## **UNIVERSITI TEKNOLOGI MARA**

# MULTIASPECT SOFT SETS AND ITS GENERALIZATIONS

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Thesis submitted in fulfillment of the requirements for the degree of **Doctor of Philosophy** 

Faculty of Computer and Mathematical Sciences

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### **CONFIRMATION BY PANEL OF EXAMINERS**

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### **AUTHOR'S DECLARATION**

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and the results of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any degree or qualification.

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#### ABSTRACT

The theory of soft sets introduced in 1999 by Molodtsov is an alternative mathematical tool for dealing with uncertainties. It basically deals with information representations of objects characterized by parameters which are defined over a single common universal set. Combinations of the theory with fuzzy sets and interval-valued fuzzy sets have resulted in the so-called fuzzy soft sets and interval-valued fuzzy soft sets. Various theoretical studies on these theories and the variants have been made. and applications of the theories in various areas particularly in the area of decision making are continuously explored. Soft sets, fuzzy soft sets and interval-valued fuzzy soft sets have greater potential in information representation should the universe sets of elements not be restricted to only a common universal set. Real life situations may involve descriptions of objects, situations or entities based on certain characteristics or attributes which may be associated with different sets of elements of different types of universal sets. In this thesis, we introduce the concepts of multiaspect soft set (MASS), multiaspect fuzzy soft set (MAFSS) and multiaspect interval-valued fuzzy soft set (MAIVFSS) which are generalizations of soft sets, fuzzy soft sets and intervalvalued fuzzy soft sets, respectively. These concepts provide platforms for information representations that allow elements from different universal sets be taken into consideration in the description of a particular object, item or entity. MASS is defined for crisp data representation while MAFSS and MAIVFSS are respectively defined for fuzzy data representation with single and interval-valued membership degrees. For each concept, the set operations are established and the algebraic properties are studied. The concepts of mapping for multiaspect soft classes, multiaspect fuzzy soft classes and multiaspect interval-valued fuzzy soft classes are presented. In addition, we put forward the axiomatic definitions of distance, distance-based similarity measures and entropy for MAFSS and MAIVFSS. We introduce weighted and nonweighted distances and similarity measures based on the Hamming distance and the Euclidean distance. Relationships between the three measures are investigated. In the final part of the thesis, we highlight the applicability of some of the introduced concepts in solving group decision making problem under MAFSS and MAIVFSS environment.

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