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Electrically and optically controlled cross-polarized wave conversion

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Abstract: Light wave propagation in third-order nonlinear media with applied external electric field is investigated. Interplay between the nonlinear electro-optic and all-optical effects is examined theoretically. Energy exchange between the orthogonal light polarizations, the cross polarization conversion, results. The assisting external field acts as either the effect-enhancing or functionality-controlling parameter. Various materials such as silica glass, silicon, other bulk and quantum well semiconductors, organic materials, and particle-doped nanostructures are referred to as possible candidates for device implementations. Numerical estimates of achievable parameters in a selected suitable material are discussed.

OCIS codes: (190.3270) Nonlinear optics; Kerr effect; (230.4320l); Nonlinear optical devices.

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1. Introduction

Photonic waveguide devices utilizing optical nonlinear effects have been studied, designed and implemented over the last several decades. Different materials have been used such as lithium niobate, organic compounds, silicon or III-V semiconductors. Mainly the second-order and the third-order nonlinearities have been employed. Traditionally, the second-order nonlinear effect has been exploited in electro-optic modulators [1], or for efficient second-harmonic generation [2], whereby the lithium niobate material has offered one of the best operating parameters. The third-order nonlinear effect has been utilized, for example, for all-optical switching and bistability in bulk as well as quantum-well-based semiconductor device structures [3], in cross-phase and self-phase modulation including four-wave mixing devices [4], or for soliton generation in optical fibers [5]. On the other hand, in fibers, it contributes rather negatively to the overall performance of high-speed long-haul communication systems [6]. Higher-order nonlinearities rarely either need to be considered or can be exploited because of their very small values. One recent example of the fifth-order nonlinearity used in a CdTe-based all-optical switching mirror is reported in [7].

The third-order nonlinearity is usually very weak in non-crystals such as glass, while the second-order effect does not exist there due to symmetry, i.e. the isotropic nature of the material. In semiconductors, especially in quantum-well structures and in nanoparticle-doped materials, the third-order nonlinearity is enhanced by many orders of magnitude. However, such enhancement is achieved at the expense of the required optical power due to the associated high absorption losses. These so-called resonant nonlinearities [8], in addition to having a very narrow range of wavelength operability, thus do not offer a good and practical switching-energy/switching-time/per-bit figure of merit as many past failed attempts to implement an efficient practical all-optical bistable switch can testify. On the other hand, the non-resonant nonlinearity such as, for example, the pure Kerr effect that exists even far from the band edge of a semiconductor and thus does not require high absorption for its utilization, is again quite weak.

Yet and despite these obvious disadvantages, the third-order nonlinearity has remained in the focus of research studies because of some indisputable attractive characteristics it offers. One is that it exists in basically all materials of interest for photonic devices, especially silicon, organics, silica glass, III-V and II-VI semiconductors, as well as in recently technologically mastered nanostructures. Second is that it may lead to the implementation of polarization-independent devices so much sought after for many years. Therefore, combining the potential advantages with achievable high optical densities in waveguides, particularly in fibers whereby long propagation distances are obtainable, still seems very attractive for the development of fiber-based or fiber-compatible functional devices. Also, the modern nanomaterials, particularly the silicon-based ones, with their high nonlinear constants promise future development of efficient integrated functional photonic devices, with silicon being the best technologically-mastered and the most application-exploited material.

Detailed and thorough theoretical treatments can be found in rather vast literature on light propagation in third-order nonlinear media, be it in semiconductors, organic materials or optical fibers. Almost exclusively, though, these treatments deal with an all-optical aspect of the third-order nonlinearity (AC Kerr effect) exploitation. It means, in essence, that the third-order nonlinear effect, generally described by a tensor-vector product of $\chi^{(3)}EEE$, where $\chi^{(3)}$ is the nonlinear susceptibility tensor and E is the

electric field vector of the participating fields (e.g. optical mode, external optical control), is engaged in a device functionality as an intensity-dependent refractive index change of a waveguide material.

However, the electric field terms, E, do not necessarily have to be all contributors from optical modes or controlling optical signals, as is the case in optical solitons, four-wave mixing or all-optical switching. An externally applied electric field can play a role of one or more constituting field contributors in this term. They can also be information-carrying signals while participating in a given functionality.

The first comprehensive study of more general cases for the third-order nonlinearity in isotropic media with applied external DC (or slow-varying) fields (DC Kerr effect) is found in [9]. A number of interesting situations can occur; for example, a propagating mode having arbitrary polarization components. In this case, the electric field modulates the different polarization mode components and can cause birefringence, which is known in fibers as polarization mode dispersion. Use of external electric fields in poling techniques or exploitation of an internal electrostatic field in optical fibers have also been used in efficient second-harmonic generation [10] where values of the second-order nonlinearity close to that of the lithium niobate were obtained. A well-known electrically-controlled optical beam shutter can be implemented utilizing the electro-optic Kerr effect.

In this paper, a different situation is investigated theoretically. The focus is on the effect of an external assisting electric field on the character of the third-order nonlinearity in terms of its role in determining the properties of propagating waves or modes, namely their longitudinal development along the propagating direction. The assisting external fields can, in fact, turn the third-order nonlinearity into a second-order-like one, as if one were dealing with an electro-optic-like effect. This is, of course, the quadratic electro-optic effect discovered by J. Kerr in 1875, when it was shown that a biasing DC electric field can induce a birefringence in optically isotropic media [11]. This birefringence is proportional to the square of the DC electric field.

However, if the optical field is significantly intense, the all-optical effects start to appear as well. Therefore in such a case, both the quadratic electro-optic and the all-optical nonlinear effects act instantaneously together. Consequently, these instantaneous actions cause an interplay regime. Such a mutual co-existence of the two effects has not been studied systematically. Usually, one effect is considered dominant while the other is neglected. This proved to be a good approximation in all-optical switching that has been investigated quite thoroughly in recent years whereby the electro-optic Kerr effect plays no role. Similarly, in an optical beam shutter or electro-optic intensity modulation, the optical Kerr effect needs not to be included. In this study, we consider both effects as being the contributors to the overall functionality and thus show that this induced interplay leads to a power exchange between the light polarization modes as well as to other interesting situations.

The origin of this interplay regime can be understood by realizing that coupling between the two light polarization modes takes place via the all-optical cross phase polarization modulation effect and via birefringent modulation due to the quadratic electro-optic effect. Thus intuitively, this will lead to an energy exchange between the light polarization modes (nonlinear polarization rotation). This energy exchange is of the same sort of mechanism as in two coupled waveguides where spatial modes are coupled in a near-waveguides structure. The energy is exchangeable once there is a phase mismatch in the coupled waveguides modes.

Utilizing the nonlinear polarization rotation has been intensively studied for pulse reshaping [12]-[13], light modulation [14], intensity discrimination [15]-[16], optical logic gating [12], and mode locking in fiber lasers [17]-[18]. Although the analysis during this paper is carried out for isotropic media, which we consider to have applicability advantages, the same approach is workable for birefringent crystals whereby the external field may either enhance the desired effect or act as an adaptive control. This, and different other configurations, may lead to novel photonic device designs.

Some considerations are provided later of various practical material candidates. Silicon, silicon nanocrystals, silica glass, bulk and quantum well GaAs, bulk CdTe semiconductor, and SiO_2-based CdTe-nanoparticle-doped, some plastics, and other materials are mentioned. It is known that the size, the density and the shape of nanoparticles embedded in a base material can modify molecular [19] and/or atomic [20] polarizability. This leads to a drastically enhanced third-order nonlinearity [21], the effect of which can now be controlled by an external electric field. Our investigation thus addresses another possible exploitation of new material nanostructuralities in novel functional photonic devices.

2. Theory

The interaction of a light wave with a propagating medium can be described by solving the nonlinear wave equation. In this paper, where a light wave is considered to propagate in a third-order nonlinear medium with an applied external electric field (slow-varying compared to the optical frequency), the general nonlinear wave equation for polarization components produces nonlinear coupled differential equations. Generally, such a set of equations does not have an analytical solution. However, to approach this challenge yet analytically, the following approximate methodology is adopted. First, a perturbation approach is employed such that the coupled nonlinear differential equations are solved approximately by neglecting certain terms for a given special case of a small nonlinearity. Second, the obtained solution considered a first iteration, is then used again to find an analytical solution of the second iteration step, which leads to amplitude constants becoming functions. Third, the obtained solutions being considered local spatial quantities analytically described, are then expressed in an infinitesimal limit in order to arrive at the desired (second-order approximate) global solution.

2.1 Governing equations

Propagation of light in a nonlinear medium is described by Maxwell's equations for the field vectors, $\boldsymbol{E}$ and $\boldsymbol{H}$, in a notoriously known form [22]:

$$\nabla \times \boldsymbol{H} = \frac{\partial(\varepsilon_0 \boldsymbol{E} + \boldsymbol{P})}{\partial t}$$
$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}. \tag{1}$$

The polarization, $\boldsymbol{P}$, can be written out consisting of linear and nonlinear terms:

$$\boldsymbol{P} = \varepsilon_0(\chi^{(1)}\boldsymbol{E} + \chi^{(2)}\boldsymbol{EE} + \chi^{(3)}\boldsymbol{EEE} \ldots) = \varepsilon_0 \chi^{(1)}\boldsymbol{E} + \boldsymbol{P}_{\boldsymbol{NL}}, \tag{2}$$

where $\chi^{(i)}$ are the linear, $i = 1$, and the nonlinear, $i = 2,3,\ldots$, susceptibilities, respectively. Combining Eqs. (1) and (2) yields the known vector wave equation:

$$\nabla \times \nabla \times \boldsymbol{E} = -\mu_0 \varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \boldsymbol{P}_{\boldsymbol{NL}}}{\partial t^2}, \tag{3}$$

where $\varepsilon = \varepsilon_o(1 + \chi^{(1)}) = n_L^2$ is the material permittivity and n_L is its linear refractive index. The double curl operator, $\nabla \times \nabla \times$, in Eq. (3) becomes the simpler Laplace operator, Δ, in a homogeneous medium. In a third-order nonlinear medium, the nonlinear polarization is usually written out in the Cartesian coordinates using the 81 components of the fourth-ranked susceptibility tensor, $\chi^{(3)}$, as [23]:

$$P_{i_{NL}} = \varepsilon_0 (\chi_{ixxx} E_x E_x E_x + \chi_{ixxy} E_x E_x E_y + \ldots. +$$
$$\chi_{izzy} E_z E_z E_y + \chi_{izzz} E_z E_z E_z) = \varepsilon_0 \sum_{x\,y\,z} \chi_{ijkl} E_j E_k E_l, \tag{4}$$

with $i \in \{x, y, z\}$.

For isotropic media such as, for example, silica glass used in optical fibers, the tensor $\chi^{(3)}$ has 21 nonzero components, of which only 3 are independent due to symmetry. One can obtain the following relationships [24]:

$$\begin{aligned}
&\chi_{yyzz} = \chi_{zzyy} = \chi_{zzxx} = \chi_{xxzz} = \chi_{xxyy} = \chi_{yyxx} \\
&\chi_{yzyz} = \chi_{zyzy} = \chi_{zxzx} = \chi_{xzxz} = \chi_{xyxy} = \chi_{yxyx} \\
&\chi_{yzzy} = \chi_{zyyz} = \chi_{zxxz} = \chi_{xzzx} = \chi_{xyyx} = \chi_{yxxy} \\
&\chi_{xxxx} = \chi_{xxyy} + \chi_{xyxy} + \chi_{xyyx} \\
&\chi_{yyyy} = \chi_{xxzz} + \chi_{xzxz} + \chi_{xzzx} \\
&\chi_{zzzz} = \chi_{yyzz} + \chi_{yzyz} + \chi_{yzzy} \\
&\chi_{xxxx} = \chi_{yyyy} = \chi_{zzzz}
\end{aligned} \tag{5}$$

Using Eq. (5) in Eq. (4), the nonlinear polarization can be simplified into:

$$\begin{aligned}
P_{x_{NL}} &= \varepsilon_0 \chi E_x (E_x^2 + E_y^2 + E_z^2) \\
P_{y_{NL}} &= \varepsilon_0 \chi E_y (E_x^2 + E_y^2 + E_z^2) \\
P_{z_{NL}} &= \varepsilon_0 \chi E_z (E_x^2 + E_y^2 + E_z^2),
\end{aligned} \tag{6}$$

where $\chi = \chi_{xxxx}$. The vector wave equation (3) can be simplified into a scalar wave equation under certain assumptions, for example, a TE mode propagation in a planar waveguide or a linearly-polarized mode traveling in a fiber. The latter is a well-studied subject whereby the scalar polarization is written as $P_{NL} \cong \chi E |E|^2 = \chi E \frac{2Z_0}{n_L} I$, where I is the light intensity and Z_0 is the free space impedance. This thus indicates an intensity-dependent field via the intensity-dependent refractive index, n, that is $n = n_L + n_{NL} I$, where n_{NL} is the known nonlinear refractive index coefficient for a given material. The scalar approximation leads then to the nonlinear wave equation. It can be solved approximately for some situations, the most known being the spatial soliton in optical fibers [25]. For this approximate case specifically, it is incorrectly called in literature the nonlinear Schrödinger equation due to its similar form. However, Schrödinger himself did not derive any nonlinear equation. In 1925 he formulated a linear motion equation for a free particle [26].

The third-order nonlinearity term, generally written as $P_{NL} = \varepsilon_0 \chi^{(3)} EEE$, does not necessarily have to be considered only as a refractive index change with intensity of light. An externally applied electric field can play a role of one or more constituting field contributors, E, in this term. This can lead to different situations yielding various interesting effects.

2.2 Propagation expressions

Let us assume a linearly polarized mode at an angular optical frequency ω, propagating along the z-axis, and having generally all field components, E_x, E_y and E_z:

$$E_{x,y,z} = \frac{1}{2} \boldsymbol{e}_{x,y,z} \gamma + \frac{1}{2} \boldsymbol{e}^*_{x,y,z} \gamma^* \tag{7}$$

where $\gamma = e^{j(\omega t - k z)}$, * means complex conjugate, $k = \omega n_L / c$ is the propagation constant, and c is the speed of light in vacuum. An external electric field, $\frac{1}{2} E_{ext}$, is applied parallel to the $x-$axis. The induced nonlinear polarization, using Eq. (6), is then given by:

$$P_{x_{NL}} = \varepsilon_0 \chi [\frac{1}{8} e_x e^2 \gamma^3 + \frac{1}{4}(e_x^2 + \frac{1}{2} e^2) E_{ext} \gamma^2 + (\frac{1}{4} e_x |e|^2 + \frac{1}{8} e_x^* e^2 + \frac{3}{8} e_x E_{ext}^2)\gamma + \frac{1}{2} e_x e_x^* E_{ext} + \frac{1}{4} |e|^2 E_{ext} + \frac{1}{8} E_{ext}^3]$$

$$P_{y,z_{NL}} = \varepsilon_0 \chi [\frac{1}{8} e_{y,z} e^2 \gamma^3 + \frac{1}{4} e_{y,z} e_x E_{ext} \gamma^2 + (\frac{1}{4} e_{y,z} |e|^2 + \frac{1}{8} e_{y,z}^* e^2 + \frac{1}{8} e_{y,z} E_{ext}^2)\gamma + \frac{1}{4}(e_{y,z} e_x^* + e_{y,z}^* e_x) E_{ext}], \quad (8)$$

where $e^2 = e_x^2 + e_y^2 + e_z^2$ and $|e|^2 = e_x e_x^* + e_y e_y^* + e_z e_z^*$. The complex conjugate terms are omitted in Eq. (8) for simplicity but have been taken into account in derivations.

The nonlinear polarization can be considered a source of new fields at frequencies ω, 2ω, 3ω. In a dispersive medium, which is usually the case, the terms at harmonic frequencies, 2ω, 3ω suffer from the phase mismatch unless some of the known techniques for phase matching is employed. This can include periodic perturbation of the dielectric constant or the boundary, or periodic modulation of the nonlinear properties of the waveguide material. In this work, we are interested only in the fundamental frequency terms at ω, and thus the harmonic terms as well as the dc terms are omitted in the further analysis. The non-zero components of the nonlinear polarization at ω thus are:

$$P_{x_{NL}} = \varepsilon_0 \chi (\frac{1}{4} e_x |e|^2 + \frac{1}{8} e_x^* e^2 + \frac{3}{8} e_x E_{ext}^2)\gamma = \varepsilon_0 \chi [\frac{3}{8} e_x |e|^2 - \frac{1}{8} e_x e_y e_y^* (1 - \frac{e_x^* e_y}{e_x e_y^*}) - \frac{1}{8} e_x e_z e_z^* (1 - \frac{e_x^* e_z}{e_x e_z^*}) + \frac{3}{8} e_x E_{ext}^2]\gamma$$

$$P_{y,z_{NL}} = \varepsilon_0 \chi (\frac{1}{4} e_{y,z} |e|^2 + \frac{1}{8} e_{y,z}^* e^2 + \frac{1}{8} e_{y,z} E_{ext}^2)\gamma \quad (9)$$

$$= \varepsilon_0 \chi [\frac{3}{8} e_{y,z} |e|^2 - \frac{1}{8} e_{y,z} e_x e_x^* (1 - \frac{e_{y,z}^* e_x}{e_{y,z} e_x^*}) - \frac{1}{8} e_{y,z} e_{z,y} e_{z,y}^* (1 - \frac{e_{y,z}^* e_{z,y}}{e_{y,z} e_{z,y}^*}) + \frac{1}{8} e_{y,z} E_{ext}^2]\gamma.$$

The source functions in Eq. (9) enter the nonlinear equation in Eq. (3). For weak nonlinearity, this wave equation can be solved employing the slow-varying-envelope approximation. In that case, all field component complex amplitudes are assumed to be functions of the longitudinal coordinate, i.e $e_x(z)$, $e_y(z)$, and $e_z(z)$. The spatial dependence on transversal coordinates, x, y, is omitted for simplicity and because it is not relevant in the analysis. For small nonlinearity, the longitudinal variations of the mutual power coupling factors in Eq. (9) can also be neglected; therefore, one can set approximately:

$$\frac{e_x^* e_y}{e_x e_y^*} \approx \frac{e_x^* e_z}{e_x e_z^*} \approx \frac{e_{y,z}^* e_x}{e_{y,z} e_x^*} \approx \frac{e_{y,z}^* e_{z,y}}{e_{y,z} e_{z,y}^*} \approx 1. \quad (10)$$

This drastically simplifies the otherwise coupled differential equations, yielding a practical assumption that all mode components satisfy the wave equation individually, and leads to the final source field expressions in the form:

$$P_{x_{NL}} = \varepsilon_0 \chi \frac{3}{8}(|\boldsymbol{e}|^2 + E_{ext}^2)\,\boldsymbol{e}_x \gamma$$
$$P_{y,z_{NL}} = \varepsilon_0 \chi \frac{3}{8}(|\boldsymbol{e}|^2 + \frac{1}{3}E_{ext}^2)\,\boldsymbol{e}_{y,z} \gamma. \quad (11)$$

The first term in all polarization components in Eq. (11) represents typical nonlinear phenomena that exist without the external electric field. It corresponds to self-phase modulation of each individual field component, i.e. $\boldsymbol{e}_x(\boldsymbol{e}_x \boldsymbol{e}_x^*)$, $\boldsymbol{e}_y(\boldsymbol{e}_y \boldsymbol{e}_y^*)$, and $\boldsymbol{e}_z(\boldsymbol{e}_z \boldsymbol{e}_z^*)$, as well as to cross-phase polarization modulation amongst the three polarization components, i.e. $\boldsymbol{e}_x(\boldsymbol{e}_y \boldsymbol{e}_y^* + \boldsymbol{e}_z \boldsymbol{e}_z^*)$, $\boldsymbol{e}_y(\boldsymbol{e}_x \boldsymbol{e}_x^* + \boldsymbol{e}_z \boldsymbol{e}_z^*)$, and $\boldsymbol{e}_z(\boldsymbol{e}_x \boldsymbol{e}_x^* + \boldsymbol{e}_y \boldsymbol{e}_y^*)$. The second term describes an external-field-controlled propagating field.

2.3 Field solutions

The slow-varying-envelope approximation applied to Eq. (3) whereby the second derivatives are neglected, the transversal field variations are omitted, and a harmonic field is assumed, yields a well know first-order differential equation:

$$\frac{\partial \boldsymbol{E}}{\partial z} = \boldsymbol{E}' = -j\frac{k}{2\varepsilon}\boldsymbol{P}_{NL}. \quad (12)$$

Substituting the source functions in Eq. (11) into Eq. (12) yields first-order differential equations for the envelopes of all three mode components:

$$\frac{\boldsymbol{e}_x'(z)}{\boldsymbol{e}_x(z)} = -j\frac{k}{n_L^2}\chi\frac{3}{8}(|\boldsymbol{e}|^2 + E_{ext}^2)$$
$$\frac{\boldsymbol{e}_{y,z}'(z)}{\boldsymbol{e}_{y,z}(z)} = -j\frac{k}{n_L^2}\chi\frac{3}{8}(|\boldsymbol{e}|^2 + \frac{1}{3}E_{ext}^2). \quad (13)$$

These equations integrate to yield the required approximate solutions in the form:

$$E_x = \boldsymbol{e}_x(0)\, e^{j\omega t} e^{-jk_0(n_L + n_{NL} I + n_{EXT} E_{ext}^2)z}$$
$$E_{y,z} = \boldsymbol{e}_{y,z}(0)\, e^{j\omega t} e^{-jk_0(n_L + n_{NL} I + \frac{1}{3}n_{EXT} E_{ext}^2)z}, \quad (14)$$

where $k_0 = \varepsilon_0 Z_0$, and $\boldsymbol{e}_x(0)$ and $\boldsymbol{e}_{y,z}(0)$ are the field's initial amplitudes.

The all-optical effects (self-phase and cross-phase polarization modulation) are represented by the second term in the exponent in Eq. (14). It is identical to the known relationship [5], namely:

$$n_{NL} \cdot I = \frac{3\chi Z_0}{4n_L^2} \cdot \frac{n_L}{2Z_0}|\boldsymbol{e}|^2. \quad (15)$$

The induced external-electric-field-controlled refractive index change is described by the third term in the exponent in Eq. (14), $n_{EXT} E_{ext}^2$, with:

$$n_{EXT} = \frac{3\chi}{8n_L} = \frac{n_L}{2Z_0} n_{NL}, \tag{16}$$

where n_{EXT} is called in this paper the nonlinear *electrical* refractive index coefficient to distinguish it from n_{NL}. It is simply related to the Kerr constant, K, for a DC Kerr effect via $n_{EXT} = \frac{2\pi}{k_0} K$. Eq. (16) establishes a simple practical relationship between the known parameter n_{NL}, the nonlinear *optical* refractive index coefficient, which is tabulated for many materials used in photonic structures and devices, and the new parameter n_{EXT}, the nonlinear electrical refractive index coefficient, which determines the strength of the external-electric-field-assisted effect.

For the external-electric-field-assisted effect, one can write from Eqs. (14)-(16) the refractive index change as:

$$\Delta n = n_{EXT} E_{ext}^2 = \frac{3}{8n_L} \chi E_{ext}^2 = \frac{n_L}{2Z_0} n_{NL} E_{ext}^2. \tag{17}$$

Using Eq. (17), one can express the propagation length L_π required for a π − phase shift in the form:

$$L_\pi = \frac{\lambda_0 Z_0}{n_L n_{NL} E_{ext}^2}, \tag{18}$$

where $\lambda_0 = \frac{2\pi}{k_0}$ is the wavelength of light.

It can be seen from Eqs. (14) and (17) that the refractive index change is quadratic with the applied external electric field. The effect affects all mode vector components while it is a factor of three stronger for the component parallel to the external field. It thus causes propagation mode phase changes that are different for different polarization components meaning an induced, and controlled, mode birefringence. The strength of this third-order-nonlinearity electro-optic effect is identical to the all-optical effect in that it produces the same refractive index change when the external electric field is the same as the electric field vector of the optical mode. Therefore, in the case when the optical power in a propagating mode is constant, the external-electric-field-assisted effect is the determining feature. On the other hand, assuming a reasonably practical condition that $E_{ext}^2 \square e_{x,y,z}^2$, which corresponds to approximately $1\text{-}mW$ of optical power in a $10\text{-}\mu m^2$ cross-section waveguide and about $E_{ext} > 1V/\mu m$, one can neglect the all-optical effects and thus make the external electric field the dominating control parameter.

One word of caution is in order when considering Eqs. (15) and (16). The third-order nonlinearity described phenomenologically by the susceptibility χ represents a complex combination of different, sometimes independent, often quantum physical effects at an atomic level. This is a very complicated theoretical problem of physics of materials. It is certainly beyond the scope of this paper to attempt to explain it properly and rigorously. Some aspects are not even well understood today, especially in relation to new engineered nanomaterials. The susceptibility is generally dispersive whereby appreciable nonlinear dispersion and large absorption exist close to resonances (e.g. bandedge, excitons, multiphoton absorption). Extremes would be the resonant nonlinearity at an optical frequency and a DC electro-optic Kerr effect. The values of the nonlinear constants, n_{NL} and n_{EXT} in Eqs. (15) - (17), respectively, would be quite different.

Therefore, the application of the simplified expressions in Eqs. (15) and (16) needs to be exercised carefully by first considering an actual situation being investigated. For example, for the case of a non-resonant nonlinearity at an optical frequency far from any absorption line, the susceptibility can be assumed dispersionless. This can be shown quantitatively by the classical anharmonic model. The actual value of the nonlinearity n_{NL} is then basically equal to the known Kerr constant of a given material and Eq.(16) is valid; it is well known that the pure electro-optic Kerr effect is a quasi-static limit of the

optical nonlinearity.

For cubic crystalline materials such as, for example, cadmium tellurite (CdTe, space group *Fm3m*), gallium arsenide, indium phosphide or silicon (GaAs, InP, Si, space group *F-43m*), the tensor $\chi^{(3)}$ has 21 nonzero components, of which only 4 are independent. Relationships similar to those in Eq. (5) can be derived as follows [24]:

$$\begin{aligned}
&\chi_{yyzz} = \chi_{zzyy} = \chi_{zzxx} = \chi_{xxzz} = \chi_{xxyy} = \chi_{yyxx} \\
&\chi_{yzyz} = \chi_{zyzy} = \chi_{zxzx} = \chi_{xzxz} = \chi_{xyxy} = \chi_{yxyx} \\
&\chi_{yzzy} = \chi_{zyyz} = \chi_{zxxz} = \chi_{xzzx} = \chi_{xyyx} = \chi_{yxxy} \\
&\chi_{xxxx} = \chi_{yyyy} = \chi_{zzzz}
\end{aligned} \tag{19}$$

Combining Eqs. (4) and (19) yields the nonlinear polarization components:

$$\begin{aligned}
P_{x_{NL}} &= \varepsilon_0 \chi E_x [E_x^2 + \varphi (E_y^2 + E_z^2)] \\
P_{y_{NL}} &= \varepsilon_0 \chi E_y [(E_x^2 + \varphi (E_y^2 + E_z^2)] \\
P_{z_{NL}} &= \varepsilon_0 \chi E_z [(E_x^2 + \varphi (E_y^2 + E_z^2)],
\end{aligned} \tag{20}$$

where $\chi = \chi_{xxxx}$ and $\varphi = (\chi_{xxyy} + \chi_{xyxy} + \chi_{xyyx}) / \chi_{xxxx}$.

Following the same procedure as above, one can arrive at similar expressions for the source (polarization) fields:

$$\begin{aligned}
P_{x_{NL}} &= \varepsilon_0 \chi \frac{3}{8} [|e_x|^2 + \varphi (|e_y|^2 + |e_z|^2) + E_{ext}^2] e_x \gamma \\
P_{y,z_{NL}} &= \varepsilon_0 \chi \frac{3}{8} [|e_x|^2 + \varphi (|e_y|^2 + |e_z|^2) + \frac{1}{3} E_{ext}^2] e_{y,z} \gamma.
\end{aligned} \tag{21}$$

It can be seen that the effects of self-phase modulation (proportional to $|e_x|^2$) and the cross-phase polarization modulation (proportional to $\varphi (|e_y|^2 + |e_z|^2)$) are separated and modified in their strengths as a result of two effective susceptibility coefficients (χ and $\chi \cdot \varphi$) governing the interaction. The field solutions are basically identical to those in Eq. (14) except that the light intensity is modified due to the difference between the self-phase and cross-phase modulation terms. It assumes the form:

$$I = \frac{n_L}{2 Z_0} [|e_x|^2 + \varphi (|e_y|^2 + |e_z|^2)]. \tag{22}$$

However, the actual interaction coefficient, n_{NL}, remains the same, and so as the external-electric-field-assisted effect (n_{EXT}). It is interesting to note that the dispersive properties of χ with respect to optical versus electrical fields' values, as discussed above, might be properly accounted for in a similar manner as in this case where χ and $\varphi.\chi$ appear in the equations together.

2.4 Power exchange expressions

Assuming the equalities in Eq. (10) simplifies considerably the solution to the nonlinear wave equation. However, it introduces an error since the coupling between the polarization components described by these terms has been neglected. To take this coupling into account and correct the error, at least partially

since the full solution is not analytically possible, one can evaluate the terms in Eq. (10) employing the solutions from the first iteration in Eq. (14), with assuming the amplitudes to be functions of distance (perturbation approach). Solving the differential equation in Eq. (12) again then yields the second iteration.

Analyzing only the x component and with coupling only to/from the y component of the mode for simplicity and without losing the generality of the solution, one writes:

$$P_{x_{NL}} = \varepsilon_0 \chi (\frac{1}{4} e_x |e|^2 + \frac{1}{8} e_x^* e^2 + \frac{3}{8} e_x E_{ext}^2) = \varepsilon_0 \chi [\frac{3}{8} e_x |e_x|^2 + \frac{3}{8} e_x |e_y|^2 - \frac{1}{8} e_x |e_y|^2 (1 - \frac{e_x^* e_y}{e_x e_y^*}) + \frac{3}{8} e_x E_{ext}^2], \quad (23)$$

where the common phase factor γ has been omitted for simplicity as well.

Substituting the source function in Eq. (23) into Eq. (12) yields the first-order differential equation for the envelope similar to the one in Eq. (13):

$$\frac{e_x'}{e_x} = -j \frac{k_0}{n_L} \chi \frac{3}{8} [|e_x|^2 + |e_y|^2 + E_{ext}^2 + \frac{1}{3} |e_y|^2 (\frac{e_x^* e_y}{e_x e_y^*} - 1)]. \quad (24)$$

Using the solutions from the first iteration, one can evaluate the coupling term in Eq. (24) as:

$$\frac{e_x^* e_y}{e_x e_y^*} - 1 = e^{j 2\phi z} - 1 = 2j e^{j\phi z} \sin(\phi z), \quad (25)$$

where $\phi = \frac{k_0}{n_L} \chi \frac{1}{4} E_{ext}^2$. Integrating Eq. (24) with respect to z yields:

$$\ln e_x = -j \frac{k_0}{n_L} \chi \frac{3}{8} \{\int (|e_x|^2 + |e_y|^2 + E_{ext}^2)\, dz + j \frac{2}{3} \int |e_y|^2 e^{j\phi z} \sin(\phi z)\, dz\}, \quad (26)$$

The integration in Eq. (26) cannot be performed analytically in this form because the magnitudes of light polarization modes, $|e_{x,y}|$, are functions of the distance z. To obtain a solution to Eq. (26), one can divide the whole process into a series of local processes where in each single local process, the light waves magnitudes, $|e_{x,y}|$, are $z-$independent. This kind of partition of the whole process is valid as long as a change in z is small enough so the interaction strength can be considered constant for the amplitude changes (not the phase changes, of course). The interaction strength depends on the non-linearity, χ, the external electric field, E_{ext}, and the initial light polarization magnitudes. One can thus assume that the light propagation magnitudes, $|e_{x,y}|$, are $z-$independent. Then the solution to Eq. (26) for each local process leads to a local coupled-field solution whereby evaluating the integral in Eq. (26) for constant light magnitudes is possible. Choosing the integration constant to be equal to $\frac{1}{4\phi}$ in order to satisfy a physical condition that the optical field does not grow towards infinity when the external electric field approaches zero, leads to:

$$\ln e_x = -j \frac{k_0}{n_L} \chi \frac{3}{8} \{[|e_x|^2 + |e_y|^2 + E_{ext}^2]\, z + j \frac{1}{3} |e_y|^2 [j z - \frac{1}{2\phi} (e^{j 2\phi z} - 1)]\}. \quad (27)$$

As can be seen from the local field solution in Eq. (27) above, the new factor, the last term, contributes not only a phase but also an amplitude change of the mode component, with both new contributions being determined by the power in the other, coupled mode component, in addition to the external electric-field dependence. In order to separate both the phase and amplitude terms, one can write the coupling term in the form:

$$\frac{1}{2\phi}(e^{j2\phi z}-1)=\frac{1}{2\phi}[\cos(2\phi z)-1+j\sin(2\phi z)]$$
$$=\frac{1}{2\phi}[\cos(2\phi z)-1]+jz\,\mathrm{sinc}(2\phi z). \tag{28}$$

The new phase and the new amplitude of the field can thus be expressed as:

$$e_x = e^{-j\frac{k_0}{n_L}\chi\frac{3}{8}\{|e_x|^2+|e_y|^2+E_{ext}^2-\frac{1}{3}|e_y|^2[1-\mathrm{sinc}(2\phi z)]\}z} \times e^{\frac{k_0}{n_L}\chi\frac{1}{8}|e_y|^2\cdot\frac{1-\cos(2\phi z)}{2\phi}} \tag{29}$$

It can be seen from Eq. (29) that the new term in the phase containing the sinc function disappears when $E_{ext}^2=0$ (i.e. $\phi=0$), thus leaving in only the self-phase and cross-phase polarization modulation terms, as expected. Also, the new amplitude term assumes a value of unity, as it is supposed to under the same condition of zero external electric field.

The same procedure can be applied to derive expressions for the other orthogonal components. As there is no fundamental qualitative difference between them, one can write, for example, for the $y-$ component:

$$e_y = e^{-j\frac{k_0}{n_L}\chi\frac{3}{8}\{|e_y|^2+|e_x|^2+\frac{1}{3}E_{ext}^2-\frac{1}{3}|e_x|^2[1-\mathrm{sinc}(2\phi z)]\}z} \times e^{-\frac{k_0}{n_L}\chi\frac{1}{8}|e_x|^2\cdot\frac{1-\cos(2\phi z)}{2\phi}}. \tag{30}$$

Equations (29) and (30) clearly indicate local exchange of power between both mode polarization components. It is thus interesting to examine that effect in more detail. The local mode fields in Eqs. (29) and (30) are normalized to unity; for a more general case, we introduce normalized power density quantities for both components, p_x and p_y, such that $p_x+p_y=p_T$, where p_T $[mW/\mu m^2]$ is the total (constant) power in the wave per cross-sectional area. We also define the term with the superscript *"i"* as the one associated with the *i-th* local process, i.e. $p_{x,y}^i=p_{x,y}$ (at the *i-th* process). The local power densities can then be expressed as:

$$p_x^{i+1}=\frac{n_L}{2Z_0}\left|e_x^i\right|^2=p_x^i\cdot e^{\frac{1}{2E_{ext}^2}\left|e_y^i\right|^2[1-\cos(2\phi z)]}$$
$$p_y^{i+1}=\frac{n_L}{2Z_0}\left|e_y^i\right|^2=p_y^i\cdot e^{-\frac{1}{2E_{ext}^2}\left|e_x^i\right|^2[1-\cos(2\phi z)]}. \tag{31}$$

Substituting for $|e_x|^2$ and $|e_y|^2$ in the exponents of Eq. (31) using powers p_x and p_y again, one can obtain two local coupled power equations:

$$p_x^{i+1} = p_x^i \cdot e^{\,f \cdot p_y^i}$$
$$p_y^{i+1} = p_y^i \cdot e^{\,-f \cdot p_x^i}, \tag{32}$$

where:

$$f(z, E_{ext}) = \frac{Z_0}{n_L E_{ext}^2}\,[1 - cos\,(2\phi\, z)]. \tag{33}$$

The series of these dynamic interplays between the coupled local processes constitutes the whole interaction that can be seen as a continuous chain of local interactions, whereby the output of each single local interaction is the input to the next one, and so on. Obtaining an analytical expression that describes these dynamic processes in a global manner is thus necessary to characterize the overall process behavior. As a first step to globalize the solution, one can rewrite the local coupled equation in Eq. (32) as difference local coupled equations:

$$p_x^{i+1} - p_x^i = p_x^i \cdot (e^{\,f \cdot p_y^i} - 1)$$
$$p_y^{i+1} - p_y^i = p_y^i \cdot (e^{\,-f \cdot p_x^i} - 1), \tag{34}$$

The difference in two successive local power density values, $p_{x,y}^{i+1} - p_{x,y}^i$, is very small for a very small value of $(f \cdot p_{x,y}^i)$. In the infinitezimal limit one can write $(f \cdot p_{x,y}^i) \to d(f \cdot p_{x,y})$ and thus justify the approximation $p_{x,y}^{i+1} - p_{x,y}^i \to d(p_{x,y})$. The following differential equations then replace the difference ones in (34):

$$d(p_x) = p_x \cdot d(f \cdot p_y)$$
$$d(p_y) = -p_y \cdot d(f \cdot p_x). \tag{35}$$

Integrating the two coupled equations in (35) and denoting p_x^0 and p_y^0 as initial values of p_x and p_y, respectively, yields the following global coupled power expressions:

$$p_x = p_x^0 \cdot e^{\,f \cdot p_y}$$
$$p_y = p_y^0 \cdot e^{\,-f \cdot p_x}. \tag{36}$$

Applying the condition of constant total power (assuming constant wave's transverse cross section, e.g. a guided mode), $p_x + p_y = p_T$, in Eq. (36) allows one to find the decoupled global analytical expressions for both power quantities in the form:

$$p_x = \frac{1}{f} \cdot ln[\,p_x^N\, e^{\,f \cdot p_T} + p_y^N\,]$$
$$p_y = -\frac{1}{f} \cdot ln[\,p_y^N\, e^{\,-f \cdot p_T} + p_x^N\,], \tag{37}$$

where $p_x^N = p_x^0 / p_T$ and $p_y^N = p_y^0 / p_T$ are the normalized power densities, which basically determine the initial power percentage distribution between the mode components. The power densities in Eq. (37) can also be expressed in a completely uncoupled form:

$$p_x = \frac{1}{f} ln[1 + 2\, sinh(f\ p_T / 2)\, p_x^N\, e^{f\, p_T/2}]$$
$$p_y = \frac{1}{f} ln[1 - 2\, sinh(f\ p_T / 2)\, p_y^N\, e^{-f\, p_T/2}]. \quad (38)$$

The periodic exchange of the power between the two wave polarization components (cross polarization conversion) is obvious in Eq. (38) above. It is, as expected, controlled by the strength of the third-order nonlinear susceptibility, by the applied external electric field, and by the power in the respective mode components. These three parameters combined with the interaction length dictate the overall achievable effect.

The analysis of the behavior of the solutions in Eq. (38) by examining the total differential of, for example, $dp_x = \frac{\partial p_x}{\partial f}[\frac{\partial f}{\partial (E_{ext})} d(E_{ext}) + \frac{\partial f}{\partial z} dz]$, shows that the minima in f in Eq. (33) correspond to the minima of p_x, while the maxima of f determine p_x maxima with the first one being the largest. An interplay between the external electric field, E_{ext}, and the distance, z, then produces an optimal condition for the maximum power exchange between the mode components, although there is no absolute maximum. The power density maximum condition dictated by the distance comes from the condition that $sin(2\phi z) = 0$. The maximum dictated by the external field is determined from the condition that $(2\phi z) \cdot sin(2\phi z) + \cos(2\phi z) - 1 = 0$. For the first (largest) maximum, these conditions yield a hyperbola in 3-D for $f(z, E_{ext})$, described by the two equations in the $z - E_{ext}$ coordinate system:

$$f = \frac{2 Z_0}{n_L E_{ext}^2}, \qquad z = \frac{\lambda n_L}{\chi E_{ext}^2}. \quad (39)$$

Examination of the behavior shows that the interplay between the external electric field and the propagation distance determines the maximum of the power exchange. The total optical power in the mode then determines the value of this maximum. It should be noted that, contra-intuitively, *decreasing* of the external field causes an *increase* in the maximum value of the power exchange; however, at the cost of a longer propagation distance required. This is a direct consequence of the fact that the f function does not have a global maximum, except at infinity (i.e. for $z = \infty$ or $E_{ext} = 0$), and behaves as a 3-D hyperbola according to Eq. (39).

On the other hand, for a given E_{ext}, or alternatively for a given z, a maximum of the power exchange can always be found, while the stronger the nonlinearity χ, the shorter the distance is required for the same power exchange and the same external field. The value of the power exchange maximum increases more less linearly with the total power in the mode. For example, for $p_T = 1\ mW / \mu m^2$, $E_{ext} = 1\ V / \mu m$, $\chi = 0.12\ \mu m^2 / V^2$, and an equal initial distribution of the power between the mode components, the maximum achievable power exchange is over 10 %.

3. Numerical considerations

Since it is not an objective of this paper to conduct a comprehensive devices design based on the theoretical treatment presented, no specific design examples for different materials and/or structures are provided here. However, numerical estimates of the cross-polarized conversion effect (CPC) are presented. They indicate promising features for novel photonic devices. The most attractive feature, in addition to the already-mentioned advantage that the third-order effect exists in all materials suitable for implementation of photonic devices, especially silicon and silicon-based nanomaterials, is that either novel electro-optic or electrically/optically controlled optical/electrical devices might be envisioned and eventually designed [33, 34]. For example, an electronically controlled all-optical switch is feasible that would function as a re-configurable all-optical device. Alternatively, an optically controlled electro-optic

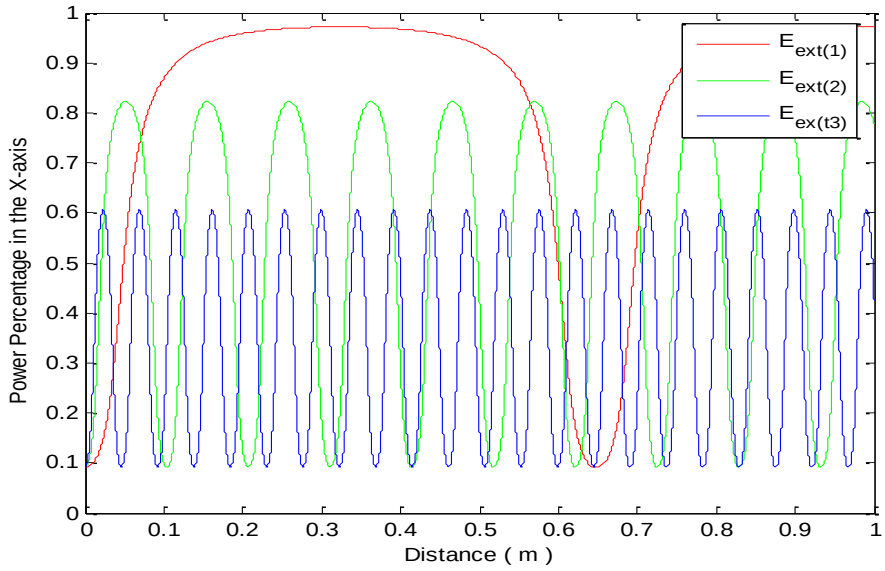

Fig. 1. The power percentage in the x-component versus distance. Constant total power and initial power percentage. The external electric fields are $E_{ext\,(1)} < E_{ext\,(2)} < E_{ext\,(3)}$.

device can be designed, the functionality of which is, for example, an optically stored information-bearing electrical signal [35].

Generally, it should be pointed out that the third-order susceptibilities have normally real and imaginary parts, χ_{ijkl}^{Re} and χ_{ijkl}^{Im}, corresponding to the phase and loss effects, respectively. They are coupled via Kramers-Kronig relation. Both contributions should be considered in numerical estimates; however, the values of χ_{ijkl}^{Im} are not usually readily available or are not even known. Thus the nonlinear loss is often not considered and only the phase effects are included in estimates or device designs.

Most materials of application interest and importance can be found in literature with their third-order nonlinearities cited or reported. For the above-mentioned reasons, the numerical calculations are, indeed, estimates as some values for given materials differ in literature as a result of various experimental methods used to evaluate them. It is known that several physical effects contribute to a material response on different time scales [27], which fact then causes the actual experimental values to vary, in addition to their wavelength dependence. Therefore, in the calculations, simulations or device designs, since the nonlinear susceptibilities are not unique quantities, rather some effective values that are available need to be used. Typical materials with known third-order susceptibilities potentially useful for photonic device design employing the external-electric-field-assisted effect theoretically investigated here, may include silicon, silica glass [5], GaAs bulk [6], CdTe bulk [7], GaAs quantum wells [8], CdTe nanocrystal [28], CdS nanocrystal [29], Si nanocrystal [30], poly (β-pinene) [31], fullerene-containing polyurethane films [32], and others.

Choosing an Si nanocrystal for its relatively strong nonlinearity, the optical power percentage in the x-component as a function of the distance is shown in figure 1 for different external electric field strengths. The total optical power and the initial power percentage are constant, that is $0.11\ [W/\mu m^2]$ and 9%, respectively.

The external electric field is the determining parameter of the CPC in this case. The strengths of the external field are $E_{ext(1)} = 0.8\ V/\mu m$, $E_{ext(2)} = 2\ V/\mu m$, and $E_{ext(3)} = 3\ V/\mu m$, respectively. It can be seen from figure 1, as mentioned before, that decreasing the external field causes an *increase* in the maximum value of CPC.

In figure 2, a normalized optical power percentage of the x-component as a function of distance is shown for different initial percentages while keeping the external field and the total power constant. The external electric field is $2\ V/\mu m$ and the total optical power is $0.101\ [W/\mu m^2]$. The initial power percentage in the $x-$component is the determining parameter of the CPC in this case. The values are $P_{I\%(1)} = 0.001,\ P_{I\%(2)} = 0.5,\ P_{I\%(3)} = 0.9,\ P_{I\%(4)} = 0.99$.

As it can be seen from figure 2, the initial power percentage plays a role in determining the range of the CPC, i.e. the depth of the power exchange. The smaller the initial power percentage in the component parallel with the direction of the external field, the stronger the CPC becomes.

In figure 3, the optical power percentage in the x-component is shown as a function of distance for different total launched optical powers, P_T, while keeping the external field and the initial power percentage constant. The external electric field is $2\ V/\mu m$, and the initial power percentage in the $x-$component is 0.2. The total launched optical power is the determining parameter of the CPC in this case with its values taken as $P_{T(1)} = 0.448\ mW/\mu m^2,\ P_{T(2)} = 0.011W/\mu m^2,\ P_{T(3)} = 0.101W/\mu m^2,\ P_{T(4)} = 1.01W/\mu m^2$.

As it can be seen from figure 3, the strength of the CPC is a function of the optical power. This feature promises electrically reconfigurable all-optical device functionality [34].

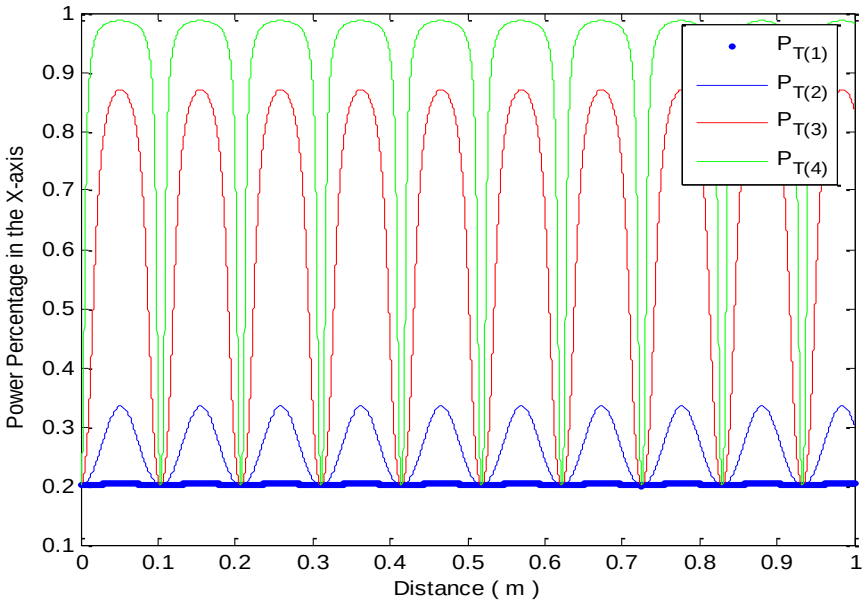

Fig. 3. The power percentage in the x-component versus distance. Constant external field and initial power percentage. The total launched optical powers are $P_{T(1)} < P_{T(2)} < P_{T(3)} < P_{T(4)}$.

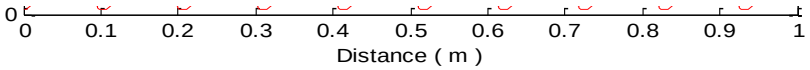

Fig. 2 The power percentage in the x-component versus distance. Constant external field and total optical power. The initial power percentages are $P_{I\%(1)} < P_{I\%(2)} < P_{I\%(3)} < P_{I\%(4)}$.

4. Summary

The presented simulated results in figures 1, 2 and 3, based on the theoretical treatment described in the previous sections and based on real-life material considerations, reveal a potential for functional photonic devices implementable and intergattable with simple structures in known well-mastered materials. An electro-optic control combined with an adaptive all-optical functionality are definitely attractive features.

5. Conclusions

The detailed theoretical treatment has been developed and a thorough analysis has been performed of optical wave propagation phase and amplitude changes due to an external electric field applied in a third-order nonlinear optical material. One of the most interesting observed features is the consequence of the all-optical and the quadratic electro-optic effects interplay. This interplay can cause the cross-polarized conversion (CPC) that can be electrically and optically controlled. The discussed features and properties of optical waves (or modes) under these conditions indicate a future potential some materials possess for functional integrated photonic devices as presented in [33]. The results obtained offer an application view at some interesting and promising characteristics. Numerical estimate considerations were briefly discussed.

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