Minimum fuel round trip from a L_2 Earth-Moon Halo orbit to Asteroid 2006 RH_120

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Abstract The goal of this paper is to design a spacecraft round trip transfer from a parking orbit to Asteroid 2006 RH_{120} , during its capture time by Earth's gravity, while maximizing the final mass or equivalently minimizing the delta-v. The parking orbit is chosen as a Halo orbit around the Earth-Moon L_2 libration point. The round-trip transfer is composed of three portions: a rendezvous transfer departing from the parking orbit to reach 2006 RH_{120}, a lock-in portion with the spacecraft following the asteroid orbit, and finally a return transfer to L_2. An indirect method based on the maximum principle is used for our numerical calculations. To partially address the issue of local minima, we restrict the control strategy to reflect an actuation corresponding to up to three constant thrust arcs during each portion of the transfer. The model considered here is the circular restricted four-body problem (CR4BP) with the Sun considered as a perturbation of the Earth-Moon circular restricted three body problem. A shooting method is applied to solve numerically this problem, and the rendezvous point to and departure point from 2006 RH_{120} are optimized using a time discretization of the trajectory of 2006 RH_{120}.

Key words: Asteroid 2006 RH_{120}, Sun perturbed Earth-Moon Bicircular Restricted Four Body Problem, Geometric optimal control, Shooting method.

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1 Introduction

In [1], the authors analyze statistically a new population of near Earth asteroids, namely the Temporarily Captured Orbiters (TCOs). They are classified by the following simultaneous conditions:

1. the geocentric Keplerian energy E_{planet} is negative;
2. the geocentric distance is less than three Earth's Hill radii ($3R_{H,\oplus} \sim 0.03$ AU);
3. the TCO makes at least one full revolution around the Earth in the Sun-Earth rotating frame.

In some of the literature they are also referred to as *minimoons* but we will use the terminology TCO here. The authors generated, pruned and integrated a very large random sample of "test-particles" from the Near Earth Object (NEO) population to calculate, among other statistics, the steady-state orbit distribution of the TCO population. Of the 10 million integrated test-particles, it was found that over 16,000 became TCOs. A consequence is that statistically, at any moment there is a one meter diameter TCO orbiting the Earth. The advantages presented by the TCOs for space missions have been discussed in several papers [1, 2, 3, 4] and we will not repeat the arguments here, but clearly it is what motivates our work. In particular, their vicinity to the Earth-Moon system is very attractive and their energy typically will help minimize the amount of thrust required by the spacecraft to reach them.

The orbits of the TCOs presented in [1] exhibit a wide range of behaviors, with capture duration going from a few weeks to a few months. In this paper, we focus on the only known TCO, namely 2006 RH_{120}. It is a few meters diameter wide asteroid and was discovered by the Catalina Sky Survey on September 2006. Its orbit from June 1st 2006 to July 31st 2007 is represented on Figures 1 (in E-M rotating frame) and 2 (in inertial frame), generated using the Jet Propulsion Laboratory's HORIZONS database which gives ephemerides for solar-system bodies. The period June 2006 to July 2007 was chosen to include the portion of the orbit during which the asteroid is considered as captured by the Earth's gravitation. We can observe that 2006 RH_{120} comes as close as 0.72 Earth-Moon distance from the Earth-Moon barycenter. This paper focuses on the design of a round trip minimum fuel transfer to 2006 RH_{120}.

We first assume the spacecraft hibernating on a periodic orbit awaiting detection of a TCO. Motivated by the successful Artemis mission and prior numerical simulations on the rendezvous transfer [5], we chose the hibernating orbit to be a Halo orbit around the Earth-Moon L_2 libration point with a z-excursion of 5000 km. This orbit is similar to the ones successfully used for the Artemis mission [6, 7]. The highest point in z-coordinate of this Halo orbit is $q_{HaloL_2} \approx (1.119, 0, 0.013, 0, 0.180, 0)$ and its period is $t_{HaloL_2} \approx 3.413$ normalized time units or 14.84 days. Figure 3 shows this Halo orbit in the EM rotating frame.

The round trip is composed of a rendezvous transfer to bring the spacecraft to 2006 RH_{120}, followed by a lock-in phase where the spacecraft travels with the asteroid and finally a return transfer to the hibernating orbit. Clearly, this optimization problem presents a very large set of variables including the departure time, the tar-

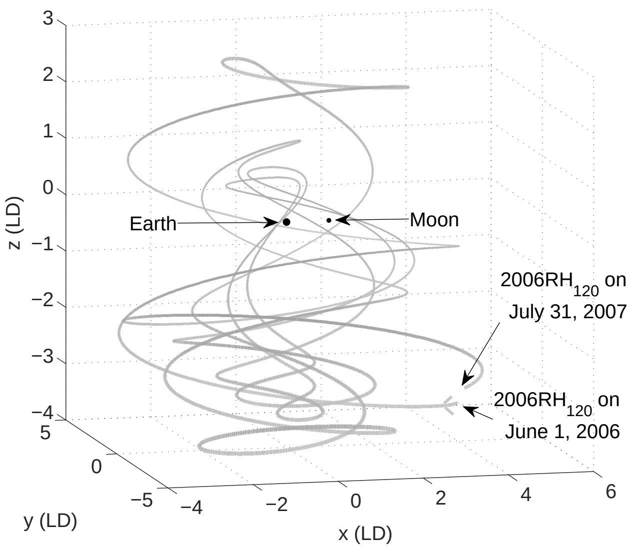

Fig. 1 *2006 RH_{120} orbit between June 1^{st} 2006 and July 31^{st} 2007 in the Earth-Moon rotating frame.*

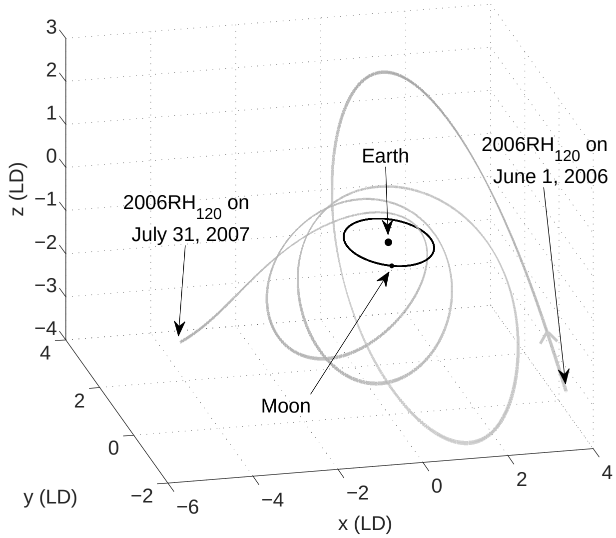

Fig. 2 *2006 RH_{120} orbit between June 1^{st} 2006 and July 31^{st} 2007 in an Earth's centered inertial frame.*

get rendezvous point, the lock-in duration on the asteroid and the return transfer duration. To simplify our approach we first decompose the round trip into a ren-

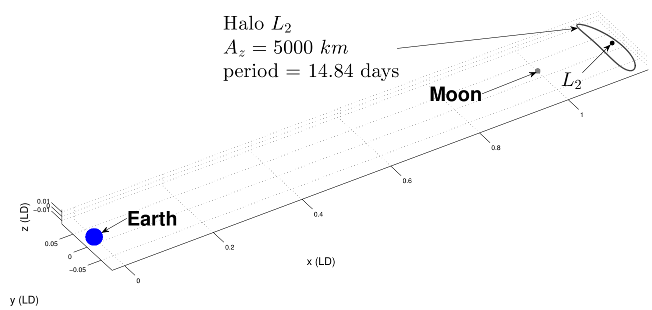

Fig. 3 *Parking Halo orbit around L_2 with a z-excursion of 5000 km, in the EM rotating frame.*

dezvous transfer and a return transfer that we address separately. Once these two optimization sub-problems are solved we will consider the global mission.

The first transfer is a rendezvous to a point on the 2006 RH_{120} orbit from a Halo orbit around the Earth-Moon L_2 libration point. The departing time of the spacecraft from the hibernating orbit depends on the detection time of the TCO, and even though it is a fixed value for 2006 RH_{120} since it was officially detected we will vary this parameter to analyze its impact on the fuel consumption. Additionally to a discretization of transfer duration, a discretization of the TCO orbit will be used to optimize the rendezvous point which will results in more than 5000 optimization problems to be solved.

The second optimization problem is the return transfer from 2006 RH_{120} to the L_2 libration point of the Earth-Moon system (rather than to the Halo orbit to reduce the number of calculations, it can be expanded easily). This problem will be solved with fixed transfer durations and we will study the influence of the departing point on the 2006 RH_{120} orbit and of the transfer duration on the fuel consumption. As for the rendezvous transfer, this produces more than 5000 return transfers to be calculated.

The global round trip will be analyzed based on the two sub-problems, the rendezvous transfer and the return transfer. We can connect the best transfers together in order to minimize the fuel consumption with the additional constraint that the return trip has to start after the forward trip ended. This will provide us information about the lock-in phase as well and its optimal duration for mission planning purpose.

In order to solve the optimization problems associated with our mission, we use indirect shooting methods [3, 4, 5, 12]. The main difficulty of these methods is the initialization of the algorithm and existence of numerous local minima. To partially reduce the number of local minima, we fix the control structure to be composed of at most three constant thrust arcs with 2 ballistic arcs in between. The reason to impose this control structure is twofold. First, preliminary calculations on a set of random TCOs with a free control structure for a similar control problem in [5] provided results mimicking impulse transfers with at most three impulses. In extremely rare instances adding a fourth switching or more would reduce the cost. Second, since the parking orbit is not a periodic orbit of the CR4BP we have to impose an initial impulse to leave the Halo orbit. Indeed, starting the transfer with a ballistic arc is extremely unlikely to be efficient or even possible since the time duration of the transfer is fixed. This very strongly suggests a strategy with one impulse to leave the hibernating orbit, a second one to redirect the spacecraft toward the rendezvous point and a final one to match the position and the velocity of the asteroid at rendezvous. We consider a chemical propulsion spacecraft with maximum thrust $T_{\max} = 22\ N$, specific impulse $I_{\text{sp}} = 230s$, and initial mass $m_0 = 350\ kg$.

The novelty of this work is at least threefold. First, the target object is a TCO and differs from the typically periodic orbits considered in the literature of minimum fuel transfers. Second, we consider synchronized transfers to produce a global round trip mission and add a practical detection constraint. Notice that the existence of an efficient round trip transfer also enables the possibility of a multiple rendezvous scenario with successives TCOs which would maximize the use of the spacecraft. Third, the calculation of all the possible rendezvous transfers with respect to departure time and rendezvous point and all possible return transfers with respect to the departure point on the TCO trajectory, makes this study very comprehensive rather than focused on a given rendezvous point or departure point from the asteroid. The trade-off of our work is the restriction to a specific three thrust arc control strategy.

As a final comment, we would like to emphasize that the techniques presented here can be applied to any TCOs. Asteroid 2006 RH_{120} was chosen as a test-bed TCO for our work to illustrate the algorithm since it is the only discovered one at this time. As of now exploratory work is being conducted with the synthetic TCOs calculated in [1], but in [2] the authors predicts that the Large Synoptic Survey Telescope (LSST) could detect about 1.5 TCOs/lunation, which amount to a dozen per year. This would provide ample population candidates for a real asteroid space mission.

The outline of the paper is as follows: Section 2 presents the equations of motion used as the dynamics of the optimal control problems. Section 3 gives the exact formulation of the two optimal control problems as well as the necessary conditions satisfied by the solutions of the problems. This Section also presents the numerical method used for the calculations. Section 4 provides the numerical results for the two optimal transfers and discusses the complete round trip problem. Finally we conclude on possible future works.

2 Equations of motion

During its mission, the spacecraft will stay in some vicinity of the Earth and Moon gravitational fields which suggests that the Earth-Moon circular restricted three body problem (CR3BP), see [8], for the equations of motion might provide a good approximation. In the CR3BP setting, the spacecraft is assumed to move in the gravitational fields of 2 primaries P_1 and P_2, of respective masses M_1 and M_2. In addition, the two primaries are assumed to follow a circular orbit around their barycenter and the spacecraft is considered to have negligible mass. A normalization to obtain a dimensionless system is introduced by setting the mass unit to $M_1 + M_2$, the unit of length as the constant distance between the two primaries and the unit of time so that the period of the primaries around their baryenter is 2π. This leads to the introduction of $\mu = M_1/(M_1 + M_2)$, the only parameter of the model. Table 1 gives numerical values of some of the parameters of the CR3BP model.

CR3BP parameters		Sun Perturbed parameters	
μ	0.01215361914	μ_S	$3.289 \cdot 10^5$
1 norm. dist. (LD)	384400 km	r_S	$3.892 \cdot 10^2$
1 norm. time	104.379 h	ω_S	-0.925 rad/norm. time

Table 1 Numerical values for the CR3BP and Sun Perturbed CR3BP.

Finally, we introduce a rotating reference frame, centered at the center of mass and so that the x-axis is oriented from P_1 to P_2. The y-axis of the rotating frame is taken orthogonal to the x-axis in the orbital plane of the 2 primaries and the z-axis completes the frame. In this reference frame, the potential energy of a spacecraft of position and velocity $q = (x, y, z, \dot{x}, \dot{y}, \dot{z})$ is given by

$$\Omega_3(x,y,z) = \frac{x^2+y^2}{2} + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} + \frac{\mu(1-\mu)}{2},$$

with ρ_1 (resp. ρ_2) the distance from the spacecraft to the first (resp. second) primary, that is

$$\rho_1 = \sqrt{(x-\mu)^2 + y^2 + z^3}, \quad \rho_2 = \sqrt{(x-1+\mu)^2 + y^2 + z^2}.$$

The uncontrolled motion of the spacecraft is then given by

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega_3}{\partial x}, \qquad \ddot{y} + 2\dot{x} = \frac{\partial \Omega_3}{\partial y}, \qquad \ddot{z} = \frac{\partial \Omega_3}{\partial z}. \tag{1}$$

It is well known that there exists 5 equilibrium points to this system, the so called Lagrange points L_1, L_2, L_3, L_4 and L_5. The points $L_{1,2,3}$ are distributed along the x-axis of the frame while L_4 and L_5 form an equilateral triangle with the two primaries in the xy-plane. We will focus on the libration point L_2 motivated by the existence of periodic orbits around this point that can be used as hibernating location for the spacecraft awaiting detection of a TCO. Notice that we could also choose L_1 and L_3 but preliminary computation suggested that L_2 is a better choice. Halo orbits around L_2 are periodic orbits that are isomorphic to circles, see [8] for the existence of and how to compute them.

Even though the TCO's orbit is in a vicinity of the Earth-Moon system during its capture it can be as far as 5 normalized distance from the CR3BP origin and preliminary calculations showed that efficient transfers might require for the spacecraft to go to an even further distance from the CR3BP origin to maximize the thrust impact on the spacecraft's motion. For this reason, to make our model more accurate, we use an extension of CR3BP as in [9]. We consider a Sun perturbed Earth-Moon CR3BP in which the Sun is assumed to follow a circular orbit around the Earth-Moon center of mass without modifying their circular orbits. In this case the potential energy of the spacecraft is $\Omega_4 = \Omega_3 + \Omega_S$ with

$$\Omega_S(x,y,z,\theta) = \frac{\mu_S}{r_S} - \frac{\mu_S}{\rho_S^2}(x\cos\theta + y\sin\theta), \tag{2}$$

where θ is the time dependent angular position of the Sun in the rotating frame, r_S is the constant distance from the Sun to the center of the reference frame, ρ_S is the distance from the spacecraft to the Sun ($\rho_S = \sqrt{(x-r_S\cos\theta)^2 + (y-r_S\sin\theta)^2 + z^2}$) and μ_S is the Sun's normalized mass ($\mu_S = M_{\text{Sun}}/(M_1+M_2)$). As the Sun is assumed to follow a circular orbit in the rotating frame, its angular position is $\theta(t) = \theta_0 + t\omega_S$ with ω_S the angular velocity of the circular orbit and θ_0 the angular position of the Sun at time 0. The equations of motion take the same form as in (1) but with the perturbed potential Ω_4 replacing Ω_3. The values of the new parameters are given in Table 1.

Since the spacecraft is equipped with thrusters, we assume they can produce a thrust of at most $T_{\max}$ Newton in any direction of $\mathbb{R}^3$. We introduce $u = (u_1,u_2,u_3) \in \bar{B}(0,1) \subset \mathbb{R}^3$ the thrust direction, and $m(\cdot)$ the mass of the spacecraft, the controlled equations of motion is an affine control system

$$\dot{q}(t) = F_0(q(t)) + \frac{\tilde{T}_{\max}}{m(t)}\sum_{i=1}^{3} F_i u_i(t) \tag{3}$$

where the drift is given by:

$$F_0(q) = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ 2\dot{y} + x - \frac{(1-\mu)(x+\mu)}{\rho_1^3} - \frac{\mu(x-1+\mu)}{\rho_2^3} - \frac{(x-r_S\cos\theta)\mu_S}{\rho_S^3} - \frac{\mu_S\cos\theta}{r_S^2} \\ -2\dot{x} + y - \frac{(1-\mu)y}{\rho_1^3} - \frac{\mu y}{\rho_2^3} - \frac{(y-r_S\sin\theta)\mu_S}{\rho_S^3} - \frac{\mu_S\sin\theta}{r_S^2} \\ -\frac{(1-\mu)z}{\rho_1^3} - \frac{\mu z}{\rho_2^3} - \frac{z\mu_S}{\rho_S^3} \end{pmatrix}, \tag{4}$$

and the vector field F_1 (resp. F_2 and F_3) is the vector of the canonical base e_4 (resp. e_5 and e_6) of $\mathbb{R}^6$. Here $\tilde{T}_{\max}$ is the maximum thrust expressed in normalized units. To complete the model, the mass decreases proportionally to the delivered thrust

$$\dot{m}(t) = -\frac{\tilde{T}_{\max}}{I_{\text{sp}}g_0}\|u(t)\|, \tag{5}$$

where I_{sp}, the specific impulse, and g_0, the gravitational acceleration at Earth's sea level, are parameters dependent on the considered thruster. For our numerical tests, we use

$$T_{\max} = 22\ N,\ I_{sp} = 230\ s,\ g_0 = 9.80665\ m/s^2,\ m_0 = 350\ kg.$$

Note however that with our fixed control structure, it is actually quite easy to change those parameters using a continuation to consider for instance a solar electric propulsion with a smaller $T_{\max}$ but a higher I_{sp}. Indeed, the main obstacle to a continuation that is based solely on the thruster parameters would be the possible change in the control structure since there is no smooth continuation path between a minimum fuel transfer with n switchings and another with $n \pm 1$ switchings.

3 Problem statement

The aim of this paper is to design minimum fuel transfer to 2006 RH_{120} from and back to an hibernating location of the spacecraft including a rendezvous period during which the spacecraft travels with the TCO. Since we want this round trip to take into account various synchronization constraints, it becomes complex when written as a single optimal control problem. To avoid this issue and obtain more general results on each portion of the global transfer, we decide first to decouple the rendezvous and return transfers. With our choice to solve these two problems for various departure times, rendezvous points, return departure times and return transfer durations, it then becomes straightforward to pair the rendezvous and return transfer using a lock-in phase between the spacecraft and the asteroid into a complete round transfer.

In this section, we introduce the rendezvous and return transfers as optimal control problems. We consider a synchronized rendezvous from a parking orbit to 2006 RH_{120}, whose orbit is shown in Figures 1 and 2. The second problem is the return transfer from 2006 RH_{120} to the Earth-Moon L_2 libration point. We also provide the necessary conditions for a control strategy and its associated trajectory to be optimal.

3.1 Rendezvous transfer

The objective of the first portion of the spacecraft round trip transfer is to rendezvous with 2006 RH_{120}. We introduce t_c to represent the capture time of the asteroid, corresponding to June 1^{st} 2006, and fix it as the origin of our mission time frame: $t_c = 0$. We make some assumptions for our calculations.

Assumption 3.1 *At the capture time t_c of asteroid 2006 RH_{120} the spacecraft is hibernating on a CR3BP-periodic Halo orbit around the Earth-Moon L_2 libration* *point, with a z-excursion of* 5000 *km. We fix the position and velocity of the spacecraft at t_c to be $q_{HaloL_2} \approx (1.119, 0, 0.013, 0, 0.180, 0)$ which corresponds to the z-highest point of the Halo orbit. This is an arbitrary choice and can be altered in future work.*

We denote by t_{start} the departing time of the spacecraft from the hibernating orbit. The position and velocity q_{start} of the spacecraft at t_{start} are determined as the result of the CR3BP uncontrolled dynamic, see eqs. (1), integrating from q_{HaloL_2} at $t_c = 0$ to t_{start} to guarantee the spacecraft departs from its correct location on the L_2-Halo periodic orbit. Our algorithm treats t_{start} as an optimization variable of the rendezvous problem and will discretize the departure time of the spacecraft to analyze its impact on the final mass. The detection time t_d^{RH} is a practical mission constraint and in section 4 we discuss how our results provide information regarding ideal windows of detection for 2006 RH_{120} corresponding to the best transfers. 2006 RH_{120} was actually discovered on September $14th$, 2006 but to gain insights on future studies with other TCOs we consider it as a possible parameter of the problem associated to the starting time of the mission.

The rendezvous point between the spacecraft and the asteroid is a position and velocity q_f^{rdv} on the 2006 RH_{120} orbit corresponding to a time $t_{\text{rdv}}^{RH} > t_{\text{start}}$. Our algorithm treats the rendezvous point as an optimization variable as well and includes a discretization of the 2006 RH_{120} orbit to analyze the impact of the rendezvous point on the fuel consumption. We also add as a constraint that $t_{\text{rdv}}^{RH} \leq$ July 31^{st} 2007 which is equivalent to say that the rendezvous must take place before the asteroid leaves the Earth gravitational field.

To reduce the complexity of the optimization problem we fix the structure of the thrust for the candidates trajectories to optimality. Our choice is motivated by the desire to mimic an impulse strategy with at most three boosts. One to depart from the Halo orbit, one to redirect the spacecraft to the rendezvous point on 2006 RH_{120} orbit and one to match the position and velocity of the spacecraft and asteroid at the end. Prior numerical calculations have shown that such strategy provides good fuel efficient transfers. This structure will be assume for the return portion of the round trip transfer as well.

Assumption 3.2 *For our transfers, we restrict the thrust strategy $u(\cdot) : [t_{\text{start}}, t_{\text{rdv}}^{RH}] \to \bar{B}(0,1) \subset \mathbb{R}^3$, i.e. the control, to have a piecewise constant norm with at most three switchings:*

$$\|u(t)\| = \begin{cases} 1 & \text{if } t \in [t_{\text{start}}, t_1] \cup [t_2, t_3] \cup [t_4, t_{\text{rdv}}^{RH}] \\ 0 & \text{if } t \in (t_1, t_2) \cup (t_3, t_4) \end{cases}, \tag{6}$$

where t_1, t_2, t_3, t_4 are called the switching times and satisfy $t_{\text{start}} < t_1 < t_2 \leq t_3 < t_4 < t_{\text{rdv}}^{RH}$. We denote by $\mathscr{U}$ the set of measurable functions $u : [t_{\text{start}}, t_{\text{rdv}}^{RH}] \to B^3(0,1)$ satisfying (6) *for some switching times $(t_{\text{start}}, t_1, t_2, t_3, t_4, t_{\text{rdv}}^{RH})$.*

In other words, we impose a control strategy with at most three thrust arcs and two ballistic arcs.

We denote by $\xi = (q, m)$ the state and by $f(t, \xi, u)$ the controlled Sun perturbed CR3BP dynamics (3) including the mass evolution (5). The optimal control problem is now written as follows:

$$(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}} \begin{cases} \min_{t_1,t_2,t_3,t_4,u(\cdot)} \int_{t_{\text{start}}}^{t^{RH}_{\text{rdv}}} \|u(t)\| dt \\ s.t.\ \dot{\xi}(t) = f(t,\xi(t),u(t)),\ a.e.\ t \in [t_{\text{start}}, t^{RH}_{\text{rdv}}] \\ \xi(t_{\text{start}}) = (q_{\text{start}}, m_0) \\ q(t^{RH}_{\text{rdv}}) = q^{RH}_{\text{rdv}} \\ t_{\text{start}} < t_1 < t_2 \leq t_3 < t_4 < t^{RH}_{\text{rdv}} \\ u(.) \in \mathscr{U} \end{cases} \tag{7}$$

To analyze the impact of our variables on the fuel consumption a direct optimization with respect to $(t_{\text{start}}, t^{RH}_{\text{rdv}})$ in addition to the other parameters $(t_1,t_2,t_3,t_4,u(\cdot))$ would produce a large number of local extrema since our approach is a variational one. To address this issue, we discretize the set of departure and duration time of the rendezvous transfer $(t_{\text{start}}, t^{RH}_{\text{rdv}})$ and solve $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$ for the finite number of discretized combinations. This produces an approximation of the optimal transfer with respect to $(t_{\text{start}}, t^{RH}_{\text{rdv}})$ and $(t_1,t_2,t_3,t_4,u(\cdot))$ that becomes more accurate as the discretization on $(t_{\text{start}}, t^{RH}_{\text{rdv}})$ is refined. We use a 15 days discretization of t_{start}, from June 1st 2006 to 360 days later and a one day discretization on t^{RH}_{rdv} from June 1st 2006 to July 31st 2007, with the additional constraint that $t^{RH}_{\text{rdv}} > t_{\text{start}}$. Note that by fixing the departure and duration time we fix the rendezvous point q^{RH}_{rdv}. Finally, as it seems unlikely that a very short transfer would produce a reasonable fuel consumption, we add a constraint for the transfer duration $t^{RH}_{\text{rdv}} - t_{\text{start}}$ to be greater than 7 days. Our choice of discretization leads to 5975 different $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$ to be solved.

To summarize our optimization algorithm, here is an example of how we would chose the best rendezvous transfer based on a given detection time t^{RH}_d, say September 14th 2006. Notice that after the discovery of the TCO, the calculation of its orbit to obtain a high enough accuracy to permit a rendezvous is not immediate and require some period of time. In other words, practically we have a constraint that is expressed as $t_{\text{start}} > t^{\text{RH}}_d + t_{calc}$ where t_{calc} is the time to run the TCO's orbit calculation. However, preliminary orbits obtained rapidly after detection are interesting to produce preliminary mission scenarios and therefore should not be neglected. For this reason, our algorithm allows to merge the detection time and the departure of the spacecraft from the hibernating orbit.

Step 1: Solve $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{\text{RH}}_{\text{rdv}}}$ for all t_{start} and $t^{\text{RH}}_{\text{rdv}}$ satisfying:

(i) $t_{\text{start}} \in [\![t_c, t_c + 360 \text{ days}]\!]$ by step of 15 days
(ii) $t^{\text{RH}}_{\text{rdv}} \in [\![t_{\text{start}} + 7 \text{ days, July } 31^{\text{st}}\ 2007]\!]$ by step of one day.

Step 2: Select the $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{\text{RH}}_{\text{rdv}}}$ with the best final mass among the ones with $t_{\text{start}} \geq$ September 14th 2006.

The first step is done without any consideration for the detection time and therefore is only performed once. Step 2 is an instantaneous step as it only needs to do a direct comparison of the results of Step 1. Moreover, note that once Step 2 is performed, it is always possible to locally refine the discretization of t_{start} and $t_{\text{rdv}}^{\text{RH}}$ around the selected values, in order to improve the final mass.

3.2 Return transfer

After the rendezvous transfer, the spacecraft will drift with the asteroid in a lock-in configuration. The optimal duration of this drift will be determined as we combine the rendezvous transfer with the return one. In this section we focus on the return transfer, it is the portion of the transfer that see the spacecraft departing from the 2006 RH_{120} orbit to return to a neighborhood of the Earth-Moon system. For simplicity, we chose to aim directly at the Earth-Moon L_2 libration point rather than the Halo orbit. Returning to the Halo orbit will involve a very significant higher number of calculations and the difference in fuel consumption would be minimal but it should be a topic for further study. The position and velocity of the L_2 point are given by $q_{L_2} \approx (1.15569383, 0, 0, 0, 0, 0)$.

We denote by q_{start}^{RH} the departing point of the return transfer on the 2006 RH_{120} orbit. It is associated to a departure time that we introduce as $t_{\text{start}}^{\text{RH}}$. Clearly, the spacecraft can depart the asteroid orbit only after its rendezvous which implies that $q_{\text{start}}^{RH} > q_{\text{rdv}}^{RH}$. Here again we assume the control $u(.)$ to be admissible if it consists of three thrust arcs and two ballistic ones, i.e. $u(.) \in \mathscr{U}$. The final time of the return transfer is denoted by t_f and satisfies $t_f > t_{\text{start}}^{\text{RH}}$. The optimal control for the return transfer is now:

$$(OCP)_{t_{\text{start}}^{\text{RH}}, t_f}^{\text{return}} \begin{cases} \min_{t_1, t_2, t_3, t_4, u(\cdot)} \int_{t_{\text{start}}^{\text{RH}}}^{t_f} \|u(t)\| dt \\ s.t.\ \dot{\xi}(t) = f(t, \xi(t), u(t)),\ a.e.\ t \in [t_{\text{start}}^{\text{RH}}, t_f] \\ \xi(t_{\text{start}}^{\text{RH}}) = (q_{\text{start}}^{RH}, m_0^{\text{RH}}) \\ q(t_f) = q_{L_2} \\ t_{\text{start}} < t_1 < t_2 \leq t_3 < t_4 < t_f \\ u(.) \in \mathscr{U} \end{cases} \tag{8}$$

Remark 3.1 *The initial mass m_0^{RH} is obtained from the final mass m_f^{rdv} of the rendezvous transfer portion. Depending on the specifics of the mission when the spacecraft is locked-in with the asteroid, we have that m_0^{RH} is equal to, less than (if the mission leaves some equipment or consumes some fuel) or greater than (if the mission brings back samples for example) m_f^{rdv}. We don't expect m_0^{RH} to play a large role in the fuel consumption, therefore for simplicity we chose to set it to 300 kg, that is 50 kg less than the mass m_0 of the spacecraft at the beginning of the rendezvous transfer.*

As for the rendezvous transfer, we discretize the optimization variables to study their impact on the fuel consumption. We also use a discretization of $(t_{\text{start}}^{\text{RH}}, t_f - t_{\text{start}}^{\text{RH}})$ and solve $(OCP)_{t_{\text{start}}^{\text{RH}}, t_f}^{\text{return}}$ for all the pairs of this discretization. The discretization on $t_{\text{start}}^{\text{RH}}$ is the same as on t_{rdv}^{RH}, so a step of one day from June 1st 2006 to July 31st 2007, while the discretization on the transfer duration $t_f - t_{\text{start}}^{\text{RH}}$ is with a step of 30 days from 30 to 360 days. This leads to 5112 different $(OCP)_{t_{\text{start}}^{\text{RH}},t_f}^{\text{return}}$. At the end, when analyzing the complete round transfer there will be a constraint on the relation between the rendezvous time and the departing time for the return mission but to gain insight on the problem we proposed to decouple them completely at first.

Figure 4 gives a schematic explanation of the whole round trip.

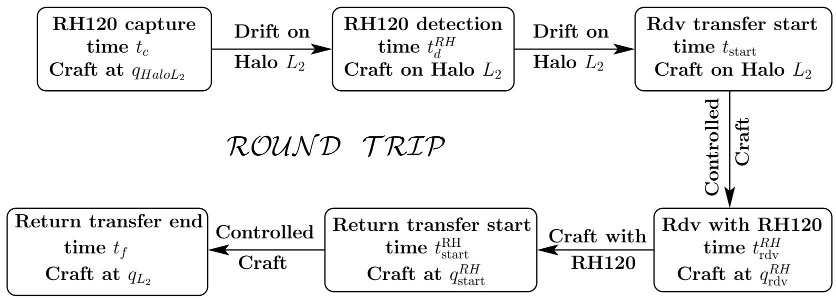

Fig. 4 *Schematic explanation of the chronology of the round trip transfer.*

3.3 Necessary conditions for optimality

The maximum principle, see [11], provides first order necessary conditions for a control and associated trajectory to be optimal. In this section we apply the maximum principle to our optimization problems.

Let us first focus on the rendezvous transfer. We denote by $\xi(t) = (q(t), m(t)) \in \mathbb{R}^6 \times \mathbb{R}_+$ the state of $(OCP)_{t_{\text{start}},t_{\text{rdv}}^{RH}}^{\text{rdv}}$, with $q(t) = (r(t), v(t))$ the position and velocity of the vehicle and $m(t)$ its mass, at time t. For $(OCP)_{t_{\text{start}},t_{\text{rdv}}^{RH}}^{\text{rdv}}$, the maximum principle introduces an adjoint state $(p^0, p_\xi(\cdot))$ defined on $[t_{\text{start}}, t_{\text{rdv}}^{RH}]$ and the so called Hamiltonian H defined by

$$H(t, \xi(t), p^0, p_\xi(t), u(t)) = p^0\|u(t)\| + \left\langle p_\xi(t), \dot{\xi}(t) \right\rangle, \text{ for a.e. } t \in [t_{\text{start}}, t_{\text{rdv}}^{RH}], \quad (9)$$

where $\langle,\rangle$ is the standard inner product. One of the condition of the maximum principle is that the optimal control maximizes the Hamiltonian. This maximization gives directly that the optimal control must be a multiple of the vector $p_v(\cdot)$ which translates into the following condition:

$$u(t) = \|u(t)\| \frac{p_v(t)}{\|p_v(t)\|}, \text{ for a.e. } t.$$

Without any constraint on the structure of the control, the maximization of the Hamiltonian leads to the definition of the switching function ψ

$$\psi(t) = p^0 + T_{\max}\left(\frac{\|p_v(t)\|}{m(t)} - \frac{1}{I_{\text{sp}}g_0}p_m(t)\right), \tag{10}$$

and the sign of $\psi(t)$ would give either $\|u(t)\| = 1$ if $\psi(t) > 0$ or $\|u(t)\| = 0$ is $\psi(t) < 0$. However, since in $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$ the control structure is constrained to have at most three maximum thrust arcs, a rewriting of the optimal control problem following a similar approach than in [13], implies that the switching function cancels at the constrained switching times but does not prescribe $\|u(t)\|$ anymore. The following theorem gives all the necessary conditions obtained from the maximum principle applied to $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$.

Theorem 3.1 *If $(q(\cdot),m(\cdot),u(\cdot)) : [t_{\text{start}},t^{RH}_{\text{rdv}}] \to \mathbb{R}^7 \times B^3(0,1)$ and $(t_1,t_2,t_3\ ,t_4) \in \mathbb{R}^4_+$ is an optimal solution of $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$, then there exists an absolutely continuous adjoint state $(p^0, p_\xi(\cdot)) = (p^0, p_r(\cdot), p_v(\cdot), p_m(\cdot)) \in \mathbb{R}^- \times \mathbb{R}^7$ defined on $[t_{\text{start}},t^{RH}_{\text{rdv}}]$ and such that:*

(a) $(p^0, p_\xi(\cdot)) \neq 0$, $\forall t \in [t_{\text{start}},t^{RH}_{\text{rdv}}]$, and $p^0 \leq 0$ is constant.
(b) The state and adjoint state satisfy the Hamiltonian dynamics:

$$\begin{aligned}\dot{\xi}(t) &= \frac{\partial H}{\partial p_\xi}(t,\xi(t),p^0,p_\xi(t),u(t)),\ \textit{for a.e. } t \in [t_{\text{start}},t^{RH}_{\text{rdv}}]\\ \dot{p}_\xi(t) &= -\frac{\partial H}{\partial \xi}(t,\xi(t),p^0,p_\xi(t),u(t)),\ \textit{for a.e. } t \in [t_{\text{start}},t^{RH}_{\text{rdv}}]\end{aligned} \tag{11}$$

(c)

$$u(t) = \frac{p_v(t)}{\|p_v(t)\|},\ \forall t \in [t_{\text{start}},t_1] \cup [t_2,t_3] \cup [t_4,t^{RH}_{\text{rdv}}] \tag{12}$$

(d) $\psi(t_1) = \psi(t_2) = \psi(t_3) = \psi(t_4) = 0$
(e) $p_m(t^{RH}_{\text{rdv}}) = 0$

Condition (e) is the final transversality condition and comes from the fact that the final mass is free. In case we would also consider a free initial time t_{start}, we would obtain an initial transversality condition of the form

$$\left\langle p_q(t_{\text{start}}), F_0^{\text{CR3BP}}(q(t_{\text{start}}))\right\rangle = 0,$$

where $F_0^{\text{CR3BP}}(\cdot)$ is the uncontrolled dynamics of the vehicle in the CR3BP model (without the Sun perturbation).

Remark 3.2 *Notice that transversality conditions at the rendezvous with asteroid 2006 RH_{120} cannot be used because we do not have an analytic expression for its orbit. In case there is an analytic expression for the rendezvous orbit it would imply that the Hamiltonian must be zero at the rendezvous, as well as $p_q(t^{RH}_{\text{rdv}})$ should be orthogonal to $\dot{q}^{RH}_{\text{rdv}}$.*

Remark 3.3 *A state, control and adjoint state* $(\xi(\cdot), u(\cdot), p^0, p_\xi(\cdot))$ *satisfying the conditions of Theorem 3.1 is called and extremal of* $(OCP)^{\mathrm{rdv}}_{t_{\mathrm{start}}, t^{RH}_{\mathrm{rdv}}}$. *We assume here that the extremals of* $(OCP)^{\mathrm{rdv}}_{t_{\mathrm{start}}, t^{RH}_{\mathrm{rdv}}}$ *are normal, that is* $p^0 \neq 0$.

For the return transfer, the maximum principle applied to $(OCP)^{\mathrm{return}}_{t^{\mathrm{RH}}_{\mathrm{start}}, t_f}$ gives the same necessary conditions as in Theorem 3.1, with t_{start} and $t^{\mathrm{RH}}_{\mathrm{rdv}}$ replaced by $t^{\mathrm{RH}}_{\mathrm{start}}$ and t_f.

3.4 Numerical method

For our numerical calculations we assume than the extremals are normal, i.e. $p^0 \neq 0$ and we can normalize it to -1. A study of the existence of abnormal extremals is out of the scope of this paper since it is mainly oriented towards giving an example of fuel efficient round trips to a TCO, here 2006 RH_{120}.

Both optimal control problems are solved using a shooting method, based on the necessary conditions. The shooting method consists in rewriting the necessary conditions of the maximum principle as the zero of a nonlinear function, namely the shooting function. Using the necessary conditions, in particular the Hamiltonian dynamics (11) and the maximization of the control (12), $(\xi(t), p_\xi(t))$ is completely defined by its initial value $(\xi_0, p_{\xi,0})$ at times t_{start} (respectively $t_{\mathrm{start,rdv}}$ for the return transfer) and by the switching times (t_1, t_2, t_3, t_4). Then, fixing ξ_0, we denote by S the shooting function:

$$S(p_{\xi,0}, t_1, t_2, t_3, t_4) = \begin{cases} q(t^{RH}_{\mathrm{rdv}}) - q^{\mathrm{rdv}} \\ \psi(t_i),\ i = 1,2,3,4 \\ p_m(t^{RH}_{\mathrm{rdv}}) \end{cases} \qquad S \in \mathbb{R}^{11}, \tag{13}$$

replacing for the transfer return problem $(OCP)^{\mathrm{return}}_{t^{\mathrm{RH}}_{\mathrm{start}}, t_f}$ respectively t^{RH}_{rdv} by t_f and q^{rdv} by q_{L_2}. It follows that if we find $(p_{\xi,0}, t_1, t_2, t_3, t_4)$ such that $S(p_{\xi,0}, t_1, t_2, t_3, t_4) = 0 \in \mathbb{R}^{11}$, then the associated $(\xi(\cdot), u(\cdot), -1, p_\xi(\cdot))$ satisfies the necessary conditions of Theorem 3.1.

In this paper, the numerical results have been obtained by computing the shooting function using the adaptative step integrator DOP853, see [14]. To find a zero of S, we used the quasi-Newton solver *HYBRD* of the Fortran *minpack* package. Since $S(\cdot)$ is nonlinear, the Newton method is very sensitive to the initial guess and seldom converges. To address this initialization sensitivity, we use two initialization techniques, described below.

The first initialization technique is a direct approach, see [10], consisting in discretizing the state ξ and control u in order to rewrite the optimal control problem as a nonlinear parametric optimization problem (NLP). In (NLP) the dynamic has been discretized using a fixed step fourth order Runge-Kutta scheme. The size of (NLP) depends on the size of the discretization. This (NLP) is solved using the modeling language *Ampl*, see [15], and the optimization solver *IpOpt*, see [16]. Once a solution of (NLP) is obtained, we use the value of the Lagarange multipliers associated to the discretization of the dynamic at initial time, as initial guess for $p_{\xi,0}$. The other unknowns are directly transcribable from (NLP). This approach cannot be used to solve our optimal control problems because in order to have a sufficiently accurate solution, each (NLP) should be solved with a very refined time discretization which would yield very long execution times of a few hours. We thus use this initialization technique when the second one fails.

The second initialization technique is a continuation from the known solution of one optimal control problem to another. For instance, let's say that we know the solution of a $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$, then we can reasonably hope that in some, if not most, of the cases, this solution is *connected* to the solution of a nearby $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}+\delta t}$. To follow the connection between these two problems, we can use elaborate continuation methods, like in [17] or [12]. Here, we chose to use a linear prediction continuation, which doesn't require the computation of the sensitivity of the shooting function but is nevertheless enough for our purpose. A solving with the continuation method usually takes a few seconds on a standard laptop, which explains why we prefer this method to the direct one.

Typically, the direct approach is used on one case and the continuation method enables us to solve tens or hundreds of other close cases. If the continuation method fails for a case, we then use the direct approach on it to be able to initiate again the continuation. In order to limit the number of local minima we add direct approach solvings and continuations from other neighbors to try to improve the solution in terms of final mass. This *local minima trimming* is based on two heuristics. The first one is a selection of locally optimal cases with the assumption that the evolution of the final mass should be more or less continuous with respect to the rendezvous point on 2006 RH_{120} (for the forward trip) or to the departure point on 2006 RH_{120} (for the return trip). For instance, for the forward trip, if two transfers with a comparable duration and neighbor rendezvous points exhibit a large final mass difference (say more than 10 kg), we launch a direct approach with the rendezvous point corresponding to the lower final mass. The second heuristic is a random selection of transfers and is used sparsely. This second round of calculations is essential and allowed us to greatly improve the solutions computed on the first solving round. Considering the large number of optimal control problems we need to solve and the trimming of local minima, this process takes several days of computation.

4 Numerical results

In this section, we provide results for the rendezvous and return transfers in the form of the best transfer and the evolution of the criterion with respect to the discretization of the initial and final times. We then provide a discussion on how these results can be combined to design a global round trip mission.

Notice that since our numerical approach is a variational one, and despite the restriction of the control structure, it is not possible to guarantee that the proposed trajectories are indeed optimal. Even the use of a (second order) sufficient condition, see [18], would only gives a proof of local optimality. It is thus likely that among the 5945 rendezvous trips and the 5112 return trips, there are some local minima that can still be improved by finding other better local or ideally global minima.

4.1 Rendezvous transfer

As mentioned in the prior section, we use a discretization on the starting time and the duration of the mission. t_{start} is discretized from 0 to 360 days by step of 15 days. For t^{RH}_{rdv}, we use a 1 day discretization of 2006 RH_{120} trajectory from June 1$^{\text{st}}$ 2006 to July 31$^{\text{st}}$ 2007 while satisfying the constraint that $t^{RH}_{\text{rdv}} \geq t_{\text{start}} + 7$ days. This gives a total of 5975 combinations and thus solving for $(t_{\text{start}}, t^{RH}_{\text{rdv}})$. We leave from a Halo Orbit around L_2 with a z-excursion of 5000 km and a position q_{HaloL_2} on June 1$^{\text{st}}$ 2006 .

The best rendezvous transfer using the restricted thrust structure we imposed is represented on Figures 5 and 6. Table 2 summarizes the main features of this best rendezvous transfer.

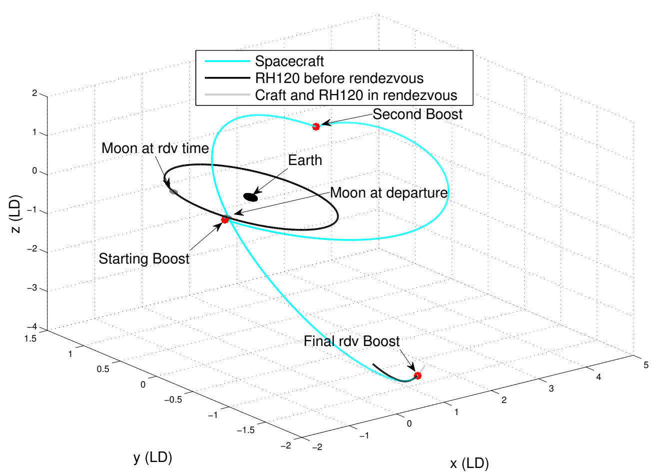

Fig. 5 *Best rendezvous transfer to 2006 RH_{120} in a geocentric inertial frame.*

This transfer is obtained from a departure time for the spacecraft from the hibernating orbit of 15 days, i.e. $t_{\text{start}} = 15$ *days*, which implies that detection of the asteroid should occur even before capture time to allow for a precise calculation of 2006 RH_{120} orbit before the mission. The rendezvous between the space-

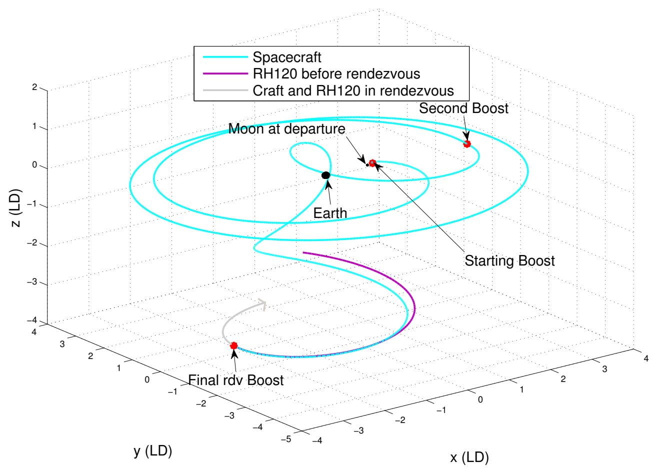

Fig. 6 *Best rendezvous transfer to 2006 RH_{120} CR3BP rotating frame.*

Best rendezvous to 2006 RH_{120}

Parameter	Symbol	Value
Departure date	t_{start}	06/16/2006
Arrival date	t_{rdv}^{RH}	10/27/2006
Final position		$(-1.958, 0.401, -3.992)$
Final velocity		$(0.224, 1.728, -0.029)$
Final Mass	m_f	280.855 kg
Delta-V	ΔV	496.43 m/s
Max dist. to Earth		1714 Mm (4.46 LD)
Min dist. to Earth		366080 km (0.95 LD)

Table 2 *Table summarizing the best rendezvous transfer to 2006 RH_{120}.*

craft and 2006 RH_{120} occurs on October 27^{th} 2006 which is 148 days after capture, i.e. $t_{\text{rdv}}^{RH} = 148\ days$. The duration of the rendezvous transfers is therefore 133 days. The point on 2006 RH_{120} at which the rendezvous occurs is given by $q_f^{\text{rdv}} \approx (-1.958, 0.401, -3.992, 0.224, 1.728, -0.029)$. As it can be observed, the rendezvous point corresponding to the best transfer is far from the Earth and Moon orbital plane and is not the closest one to the departure point as it is 5.08 LD from the L_2 libration point. A possible explanation is that as the spacecraft moves away from the influence of the two primaries, the thrusters have a larger impact on the motion of the vehicle. It is also important to note that this rendezvous point is not simply the closest one in terms of distance or energy. Figure 7 illustrates this fact , it represents the best final mass obtained for each rendezvous point on 2006 RH_{120} and the distance from these rendezvous point to the L_2 libration point. It can be observed that there is first a step increase in the final mass for the rendezvous with 2006 RH_{120} that occur near capture time, the reason is that these transfers correspond by design to rendezvous transfers with a short duration. The best transfers are obtained between 120 and 170 days, then the final mass is rather constant with some fluctuations most likely due to the existence of local minima. This observation is good in terms of the design of a real mission, it provides us with a lot of flexibility over the departure times for the spacecraft from its hibernating orbit.

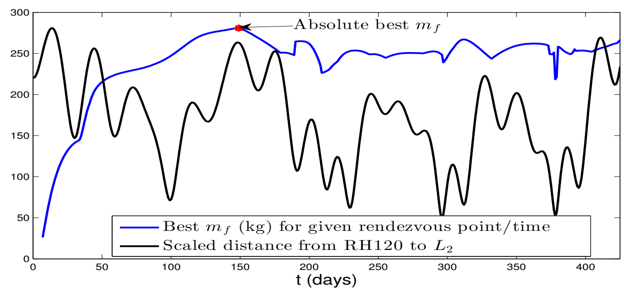

Fig. 7 *Best final mass among all t_{start} with respect to the rendezvous time. Also pictured is the distance from the rendezvous point to L_2, this distance has been scaled.*

The final mass of the spaceraft for the best overall rendezvous is $m_f \approx 280.855\,kg$ which corresponds to a $\Delta V \approx 496.43 m/s$, where we compute ΔV such that $m_f = m_0 e^{-\frac{\Delta V}{I_{\text{sp}} g_0}}$. Notice that in [5] we have obtained a better transfer corresponding to a ΔV of $203.6 m/s$, however for this mission the rendezvous would take place on June 26th 2006 and the duration would be about 415 days which requires to detect and launch the mission about 14 months before June 1st 2006 creating an unrrealistic scenario. Note that the spacecraft performs only one revolution around the Earth in the inertial reference frame.

Finally, let us comment on the thrust strategy for this best rendezvous transfer. The norm of the control is shown on Figure 8. The three thrust arcs last respectively 16.44 min, 1.62 hours and 4.23 min and the two ballistic arcs last 68.70 and 64.25 days. The second thrust takes place approximatively in the middle of the transfer, but notice that it is typically not the case (see the best return transfer below). It can also be observed that the position and velocity of the spacecraft at the beginning of the second thrust arc is $q(t_2) = (3.286, -0.141, -0.012, -0.476, -3.185, 0.012)$, which is at 3.29 lunar distance from the EM barycenter, that is 1.26 million km.

Figure 9 gives the evolution of the final mass with respect to t_{rdv}^{RH} and t_{start}. To read the graph, notice that the scale for the t_{start} need to be multiplied by 15 (the discretization rate) to justify the void region for which no rendezvous transfer are associated. It also reflects the fact that $t_{\text{rdv}}^{RH} \geq t_{\text{start}} + 7 days$.

Figure 10 is a selection of the evolution of the final mass and ΔV with respect to the rendezvous time $t_{\text{rdv}}^{\text{RH}}$ for various starting dates t_{start}. So it represent a sectional view of 3D Figure 9.

From Figures 9 and 10, we can see that there is a first gradual increase of the final mass with respect to the transfer duration $t_{\text{rdv}}^{RH} - t_{\text{start}}$. However, this increased

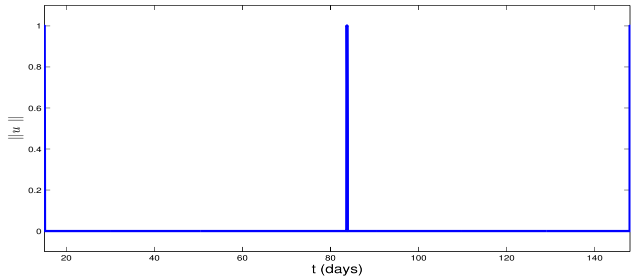

Fig. 8 *Norm of control for the rendezvous transfer to 2006 RH_{120}.*

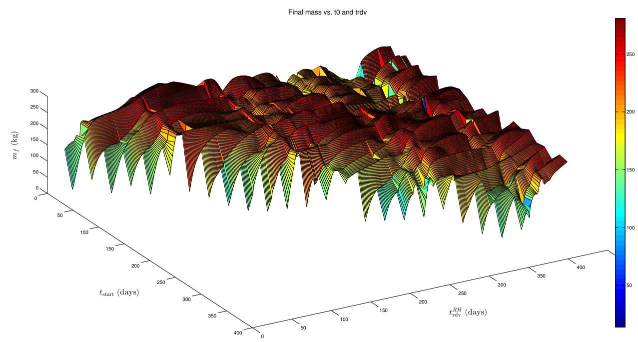

Fig. 9 *Evolution of final mass for $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$ with respect to the rendezvous and starting dates.*

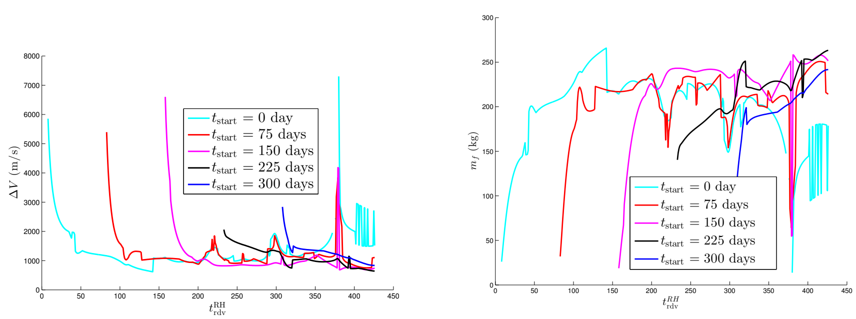

Fig. 10 *Evolution of ΔV (left) and final mass (right) for $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{RH}_{\text{rdv}}}$ with respect to the rendezvous date and various starting dates $t_{\text{start}} \in \{0, 75, 150, 225, 300\}$ days.*

efficiency of the transfer stops after 30 to 120 days, depending on the starting date. It suggests that after a period of about 2 months, the final mass is typically less sensitive to an increase in transfer duration and depends more heavily on the rendezvous point.

Figure 11 provides the number of rendezvous transfers corresponding to a given final mass range, as a histogram. We can observe that the majority of transfers provide a final mass above 200kg, this is very good to design an actual mission since it implies that there is flexibility with respect to the departure time and duration of the transfer to produce an efficient transfer.

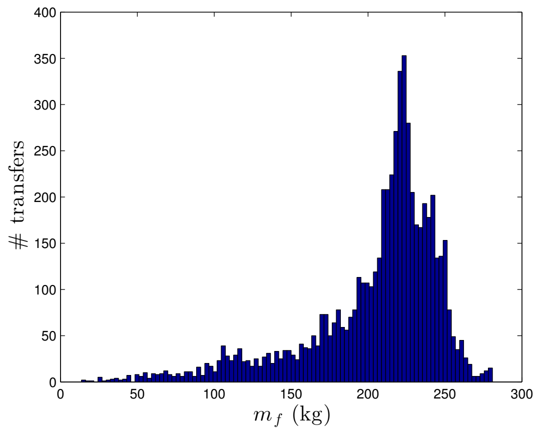

Fig. 11 *Number of rendezvous transfers per final mass range. Most rendezvous transfers provide a final mass above 200kg with a pick at around 230kg.*

Based on our discretization, we considered 25 departure times for the spacecraft for its hibernating location. Table 3 gives a quick overview of the best rendezvous transfers for each of those departure time t_{start}. Officially, 2006 RH_{120} has been detected on September 14^{th} 2006 which is 105 days after its capture by Earth gravity, that is $t_d^{\text{RH}} = 105$. Using Table 3, we see that this gives that the best departure time t_{start} satisfying $t_{\text{start}} \geq t_d^{\text{RH}}$ is $t_{\text{start}} = 180$ days after June 1^{st} 2006. Notice that under this scenario, the 75 days between the detection time and the departure of the spacecraft for the rendezvous mission ensure that the celestial mechanic computations required to predict 2006 RH_{120}'s orbit with enough precision can be completed. This rendezvous transfer provides a final mass of 267.037 kg, or equivalently $\Delta V = 610.224$ m/s, and a rendezvous date 312 days after capture, that is April 9th 2007. In particular, we will see that this rendezvous transfer could be combined with the best return transfer given in the following section. If in case of practical consideration the departure of the spacecraft should be delayed, it can be observed from Table 3 that the mass loss can be minimized since for instance for $t_{\text{start}} = 285$ days we have that the final mass is 266.525 kg which is not even a one kilo difference from a departure 180 days after capture. However, this late rendezvous time might seriously compromise the efficiency of the return transfer. Clearly, an early detection of the TCO or timely departure of the spacecraft once the asteroid orbit has been determined is much preferable for a fuel efficient round trip transfer.

t_{start} (d.)	t_{rdv}^{RH} (d.)	m_f (kg.)	t_{start} (d.)	t_{rdv}^{RH} (d.)	m_f (kg.)
0	141	265.831	195	392	255.172
15	148	280.855	210	363	262.025
30	111	234.909	225	425	263.304
45	138	251.379	240	359	240.076
60	146	231.103	255	425	260.92
75	414	250.772	270	416	256.618
90	273	250.608	285	425	266.525
105	414	250.02	300	425	241.721
120	290	252.547	315	425	246.111
135	390	245.707	330	407	254.773
150	380	258.222	345	425	251.158
165	314	244.521	360	425	245.369
180	312	267.037			
Avg	-	253.091	σ	-	11.136

Table 3 *Best rendezvous dates and final mass for the 25 different t_{start}. Notice that the average final mass is* 253.091 ± 11.136.

4.2 Return transfer from 2006 RH$_{120}$ to L_2

To get a global idea of the impact of the choice of departure time from asteroid 2006 RH_{120} and duration of the transfer we study at first the return transfer as completely decoupled from the rendezvous transfer. In an unrealistic way we will assume the spacecraft can depart 2006 RH_{120} as soon as June 1$^{\text{st}}$ 2006 and we use a 1 day discretization of the 2006 RH_{120} orbit. However, to keep the number of calculations under control we use a 30 days discretization for the transfer duration $t_f - t_{\text{start}}^{\text{RH}}$, from 30 to 360 days. This gives a total of 5112 combinations for $(t_{\text{start}}^{\text{RH}}, t_f - t_{\text{start}}^{\text{RH}})$. Since the mass at the departure from 2006 RH_{120} is unknown before hand, in order to be able compare all the return trips we choose arbitrarily to set the initial mass of the return trip to 300 kg, which is 50 kg less than the initial mass of the rendezvous transfer.

The best return transfer to L_2 under our thrust restrictions is shown on Figure 12 and Table 4 summarizes the main features of the best return transfer.

Best return trip from 2006 RH$_{120}$

Parameter	**Symbol**	**Value**
Departure date	$t_{\text{start}}^{\text{RH}}$	06/01/2007
Arrival date	t_f	01/27/2008
Initial position		$(0.238, -0.598, -2.228)$
Initial velocity		$(-0.947, -0.477, 0.496)$
Final Mass	m_f	250.712 kg
Delta-V	ΔV	404.815 m/s
Max dist. to Earth		2031 Mm (5.28 LD)
Min dist. to Earth		265520 km (0.69 LD)

Table 4 *Table summarizing the best return transfer from 2006 RH$_{120}$ to L_2.*

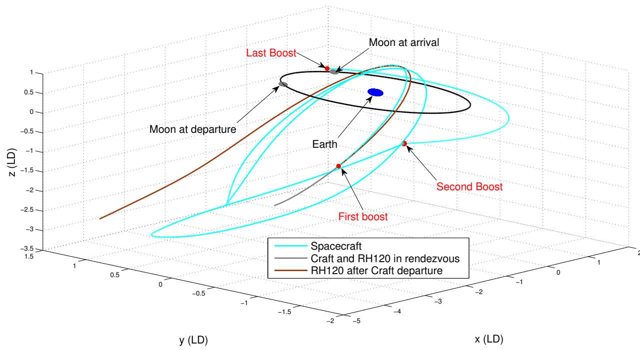

Fig. 12 *Best return transfer from 2006 RH$_{120}$ to Earth-Moon L$_2$ in a geocentric inertial frame.*

The starting date of the best return transfer is 365 days after June 1$^{\text{st}}$ 2006, which corresponds to June 1st 2007, and the transfer duration is 240 days. Notice that this departure date occurs shortly before 2006 RH$_{120}$ escape Earth gravity and after the best rendezvous transfer found previously which makes it an ideal candidate for a complete round trip. The final mass for this transfer is $m_f \approx 250.712$ kg, which is equivalent to $\Delta V \approx 404.815$ m/s which is comparable (slightly better) to the ΔV of the best rendezvous transfer.

Figure 13 gives the norm of the control associated to the best return transfer. This thrust strategy has three thrust arcs lasting respectively 2.15 min, 1.32 hours and 3.06 min. The two ballistic arcs durations are respectively 213.788 and 26.15 days, and contrary to the best rendezvous transfer the second thrust arc does not occur in the middle of the transfer but rather near the end. However, as it was the case for the rendezvous transfer the second thrust arc is the longest one.

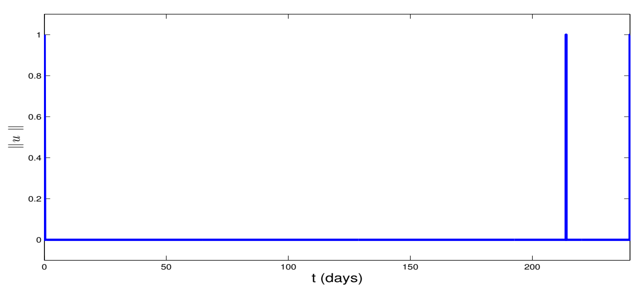

Fig. 13 *Norm of control for the best return transfer from 2006 RH$_{120}$.*

Figure 14 gives the evolution of the final mass with respect to t_{rdv}^{RH} for various choices of $t_{\text{start}}^{\text{RH}}$. We can see that for the return trip, there is not a large difference of final mass with respect to neither the different possible starting dates nor the different transfer times.

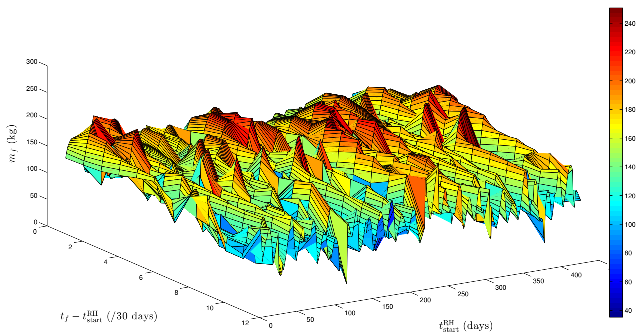

Fig. 14 *Evolution of final mass for* $(OCP)^{\text{return}}_{t^{\text{RH}}_{\text{start}},t_f}$ *with respect to the starting date and transfer time.*

Figure 15 is a selection of the evolution of the final mass and ΔV with respect to the departure date $t^{\text{RH}}_{\text{start}}$ for various transfer durations $t_f - t^{\text{RH}}_{\text{start}}$. So it represent a sectional view of 3D Figure 14.

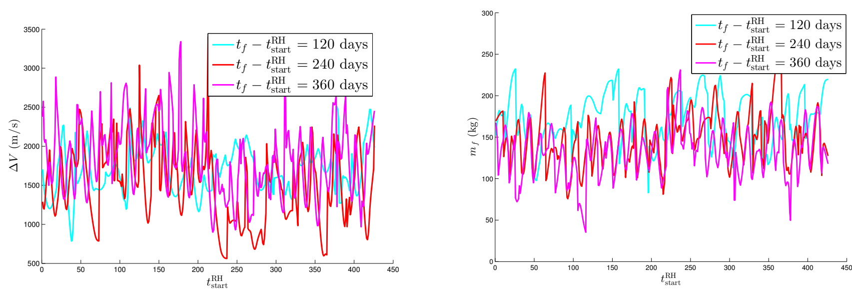

Fig. 15 *Evolution of* ΔV *(left) and final mass (right) for* $(OCP)^{\text{return}}_{t^{\text{RH}}_{\text{start}},t_f}$ *with respect to the starting date and various transfer durations* $t_f - t^{\text{RH}}_{\text{start}} \in \{120, 240, 360\}$ *days*

From the evolution of the final mass with respect to the transfer duration, it seems that allowing more time for the transfer does not always gives a more efficient return transfer. It is however possible the optimal control problem from a fixed duration one to one with a maximum allowed duration would give better results. Indeed, for the return transfer, it would make sense to be more lax with respect to the transfer duration than for the synchronized rendezvous transfer. This remark is partially illustrated by the results from [5] where the transfer duration is free, albeit these results are for a rendezvous type transfer.

Figure 16 gives the number of rendezvous transfers corresponding to a give final mass range, as a histogram. Contrary to the rendezvous transfers, it reflects more a density with standard normal distribution and most of the return transfers give an average final mass of around 160 kg. This distribution does not provide us with as much flexibility regarding which transfer to use as it was the case for the rendezvous transfers since we then had many more transfers with a final mass close to the best one.

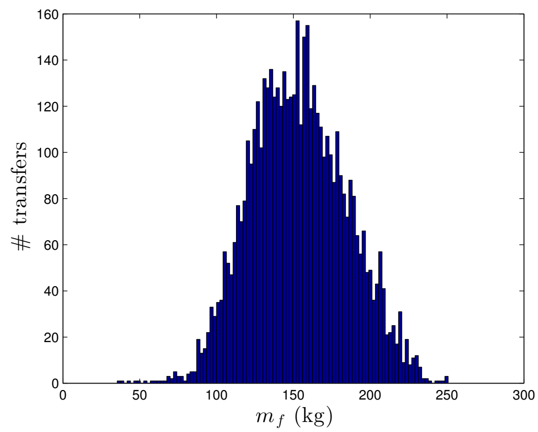

Fig. 16 *Number of return transfers per final mass range. Notice the similar shape to a density of a standard normal distribution. The largest pick is at around 170kg.*

Table 5 gives a quick overview of the best return trips for each transfer duration $\Delta t^f = t^f - t^{RH}_{start}$. It can be observed that the best transfers take place for durations between 120 and 240 days, but that augmenting the duration beyond 240 days does not provide more fuel efficiency. It also seems like that except for the return transfers lasting less than 150 days, all the others depart from a late date on 2006 RH_{120}. That is good because we cannot hope that a rendezvous transfer arrives to early on 2006 RH_{120}.

Δt^f (d.)	t^{RH}_{start} (d.)	m_f (kg.)	Δt^f (d.)	t^{RH}_{start} (d.)	m_f (kg.)
30	37	211.681	210	271	231.035
60	18	225.091	240	365	250.712
90	149	220.053	270	221	205.216
120	25	232.328	300	271	207.765
150	154	236.009	330	218	201.274
180	236	233.768	360	236	231.093

Table 5 *Best starting date for return trip for the 12 different transfert duration $\Delta t_f = t^f - t^{RH}_{start}$. Mean value of final mass is 223.835 kg and standard deviation is 14.843 kg.*

4.3 Complete round trip mission

Since the aim of this paper is to design a round trip mission to 2006 RH_{120}, we need to combine a rendezvous transfer with a return transfer in a realistic way. This implies that we need to take into account some practical constraints such that the fact that the return transfer has to start after the end of the rendezvous transfer, that is $t_{\text{rdv}}^{RH} < t_{\text{start}}^{\text{RH}}$. This means that the spacecraft stays with 2006 RH_{120} for $t_{\text{start}}^{\text{RH}} - t_{\text{rdv}}^{R}H$ days. We prefer to think of the lock-in duration between the spacecraft and 2006 RH_{120} to be a consequence of our calculation rather than a fixed value by the user. Our calculations determine what the ideal lock-in duration should be and the only constraint would be to check that it corresponds to a realistic time for the science part of the mission.

From our prior calculations, it can be observed that the best rendezvous and return transfers satisfy the time constraint, therefore we only need to modify the initial mass of the return transfer to match our desired scenario. We chose to simply impose that the mass at the end of the rendezvous transfer equals the mass at the departure of the return transfer, in other word there is no loss or addition of mass during the lock-in phase. This is an arbitrary choice, and we could for instance also decide that some equipment was left on the asteroid or some material collected form the asteroid that would alter the departure mass in a different way. Based on our choice, the return transfer must starts with an initial mass of 280.855 kg instead of the $300kg$ prescribed previously. This modification is addressed easily through a continuation on the previous best return transfer. It provides a return transfer that is nearly the same as the one with the higher mass. Table 6 gives the main features of the return trip, while Table 2 of the rendezvous section still gives the main features of the rendezvous transfer. Figure 17 shows the entire round trip transfer in a Geocentric inertial reference frame.

Best return trip from 2006 RH_{120} for the round trip mission

Parameter	**Symbol**	**Value**
Stay on 2006 RH_{120}	$t_{\text{start}}^{\text{RH}} - t_{\text{rdv}}^{RH}$	217 days
Departure date	$t_{\text{start}}^{\text{RH}}$	06/01/2007
Arrival date	t_f	01/27/2008
Initial position		$(0.238, -0.598, -2.228)$
Initial velocity		$(-0.947, -0.477, 0.496)$
Final Mass	m_f	234.713 kg
Delta-V	ΔV	404.814 m/s
Max dist. to Earth		2031 Mm (5.28 LD)
Min dist. to Earth		265519 km (0.69 LD)

Table 6 *Table summarizing the best return transfer from 2006 RH_{120} to L_2, after pairing with the rendezvous transfer (so $m_0 = 280.855$ kg).*

As mentioned in section 4.1, the best round trip transfer requires to detect 2006 RH_{120} at or almost immediately after capture which is not an ideal scenario especially given the fact that 2006 RH_{120} was actual detected 105 days after June

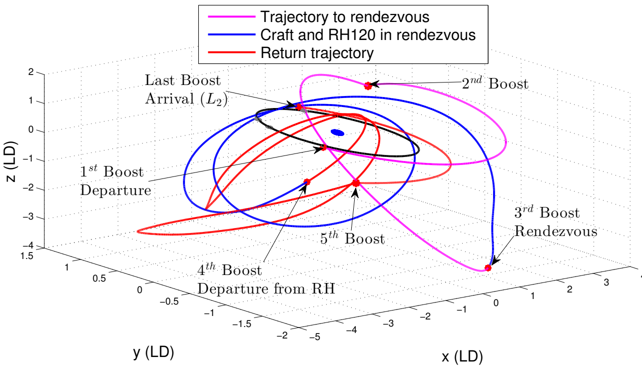

Fig. 17 *Best round trip transfer from 2006 RH_{120} to Earth-Moon L_2 in a geocentric inertial frame*

1^{st} 2006. This suggests that additional scenarii should be analyzed. Moreover, the round trip transfer should also allow for the spacecraft ample time to perform its mission on the TCO. We denote by $\delta t_{\text{mission}}$ the minimum time the spacecraft need to stay on 2006 RH_{120} to complete the science aspect of the mission. This constraint can be expressed as $t^{\text{RH}}_{\text{start}} \geq t^{\text{RH}}_{\text{rdv}} + \delta t_{\text{mission}}$. Table 7 gives a sample of the best round trip transfers for various t^{RH}_{d} and $\delta t_{\text{mission}}$. Since the best return transfer departs one year after June 1^{st} 2006 it can be used in almost all scenarii but the last one when the rendezvous portion ends 395 days after June 1^{st} 2006. For instance if we assume that the detection occurs only 210 days after June 1^{st} 2006 , and that we need only 30 days for the lock-in phase, the rendezvous transfer can be chosen as taking 102 days to be combined with the best overall return transfer. However, if we impose a 60 days lock-in constraint for the spacecraft and the asteroid we need to chose a different rendezvous transfer reaching 2006 RH_{120} in 95 days. We can observe that the longer the lock-in phase the more expensive the round trip transfer becomes. Another way to look at our calculations would be to design efficient round trip transfers and deduce from these data the ideal windows for detection and lock-in phases. This would provide additional information for the overall design of a mission to TCOs.

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t_d^{RH}	$\delta t_{\text{mission}}$	rendezvous transfer			return transfer			round trip	
		t_{start}	$t_{\text{rdv}}^{\text{RH}}$	ΔV (m/s)	$t_{\text{start}}^{\text{RH}}$	$t_f - t_{\text{start}}^{\text{RH}}$	ΔV (m/s)	Total δV (m/s)	Duration
0	30	15	148	496.43	365	240	404.82	901.25	373
30	30	180	312	610.22	365	240	404.82	1.01504	372
30	60	180	305	684.66	365	240	404.82	1089.48	365
210	30	210	312	732.23	365	240	404.82	1137.05	342
210	60	210	305	809.61	365	240	404.82	1214.43	335
240	30	255	319	892.82	365	240	404.82	1297.63	304
240	60	240	305	1010.88	365	240	404.82	1415.70	305
270	30	270	335	1034.27	365	240	404.82	1439.09	305
300	30	330	395	936.80	425	120	704.06	1640.85	185

Table 7 *Table summarizing the best round trips with detection and mission duration constraints, with* $m_0 = 350$ *kg and* $m_0^{\text{RH}} = 300$ *kg. All times are expressed in days.*

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