Transactions of the VŠB – Technical University of Ostrava, Mechanical Series
No. 2, 2008, vol. LIV,
article No. 1620

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A TOOL FOR ANALYSING SHOCKS

NÁSTROJE K ANALÝZE RÁZŮ

Abstract

The paper deals with evaluation the shock response spectrum (SRS), which is employed to analysis of impulse signals. This kind of spectra is a set of the minimum or maximum values of the time response produced by the single degree-of-freedom (SDOF) system, which is tuned to a set of resonance frequencies, to a shock signal. The response is evaluated as an output of the second order infinite response filter (IIR) with an input, which is the sequence of the acceleration signal samples. The theory is illustrated by examples.

Abstrakt

Příspěvek pojednává o výpočtu rázového spektra, které je použito k analýze impulsních signálů. Tento druh spektra je složen z množiny maximálních nebo minimálních hodnot časových odezev na impulsní buzení, získaných ze soustavy s jedním stupněm volnosti, která je naladěná na množinu rezonančních frekvencí. Odezva je vypočtena jako výstup IIR filtru s impulsním signálem ve formě posloupnosti hodnot vzorků signálů zrychlení.

1 INTRODUCTION

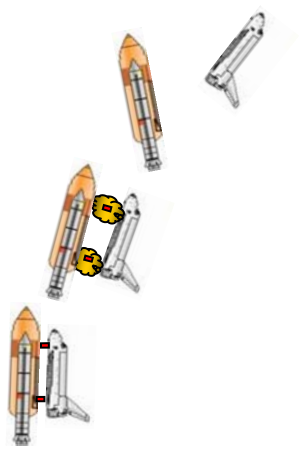

Fig. 1 Shock acting on a mechanical structure

The reliability of the technical equipment and protection against their failure is one of the basic conditions of their use. In our paper we would like to focus reader's attention to one particular area of the mechanical structure loading by force shocks. Especially high importance has analysis of the shock effect on such equipments, which are expensive and can not be repaired when the failure appears, i.e. a space lab.

The failure can be caused by an action lasting a very short moment – a shock. Let ´s explore this action, for example at the space lab. The space lab is getting rid of its supporting tanks by launching supporting tanks which will cause the throw off of the supporting tanks from the space lab. The shock, which will arise at this launch, can damage the equipment in the space.

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Due to the impact of the shock, some mechanical parts can start to vibrate, their acceleration will increase over their mechanical limit and the mentioned parts will be damaged. During this can happen that a physical damage does not appear, but a failure based on short connection. Shock response spectrum (SRS) analysis is described by the Czech standard ČSN 18431-4 and is based on the evaluation of a response of the single-degree-of-freedom (SDOF) systems to the given acceleration signal.

To calculate the shock response spectrum (SRS), the acceleration signal to be analyzed is applied to the bases of a set of SDOF systems, characterized by their natural frequencies and Q-factor value. The responses are calculated, and the maxima of each time response as a function of the natural frequencies create the shock response spectrum. In a basic version of the shock response spectrum, the maximum of the absolute value of the response as a function of time is evaluated. In the calculation of SRS, the natural frequencies are selected in a logarithmic scale. The same Q-factor is employed for all SDOF systems. The number of natural frequencies depends on the Q-factor (or damping). For a common Q-factor equaled to 10, which corresponds to a damping ratio of 5%, six frequencies per octave at minimum is required, which is corresponding to 20 frequencies per decade. A finer resolution is required if a smaller value of damping is assumed.

As an example, Figure 8 shows the (maximax) shock response spectrum for a half sine pulse with duration of 11 milliseconds and amplitude of 10 g.

Symbols (and abbreviated terms)

a	acceleration in Laplace-domain $[(m/s^2)\cdot s]$
c	damping constant in SDOF system [N/(m/s)]
f_n	natural frequency for SDOF system [Hz]
f_s	sampling frequency, sampling rate [Hz]
$H(z)$	transfer function in z domain
k	spring constant in SDOF system [N/m]
m	mass in SDOF system [kg, N/(m/s2)]
Q	Q factor, resonance gain
s	Laplace variable, complex frequency [rad/s]
T	sampling time interval [s]
α	digital filter denominator coefficient
β	digital filter numerator coefficient
ω_n	natural frequency (in radians/sec) [rad/s]
ζ	damping ratio, fraction of critical damping

2 SHOCK RESPONSE SPECTRUM FUNDAMENTALS

A shock response spectrum is defined as the response to a given acceleration in a set of “single-degree-of-freedom”, SDOF, mass-damper-spring oscillators. The given acceleration is applied to the base of all mentioned oscillators.

The oscillators and the maximum values of the response of each oscillator versus the natural frequency forms the shock response spectrum, see Figure 2 and 3.

Each single-degree-of-freedom system has a unique set of parameters; mass ***m***, damping constant ***c*** and spring constant ***k***. The parameters of the system are the conventional ones. The transfer function relating the base acceleration a_1 to the mass acceleration a_2 can be written in the form

$$\frac{a_2}{a_1} = \frac{cs + k}{ms^2 + cs + k} \tag{1}$$

where ***s*** is the Laplace variable (complex quantity) in rad/s. The single-degree-of-freedom system is normally characterized by its (undamped) natural frequency, f_n, in Hz, the dimensionless damping ratio ζ and the resonance gain Q (Q-factor). All these parameters can be evaluated as a function of mass, stiffness and damping coefficient

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{2}$$

$$Q = \frac{\sqrt{km}}{c} \tag{3}$$

$$\varsigma = \frac{1}{2Q} = \frac{c}{2\sqrt{km}} \tag{4}$$

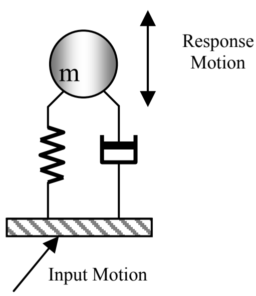

Fig. 2 Single-degree-of-freedom system

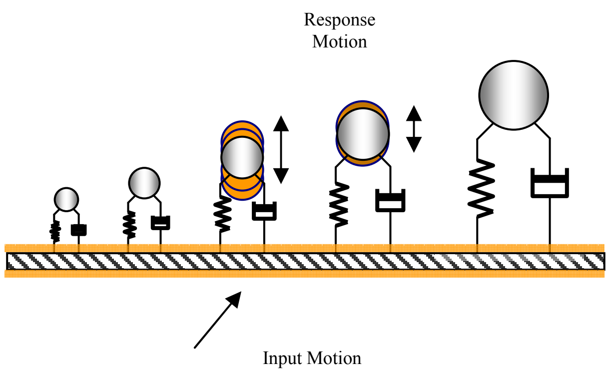

Fig. 3 SDOF systems in resonances

2.1 Filter coefficients for absolute acceleration response

Let the following quantities be designated as sampling frequency f_s [Hz], sampling time interval $T = \frac{1}{f_s}$ [seconds], natural frequency f_n Hz, natural angular frequency $\omega_n = 2\pi f_n$ [rad/s], resonance gain Q.

In contrast to the formula (1) the transfer function may be stated as a function of the natural frequency and Q-factor:

$$H(s) = \frac{a_2(s)}{a_1(s)} = \frac{\frac{\omega_n s}{Q} + \omega_n^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \tag{5}$$

If the continuous time response is replaced by sampled response, the transfer function (5) is turned into the transfer function in Z-transform. The transfer function (5) is approximated by a digital filter:

$$H(z) = \frac{a_2(z)}{a_1(z)} = \frac{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}, \tag{6}$$

which coefficients are as follows:

$$\beta_0 = 1 - \exp(-A)\sin(B)/B \tag{7}$$

$$\beta 1 = 2\exp(-A)\{\sin(B)/B - \cos(B)\} \tag{8}$$

$$\beta_2 = \exp(-2A) - \exp(-A)\sin(B)/B \tag{9}$$

$$\alpha_1 = -2\exp(-A)\cos(B) \tag{10}$$

$$\alpha_2 = \exp(-2A) \tag{11}$$

where

$$A = \frac{\omega_n T}{2Q} \tag{12}$$

$$B = \omega_n T \sqrt{1 - \frac{1}{4Q^2}} \tag{13}$$

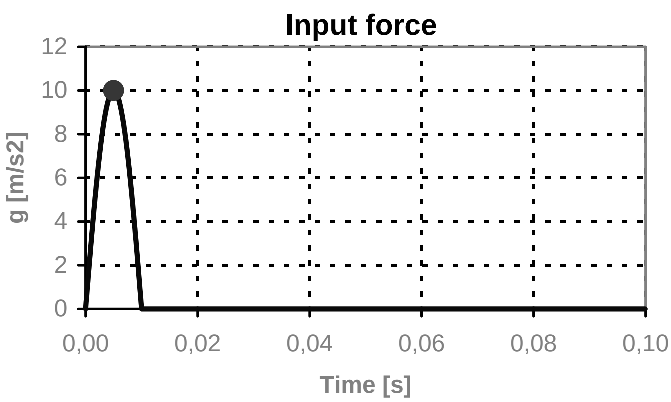

Fig. 4 Input shock force

We compute the responses for the following SDOF natural frequencies:

f = (1,2,3,4,5,6,7,8,9,10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000) [Hz]

The particular results are shown in Figures 4, 5, and 6 as time responses of the SDOF to the excitation. The shock response spectrum for half the sine pulse is shown in Figure 7. The axis *x* is corresponding to the SDOF resonant frequency and the axis *y* is corresponding to the maximum value of the appropriate time response, which were estimated using calculation.

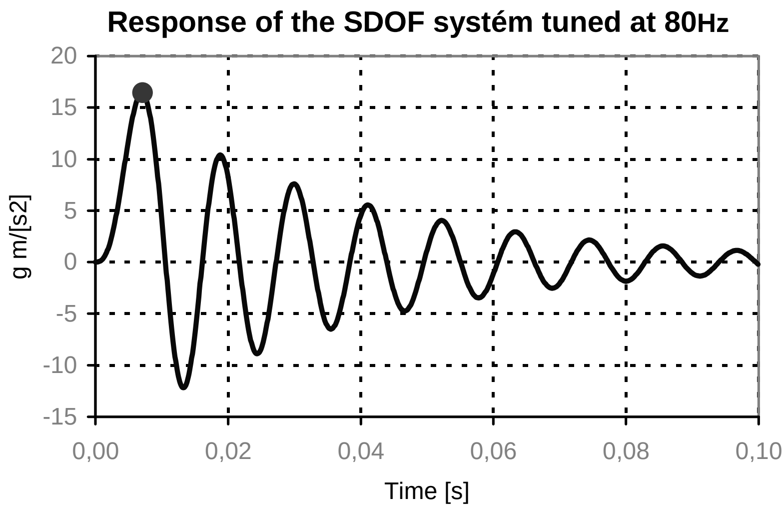

Fig. 5 Response of the SDOF system tuned on 80 Hz to the shock impulse shown in Figure 4

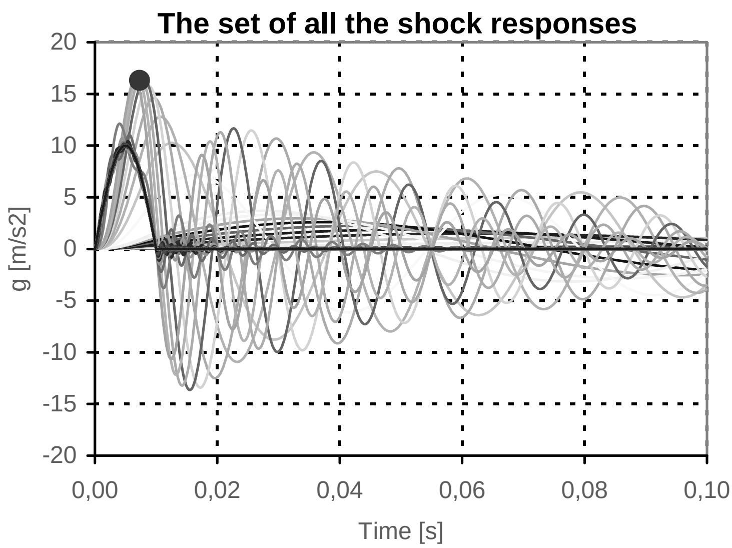

Fig. 6 The set of all the shock responses

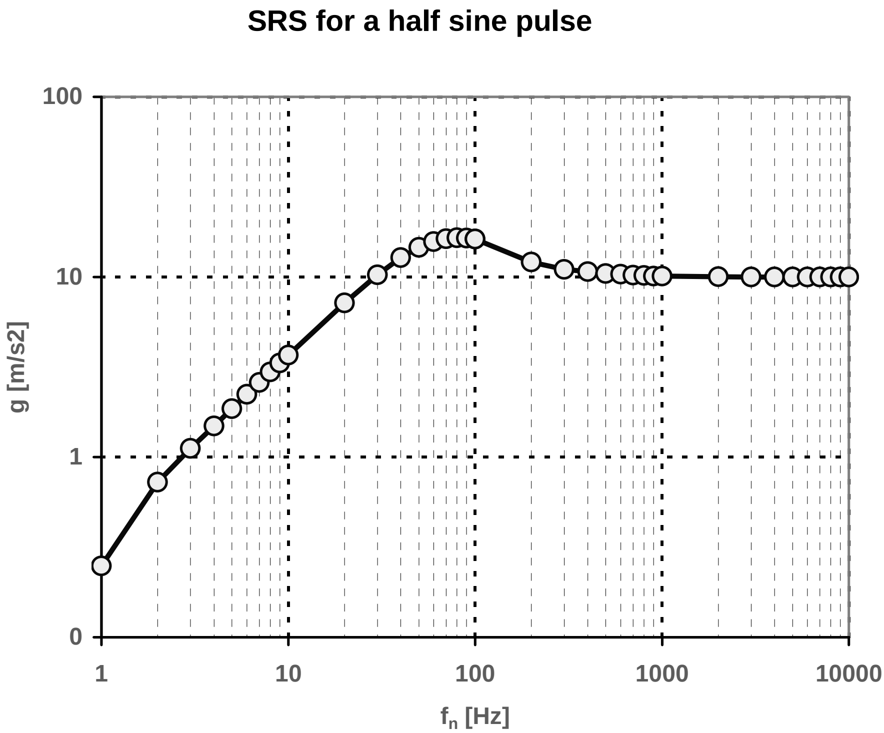

Fig. 7 Shock Response Spectrum

3 CONCLUSIONS

The paper presents the method of evaluation the shock response spectrum, which is characterizing time responses to the shocks, namely their maxima or minima as a function of the natural frequencies of a set of the single-degree-of-freedom systems. The vibration (or shock) is recorded in digital form, commonly as acceleration signal. The single-degree-of-freedom systems are approximated by an IIR digital filter, which response to the sampled acceleration signal can be easily evaluated. This shock response spectrum shows how the individual component of the impulse signal excites the mechanical structure to resonate.

Acknowledgement

This paper was written based on work done and financial support of the E-learning Elements for Support of Engineering Education project No. CZ.O4.01.3/3.2.15.2/0326.

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