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Predictive Feedback Control and Fitts' Law

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Abstract Fitts' law is a well established empirical formula, known for encapsulating the "speed-accuracy trade-off". For discrete, manual movements from a starting location to a target, Fitts' law relates movement duration to the distance moved and target size. The widespread empirical success of the formula is suggestive of underlying principles of human movement control. There have been previous attempts to relate Fitts' law to engineering-type control hypotheses and it has been shown that the law is exactly consistent with the closed-loop step-response of a time-delayed, first-order system. Assuming only the operation of closed-loop feedback, either continuous or intermittent, this paper asks whether such feedback should be predictive or not predictive to be consistent with Fitts law. Since Fitts' law is equivalent to a time delay separated from a first-order system, known control theory implies that the controller must be predictive. A predictive controller moves the time-delay outside the feedback loop such that the closed- loop response can be separated into a time delay and rational function whereas a non-predictive controller retains a state delay within feedback loop which is not consistent with Fitts' law. Using sufficient parameters, a high-order non-predictive controller could approximately reproduce Fitts' law. However, such high-order, "non-parametric" controllers are essentially empirical in nature, without physical meaning, and therefore are conceptually inferior to the predictive controller. It is a new insight that using closed-loop feedback, prediction is required to physically explain Fitts' law. The implication is that prediction is an inherent part of the "speed-accuracy trade-off".

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1 Introduction

In the more than fifty years since its genesis, Fitts' law (Fitts 1954), has entered the textbooks (Wickens and Hollands 2000) as the standard empirical relationship between movement time, distance moved and target size applicable to a wide range of human movement situations (Plamondon and Alimi 1997, Table 1). It has also become a standard tool in the field of Human Computer Interaction (Soukoreff and MacKenzie 2004).

The fact that Fitts' law has such wide applicability implies that any model of human motion control must account for its predictions. As discussed by Plamondon and Alimi (1997), and the numerous peer comments appended, there are a number of hypotheses that are supported by Fitts' law. One of the hypotheses mentioned by Plamondon and Alimi (1997, 2.1.3) is a simple feedback control model (attributed to Connelly (1984)) which has also (apparently independently) been noted by Phillips and Repperger (1997) and by Cannon (1994). In particular, as discussed by Cannon (1994) and by Jagacinski and Flach (2003), rewriting Fitts' law using natural logarithms converts the two parameters of Fitts' law into the two parameters of a the step response of a first order system with time-constant T delayed by a time t_d.

An early contribution to the engineering literature on the feedback control of *time-delay* systems was provided by Smith (1959) and extended by Marshall (1979). The key result is that *predictive feedback control of time-delay systems moves the time-delay outside the feedback loop*. This not only simplifies design, but gives a closed-loop system where the time-delay is separated from the rest of the system dynamics. Smith's predictor has a number of drawbacks, in particular the inability to control unstable systems, but the basic idea was seminal and the problems can be overcome (Gawthrop et al. 1996). Smith's predictor has been suggested as a basis for human movement control (Miall et al. 1993b; Miall and Wolpert 1996; Wolpert et al. 1998) (although some doubts have been recently expressed (Miall and Jackson 2006)). State-space based predictive control was developed by Kleinman (1969) and appears in the text book of Sage and Melsa (1971). Again, this has been suggested as a basis for human motor control(McRuer 1980; Wickens and Hollands 2000; Miall and Wolpert 1996; Van Der Kooij et al. 1999).

However, even though Fitts' law and predictive control occur in the same chapter of the textbook of Wickens and Hollands (2000), the use of the predictive control as a theoretical underpinning of Fitts' law appears to have gone unnoticed hitherto. This paper demonstrates the fact that the feedback control-theoretical interpretation of Fitts' law implies that the underlying closed-loop feedback system has the property that the time-delay is separated from the rest of the system dynamics and thus Fitts' law has a predictive control interpretation.

Feedback control systems can be represented in either state-space or transfer function form. Plamondon and Alimi (1997) use a state-space formulation whereas Cannon (1994) and Phillips and Repperger (1997) use a transfer function approach. The choice of representation is not a fundamental issue but rather a matter of convenience: either representation can be converted into the other. This paper uses a state-space approach.

Similarly, there is a dichotomy between optimal control and other control design methods. Optimal control is often associated with state-space methods (Kwakernaak and Sivan 1972), but can equally well be associated with transfer-function methods (Newton et al. 1957). However both optimal and non-optimal approaches ultimately lead to the same form of feedback control and, in some circumstances, a feedback control system can be associated with an optimisation criteria (Kalman 1964) even if it was not explicitly designed to be optimal.

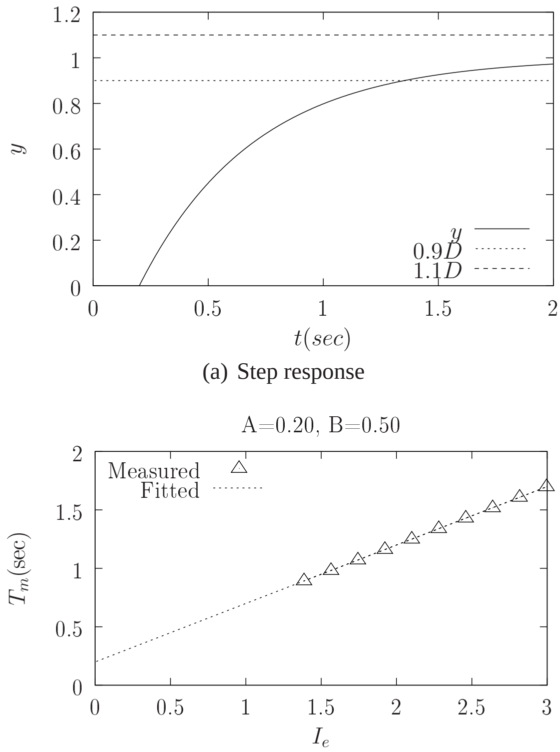
Open-loop optimal control with a *minimum-variance end-point criteria* and *signal-dependent* noise has been considered by Harris and Wolpert (1998) and shown to give movement trajectories consistent with Fitts' law. It is not clear how this result relates to the *closed-loop* explanation given in this paper.

Continuous feedback is not the only possible feedback mechanism associated with human movement. Craik (1947) introduced the idea of *Intermittent Control* in the context of human movement and more recent developments have been reported (Beggs and Howarth 1972; Neilson et al. 1988; Miall et al. 1993a; Doeringer and Hogan 1998; Neilson 1999; Bhushan and Shadmehr 1999a; Lakie et al. 2003; Neilson and Neilson 2005; Loram et al. 2006). Intermittency is related to control using a series of submovements (Meyer et al. 1990; Doeringer and Hogan 1998). Intermittent control has also been discussed in the engineering literature (Ronco et al. 1999; Gawthrop and Wang 2006; Furuta et al. 2005). The argument of this paper applies to both continuous and intermittent feedback.

This paper is primarily concerned with the dichotomy between predictive and non-predictive control. Assuming feedback control, this paper shows that predictive control is consistent with Fitts' law whereas non-predictive control is not.

The outline of the paper follows. Section 2 gives the control theoretic interpretation of Fitts' law. Section 3 emphasises the implications of the result of Section 2 for feedback control and gives the main results concerning predictive control in both continuous and intermittent form together with an example. Section 4 concludes the paper.

2 Fitts' law and Step response



(a) Step response

(b) Fitts' diagram

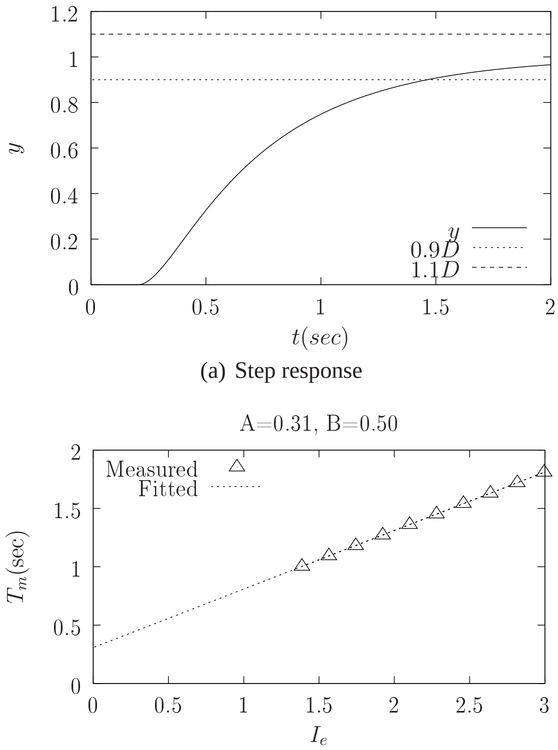
Fig. 1 Delayed first-order response (Time constant $T = 0.5$; time delay $t_d = 0.2$). As expected, Fitts' law gives an exact match to a first-order + delay response. The line has slope $T = 0.5$ and crosses the vertical axis at $t_d = 0.2$

This section brings together previous results (Connelly 1984; Cannon 1994; Phillips and Repperger 1997; Jagacinski and Flach 2003) on the control theoretic interpretation of Fitts' law. Following, for example, Wickens and Hollands (2000, chapter 10) Fitts' model can be expressed as:

$$T_m = a + bI_d \tag{1}$$

$$I_d = \log_2 \frac{2D}{W} \tag{2}$$

where T_m is the movement time, D the distance moved, W the target width and I_d the index of difficulty. a and b are the two parameters which are adjusted to fit the data.



(a) Step response

(b) Fitts' diagram

Fig. 2 Delayed second-order response. This second order+delay response approximates the first-order + delay response of Figure 1(a) because the second time constant $T_2 = 0.1$ is small compared to $T = 0.5$. In terms of the Fitts' law interpretation, the estimated time constant $T = 0.5$ is correct; the estimated delay $\hat{t}_d \approx t_d + T_2$.

It is a mathematical fact that $\log_2 x = \alpha \log_e x$ where the constant $\alpha = \log_2 e \approx 1.44$. Hence (1) can be rewritten as:

$$T_m = A + BI_e \tag{3}$$

$$I_e = \log_e \frac{2D}{W} \tag{4}$$

where $A = \alpha a$ and $B = \alpha b$. Of course, the two representations of Fitts' law are equivalent. However, as discussed in the literature, the use of $\log_2$ has information theory connotations whereas, as will be shown here, using natural (or Naperian) logarithms ($\log_e$ or ln) reveals the control-theoretic aspects of Fitts' law.

In control engineering, systems are often approximated by a rational transfer function plus time delay model which has the transfer function:

$$\frac{Y(s)}{R(s)} = G_c(s) = e^{-st_d} \frac{b_c(s)}{a_c(s)} \tag{5}$$

where e^{-st_d} is the transfer function of a time-delay t_d and $\frac{b_c(s)}{a_c(s)}$ is the ratio of two polynomials in s: a rational transfer function.

For exact consistency with Fitts' law, we consider the special case of (5) where

$$G_c(s) = \frac{e^{-st_d}}{1+sT} \tag{6}$$

where the two parameters are the *time-delay* t_d and the *time-constant* T. Given a step reference signal $r(t)$ of the form

$$r(t) = \begin{cases} 0 & t < 0 \\ D & t \geq 0 \end{cases} \tag{7}$$

the resultant *step-response* $y(t)$ is

$$y(t) = \begin{cases} 0 & t < t_d \\ D\left(1 - e^{-\frac{t-t_d}{T}}\right) & t \geq t_d \end{cases} \tag{8}$$

Figure 1(a) shows such a response with $D = 1$, $t_d = 0.2$sec and $T = 0.5$sec.

Suppose that this step response represents movement towards a target of width W centred at D. Then, given the monotonic nature of the step response, the target is hit when $y = D - \frac{W}{2}$ which, from (6) occurs when $t = T_m$ where

$$\frac{W}{2} = De^{-\frac{T_m - t_d}{T}} \tag{9}$$

and T_m is the movement time. Taking natural logarithms and rearranging:

$$T_m = t_d + T \ln \frac{2D}{W} \tag{10}$$

this corresponds to the natural log version of Fitts' formula (3) if:

$$A = t_d \tag{11}$$

$$B = T \tag{12}$$

2.1 Example

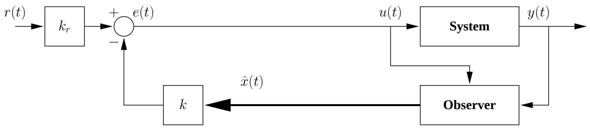
Consider the particular case of (6) with time constant $T = 0.5$ and time delay $t_d = 0.2$. Figure 1(a) shows the corresponding step response $y(t)$ plotted against time.

Figure 1(b) shows values extracted from Figure 1(a) for 10 values of W logarithmically spaced from $W = 0.1$ to $W = 0.5$. In particular, the result for $W = 0.1$ corresponds to $I_e = \ln 20 \approx 3$ and thus $T_m = 0.2 + 0.5I_e \approx 1.7$.

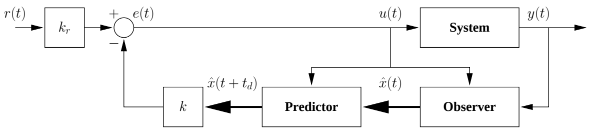
In practice, the step response transfer function need not be exactly of the first-order+delay form of (6). As an example, consider the *second*-order + delay transfer function:

$$G_c(s) = \frac{e^{-st_d}}{(1+sT)(1+sT_2)} \tag{13}$$

where $T_2 \ll T$. As illustrated in Figure 2, such a system can be approximated by one of thye form of (6) with the delay replaced by $t_d + T_2$.



(a) Conventional



(b) Predictive

Fig. 3 Feedback control. (a) Conventional state-feedback control uses a state *observer* (an optimal version of which is called a Kalman filter) to give an estimate $\hat{x}(t)$ of the current system state $x(t)$ using a forward model of the system together with the measured system output $y(t)$ and the system input $u(t)$. The estimated state $\hat{x}(t)$ is multiplied by a (vector) *state-feedback gain k* and used as a feedback signal. The reference signal $r(t)$ is multiplied by the *reference gain* k_r and added to the feedback signal to give the system input $u(t)$. (b) Predictive control uses a *predictor* to give an estimate $\hat{x}(t+t_d)$ of the *future* state $x(t+t_d)$ of the system using a forward model, the estimated current state $\hat{x}(t)$ and the system input $u(t)$. The feedback uses the predicted state $\hat{x}(t+t_d)$; this removes the effect of a pure time-delay t_d in the system dynamics

3 Control system implications

If, in common with other authors, it is assumed that the model (6) represents the *closed-loop* response of a feedback control system, there are three properties of system represented by (6) which have interesting implications:

1. the steady-state gain is unity (that is, if r is of the form of (7), the output y settles down at a value of D);
2. the rational part of transfer function is first order and
3. the system transfer function is the product of a pure time delay and a rational transfer function.

The first property is straightforward to achieve by suitable control system design; and the second property can be approximated as discussed later. The third property is the focus of this paper; in particular, we emphasise that the use of *predictive control* leads to a closed-loop system with property 3.

It is assumed that the controlled system is a *delay- differential* system of the form

$$\begin{cases} \dot{x}(t) & = Ax(t) + Bu(t-t_d) \\ y(t) & = Cx(t) \end{cases} \tag{14}$$

$x(t)$ is the system state, y and u the system output and input respectively and t_d is the system time-delay. A is an $n \times n$ matrix, B an $n \times 1$ column vector and C an $1 \times n$ row vector. x is an $n \times 1$ column vector – the system state. t_d is the system time delay. Typically, but not necessarily, $n = 2$ and the elements of the state are velocity and position; this case is examined in the example of Section 3.5. There is no loss of generality in placing the delay at the input rather than the output; the latter can be accommodated by appropriately shifting the time variable.

3.1 Non-predictive control

Given a system of the form of equation (14) conventional state-space controller architecture has two main parts as outlined in Figure 3(a)

1. a state *observer* to give an estimate $\hat{x}(t)$ of the current system state $x(t)$ using a forward model of the system together with the measured system output $y(t)$ and the system input $u(t)$. Following the standard textbooks, a state observer can be written as:

$$\begin{cases} \dot{\hat{x}}(t) & = (A-LC)\hat{x}(t) + Bu(t-t_d) - Ly(t) \\ \hat{y}(t) & = C\hat{x}(t) \end{cases} \tag{15}$$

   where $\hat{x}$ is the estimated state and $\hat{y}$ the estimated output. In the sense that it contains the system matrices A, B and C, and the time-delay t_d, the observer equation (15) can be thought of as a *forward model* (Miall and Wolpert 1996; Wolpert et al. 1998; Bhushan and Shadmehr 1999b; Davidson and Wolpert 2005) of the system. The *observer gain vector L* can be chosen either to fix the eigenvalues of $A - LC$ or by optimisation – see Kwakernaak and Sivan (1972) for details. The observer also has a Bayesian interpretation (Jacobs 1974; Bays and Wolpert 2007). It is a standard result (Kwakernaak and Sivan 1972) that the design of such an observer is independent of the state feedback design. In particular, the controller can be designed as if the state estimate is correct:

$$\hat{x}(t) = x(t) \tag{16}$$

   For clarity, equation (16) will be assumed for the rest of this paper.
2. state feedback comprising a vector k multiplying the estimated state and a scalar k_r multiplying the reference $r(t)$.

$$u(t) = k_r r(t) - k\hat{x}(t) \tag{17}$$

In the delay-free case ($t_d = 0$), assumption (16), together with the system equation (14) and the controller equation (17) implies the closed-loop system:

$$\begin{cases} \dot{x}(t) & = A_c x(t) + Bk_r r(t) \\ y(t) & = Cx(t) \end{cases} \tag{18}$$

where

$$A_c = A - Bk \tag{19}$$

As discussed in the textbooks Kwakernaak and Sivan (1972) k can be chosen either from an optimal control point of view or to fix the closed-loop eigenvalues of $A - Bk$.

The closed-loop system (18) can be rewritten as the rational transfer function:

$$\frac{y}{r} = g(s) = C\left[sI - A_c\right]^{-1} Bk_r \tag{20}$$

This is of the form of (5) with $t_d = 0$.

If, on the other hand, the delay is non-zero ($t_d > 0$) the closed-loop system (18)is replaced by

$$\begin{cases} \dot{x}(t) &= Ax(t) - Bkx(t-t_d) + Bk_r r(t-t_d) \\ y(t) &= Cx(t) \end{cases} \quad (21)$$

Because (21) has a *state* delay, it cannot be written as the product of a rational transfer function and a time delay as in (5) and is therefore not consistent with Fitts' law. An example is given in Section 3.5.

However, as shown in Section 3.3, *predictive* control removes the state delay giving a closed-loop system that can be written as (5). This is the key argument of the paper.

3.2 Approximate Predictive Control

Although the purpose of this paper is to advocate *predictive* control as the control design method that best explains Fitts' law, this section looks at an approach intermediate between that of the previous and the subsequent section. In particular, an observer/state-feedback control of the form of Section 3.1 and Figure 3(a) is derived which explicitly takes account of the time delay by replacing the time-delay by a rational transfer function approximation.

As discussed by Marshall (1979), the *transcendental* transfer function e^{-st_d} can be approximated by a number of forms of *rational* transfer function. One of these is:

$$e^{-st_d} \approx \frac{1}{(1+s\frac{t_d}{N})^N} \quad (22)$$

The approximation improves with increasing N. The transfer function of (22) has an N-dimensional state-space representation of the form:

$$\begin{cases} \dot{x_d}(t) &= A_d x_d(t) + B_d u(t) \\ u_d(t) &= C_d x(t) \end{cases} \quad (23)$$

where the state x_d has N components and $u_d(t) \approx u(t-t_d)$. A rational approximation to the delay-differential system (14) is then given by combining (14) and (23) to give:

$$\begin{cases} \dot{x_a}(t) &= A_a x_a(t) + B_a u(t) \\ y_a(t) &= C_a x_a(t) \end{cases} \quad (24)$$

where

$$A_a = \begin{bmatrix} A & BC_d \\ \mathbf{0}_{N\times n} & A_d \end{bmatrix} \quad (25)$$

$$B_a = \begin{bmatrix} \mathbf{0}_{n\times 1} \\ B_d \end{bmatrix} \quad (26)$$

$$C_a = \begin{bmatrix} C \ \mathbf{0}_{1\times N} \end{bmatrix} \quad (27)$$

where $\mathbf{0}_{N\times n}$ is the $N \times n$ zero matrix.

A controller can now be designed for the $n+N$-dimensional *approximate* system (22) – using the methods of Section 3.1 – but applied to the *actual* system (14). An example is given in Section 3.5.

However, although this approach provides a method for implementing control for a time-delay system, it is conceptually cumbersome compared with the conceptual clarity of the predictive controller to be discussed in Section 3.3.

3.3 Predictive Control

This section shows that a class of predictive state-space controllers give closed-loop responses that approximate Fitts' law. As outlined in Figure 3(b), these controllers have three parts: a state *observer*, a state *predictor* and state *feedback*.

Kleinman (1969) showed how a state predictor can be written in state-space form. A simpler approach, given by Sage and Melsa (1971), is given by the following formula:

$$\hat{x}(t+t_d|t) = e^{At_d}\hat{x}(t) + \int_0^{t_d} e^{At'} Bu(t-t')dt' \quad (28)$$

where $\hat{x}(t+t_d|t)$ is the predicted state at time $t+t_d$ based on data available at time t. As indicated in Figure 3(b), the predictor (28) has two inputs: $\hat{x}(t)$ from the observer and $u(t)$ the system input.

As shown by Sage and Melsa (1971), the error $\tilde{x}(t+t_d)$ given by:

$$\tilde{x}(t+t_d) = \hat{x}(t+t_d|t) - x(t+t_d) \quad (29)$$

is not dependent on the state x and therefore does not affect stability or the response to $r(t)$. In the same spirit as (16) it is assumed that $\tilde{x}(t+t_d) = 0$ for the rest of this paper and so

$$\hat{x}(t+t_d|t) = x(t+t_d) \quad (30)$$

The *predictive* state-feedback controller corresponding to (17) is of the form:

$$u(t) = k_r r(t) - k\hat{x}(t+t_d|t) \quad (31)$$

where $r(t)$ is the reference signal, k_r is a scalar gain and k is the *feedback gain* an n-dimensional row vector.

Equations (15), (28) and (31) form the feedback controller which, by construction, is realisable.

Substituting equations (30) and (31) into the system equation (14) gives the *closed-loop system*:

$$\begin{cases} \dot{x}(t) &= A_c x(t) + Bk_r r(t-t_d) \\ y(t) &= Cx(t) \end{cases} \quad (32)$$

where A_c is given by (19). Unlike (21), (32) has no *state* delay term $x(t-t_d)$.

The key point here is that, due to the predictive term in the controller equation (31), the closed-loop system (32) is such that the time delay only occurs at the input reference signal; the delay is moved outside the loop. Predictive control thus satisfies property 3. To emphasise this point, the closed-loop system (32) can be rewritten as the transfer function representation (5) with:

$$\frac{b_c(s)}{a_c(s)} = C_c\,[sI - A_c]^{-1}\,Bk_r \tag{33}$$

The steady-state gain of the closed-loop system (32) (from r to y) is:

$$g_{ss} = -CA_c^{-1}Bk_r = -CA_c^{-1}Bk_r \tag{34}$$

Equation (34) can be used to choose k_r so that $g_{ss} = 1$ thus satisfying property 1.

Property 2 will not be exactly satisfied unless $n = 1$. However, as discussed in Figure 2 of Section 3.5, it can be approximately satisfied in the case $n = 2$ by suitable choice of the feedback gain k.

3.4 Intermittent predictive control

As discussed in the Introduction, *intermittent* control has a long history in the context of human motion control and a somewhat shorter history in engineering motion control. This section gives a brief introduction to a particular form of intermittent control based on Gawthrop and Wang (2006) to which the reader is referred for more detail. The main result of this section is that predictive intermittent control, like predictive continuous control, is also consistent with Fitts' law.

The intermittent controller discussed by Gawthrop and Wang (2006) (in turn based on earlier work (Ronco et al. 1999; Gawthrop and Ronco 2000, 2002; Gawthrop 2002, 2004), generates a sequence of open-loop control trajectories each of which lasts for a time Δ_{ol}.

At each time $t_i = i\Delta_{ol}$, a state measurement is taken and used to generate the trajectory parameter vector U_i given by:

$$U_i = U(t_i) = K_r r(t) - K\hat{x}(t_i|t_{i-1}) \tag{35}$$

where the prediction $\hat{x}(t_i|t_{i-1})$ is once again given by (28).

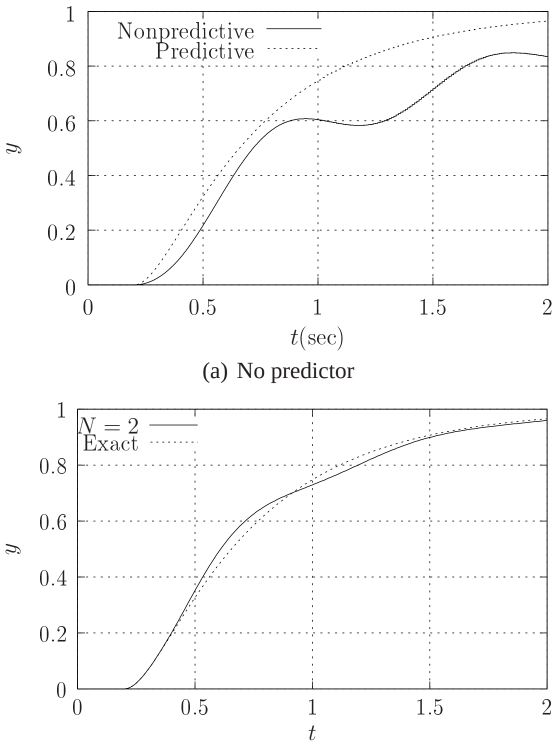
The trajectories are a weighted sum of *basis functions* which, in the special case considered here, can be written as the states x_u of the unforced dynamic system:

$$\begin{cases} \frac{dx_u}{d\tau}(\tau) & = A_c x_u(\tau) \\ x_u(0) & = x_{u0} \end{cases} \tag{36}$$

The corresponding *open-loop* control signal is then

$$u(t) = u(\tau + t_i) = x_u^T(\tau)U_i \tag{37}$$

As discussed by Gawthrop and Wang (2006), in the absence of disturbances and state-error, an appropriate choice of K and K_r makes the control signal generated by (37) identical to that generated by (31). It therefore follows that, in these circumstances, the conclusions of Section 3.3 pertaining to *continuous* predictive control are equally applicable to *intermittent* predictive control.



(a) No predictor

(b) Approximate predictor

Fig. 4 Non-predictive control step responses. (a) compares the non-predictive and predictive controllers; the predictive controller has the correct reponse of a pure time delay followed by an exponetial; the non-predictive controller has a more oscillatory response. (b) compares the approximate-predictive ($N = 2$) and predictive controllers. In this case, the approximate-predictive response closely follows that of the predictive control response; this improves with increasing N.

The introduction of disturbances and state-error does however, have different effects on continuous and intermittent control. As mentioned in the Conclusions, this could form the basis of an experiment to distinguish these two possibilities.

3.5 Example

Consider the system of the form of (14) where:

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{38}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{39}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{40}$$

$$t_d = 0.2 \tag{41}$$

which corresponds to the transfer function $G(s) = \frac{e^{-0.2s}}{s^2}$. This system corresponds to the the motion of an inertia of unit mass driven by a force delayed in time by 0.2sec where the system output y is position and the system input u is the force.

Using standard state-space method known as “pole placement” Kwakernaak and Sivan (1972) the feedback gain k was chosen to give two (stable) closed-loop poles at $s = -2$ and $s = -10$ giving:

$$k = \begin{bmatrix} 12 & 20 \end{bmatrix} \tag{42}$$

$$k_r = 20 \tag{43}$$

predictive control gives the closed-loop system of the form of (5)&(33) where

$$G_c(s) = e^{-0.2s} \frac{1}{(1+0.5s)(1+0.1s)} \tag{44}$$

Noting that $e^{-st_d} \approx 1 - st_d \approx \frac{1}{1+st_d}$, it follows that $G_c(s)$ of (44) can be approximated by:

$$G_c(s) \approx e^{-0.3s} \frac{1}{1+0.5s} \tag{45}$$

This result corresponds to Figure 2.

On the other hand, *non-predictive* control gives an unstable closed-loop system for the design parameters of (42). With some experimentation it was found that reducing the gains to

$$k = \begin{bmatrix} 6 & 5 \end{bmatrix} \tag{46}$$

$$k_r = 5 \tag{47}$$

gave a stable system with the step response of Figure 4(a). The initial response is approximately the same as that of the non-predictive control, but the time delay causes oscillations which lead to a quite different response.

Using the approach of Section 3.2, a high order controller was designed. As noted in Section 3.2, closed-loop poles corresponding to the approximate delay cannot be chosen arbitrarily. For this example, the N poles corresponding to the approximate delay were left at the open-loop positions of $s = -\frac{N}{t_d}$, the remaining $n = 2$ poles were chosen as for the predictive controller at $s = -2$ and $s = -10$. The results for $N = 1$, $N = 2$ and $N = 4$ appear in Figure 4(b). Figure 4(b) illustrated that it is, indeed, possible to approximate the response of a predictive controller using a high-order without an explicit predictor. However, although such a controller may be convenient for implementation, it lacks the conceptual clarity of the underlying predictive controller.

4 Discussion and Conclusion

Assuming the existence of closed-loop feedback, either continuous or intermittent, we have considered whether such feedback should be predictive or not predictive to be consistent with Fitts’ law. Since Fitts’ law is equivalent to a time delay separated from a first-order system, we have demonstrated, using known control theory, that the controller must be predictive. A non-predictive controller retains a state delay within feedback loop which is not consistent with Fitts’ law.

Whilst any predictive controller can, as discussed in Section 3.2, be approximated by a controller without an explicit predictor, such as a rational transfer function, such a controller would be of high-order. Such ”non-parametric” controllers are essentially empirical in nature, and, although useful for implementation are without physical meaning when divorced from the underlying predictive controller. On the other hand a predictive controller offers an exact conceptual explanation of Fitts’ law with a clear physical interpretation. This new insight supports the idea that predictive mechanisms underlie the empirical ”speed-accuracy trade-off” known as Fitts’ law.

The relation between predictive, feedback control and human neurophysiology. The predictive control model (Fig 3(b)) assumes (i) that control is exercised on the basis of feedback between the intended and actual final position and (ii) that the motor system estimates system states (e.g. position and velocity of hand, or the length/tension of muscle actuators) at a time in the future in order to counter the time delay present in the neuromuscular system. It is commonplace to assume that motor control is exercised on the basis of feedback and generally it is uncontroversial to assume feedback between the intended and actual state, i.e. position and velocity, of the hand. There are issues as to whether feedback is continuous or intermittent, though as we argued, whether feedback is continuous or intermittent does not alter the case for predictive control. Many (including Connelly (1984),Cannon (1994), Jagacinski and Flach (2003) and Phillips and Repperger (1997)) though not all (Harris and Wolpert 1998), explanations of Fitts law have assumed feedback.

Whether or not the motor control system uses prediction has been subject to considerable debate. Evidence for prediction in the motor control system has been steadily increasing (Davidson and Wolpert 2005). Usually, prediction refers to forward models which estimate internal and external states of the body from which motor commands are derived (Miall and Wolpert 1996; Wolpert et al. 1998; Davidson and Wolpert 2005; Bays and Wolpert 2007) and such prediction is associated with the cerebellum. The predictive model in this paper (Fig 3(b)) goes beyond the usual forward model: in addition to the usual forward model (observer in Figs 3(a) and 3(b)) there is an explicit prediction of the future state (predictor in Fig 3(b)) which eliminates from the feedback loop the time delay inherent in the neuromuscular system. Currently, there is less neurophysiological evidence for such predictors. In the past, authors have advocated Smith predictors (Miall et al. 1993b; Miall and Wolpert 1996; Wolpert et al. 1998) for this role of removing the inherent time delay from the feedback loop, though more recently they have provided neurophysiological evidence against the adaptations expected for a Smith predictor (Miall and Jackson 2006). Unlike the Smith predictor which cannot stably predict unstable systems, the predictive control model implemented in this paper is stable and is proposed as a basis for neurophysiological control.

As discussed in the Introduction Craik (1947) introduced the idea of *Intermittent Control* to describe human movement control. As discussed in and Section 3.4, intermittent control is closely related to predictive control and, in some circumstances is indistinguishable from it. Thus the conclusions of this paper do not distinguish between predictive and intermittent control. However, experiments designed to probe the differences between the two, in particular those focused on exploiting the intermittent open-loop nature of intermittent control could resolve this issue. For example a set of perturbation signals where an initial pulse is followed by another within, say, 200ms, could be used.

Scientific predictions from the predictive feedback model which are different from previous models. Most previous models have assumed feedback. However, *open-loop* (that is, no feedback) optimal control with a minimum-variance endpoint criteria and signal dependent noise has been considered by Harris and Wolpert (1998) and had been shown to give hand movement trajectories consistent with Fitts' law for durations up to 0.7s. If the target were moved during the movement, this open loop theory would predict no alteration of trajectory. In contrast, the predictive feedback model predicts that the hand trajectory would respond to online changes in target position after a certain delay e.g. 0.2s.

The predictive feedback control model predicts that the closed-loop system is the product of a rational transfer function and a pure time-delay - the time delay is taken outside the closed-loop system. In contrast, previous non-predictive feedback models (e.g. Connelly (1984),Cannon (1994), Jagacinski and Flach (2003) and Phillips and Repperger (1997)) would predict a closed-loop system with a non-rational transfer function arising from the time-delay embedded in the closed-loop.

The biological meaning of this new insight can be illustrated with an experiment. Prediction solves the problem of having time delays in the feedback loop, so the difference between a predictive and non-predictive controller will be heightened when the load to be controlled is unstable and the feedback time delays are large. Consider a Fitts type experiment where a subject is asked to move an unstable load as quickly as possible to a newly presented target. The load is unstable, so the control must use feedback as without feedback, the unstable load cannot be stabilised. If an artificial delay is inserted between visual feedback of the load position and application of force to move the load, the closed-loop system will become more oscillatory and unstable. If the delay is great enough, a feedback controller without prediction will be unable to stabilise the load in its movement to the target. On the other hand, with sufficient training, we suggest that the hypothesised predictive controller can be tuned to predict the added time delay such that the load can be moved stably to the target. Moreover, our results imply that Fitts' law then applies but with the A parameter of (3) increased by the added delay.

As we have shown in this paper, predictive control gives the simplest feedback control parameterisation of Fitts law. If this insight has biological as well mathematical validity, then experiments of the kind discussed in this section will show human visuo-manual control to be predictive.

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