

**UNIVERSITI TEKNOLOGI MARA**

**ON POSITIVE SOLUTIONS FOR  
SINGULAR BOUNDARY VALUE  
PROBLEMS OF DIFFERENTIAL AND  
DIFFERENCE EQUATIONS**

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Thesis submitted in fulfillment  
of the requirements for the degree of  
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## AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged or referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any other degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

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## ABSTRACT

This thesis is concerned with the existence and multiplicity of positive solutions to singular boundary value problems (BVPs) of differential and difference equations. By using the Krasnoselskii fixed point theorem on compression and expansion in cone, sufficient conditions for the existence of positive solutions are established for a singular system of first-order differential equations and singular second-order BVPs of difference equations. Our results give an almost complete structure of the existence of positive solutions for the problems studied with an appropriately chosen parameter. By choosing appropriate cone, the singularity of the equations is essentially removed and the associated positive operator becomes well defined for certain ranges of functions even when  $e_i$  is negative. By employing the Krasnoselskii fixed point theorem in cone, the existence and multiplicity of positive periodic solutions for a singular system of first-order ordinary differential equations is established. As an extension, the discrete analogue of singular differential problems of second-order BVPs with a parameter is derived. The existence of positive solutions is obtained by applying the Krasnoselskii fixed point theorem in cone. The result is then extended to a singular discrete system of second-order two-point BVPs. Also the existence of positive solutions is investigated for a singular discrete system of second-order multi-point BVPs.

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