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Damped Bloch Waves in Lattices Metamaterials with Inertial Resonators

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Abstract

The present paper is focused on the acoustic behaviour of periodic beam-lattices metamaterials containing inertial viscoelastic resonators connected with elastic slender ligaments. A simplified model is considered where the ligaments are considered as massless and the viscoelastic resonators are contained inside rigid rings located at the lattice nodes. Firstly, a Lagrangian model is formulated in order to assess the influence of the dynamic and viscoelastic properties of the resonators on the acoustic behaviour. An equivalent generalized micropolar model is obtained through a continualization of the discrete model and the constitutive tensors and the equation of motion are formulated. The propagation of harmonic waves is assumed and the Christoffel equation for both the discrete and the continuum model are obtained. It is shown that the hermitian matrix governing the Christoffel equation of the Lagrangian model is approximated by the corresponding one from the micropolar model with an error $O\left(|\mathbf{k}|^3\right)$.

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Keywords: Lattice metamaterials; inertial resonators; viscoelasticity; dispersive waves; Lagrangian model; generalized micropolar model

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1. Introduction

The propagation of elastic waves may be strongly affected by periodic arrangement of scatterers in the material microstructure. This has spurred many researches on new materials such as phononic crystals and metamaterials for the control of dispersive waves. In fact, the periodicity of the material microstructure may lead to destructive interferences (Bragg periodic scattering and Mie localized scattering) inducing attenuation of the amplitude of the travelling waves for some bands of frequencies called acoustic wave spectral gap or band gaps. In this respect, the complex band structure associated to damped Bloch waves in periodic materials is analysed in [1-3], a high-frequency homogenization in micropolar continua for chiral metamaterials have been proposed in [4] and optimal design of auxetic hexachiral metamaterials are investigated in [5,6].

In the present paper, a periodic beam-lattice metamaterial containing inertial viscoelastic resonators connected to elastic slender ligaments is formulated. The complex Floquet-Bloch spectrum is determined and the complex modes are identified. The real part band structure and its imaginary part characterize the attenuation and propagation modes of dispersive waves, respectively. Moreover, a high-frequency higher-order homogenization in micropolar continua is proposed. By approximating the ring displacements of the discrete model as a continuum field and through a continualization of the equation of motion of the discrete model, a generalised micropolar equivalent continuum is derived, together with the overall equation of motion and the constitutive equation. Finally, the validity limits of the generalized micropolar model are obtained by comparing the hermitian matrix of the Christoffel equation with the corresponding one from the discrete model.

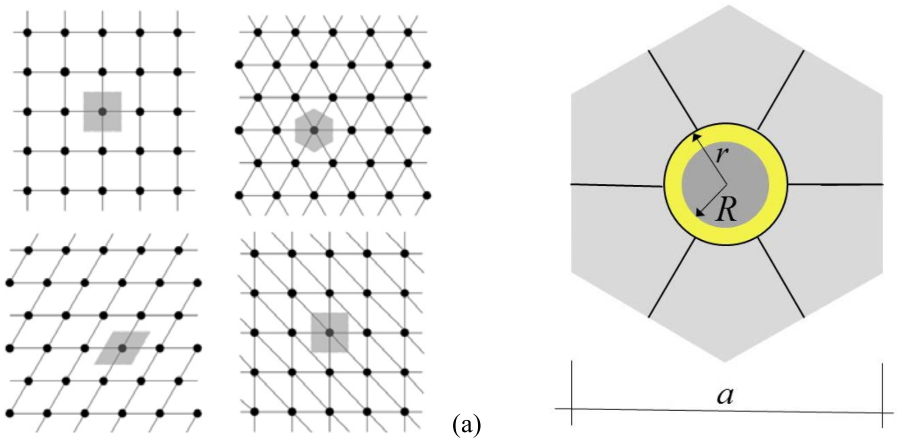

Fig. 1. (a) beam-lattices metamaterials and periodic cells; (b) inertial resonator.

2. Lagrangian model of the lattice metamaterial

Let us consider the 2-D quadrilateral and triangular beam-lattice metamaterials characterized by the periodic cells shown in Figure 1(a). Each cell is made up of a ring with mean radius r and n (=4,6) slender ligaments of length l, section width w and unit thickness, rigidly connected to the rings. A heavy disk with external radius R shown in Figure 1(b) (in dark grey), is located inside the ring through a soft viscoelastic annulus (in yellow). This inclusion plays the role of low-frequency resonator. The Young modulus of the ligaments is denoted by E_s, while the translational and the rotatory inertia of the rings are M_1 and J_1, respectively. The soft viscoelastic coating inside the resonator is characterized by the translational and rotational relaxation functions $k_d(t)$ and $k_\theta(t)$, respectively, while the translational and rotatory inertia mass density of the internal resonator are M_2 and J_2, respectively. The motion of the rigid ring is denoted by the displacement vector $\mathbf{u}$ and the rotation ϕ, respectively, while the motion of the internal resonator is denoted by the displacement vector $\mathbf{v}$ and the rotation θ. The resulting Lagrangian model is characterised by six Dofs per node and the equation of motion of the reference cell are written as a system of six integral-ordinary differential equations

$$E_s\left(\frac{w}{l}\right)\sum_{i=1}^{n}\left\{\left[(\mathbf{d}_i\otimes\mathbf{d}_i)+\left(\frac{w}{l}\right)^2(\mathbf{t}_i\otimes\mathbf{t}_i)\right](\mathbf{u}_i-\mathbf{u})-\frac{a}{2}\left(\frac{w}{l}\right)^2\mathbf{t}_i(\phi+\phi_i)\right\}+\int_{-\infty}^{t}k_d(t-\tau)\frac{d}{d\tau}(\mathbf{v}-\mathbf{u})d\tau-M_1\ddot{\mathbf{u}}=\mathbf{0}\,, \tag{1}$$

$$E_s\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\left\{\frac{a}{2}\mathbf{t}_i\cdot(\mathbf{u}_i-\mathbf{u})-\frac{a^2}{4}(\phi+\phi_i)+\frac{l^2}{12}(\phi_i-\phi)\right\}+\int_{-\infty}^{t}k_\theta(t-\tau)\frac{d}{d\tau}(\theta-\phi)d\tau-J_1\ddot{\phi}=0\,, \tag{2}$$

$$\int_{-\infty}^{t}k_d(t-\tau)\frac{d}{d\tau}(\mathbf{v}-\mathbf{u})d\tau+M_2\ddot{\mathbf{v}}=\mathbf{0}\,, \tag{3}$$

$$\int_{-\infty}^{t}k_\theta(t-\tau)\frac{d}{d\tau}(\theta-\phi)d\tau+J_2\ddot{\theta}=0\,, \tag{4}$$

where the unit vector $\mathbf{d}_i$ represents the i-th ligament orientation, $\mathbf{t}_i=\mathbf{e}_3\times\mathbf{d}_i$ is the unit vector normal to $\mathbf{d}_i$ and $\mathbf{u}_i$, ϕ_i are the displacement and the rotation of the adjacent i-th ring, respectively.

By applying the bilateral Laplace transform $\left(\mathcal{L}(\bullet)=\int_{-\infty}^{+\infty}(\bullet)e^{st}dt\right)$ to the equation of motion (1)-(4) and by imposing the Floquet-Bloch conditions in the Laplace space ($\mathcal{L}(\mathbf{u}_i)-\mathcal{L}(\mathbf{u})=\hat{\mathbf{u}}\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}-1\right]$, $\mathcal{L}(\phi_i)\pm\mathcal{L}(\phi)=\hat{\phi}\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}\pm1\right]$, with $\mathbf{x}_i$ the vector position of the centre of the i-th ring and $\hat{\mathbf{u}}$, $\hat{\phi}$ the displacement and rotation in Bloch-Laplace space) the generalized Christoffel equation is obtained in terms of the complex angular frequency s and the wave vector $\mathbf{k}$. It may be noted that the assumption of a bilateral Laplace transform is here justified under the assumption of a continuity prolongation of the integral kernel as a zero function in the negative t-time domain. The Floquet-Bloch spectrum $s(\mathbf{k})$ is obtained by solving the transcendental characteristic equations associated to the generalized Christoffel equation, from which six dispersive branches are obtained in the irreducible Brillouin zone. The real part $\mathrm{Re}\left[s(\mathbf{k})\right]$ and the imaginary part $\mathrm{Im}\left[s(\mathbf{k})\right]$ of the complex angular frequency characterize the attenuation and propagation modes of dispersive waves, respectively.

The equation of motion in the Laplace space may approximated by replacing the Laplace transform of the viscoelastic terms $k_d*(\dot{\mathbf{v}}-\dot{\mathbf{u}})$ and $k_\theta*(\dot{\theta}-\dot{\phi})$ ($*$ denoting the convolution product) with their first order Taylor polynomials $\mathcal{L}(k_d(t))s\mathcal{L}(\mathbf{v}-\mathbf{u})\approx\left(\hat{k}_d^0+\hat{k}_d^1s+O(s)\right)\mathcal{L}(\mathbf{v}-\mathbf{u})$ and $\mathcal{L}(k_\theta(t))s\mathcal{L}(\theta-\phi)\approx\left(\hat{k}_\theta^0+\hat{k}_\theta^1s+O(s)\right)\mathcal{L}(\theta-\phi)$ equivalent to that obtained by a discrete model with classical viscous damping [2, 3]. The governing equation in the Laplace space (or Christoffel equation) takes the simplified form in analogy to the case of classic damped discrete models:

$$E_s\left(\frac{w}{l}\right)\sum_{i=1}^{n}\left\{\left[(\mathbf{d}_i\otimes\mathbf{d}_i)+\left(\frac{w}{l}\right)^2(\mathbf{t}_i\otimes\mathbf{t}_i)\right]\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}-1\right]\hat{\mathbf{u}}-\frac{a}{2}\left(\frac{w}{l}\right)^2\mathbf{t}_i\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}+1\right]\hat{\phi}\right\}+\left(\hat{k}_d^0+\hat{k}_d^1s\right)(\hat{\mathbf{v}}-\hat{\mathbf{u}})+M_1s^2\hat{\mathbf{u}}=\mathbf{0}, \tag{5}$$

$$E_s\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\left\{\frac{a}{2}\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}-1\right]\mathbf{t}_i\cdot\hat{\mathbf{u}}-\frac{a^2}{4}\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}+1\right]\hat{\phi}+\frac{l^2}{12}\left[e^{i\mathbf{k}\cdot\mathbf{x}_i}-1\right]\hat{\phi}\right\}+\left(\hat{k}_\theta^0+\hat{k}_\theta^1s\right)\left(\hat{\theta}-\hat{\phi}\right)+J_1s^2\hat{\phi}=0\,, \tag{6}$$

$$\left(\hat{k}_d^0+\hat{k}_d^1s\right)(\hat{\mathbf{u}}-\hat{\mathbf{v}})+M_2s^2\hat{\mathbf{v}}=\mathbf{0}\,, \tag{7}$$

$$\left(\hat{k}_\theta^0+\hat{k}_\theta^1s\right)\left(\hat{\phi}-\hat{\theta}\right)+J_2s^2\hat{\theta}=0\,. \tag{8}$$

In the equivalent matrix form the Christoffel equation is written

$$\mathbf{C}_{Lag}(\mathbf{k},s)\hat{\mathbf{U}} = \begin{bmatrix} \mathbf{A}-s^2M_1\mathbf{I}_2 & \mathbf{a}^+ & -(\hat{k}_d^0+\hat{k}_d^1 s) & \mathbf{0} \\ \mathbf{a}^- & b-s^2J_1 & \mathbf{0} & -(\hat{k}_\theta^0+\hat{k}_\theta^1 s) \\ -(\hat{k}_d^0+\hat{k}_d^1 s) & \mathbf{0} & (\hat{k}_d^0+\hat{k}_d^1 s)-s^2M_2 & \mathbf{0} \\ \mathbf{0} & -(\hat{k}_\theta^0+\hat{k}_\theta^1 s) & \mathbf{0} & (\hat{k}_\theta^0+\hat{k}_\theta^1 s)-s^2J_2 \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{u}} \\ \hat{\phi} \\ \hat{\mathbf{v}} \\ \hat{\theta} \end{Bmatrix} = \mathbf{0}, \tag{9}$$

being

$$\begin{aligned} \mathbf{A} &= E_s\left(\frac{w}{l}\right)\sum_{i=1}^{n}\left\{\left[(\mathbf{d}_i\otimes\mathbf{d}_i)+\left(\frac{w}{l}\right)^2(\mathbf{t}_i\otimes\mathbf{t}_i)\right]\left[1-\cos(\mathbf{k}\bullet\mathbf{x}_i)\right]\right\}+\left(\hat{k}_d^0+\hat{k}_d^1 s\right)\mathbf{I}_2 \quad , \\ \mathbf{a}^+ &= i\frac{E_s a}{2}\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\sin(\mathbf{k}\bullet\mathbf{x}_i)\mathbf{t}_i \quad , \\ \mathbf{a}^- &= -i\frac{E_s a}{2}\left(\frac{w}{l}\right)^3\sin(\mathbf{k}\bullet\mathbf{x}_i)\mathbf{t}_i^T \quad , \\ b &= \frac{E_s a^2}{4}\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\left\{\left[1+\cos(\mathbf{k}\bullet\mathbf{x}_i)\right]+\frac{l^2}{3a^2}\left[1-\cos(\mathbf{k}\bullet\mathbf{x}_i)\right]\right\}+\left(\hat{k}_\theta^0+\hat{k}_\theta^1 s\right) \quad . \end{aligned} \tag{10}$$

The Floquet-Bloch spectrum $s(\mathbf{k})$ is obtained by solving the characteristic equation (9) from which six dispersive branches are obtained in the irreducible Brillouin zone. The real part $\mathrm{Re}\left[s(\mathbf{k})\right]$ and the imaginary part $\mathrm{Im}\left[s(\mathbf{k})\right]$ of the complex angular frequency characterize the attenuation and propagation modes of dispersive waves, respectively. It is worth to note that if a wave vector with real components is considered, the solution of the characteristic equation provides propagating modes and/or modes of temporal damping [7]. In the more general case of complex components of the wave vector, both spatial and temporal damping are considered [7,8].

3. Generalized micropolar continuum

An approximation to the description of motion of the discrete model is obtained by introducing continuous fields of displacement and rotation to describe the generalized displacement of rings and resonators. The displacement vector and the rotation of the ring of the *i*-th neighbouring cell may be approximated through a second-order Taylor expansion in terms of: i) the first and second macro-displacement gradient $\mathbf{H}=\nabla\mathbf{u}$ and $\nabla\mathbf{H}$; ii) the curvature $\boldsymbol{\chi}=\nabla\phi$ and the gradient of the curvature $\nabla\boldsymbol{\chi}$:

$$\begin{aligned} \mathbf{u}_i &\simeq \mathbf{u}+\mathbf{H}\mathbf{x}_i+\frac{1}{2}\nabla\mathbf{H}:(\mathbf{x}_i\otimes\mathbf{x}_i), \\ \phi_i &\simeq \phi+\boldsymbol{\chi}\cdot\mathbf{x}_i+\frac{1}{2}\nabla\boldsymbol{\chi}:(\mathbf{x}_i\otimes\mathbf{x}_i), \end{aligned} \tag{11}$$

By substituting the above expansion in the Laplace transform of the equation of motion of the discrete model (5)-(8), the macroscopic equation of motion in the Laplace domain is obtained as the governing equation of a generalized micropolar continuum (see also [5])

$$\begin{cases} div\left(\mathbb{E}_s\hat{\mathbf{\Gamma}}\right)+\left(\widehat{K}_d^0+\widehat{K}_d^1 s\right)\left(\hat{\mathbf{v}}-\hat{\mathbf{u}}\right)+\rho_1 s^2\hat{\mathbf{u}}=\mathbf{0} \ , \\ -\delta_{3jh}\left(\mathbf{e}_j\otimes\mathbf{e}_h\right):\mathbb{E}_s\hat{\mathbf{\Gamma}}+\mathbf{E}_s:\nabla\hat{\boldsymbol{\chi}}+\left(\widehat{K}_\theta^0+\widehat{K}_\theta^1 s\right)\left(\hat{\theta}-\hat{\phi}\right)+I_1 s^2\hat{\phi}=0 \ , \\ \left(\widehat{K}_d^0+\widehat{K}_d^1 s\right)\left(\hat{\mathbf{u}}-\hat{\mathbf{v}}\right)+\rho_2 s^2\hat{\mathbf{v}}=\mathbf{0} \ , \\ \left(\widehat{K}_\theta^0+\widehat{K}_\theta^1 s\right)\left(\hat{\phi}-\hat{\theta}\right)+I_2 s^2\hat{\theta}=0 \ , \end{cases} \tag{12}$$

being $\hat{\mathbf{\Gamma}}$ the Laplace transform of the non-symmetric micropolar strain tensor, $\widehat{K}_d^\alpha=\widehat{k}_d^\alpha/A_{cell}$ and $\widehat{K}_\theta^\alpha=\widehat{k}_\theta^\alpha/A_{cell}$ $(\text{with } \alpha=0,1)$ the overall constitutive parameters of the resonator, $\rho_1=M_1/A_{cell}$ and $\rho_2=M_2/A_{cell}$ the overall mass densities, $I_1=J_1/A_{cell}$ and $I_2=J_2/A_{cell}$ the micro-rotatory inertia terms $I_1=J_1/A_{cell}$ and $I_2=J_2/A_{cell}$ of the resonator. Finally, the fourth and the second order elastic tensors in equation (12) are given in the form

$$\mathbb{E}_s=\frac{E_s a^2}{2A_{cell}}\left(\frac{w}{l}\right)\sum_{i=1}^{n}\left[\left(\mathbf{d}_i\otimes\mathbf{d}_i\otimes\mathbf{d}_i\otimes\mathbf{d}_i\right)+\left(\frac{w}{l}\right)^2\left(\mathbf{t}_i\otimes\mathbf{d}_i\otimes\mathbf{t}_i\otimes\mathbf{d}_i\right)\right], \tag{13}$$

$$\mathbf{E}_s=-\frac{E_s a^4}{24A_{cell}}\left(\frac{w}{l}\right)^3\left[3-\left(\frac{l}{a}\right)^2\right]\sum_{i=1}^{n}\left(\mathbf{d}_i\otimes\mathbf{d}_i\right) \ . \tag{14}$$

The Christoffel equation (in matrix form) of the generalized micropolar continuum takes the same structure as the one of the Lagrangian model

$$\mathbf{C}_{Hom}\left(\mathbf{k},s\right)\hat{\mathbf{U}}=\begin{bmatrix} \mathbf{A}_{Hom}-s^2\rho_1\mathbf{I}_2 & \mathbf{a}_{Hom}^{+} & -\left(\widehat{K}_d^0+\widehat{K}_d^1 s\right) & \mathbf{0} \\ \mathbf{a}_{Hom}^{-} & b_{Hom}-s^2 I_1 & \mathbf{0} & -\left(\widehat{K}_\theta^0+\widehat{K}_\theta^1 s\right) \\ -\left(\widehat{K}_d^0+\widehat{K}_d^1 s\right) & \mathbf{0} & \left(\widehat{K}_d^0+\widehat{K}_d^1 s\right)-s^2\rho_2 & \mathbf{0} \\ \mathbf{0} & -\left(\widehat{K}_\theta^0+\widehat{K}_\theta^1 s\right) & \mathbf{0} & \left(\widehat{K}_\theta^0+\widehat{K}_\theta^1 s\right)-s^2 I_2 \end{bmatrix}\begin{Bmatrix}\hat{\mathbf{u}}\\ \hat{\phi}\\ \hat{\mathbf{v}}\\ \hat{\theta}\end{Bmatrix}=\mathbf{0} \ , \tag{15}$$

being

$$\begin{aligned} \mathbf{A}_{Hom}&=\frac{E_s a^2}{2A_{cell}}\left(\frac{w}{l}\right)\sum_{i=1}^{n}\left\{\left[\left(\mathbf{d}_i\otimes\mathbf{d}_i\otimes\mathbf{d}_i\otimes\mathbf{d}_i\right)+\left(\frac{w}{l}\right)^2\left(\mathbf{t}_i\otimes\mathbf{t}_i\otimes\mathbf{d}_i\otimes\mathbf{d}_i\right)\right]:\left(\mathbf{k}\otimes\mathbf{k}\right)\right\}+\left(\widehat{K}_d^0+\widehat{K}_d^1 s\right)\mathbf{I}_2 \ , \\ \mathbf{a}_{Hom}^{+}&=i\frac{E_s a^2}{2A_{cell}}\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\left(\mathbf{t}_i\otimes\mathbf{d}_i\right)\mathbf{k} \ , \\ \mathbf{a}_{Hom}^{-}&=-i\frac{E_s a^2}{2A_{cell}}\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\left(\mathbf{d}_i\otimes\mathbf{t}_i\right)\mathbf{k} \ , \\ b_{Hom}&=\frac{E_s a^2}{4A_{cell}}\left(\frac{w}{l}\right)^3\sum_{i=1}^{n}\left\{2-\frac{1}{2}\left[a^2-\frac{l^2}{3}\right]\left(\mathbf{d}_i\otimes\mathbf{d}_i\right):\left(\mathbf{k}\otimes\mathbf{k}\right)\right\}+\left(\widehat{K}_\theta^0+\widehat{K}_\theta^1 s\right) \ . \end{aligned} \tag{16}$$

The accuracy obtained by the continuum formulation may be appreciated by noting the following property of the hermitian matrix $\mathbf{C}_{Lag}\left(\mathbf{k},s\right)=A_{cell}\mathbf{C}_{Hom}\left(\mathbf{k},s\right)+O\left(\left|\mathbf{k}\right|^3\right)$, as already obtained for block lattice [9]. In the long wavelength limit $\lambda\to\infty$, namely $\left|\mathbf{k}\right|\to 0$, the complex frequencies obtained by the two spectral problems turn out to be coincident. In this case, the starting point of the two acoustic branches $\left(\left|\mathbf{k}\right|=0, s=0\right)$ is obtained by solving problem (15). It may be seen that in the neighborhood of such state, two propagative modes take place. In addition, four branch points are obtained from which four optical branches depart. In general, these branches may be representative of propagative waves with or without temporal and/or spatial damping.

4. Conclusion

A simplified model of periodic beam-lattice containing inertial resonators has been formulated to analyse the influence of the dynamic characteristics of the inertial resonators and of their viscoelastic constitutive parameters on the acoustic behaviour. The beam-lattices is made up of a periodic array of rigid heavy rings, each one connected to the others through elastic slender massless ligaments and containing an internal resonator made of a rigid disk in a soft viscoelastic annulus. A discrete Lagrangian model has been formulated involving the inertia of the lattice rings and the elasticity of the ligaments connecting the rings. The soft viscoelastic annulus is described through two relaxation functions. The equation of motion is formulated in the Laplace space under the simplifying assumption of linearized viscoelasticity. The Christoffel equation is derived, from which the complex Floquet-Bloch spectrum is obtained. The band structure of the lattice without resonators is characterized by two acoustical branches and an optical one. When considering the presence of inertial resonators, two acoustical branches and four optical branches characterize the band structure, the latter ones being representative of propagative waves with or without temporal and/or spatial damping.

By approximating the displacement and rotation of the rings of the discrete Lagrangian model as a continuum field, an equivalent generalized micropolar continuum has been derived through a continualization procedure of the Lagrangian governing equations. The overall equation of motion and the constitutive equation of the resulting generalized micropolar model having six degrees of freedom are given in closed form. The accuracy of the dispersive function obtained through the generalized micropolar model has been analyzed and it is shown that the hermitian matrix appearing in the Christoffel equation for the Lagrangian model is approximated by the corresponding one from the micropolar continuum model within an error $O\left(|\mathbf{k}|^3\right)$.

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