DEVELOPMENT OF MATHEMATICAL MEANS FOR ESTIMATION OF ECOLOGICAL AND ECONOMICAL LOSSES FROM POLLUTION OF ATMOSPHERIC AIR IN ZONES OF TECHNOGENIC OBJECTS IMPACT

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ABSTRACT. – Development of mathematical means for estimation of ecological and economical losses from pollution of atmospheric air in zones of technogenic objects impact. The article describes the solution of one of the most important problems of rational use of natural resources. Modern mathematical tools for damage estimation, which are caused by atmospheric pollution to natural objects, and also methods for calculation of the cost for their renewal were developed. The solution of problem was divided into 3 stages. At the first stage it was defined basic anthropogenic sources of pollution, was illustrated conceptual behavior of pollutants in the atmosphere emitted by technological stationary point source. Choice of mathematical model that allows to determine the distribution of pollutants concentration in the air in zones of pollution by point stationary sources in the short-term discharges was proved. At the second stage it was developed mathematical tools to determine the level of objects damage, which were in the zone of pollution, depending on the intensity and duration of exposure of technogenic sources. At the third stage it was developed mathematical models to determine the recoverable amount of natural objects depending on their level of damage. Model example of developed means usage was described. Advantages of developed means over existed analogs were noticed.

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Key words: mathematical model, atmospheric air pollution, short-term discharge, damage of natural objects, economic damage.

1. INTRODUCTION

Clean atmospheric air – is one of the inviolable conditions of healthy life. However, energy-intensive, morally and physically old technological and environmental equipment, lack of treatment facilities and effective control over the activities of dangerous enterprises, low technological discipline, acute shortage of funds to ensure the normal operation of treatment facilities and structures lead to air basin pollution of population majority habitation. Pollution of surface layer of atmosphere (50-100 m) – is the most powerful, permanent factor of negative influence on people, food chain and the environment. Polluted air intensively impacts not only on humans and biota, but also on the hydrosphere, soil, vegetation, geological environment, buildings and other man-made objects. Therefore, protection of air and ozone layer is the highest priority problem of ecology; it is given attention to this problem in all developed countries (Troyanskyj and Dashkovskyj, 2004).

One of the important tasks of rational nature use is development of methods for assessment of damage magnitude for natural objects by polluted atmosphere, as well as methods for calculating the cost of their recovery.

2. LITERATURE REVIEW

Estimation of economic losses from environmental pollution has a great theoretical basis in economics and wide application with the greatest difficulty in definition of interdisciplinary types of losses (Ilicheva, 2005). In this article the interaction of economic and environmental indicators is considered, in other words economic indicators are changing due to changes in environmental parameters of air medium in zones of technogenic objects influence.

Despite on the constant improvement of methodical bases of quantitative determination of economic losses from environmental pollution, currently existing methods (Balatskiy, 1982, Byistrov et al., 1986, Hachaturov, 1987, Elahovskiy, 1988, Regulatory Document – 22/195, 1991, Golub and Strukova, 1995, Ryumina, 1999, Mkrtchan and Plyaskina, 2002, Ilicheva, 2005, Pahomova and Rihter, 2006, Bobyilev and Hodzhaev, 2010) have a number of disadvantages caused by the lack of quantitative dependencies of losses level of perception objects on the level of pollution. During calculation of economic losses such factors as intensity, consistency, duration of effects of pollution, processes of potential self-renewal of natural objects are not taken into account.

Since the concept of economic loss is the only measure of technogenic impact on the various sectors of society, its calculation requires a large set of input data, many of which are not fixed or are not accessible to formalization. Part of the social, moral, aesthetic and other losses that have some economic equivalent theoretically can be expressed using cost estimations, but it is far beyond the capabilities of modern economic system, therefore the estimated economic losses are always undervalued in relation to the actually existing (Ilicheva, 2005).

This article is dedicated to the problem solution of means development for estimation of environmental and economic losses from air pollution by technogenic objects, which do not have mentioned above disadvantages.

3. METHODS AND RESULTS OF INVESTIGATION

Solution of problem was divided into 3 stages:

1) determination of the level of air pollution depending on technogenic sources of emissions and weather conditions;
2) determination of the losses level of objects which are in the zone of pollution, depending on the intensity and duration of exposure of technogenic sources;
3) determination of cost for renewal of objects depending on their level of its losses.

3.1. Determination of the level of air pollution

Technogenic air pollution is result of practical human activity. Main sources of pollution (SP) connected with human activity are described at the fig. 1.

Investigations carried in work (Popov, 2010) showed that biggest amount of pollutants (P) are emitted by technogenic objects through chimneys, that are stationary point SP from the point of mathematical modeling view. Further material will be dedicated to such technological SP.

After getting into the air, character of such impurities (radioactive substances, toxic gas or aerosol) is determined by their own physical properties and the properties of the atmosphere in which they are located. Conceptual behavior of SP which emitted by technogenic stationary point source is shown on the fig. 2 (Popov, 2010).

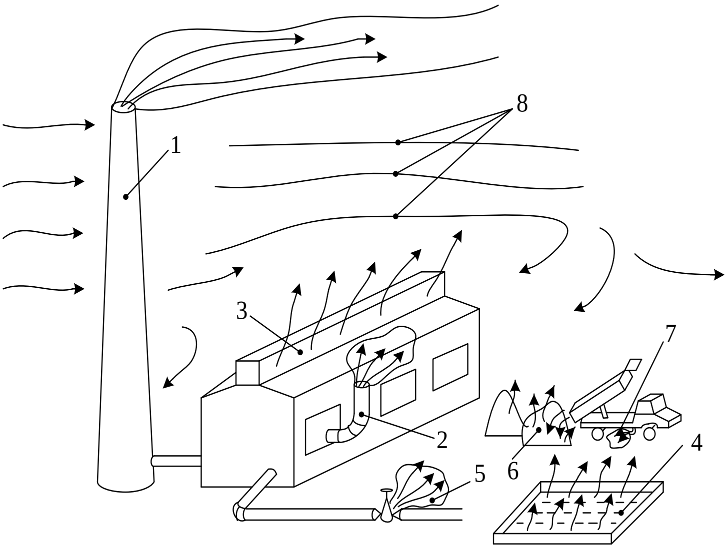

Figure. 1. Main technogenic SP of atmosphere:
1 – high chimney; 2 – low chimney; 3 – aeration flashlight of workshop; 4 – evaporation from the pool surface; 5 – leakages through unthickness of equipment; 6 – dust formation during unloading of bulk materials; 7 – car exhaust pipe; 8 – direction of air currents movement

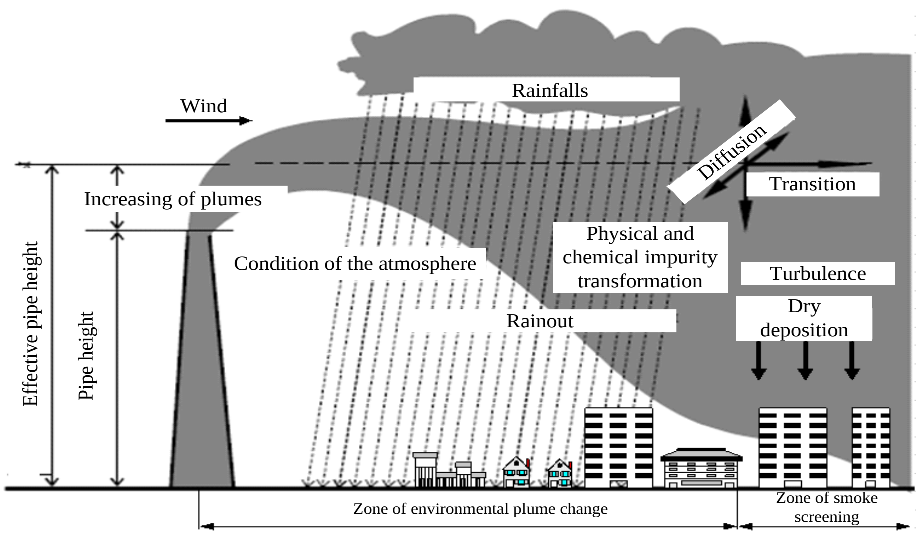

Figure. 2. Behavior of P in air medium in result of their emission by technogenic SP.

Determination of concentration distribution of P, that is emitted into the city atmosphere from technogenic SP, based on solution of mass-transfer equation in Cartesian coordinate system with certain initial and boundary conditions (Popov, 2010):

$$\frac{\partial C}{\partial t}+\frac{\partial uC}{\partial x}+\frac{\partial vC}{\partial y}+\frac{\partial\left(w-w_s\right)C}{\partial z}+\alpha C= \\ =\frac{\partial}{\partial x}k_x\frac{\partial C}{\partial x}+\frac{\partial}{\partial y}k_y\frac{\partial C}{\partial y}+\frac{\partial}{\partial z}k_z\frac{\partial C}{\partial z}+\sum_{i=1}^{n}Q_i\left(t\right)\delta\left(r-r_i\left(t\right)\right) \quad (1)$$

where $C\left(x, y, z, t\right)$ – impurities concentration, [g/m^3]; t - time, [sec]; u, v, w - components of wind field by the direction of axes x, y, z, [m/sec]; k_x, k_y, k_z – coefficients of turbulent diffusion, [m^2/sec]; w_s - velocity of gravitational deposition of particles of P, [m/sec]; α - coefficient that determines the change of pollution concentration with time due to the interaction with the environment, [1/sec]; Q_i - power of point SP, [g/sec]; $\delta\left(r-r_i\right)$ – Dirac delta function; $r_i=\left(x_i, y_i, z_i\right)$ – location of SP.

In general this equation is solved only by numerical methods, which in turn are long and complex processes. It does not allow for relevant authorities to obtain operatively necessary results of the distribution of contamination and to take fast effective management decisions to reduce the anthropogenic impact on the air. Therefore, in practice the mathematical models derived from the solution of equation (1) with the corresponding assumptions are used. Such models quickly allow to obtain desired result with error no more than 15-20%, which is acceptable for this class of problems.

In our case, we have considered the following formulation of the problem. Let at the initial time and initial zero concentration in point $\left(x_0, y_0, z_0\right)$ of infinite space $-\infty < x, y, z < \infty$ M gas units were emitted (created, exploded) instantly. Taking into account assumption that in equation (1) parameters u, v, w, k_x, k_y, k_z do not depend on coordinates, while $k_x=k_y=k_z=k$, so at wind availability and at interaction with environment distribution of P concentration will be described by following parabolic equation with certain initial and boundary conditions (Popov, 2010):

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}+\sigma C=k\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+M\delta\left(\left|r-r_0\right|\right)\delta\left(t\right), \quad (2)$$

initial conditions: $C\left(x, y, z, 0\right)=0$;

boundary conditions: $C\left(\left|r\right|\to\infty\right)\to 0$.

Solution (Popov, 2010):

$$C\left(x,y,z,t\right)=\frac{M}{8\sqrt{\left(\pi kt\right)^{3}}}\exp\left(-\frac{\left(x-ut-x_{0}\right)^{2}+\left(y-vt-y_{0}\right)^{2}+\left(z-wt-z_{0}\right)^{2}}{4kt}\right) \quad (3)$$

$$x\neq x_{0},\ y\neq y_{0},\ z\neq z_{0},\ t>0.$$

3.2. Determination of the losses level of objects

Pollution of environment leads to damage of biological and non-biological objects. Let consider object in point $\vec{r}\left(x,y,z\right)$, which during time $0\leq\tau\leq t$ was under the influence of pollutants with intensity $\varphi\left(\vec{r},t\right)=C\left(\vec{r},t\right)/C_{0}\left(\vec{r}\right)$, where $C_{0}\left(\vec{r}\right)-$ concentration of immediate death of object.

Works (Balatskiy, 1982, Hachaturov, 1987, Golub and Strukova, 1995) proposed to define level of object damage by scalar function of damage $0\leq\ddot{I}\left(\vec{r},t\right)\leq1$. If there is no damage $\ddot{I}=0$, if $\ddot{I}=1$ object is die (destroy).

Value of object damage $\ddot{I}\left(\vec{r},t\right)$ in the time moment *t* depends not only from intensity of influence in moment *t*, but on all history of influence on the gap $0\leq\tau\leq t$. To define magnitudes $\ddot{I}\left(\vec{r},t\right)$ linear functional $\varphi\left(\vec{r},t\right)$ may be used (Elahovskiy, 1988, Ryumina, 1999):

$$\ddot{I}\left(\vec{r},t\right)=\varphi\left(\vec{r},t\right)+\int_{0}^{t}H\left(t,\tau\right)\varphi\left(\vec{r},\tau\right)d\tau\,. \quad (4)$$

Function $H\left(t,\tau\right)$ is called nucleus of heredity and characterizes speed of “forgetness” in the time moment t about influence in moment τ. If object properties do not change (object is not growing), so nucleus $H\left(t,\tau\right)$ depends only on difference $t-\tau$:

$$\ddot{I}\left(\vec{r},t\right)=\varphi\left(\vec{r},t\right)+\int_{0}^{t}H\left(t-\tau\right)\varphi\left(\vec{r},\tau\right)d\tau\,. \quad (5)$$

First item in formulas (4), (5) characterizes «immediate» damage in time moment t from influence in this moment. Second item takes into account damage accumulated on time interval $0\leq\tau<t$.

Works (Elahovskiy, 1988, Ryumina, 1999, Mkrtchan and Plyaskina, 2002, Pahomova and Rihter, 2006) proposed to define damage of plant using exponential function like nucleus

$$H\left(t-\tau\right)=ce^{-\alpha\left(t-\tau\right)}, \quad (6)$$

where c, $\alpha-$ constants, which are defined experimentally.

In this case equation for damage calculation takes following form:

$$\ddot{I}\left(\vec{r},t\right)=\varphi\left(\vec{r},t\right)+c\int_{0}^{t}e^{-\alpha(t-\tau)}\varphi\left(\vec{r},\tau\right)d\tau . \qquad (7)$$

Expression (7) allows to take into account characteristics of behavior, which are character for biological objects that are able to fully or partially renewal after the termination of harmful effects.

Non-biological objects in contrast to biological objects can only accumulate damage caused by certain impulses of pollution, not depending on the time moment and order of action.

To describe damage of such object it might be used formula (4) with nucleus which depends only on one argument: $H\left(t,\tau\right)\equiv H\left(\tau\right)$, meaning formula

$$\ddot{I}\left(\vec{r},t\right)=\varphi\left(\vec{r},t\right)+\int_{0}^{t}H\left(\tau\right)\varphi\left(\vec{r},\tau\right)d\tau . \qquad (8)$$

Functions of damage for certain kinds of loaded elements of constructions in aggressive medium are proposed in works (Balatskiy, 1982, Byistrov et al., 1986, Golub and Strukova, 1995, Ryumina, 1999).

If in some time moment the level of object damage reaches 1 and object die (destroy), for this object in next time moments $\ddot{I}\left(\vec{r},t\right)=1$.

In the case of point short-term emission of pollutants with mass *M* relative intensity of pollution is

$$\begin{aligned}\varphi\left(\vec{r},t\right)&=\frac{C\left(x,y,z,t\right)}{C_0\left(x,y,z\right)}=\frac{M}{8\sqrt{\left(\pi k t\right)^3}C_0\left(x,y,z\right)}\times\\&\times\exp\left(-\frac{\left(x-ut-x_0\right)^2+\left(y-vt-y_0\right)^2+\left(z-wt-z_0\right)^2}{4kt}-\sigma t\right).\end{aligned} \qquad (9)$$

Substituting this function in expressions (4), (5), (7) or (8), it might be received dependency of object damage on time, emission volume, direction and speed of wind, coefficients of diffusion and other. Integrals in right sides of mentioned above expressions may be calculated numerically.

3.3. Determination of losses volume from pollution

Let denote $\rho_j\left(\vec{r}\right)$ – is surface density of *j* type objects allocation in point $\vec{r}$, then $\rho_j\left(\vec{r}\right)dS$ – number of such objects on the area dS.

Let $R_j\left(\ddot{I}\right)$ – is cost for renew of one *j*-type object with level of damage *П*. The total cost for object renew, located on the area *S* in time moment *t*, is defined by formula:

$$R_S(t) = \sum_j R_j(t) = \sum_j \iint_S R_j\left[\ddot{I}\ (\vec{r},t)\right]\rho_j(\vec{r})\,dS\ . \qquad (10)$$

In separate case when cost for renew is proportional to level of damage:

$$R_j\left[\ddot{I}\ (\vec{r},t)\right] = R_{0j}(\vec{r})\ddot{I}\ (\vec{r},t)\,, \qquad (11)$$

where $R_{0j}(\vec{r})$ – cost of undamaged object of j type, formula (10) is transformed into:

$$R_S(t) = \sum_j \iint_S R_{0j}(\vec{r})\ddot{I}\ _j(\vec{r},t)\rho_j(\vec{r})\,dS\ . \qquad (12)$$

If uniform objects on the area S are homogeneously distributed ($\rho(\vec{r}) = \rho_0 = const$), then formula (12) is transformed into

$$R_S(t) = R_0\rho_0 \iint_S \ddot{I}\ (\vec{r},t)\,dS\ . \qquad (13)$$

For practical calculations in formulas (10)-(13) it is convenient to change integrals with appropriate sums. Lets divide area S on n small squares in such manner to receive in i square sustainable damage of objects. Let ΔS_i $(i = 1,\ 2, \dots, n)$ – area, and $\vec{r}_i$ – radius-vector of i square; m_{ji} – number of objects of j type on i square with damage level $\ddot{I}\ _{ji}$.

Substituting double integral by sum in formula (10) total cost for renew of objects in time moment t is received:

$$R_S(t) = \sum_j R_j(t) \approx \sum_j \sum_{i=1}^{n} R_{ji}(t)\,m_{ji}\ , \qquad (14)$$

where $R_{ji}(t)$ – cost for renew of one j type object with damage level $\ddot{I}\ _{ji}$.

Formulas (10)-(14) allow to estimate economical expenditures on renew of environment in zones with different level of damage.

Application of developed mathematical means is considered on the next model example. Let in time moment $t = 0$ in point with coordinates $x_0 = 1$, $y_0 = 3$ (km) emission of P SO_2 took place in volume $M/C_0 = 0{,}01$.

Fig. 3-5 describe level of spruce damage for time moments $t = 10$, 30, 120 min, calculated by formula (7) with parameters $\alpha = 3{,}3 \cdot 10^{-6}$ sec^{-1}, $c = 5{,}5087$ sec^{-1}, and wind parameters $u = 4$ m/sec, $v = 0$ m/sec. Coefficients of diffusion $k = 0{,}001$ km^2/sec, characteristic of aerosol decomposition $\sigma = 1 \cdot 10^{-5}$ sec^{-1}. Size of area is 8×6 km.

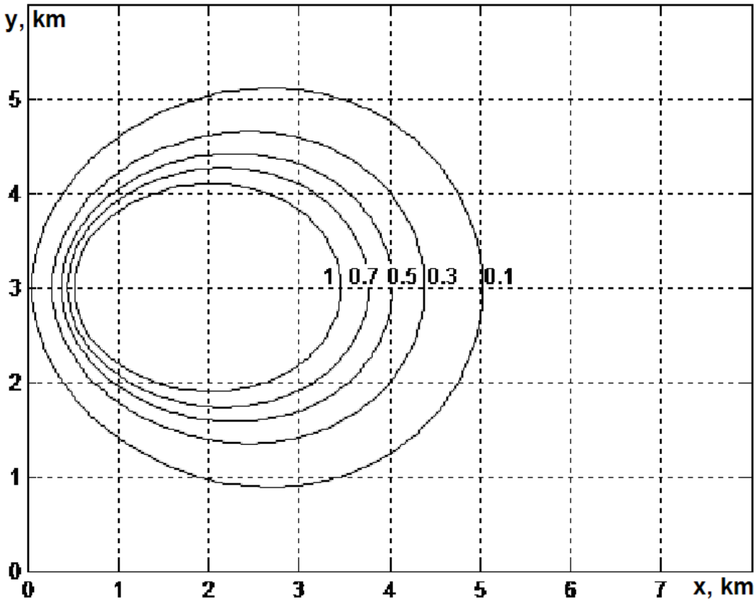

Figure. 3. Lines of function of damage level for t=10 min

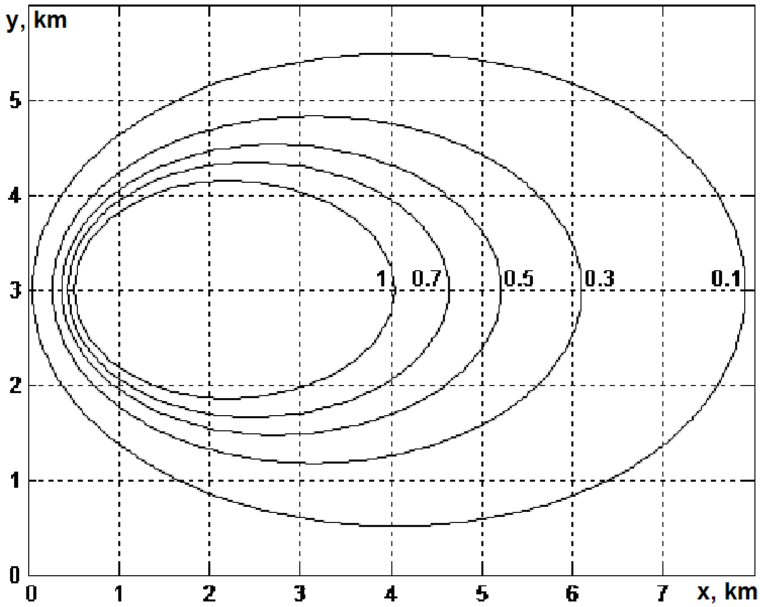

Figure. 4. Lines of function of damage level for t=30 min

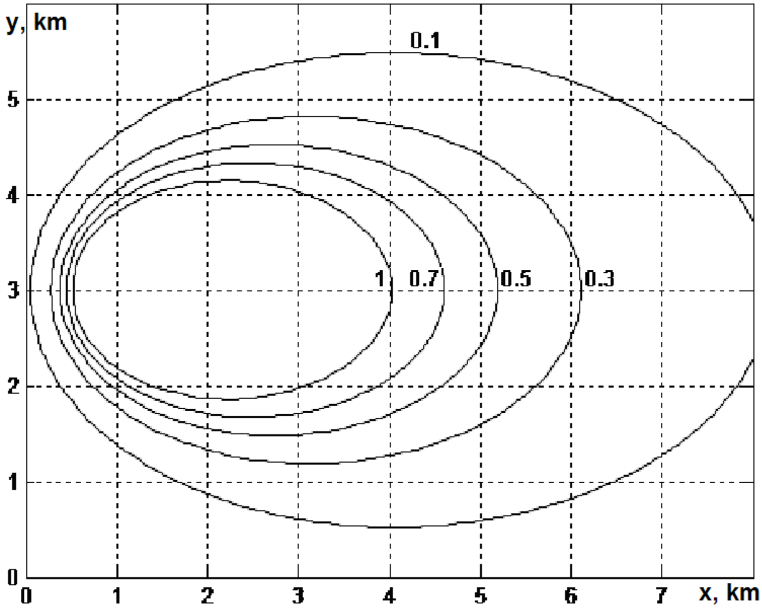

Figure. 5. Lines of function of damage level for t=120 min

On fig. 3-5 internal contour Ï $=1$ limits zone of plants death. With time changing this zone at the beginning increases, reaching maximum size, and then stays stable. Zones with damage Ï <1 reach maximum sizes and then in consequence of self-renew of plants after termination of harmful influence, decrease to contour with damage Ï $=1$.

Fig. 6 described dependencies of relative loss on time $R_S^*(t) = R_S(t)/(R_0\rho_0)$, calculated by formula (13), for square with size 8×6 km described on fig. 3-5 at different speed u of west wind $(v=0)$. In the case of wind absent $(u=0)$ it is assumed that SP is located in center of zone $x_0 = 4$, $y_0 = 3$ (km).

Table 1 described values of maximum relative loss $\max R_S^*(t)$ and time which is necessary to reach this maximum $t_{\max}$ for considered above zone at different speed of west wind. SP was located in point with coordinates $x_0 = 1$, $y_0 = 3$ (km) for all cases.

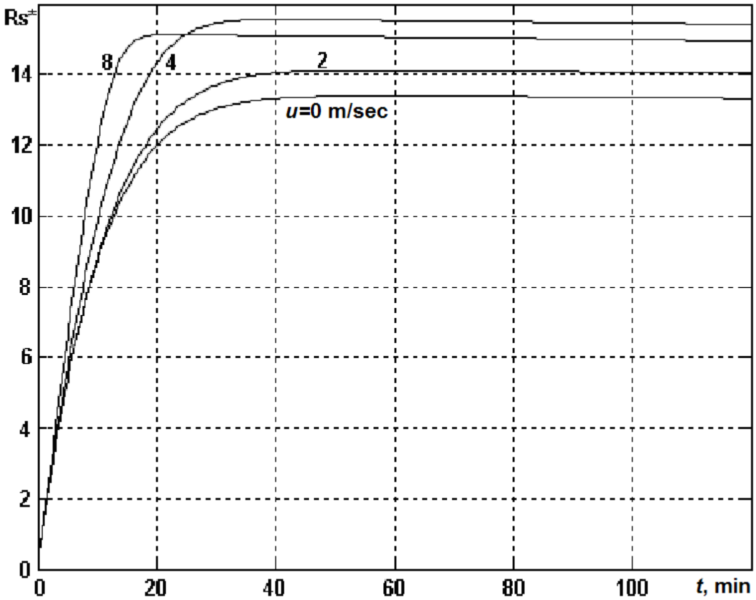

Figure. 6. Dependence of relative loss $R_S^*(t)$ on time t and wind velocity u

Table 1. Dependence of maximum relative loss in calculated zone on wind speed u

u, m/sec	0	2	4	8	10
$\max R_S^*(t)$	10,65	14,12	15,56	15,13	14,32
$t_{\max}$, min	58,5	55,5	41	22,5	18

4. CONCLUSIONS

In article mathematical model which allows to estimate level of object damage caused by short-term point emission of P in atmosphere, and also level of polluted objects damage and cost for their renew was proposed.

Proposed model has following advantages:

- it needs smallest expenditures to compare with determination of cost for renew of objects in different zones using measurement (expert estimation) of damage;
- it allows to describe zone allocation with different cost for renew of objects;
- it allows to estimate quickly and economically cost for renew in different zones and to predict cost at known mass of emission of P and atmospheric conditions;

- it allows to plan nature protection measures.

Model might be used at calculations of economic loss from objects pollution caused by short-term point emission of P in atmosphere.

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