A Best-Fit Model of Power Losses in Cold Rolled-Motor Lamination Steel Operating in a Wide Range of Frequency and Magnetization

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A procedure is described for identifying a mathematical model of core losses in ferromagnetic steel based on a minimal amount of experimental data. The new model has a hysteresis loss multiplicative coefficient variable only with frequency, a hysteresis loss power coefficient variable both with frequency and induction and a combined coefficient for eddy-current and excess losses that is, within a set frequency range, variable only with induction. Validation was successfully performed on a large number of different samples of nongrain oriented fully and semiprocessed steel alloys. Over a wide range of frequencies between 20 Hz and 2.1 kHz and inductions from 0.05 up to 2 T, the errors of the proposed model are substantially lower than those of a conventional model with fixed value coefficients.

Index Terms—**Core loss, eddy-current loss, electrical machine, Epstein test, ferromagnetic steel, hysteresis loss, iron loss.**

I. Introduction

IMPROVED models of core losses in ferromagnetic steel have continued to be an object of study for the last century, with recent developments encompassing both experimental and theoretical aspects [1]–[3]. Standard industrial practice involves constant frequency measurements of power losses on an Epstein or single sheet tester. Among the means for estimating the specific core losses at a frequency for which experiments were not performed, a best fit model represents a natural engineering choice. Yet, such an approach, which is beneficial for the cost effective numerical simulation of electromagnetic devices, and, at the same time, is a challenging task due to the nonlinearity of the material model, is largely undocumented.

Recent studies have reported promising results, obtained by employing variable coefficients for the conventional core loss models, e.g., [4] and [5]. This paper brings further original contributions by describing a systematic procedure for identifying a best fit model that covers with remarkably low errors a very wide range of frequencies between 20 and 2.1 kHz and inductions from 0.05 up to 2 T. Other aspects also discussed include the minimal requirements of experimental data, model validation and separation of loss components.

II. Experimental Data Collection

The materials studied are nongrain-oriented steels suitable for the high-volume production of rotating electrical machines.

Both fully processed and semiprocessed furnace-annealed grades were analyzed. A large number of samples of different alloys and lamination thickness were systematically investigated, but due to space limitations, only examples from a single sample of the semiprocessed type are included in this paper. Nevertheless, the methods described and the trends identified are generally applicable to the larger class of steels under

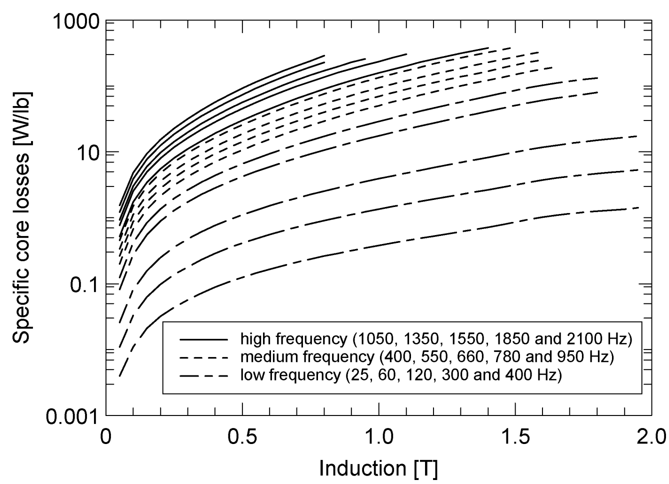

Fig. 1. Measured core loss curves of constant frequency employed for mathematical model identification. Data is grouped in three frequency ranges.

consideration. The main characteristics of the 0.022″-thick laminated example are a 3071 relative permeability, a 3.16 W/lb power loss, both at 1.5 T and 60 Hz, and a volumetric mass density of 7800 kg/m^3.

Testing was performed on a Brockhaus Messtechnik hysteresisgraph model MPG100D AC/DC coupled to an Epstein frame, which was built according to the ASTM standard for power frequency measurements. At high-frequency, part of the Epstein strips was removed from the stack in order to overcome the electric current limitations due to the amplifier and inductor combination. Small induction increments of 0.05 T were used to measure constant frequency core loss curves, some of which were employed for model identification (Fig. 1) and the reminder for model validation.

III. Mathematical Model Formulation

In the power line frequency range, one of the widely popular frequency domain models of specific core losses wFe in-

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cludes the contribution of a supplementary term due to excess (or anomalous) losses [2]. A recent study has shown that for lamination strips a very good fit, within couple of percent of error, can be achieved for frequencies f up to 400 Hz and inductions B up to 2 T only by employing variable coefficients [5]. Our attempts at extending this model at higher frequencies were unsuccessful as they resulted in unacceptably large errors.

Instead, a more traditional formulation, which considers only two components

$$w_{\mathrm{Fe}} = k_{\mathrm{h}} B^{\alpha} f + k_{\mathrm{e}} B^2 f^2 \tag{1}$$

and where the first right-hand term is associated with the hysteresis losses and the second one with the eddy-current losses, was successfully employed.

A variety of (1), having the hysteresis power loss coefficient α as a first order polynomial of B, has been used in electric motor engineering design software for more than a decade [6]. A recent systematic study, undertook by large group of researchers [4], provided additional proof that (1) could be satisfactorily employed in the study of electrical machines, and, presumably, the conclusions could be extended to other low frequency electromagnetic devices. In [4], the specific core losses were modeled using a constant value of the eddy current coefficient k_{e} and a three step approximation was introduced for the hysteresis loss coefficients k_{h} and α, with no further details being provided on the material model errors over a wide frequency range.

It should be noted that the adoption of (1) does not exclude the existence of anomalous losses, but it rather assumes that these can not be separated from the eddy-current losses, a hypothesis already advanced by other authors based on a different approach than ours [7]. In the following, although k_{e} is referred to as an eddy-current coefficient, it is, in fact, a coefficient for combined eddy-current and excess losses.

Dividing (1) by f yields a first-order polynomial equation

$$\frac{w_{\mathrm{Fe}}}{f} = k_{\mathrm{h}} B^{\alpha} + k_{\mathrm{e}} B^2 f = a + bf \tag{2}$$

the coefficients a and b of which are identifiable by linear fitting the values of the ratio between the experimental core loss data and the same f at any given B (Fig. 2). Although, in principle, a minimum of only two measurements with the same B and a different f are required, during trials it was found that five points are beneficial in improving the overall stability of the numerical procedure. Typically, a very good fit, with an r^2 in excess of 0.95, provided a and b and the later was used to compute the discrete values of k_{e} at set B.

The derivation and use of a single k_{e}, as a polynomial function of induction for the entire frequency range, concluded very large errors for w_{Fe} and success was achieved by splitting the data and performing the fitting separately on three frequency ranges, identified as low (up to 400 Hz), medium (400 to 1000 Hz) and high (Fig. 1). Accordingly, distinct third-order polynomial curves were obtained for k_{e} (Fig. 3).

Each of the three frequency intervals included five curves of core losses at constant f and variable B, so that at low values of induction five data points can be employed for fitting and at least a minimum of three points are available at higher values of induction. A total of fourteen power loss parametric data sets were considered in the example as the 400-Hz curve was specifically selected as an "overlap" (i.e., to be part of both the low and medium frequency range), in order to study the discontinuities of the model and to evaluate the potential of reducing the requirements for experimental data.

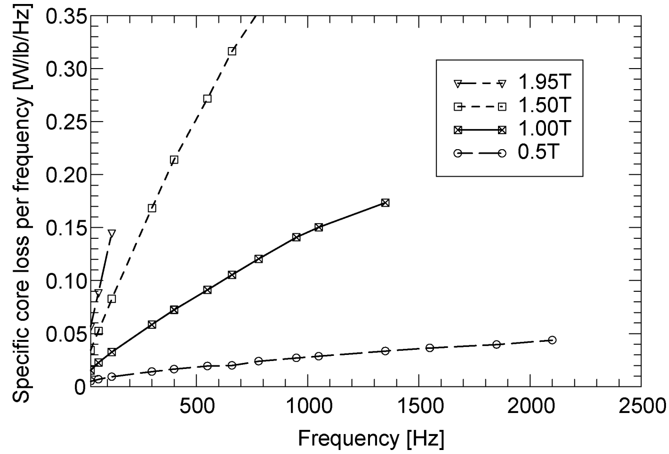

Fig. 2. Ratio of core loss and frequency w_{Fe}/f as per (2).

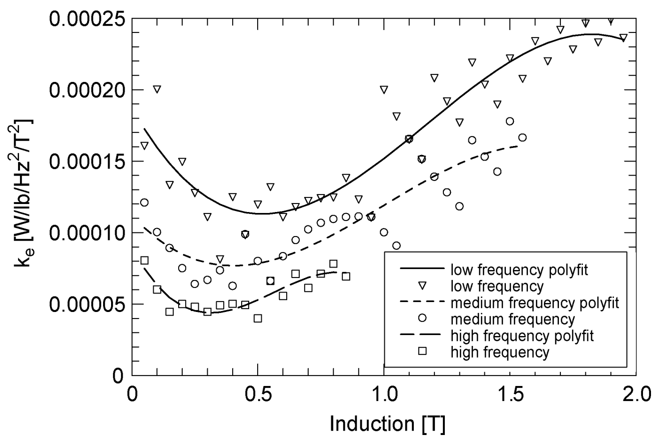

Fig. 3. Variation of the eddy-current loss component coefficient k_{e} with magnetic induction and frequency.

The values of the coefficient a were recalculated with (2) and the k_{e} polynomials

$$k_{\mathrm{e}} = k_{\mathrm{e}0} + k_{\mathrm{e}1} B + k_{\mathrm{e}2} B^2 + k_{\mathrm{e}3} B^3. \tag{3}$$

The hysteresis power coefficient α was also assumed to be a third order polynomial of B [5] and a logarithmic operator provided an equation

$$\log a = \log k_{\mathrm{h}} + (\alpha_0 + \alpha_1 B + \alpha_2 B^2 + \alpha_3 B^3) \log B \tag{4}$$

with five unknowns, k_{h}, α_0, α_1, α_2, α_3, which were solved by linear regression using all the available data from a given f and different B values.

As a result, at set constant frequency, α as a function of induction (Fig. 4), as well as a single-valued k_{h} (Fig. 5), were determined. The values of α, averaged for each frequency and plotted Fig. 5, are for information purpose only and for comparison with conventional models with fixed coefficients; their direct use, instead of the local values from Fig. 5, is not recommended as it would yield large numerical errors.

The previously described algorithm identifies variable coefficients for the mathematical model (1). More specifically, the hysteresis loss multiplicative coefficient k_{h} changes only with frequency, the hysteresis loss power coefficient α is a nonlinear function of both frequency and induction and the eddy-current

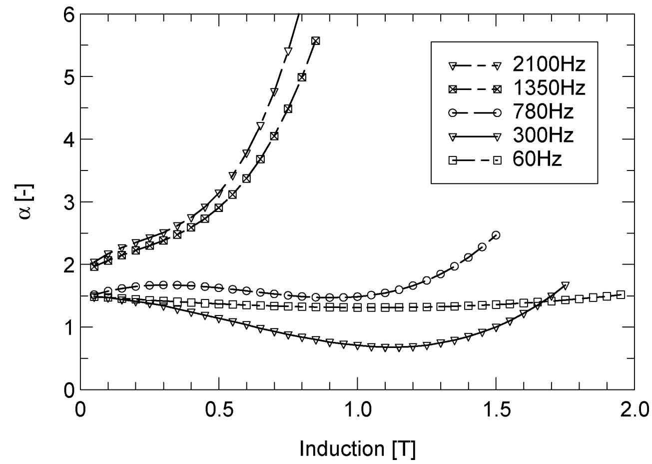

Fig. 4. Examples of the variation of the hysteresis loss power coefficient α with magnetic induction and frequency.

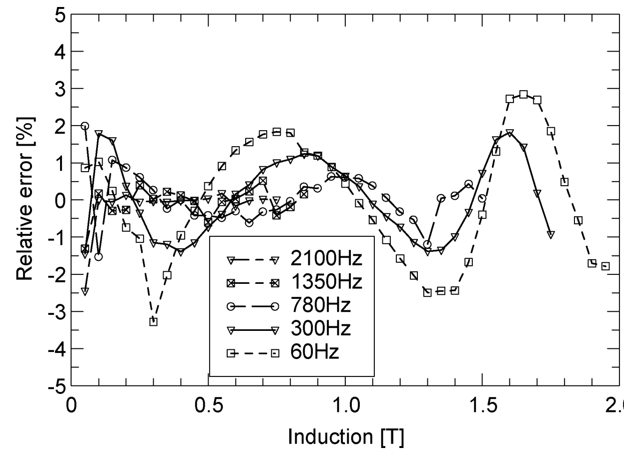

Fig. 6. Relative error between the calculated and the Epstein measured core losses at frequencies used in the numerical model identification.

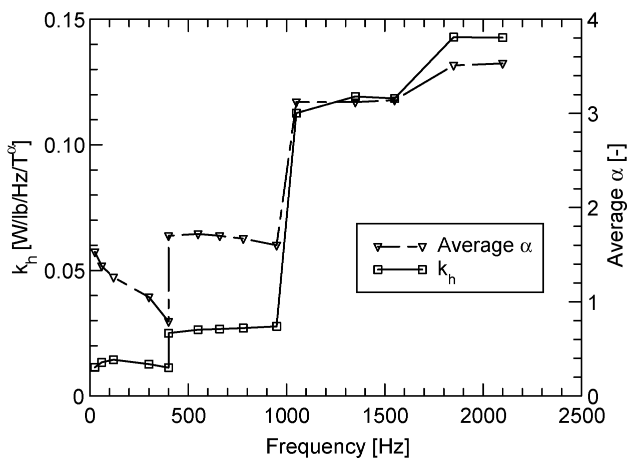

Fig. 5. Variation of the hysteresis loss multiplicative coefficient k_{h} and of the average α coefficient with frequency.

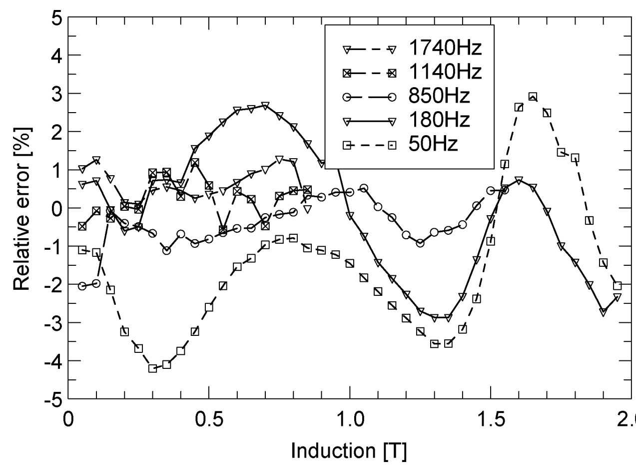

Fig. 7. Relative error between the calculated and the Epstein measured core losses at frequencies not used in the numerical model identification.

loss coefficient k_{e} is, within a set frequency range, dependent only of the induction.

IV. Model Validation and Discussions

The material model with variable coefficients can be used to calculate the power losses at the very same frequencies employed by the coefficient identification procedure. Examples of the relative errors between numerical and experimental data are plotted in Fig. 6 and are contained within a band of approximately plus and minus 3%.

As part of a validation process, errors were also calculated at other frequencies for which the experimental data was not considered in the model identification algorithm. In this case, the values of k_{e} were computed with a polynomial of B having coefficients dependent on the frequency range (Fig. 3). The hysteresis loss coefficient k_{h} was determined by linear interpolation from the curve of Fig. 5. Linear interpolation with frequency was used also used for each individual polynomial coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ at any set value of B. Again, the errors between the core losses estimated by (1) with variable coefficients and the measured data are within satisfactory limits for most practical purposes (Fig. 7).

In order to compare the new model with the conventional core loss model with constant coefficients, an example is provided. The classical eddy-current coefficient was computed with the well known formula

$$k_{\mathrm{e\_classic}} = \frac{\pi^2 \sigma \delta^2}{6\rho_V} \tag{5}$$

where σ is the material conductivity, δ is the lamination thickness and ρ_{v} is the volumetric mass density. The resultant value of $k_{\mathrm{e}} = 1.466 \times 10^{-4}$ W/lb/Hz^2/T^2, corresponds to different inductions on the low and mid frequency curves of Fig. 3 and it should be noted that in the new model k_{e} additionally incorporates the effect of anomalous losses.

Typical conventional values of α are equal to 1.6–2.2 and are comparable with those plotted in Fig. 5 for the low and medium frequency ranges of the new model. For a numerical comparison exercise, average values, one for each frequency range, were selected from the k_{h} and α curves of Fig. 5, on the understanding than any such choice would be debatable to an extent and that the physical explanation of the variability of the hysteresis loss coefficients eludes the best fit model.

The errors introduced by these constant coefficients can be very large indeed, as exemplified in Fig. 8. At low frequency the maximum absolute error exceeds 30% and the values increase significantly towards 100%, and above, at high frequency. Changing the fixed values of the coefficients would affect

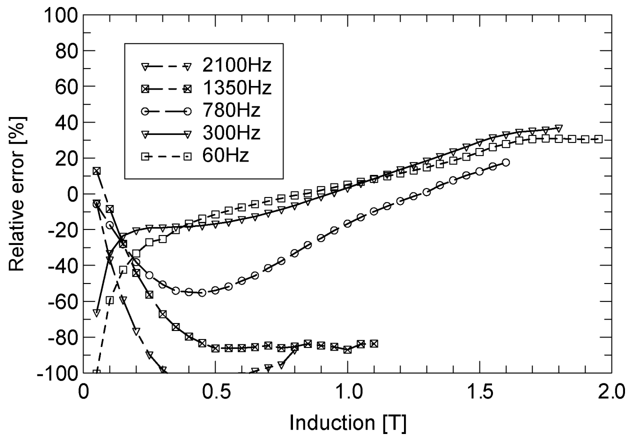

Fig. 8. Relative error between the values estimated with fixed value coefficients and Epstein measured core losses. The y-axis scale limits are twenty times larger than in Figs. 6–7.

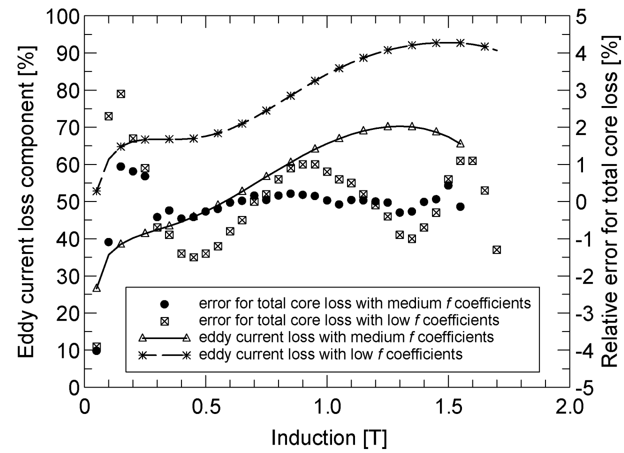

Fig. 9. Percentage of eddy current loss component and the relative errors between computed and experimental values of total core loss at the 400-Hz overlap between the low and medium frequency ranges.

the shape of the error curves, but multiple numerical trials combined with the logic supporting the variability of coefficients, as previously described, indicate there are no unique single-valued coefficients that would bring the maximum errors within an acceptable level over the entire range of induction and frequency.

The separation of loss components is of interest for several reasons. First, from a phenomenological point of view, it is important to asses, on one hand, the effect of the moving magnetic domains, which are associated with the hysteresis losses, and, on the other hand, the influence of the time varying magnetization on the eddy-current losses. Second, each of the loss components receives a different treatment in the analysis of electromagnetic devices [8], [9]. The detailed modeling of the minor hysteresis loops is beyond the scope of the newly described best fit model and to account for this effect a possible solution is the use of a correction factor as proposed for example in [8].

It is clearly shown in Figs. 3–5 that the loss coefficient curves are discontinued at the internal boundaries between the three frequencies intervals employed in the study. As mentioned before, the 400-Hz frequency was purposely selected as an overlap between the low and the mid frequency range. At this frequency, loses were calculated with the two different sets of coefficients and the results plotted in Fig. 9 indicate that, although the total losses are the same for all practical purposes, the separation of components yields different values for the split between eddy current and hysteresis losses.

Because the division between the frequency intervals is arbitrary, it can be concluded, on a more general basis, that the separation of loses according to (1) and with the described coefficient identification algorithms remains a challenge, which may be beyond the reach of a best fit mathematical model. Possible improvements, to be achieved, for example, through a preferred selection of frequency measurements, are under study.

V. Conclusion

The proposed core loss model has coefficients variable with induction and/or frequency and yields substantially smaller errors than a conventional version with fixed valued coefficients. The oscillating errors and the discontinuities identified recommend the use of a relative large amount of experimental data for model identification. Up to 15 power loss curves of constant frequency measured with a fine induction step of 0.05 T, and separated by as little as 35 Hz and as much as 300 Hz, were successfully employed. The specific core loss model described can be employed in conjunction with state of the art computational methods of the electromagnetic field, e.g., [4] and [5], and is readily applicable for the fast computational harmonic analysis of electromagnetic devices under variable voltage (flux) and frequency supply.

References

[1] I. D. Mayergoyz, *Mathematical Models of Hysteresis*. New York: Springer-Verlag, 1991.

[2] G. Bertotti, *Hysteresis in Magnetism*. New York: Academic, 1998.

[3] E. Della Torre, *Magnetic Hysteresis*. Piscataway, NJ: IEEE, 2000.

[4] H. Domeki *et al.*, "Investigation of benchmark model for estimating iron loss in rotating machine," *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 794–797, Mar. 2004.

[5] D. M. Ionel, M. Popescu, S. J. Dellinger, T. J. E. Miller, R. J. Heideman, and M. I. McGilp, "On the variation with flux and frequency of the core loss coefficients in electrical machines," *IEEE Trans. Ind. Appl.*, vol. 42, no. 3, pp. 658–668, May/Jun. 2006.

[6] J. R. Hendershot and T. J. E. Miller, *Design of Brushless Permanent-Magnet Motors*. Mentor, OH: Magna Physics, 1994.

[7] A. Boglietti, A. Cavagnino, M. Lazzari, and M. Pastorelli, "Predicting iron losses in soft magnetic materials with arbitrary voltage supply: An engineering approach," *IEEE Trans. Magn.*, vol. 2003, no. 2, pp. 981–989, Mar. 2003.

[8] J. D. Lavers, P. Biringer, and H. H. Hollitscher, "A simple method of estimating the minor loop hysteresis loss in thin laminations," *IEEE Trans. Magn.*, vol. 14, no. 5, pp. 386–388, Sep. 1978.

[9] K. Atallah, Z. Q. Zhu, and D. Howe, "An improved method for predicting iron losses in brushless permanent magnet DC drives," *IEEE Trans. Magn.*, vol. 28, no. 5, pp. 2997–2999, Sep. 1992.

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