2. Which can be decomposed into a correction for predictor unreliability multiplied by a correction for direct range restriction:

$$R_{xy} = \frac{1}{\sqrt{R_{xx}}} \times \frac{r_{xy}(U_X)}{\sqrt{1 + r_{xy}^2(U_X^2 - 1)}}$$

3. Because a validity coefficient is corrected in the same manner for predictor unreliability, criterion unreliability, or both (Spearman, 1904), we can replace the unrestricted predictor reliability (R_{xx}) with the unrestricted criterion reliability (R_{yy}). As was done by Stauffer and Mendoza, the validity coefficient is first adjusted for range restriction and then corrected for unreliability using the unrestricted reliability.

$$R_{xy} = \frac{1}{\sqrt{R_{yy}}} \times \frac{r_{xy}(U_X)}{\sqrt{1 + r_{xy}^2(U_X^2 - 1)}}$$

4. In a similar manner, the correction for indirect range restriction can be substituted for the correction for direct range restriction. As before, the validity coefficient is first corrected for range restriction and subsequently corrected for criterion unreliability with the unrestricted reliability:

$$R_{xy} = \frac{1}{\sqrt{R_{yy}}} \times \frac{r_{xy} + r_{xz}r_{yz}(U_Z^2 - 1)}{\sqrt{[1 + r_{xz}^2(U_Z^2 - 1)][1 + r_{yz}^2(U_Z^2 - 1)]}}$$

5. Substitution: R_{yy} specified in terms of r_{yy}. Obtained from Schmidt, Hunter, and Urry (1976; as corrected in Hunter, Schmidt, & Le, 2006) – with *X* variable changed to *Z* variable because selection is made on *Z*.

$$R_{yy} = 1 - \frac{1 - r_{yy}}{1 + r_{yz}^2(U_Z^2 - 1)}$$

When substituted in the adapted Stauffer and Mendoza equation from Step 4 yields:

$$R_{xy} = \frac{1}{\sqrt{1 - \frac{1 - r_{yy}}{1 + r_{yz}^2(U_Z^2 - 1)}}} \times \frac{r_{xy} + r_{xz}r_{yz}(U_Z^2 - 1)}{\sqrt{[1 + r_{xz}^2(U_Z^2 - 1)][1 + r_{yz}^2(U_Z^2 - 1)]}}$$

6. Unfactoring: $(a - b)cd = (ad - bd)c$

$$R_{xy} = \frac{r_{xy} + r_{xz}r_{yz}(U_Z^2 - 1)}{\sqrt{\left[[1 + r_{yz}^2(U_Z^2 - 1)] - (1 - r_{yy})\right][1 + r_{xz}^2(U_Z^2 - 1)]}}$$

7. Simplifying: $(1 + ab) - (1 - d) = ab + d$

$$R_{xy} = \frac{r_{xy} + r_{xz}r_{yz}(U_Z^2 - 1)}{\sqrt{[r_{yz}^2(U_Z^2 - 1) + r_{yy}][1 + r_{xz}^2(U_Z^2 - 1)]}}$$

8. Factoring out r_{yy}: $(a + b) = b\left(\frac{a}{b} + 1\right)$

$$R_{xy} = \frac{r_{xy} + r_{xz}r_{yz}({U_Z}^2 - 1)}{\sqrt{r_{yy}\left[\frac{{r_{yz}}^2}{r_{yy}}({U_Z}^2 - 1) + 1\right][1 + {r_{xz}}^2({U_Z}^2 - 1)]}}$$

9. Pulling r_{yy} out from under radical: $\sqrt{abc} = \sqrt{a}\,\sqrt{bc}$ and reordering $(ab) = (ba)$ in denominator

$$R_{xy} = \frac{r_{xy} + r_{xz}r_{yz}({U_Z}^2 - 1)}{\sqrt{r_{yy}}\sqrt{[1 + {r_{xz}}^2({U_Z}^2 - 1)]\left[\frac{{r_{yz}}^2}{r_{yy}}({U_Z}^2 - 1) + 1\right]}}$$

10. Moving $\sqrt{r_{yy}}$ into numerator: $\frac{a+b}{cd} = \frac{\frac{a}{c}+\frac{b}{c}}{d}$ and reordering $(a + b) = (b + a)$ in last section of denominator

$$R_{xy} = \frac{\frac{r_{xy}}{\sqrt{r_{yy}}} + r_{xz}\left(\frac{r_{yz}}{\sqrt{r_{yy}}}\right)({U_Z}^2 - 1)}{\sqrt{[1 + {r_{xz}}^2({U_Z}^2 - 1)]\left[1 + \frac{{r_{yz}}^2}{r_{yy}}({U_Z}^2 - 1)\right]}}$$

11. Which equals Equation 6 from our manuscript.

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