Sborník vědeckých prací Vysoké školy báňské – Technické univerzity Ostrava
číslo 1, rok 2009, ročník LII, řada hutnická
článek č. 1440

NORMALITY TEST OF COOLED SOLID TIME CONSTANT

I. KOPAL*, I. RUŽIAK, M. MOKRYŠOVÁ, J. KUČEROVÁ

Institute of Material and Technological Research, Faculty of Industrial Technologies, Alexander Dubček University of Trenčín; Púchov, Slovak Republic, contact author: kopal@fpt.tnuni.sk

ABSTRACT: The article deals with the practical identification of the thermal time constant of a first-order thermal system of a solid cooled in the fluid. The normality test for an empirical data set produced by calculations of the thermal time constant from a linearized time history of the cooled solid surface temperature in the process of its cooling is discussed. Article also introduces an algorithm of identification of an optimal time interval for reliable most probable value of time constant estimation.

KEY WORDS: first-order exponential model, thermal time constant, Lilliefors test for normality

1. INTRODUCTION

In the heat transfer analysis some solids are observed to behave like a "lump" whose entire temperature remains essentially uniformed at all times during a heat transfer process. The temperature of such solids can be taken to be a function of time only. It can be represented by a solid surface temperature time history. Heat transfer analysis which utilizes this idealization is very well known as lumped system analysis. It is applicable only when the *Biot number* (the ratio of internal thermal resistance within the solid to external thermal resistance at the surface of the solid) is less than or equal to *0.1*.

In the case of moveless solid which is cooled without heat sources or absorbents in a stagnant incompressible fluid with negligible volume of extensibility, the solid surface temperature time history can be described by a first-order exponential model in the form of

$$T(t) = T_\infty + (T_0 - T_\infty)e^{-\frac{t}{\tau}}, \tag{1}$$

with the thermal time constant of cooled solid thermal system τ, fluid temperature T_∞ and maximum solid temperature T_0. Thermal time constant, which represents very notable characteristic of a cooling process kinetics frequently exploited in various ranges of material engineering [1, 2], can be computed from a linearized temperature time history (1) by using equation [3, 4]

$$\ln\frac{T(t) - T_\infty}{T_0 - T_\infty} = -\frac{1}{\tau}t. \tag{2}$$

However, the time constant computation from an experimental temperature time history of the cooled solid usually produces a time series of time constant values with unknown mean and variance instead of one constant value, because of presence of a noise. Time window in which this time series comes from a normal probability distribution can be used for practical time constant estimation. A *Lilliefors test* is a suitable parametric statistical test for finding out in which time window does the time constant have a normal distribution.

2. LILIEFORS TEST FOR NORMALITY

The *Lilliefors test* for normality evaluates the null hypothesis H_0 which says that the random sample from a population with a particular population probability distribution has a normal distribution with unspecified mean μ and variance σ^2 against the alternative hypothesis H_1 that the sample does not have a normal distribution. Like most statistical tests, this test for normality defines a criterion and gives its sampling distribution. When the probability distribution associated with this criterion is smaller than given α-level, the alternative hypothesis is accepted (i.e., we conclude that the sample does not come from a normal distribution).

An interesting peculiarity of the Lilliefors test is the technique used to derive the sampling distribution of defined criterion. In general, mathematical statisticians derive the sampling distribution of criterion by using analytical techniques. However, in this case this approach failed and consequently, Lilliefors decided to calculate an approximation of the sampling distribution by using the *Monte-Carlo technique.* Essentially, the procedure consists of extracting a large number of samples from a normal population and computing the value of the criterion for each of these samples. The empirical distribution of the values of the criterion gives an approximation of the sampling distribution of the criterion under the null hypothesis [5].

3. ALGORITHM

In the first step of test the only part selected for next analysis is the one, which is of the linearized temperature time history and which doesn't produce complex values. The time series of τ_i is computed by using equation (2). Extreme τ_i values and all outliers, according to the rule

$$if \quad |\tau_i - \mu| > \sigma \quad then \quad \tau_i = \mu, \qquad (3)$$

are removed. Each of the τ_i value is then transformed into *Z-scores*

$$Z_i = \frac{\tau_i - \mu}{\sigma} \qquad (4)$$

by subtracting the sample mean and dividing by the sample standard deviation, so that the mean of the *Z-score* series is *0* and the standard deviation is *1.0*. The empirical cumulative distribution function of this *Z-score* series

$$S(Z_i) = \frac{1}{n}\sum_{i=1}^{n} \xi_i (Z_i \le z), \qquad (5)$$

with characteristic function

$$\xi_i(z) = \begin{cases} 0 & if \quad Z_i > z \\ 1 & if \quad Z_i \le z, \end{cases} \qquad (6)$$

is computed [6].

Similarly, the distribution function of the standard normal distribution

$$F(Z_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_i} \exp\left(-\frac{1}{2} Z_i^2\right) dZ \qquad (7)$$

is obtained at the same probability points [7]. The maximum difference of the two distribution functions at any point, defined as supremum

$$D = \sup_{z_i} |F(Z_i) - S(Z_i)|, \qquad (8)$$

is then computed. The H_0 hypothesis can be rejected at the significance level α if this maximum difference exceeds the $D_{(1-\alpha)}$ quantile, or *p-quantile*, in a table of quantiles of the *Lilliefors test* statistics [8] defined via distribution function (7) as

$$p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_p} \exp\left(-\frac{1}{2} Z_i^2 \right) dZ . \qquad (9)$$

Test is repeated for each possible time interval of analyzed τ_i series containing four values, marginally. Finally, the most probable time constant value is computed as a median of τ_i values in that time window wherein the null hypothesis cannot be rejected and wherein the difference between median and mean of τ_i is at minimum.

4. CONCLUSION

The algorithm was tested on temperature time history simulating the cooling process of a slab made of a rubber blend for tire industry. The relative difference between the estimated time constant and its value initially embedded into the model (1) was less than *0.1 %*. Accordingly, we can conclude, that described algorithm based on the *Lilliefors test* for normality represents a very suitable tool for identification of optimal time window for the most probable thermal time constant estimation from the linearized temperature time history of the solid cooled in the fluid.

5. REFERENCES

[1] KOŠTIAL, P.: Fyzikálne základy materiálového inžinierstva I., ZUSI, Žilina, Slovakia, 2000.
[2] KOŠTIAL, P.: Fyzikálne základy materiálového inžinierstva II., ZUSI, Žilina Slovakia, 2002.
[3] LETKO, I. et al.: Priemyselné technológie I., Zusi, Žilina Slovakia, 2002.
[4] LETKO, I. et al.: Priemyselné technológie II., Zusi Žilina 2002.
[5] SHESKIN, D.J.: Handbook of Parametric and Nonparametric Statistical Procedures, Chapman & Hall, NY, 2007.
[6] MATEJIČKA, L., PIATKA, Ľ.: *Mathematical-Physical Series*, 11, 1997, 31 - 47, VŠDS, Žilina, Slovakia.
[7] PIATKA, Ľ., MATEJIČKA, L.: *Mathematical-Physical Series*, 10, 1995, 45 - 54, VŠDS, Žilina, Slovakia.
[8] CONOVER, W.: Practical Nonparametric Statistics, 2nd Edition, John Wiley & Sons, New York, 1980.