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## LEVEL CROSSING RATE OF MACRODIVERSITY WITH THREE MICRODIVERSITY SC RECEIVERS OVER GAMMA SHADOWED NAKAGAMI-M CHANNEL

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### ABSTRACT

Macrodiversity system with macrodiversity selection combining (SC) receiver and three microdiversity SC receivers operating over Gamma shadowed Nakagami-m multipath fading environment is considered in this work. Level crossing rate of signals at outputs microdiversity SC receivers are calculated and by using these formulas, closed form expression for average level crossing rate of macrodiversity SC receiver output

signal is evaluated. Average fade duration of proposed macrodiversity system can be calculated as ratio of outage probability and level crossing rate. By using obtained results the influence of Nakagami-m multipath fading severity parameter, Gamma shadowing severity parameter and shadowing correlation parameter on average level crossing rate is analyzed and discussed.

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**Key words:** gamma shadowed, level crossing rate, Nakagami-*m*, macrodiversity, microdiversity, selection combiner.

### 1. INTRODUCTION

Macrodiversity techniques can be used to reduce simultaneously long term fading and short term fading effects on system performance of wireless communication systems. Macrodiversity reception has application in cellular mobile radio systems. Macrodiversity system is consisting of macrodiversity SC receiver and two or more microdiversity receivers. Macrodiversity receiver combines signals with multiple antennas at base station resulting in reduction of short term fading effects and macrodiversity receiver combines signals with two or more base stations resulting in reduction of long term fading effects on system performance (Stuber, 2003), (Goldsmith, 2005).

Level crossing rate is important the second order performance measure of wireless communication system which can be calculated as average value of random process the first derivative. Average fade duration is defined as average time that resultant signal bellow specified level and can be calculated as ratio of outage probability and level crossing rate (Lee, 1993). Outage probability is the first order performance measure of wireless system and is defined as probability that resultant signal falls bellow of specified threshold. Nakagami-*m* distribution can be

used to describe signal envelope variation in fading channels. This distribution has one parameter *m*. When parameter *m* goes to one, Nakagami-*m* distribution reduces to Rayleigh distribution and when *m* goes to infinity Nakagami-*m* multipath fading channel becomes no fading channel. Long term fading channel can be described by using Gamma distribution (Proakis, 2001), (Panic et al., 2013).

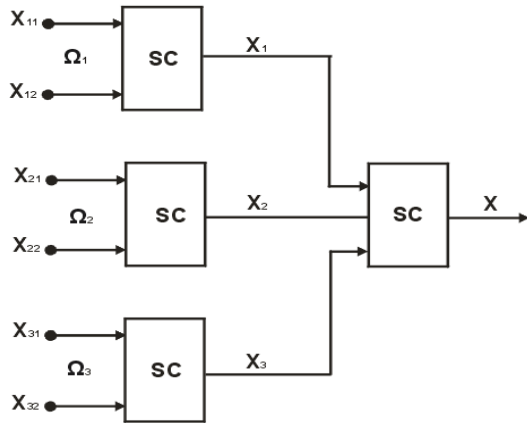
There are more works in open technical literature consisting performance of wireless macrodiversity communication systems operating over composite fading channels in the presence long term fading and short term fading. In (Stefanovic et al., 2011) macrodiversity system with macrodiversity SC receiver and two microdiversity MRC receivers operating over Gamma shadowed Nakagami-*m* multipath fading channel and the second order performance of proposed wireless communication system such as the average level crossing rate and the average fade duration are calculated. Average fade duration and average level crossing rate of wireless communication system in the presence Gamma long term fading and Rician short term fading are evaluated and the influence of Rician factor on level crossing rate is analyzed in (Sekulovic & Stefanovic, 2012) and

(Panic et al., 2014). In (Stefanovic et al., 2014) macrodiversity system with two microdiversity SC receivers in the presence Gamma large scale fading and  $k$ - $\mu$  small scale fading is considered and the second order performance measures are evaluated.

In this paper, wireless microdiversity communication system with macrodiversity SC receiver and three microdiversity SC receivers operating over Gamma shadowed Nakagami- $m$  multipath fading channel. Average level crossing rates signals at outputs of microdiversity SC receivers are calculated and by using these formulas, closed form expression of macrodiversity SC receiver output signal is calculated. By using this expression average fade duration of proposed macrodiversity system can be evaluated. To the best author's knowledge, average level crossing rate of macrodiversity reception with macrodiversity SC receiver and three microdiversity SC receivers in the presence Gamma shadowing and Nakagami- $m$  multipath fading.

## 2. LEVEL CROSSING RATE SIGNALS AT OUTPUTS OF MICRODIVERSITY SC RECEIVERS

Wireless macrodiversity communication system considered in this paper is shown at Fig. 1. Macrodiversity reception system consists of macrodiversity selection combining receiver and three microdiversity SC receivers. Macrodiversity SC receiver selects microdiversity SC receiver with the highest signal envelope average power to provide service to user. Microdiversity SC receivers have two inputs.



**Fig. 1** Macrodiversity system.

Signal envelope at inputs and outputs of microdiversity receivers and at inputs and output of

macrodiversity SC receiver are denoted at Figure 1. Signal envelopes average power at inputs of microdiversity SC receivers are denoted with  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$ . Random variables  $x_{ij}$ ,  $i=1,2$ ,  $j=1,2$  follow Nakagami- $m$  distribution (Djordjevic et al., 2015):

$$p_{x_{ij}}(x_{ij}) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega_i} \right)^m x_{ij}^{2m-1} e^{-\frac{m}{\Omega_i} x_{ij}^2}, \quad x_{ij} \geq 0 \quad (1)$$

where  $m$  is Nakagami- $m$  multipath fading severity parameter. Cumulative distribution function of  $x_{ij}$  is:

$$F_{x_{ij}}(x_{ij}) = \frac{1}{\Gamma(m)} \gamma \left( m, \frac{m}{\Omega_i} x_{ij}^2 \right), \quad x_{ij} \geq 0 \quad (2)$$

Joint probability density function of  $x_i$  and  $\dot{x}_i$  is:

$$\begin{aligned} p_{x_i, \dot{x}_i}(x_i, \dot{x}_i) &= p_{x_{i1}, \dot{x}_{i1}}(x_i, \dot{x}_i) F_{x_{i2}}(x_i) + \\ &+ p_{x_{i2}, \dot{x}_{i2}}(x_i, \dot{x}_i) F_{x_{i1}}(x_i) = \\ &= 2 p_{x_{i1}, \dot{x}_{i1}}(x_i, \dot{x}_i) F_{x_{i2}}(x_i) = \\ &= \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega_i} \right)^m x_i^{2m-1} e^{-\frac{m}{\Omega_i} x_i^2} \times \\ &\times \frac{1}{\sqrt{2\pi} \beta_i} e^{-\frac{\dot{x}_i^2}{2\beta_i}}, \quad i = 1, 2, 3 \end{aligned} \quad (3)$$

where  $\beta_i = \pi^2 f_m^2 \frac{\Omega_i}{m}$ ,  $f_m$  is maximal Doppler frequency.

Average level crossing rate of microdiversity SC receivers output signals is (Djosic et al., 2015):

$$\begin{aligned} N_{x_i} &= \int_0^\infty d\dot{x}_i \cdot \dot{x}_i \cdot p_{x_i, \dot{x}_i}(x_i, \dot{x}_i) = 2 N_{x_{i2}} F_{x_{i2}}(x_i) = \\ &= \frac{4\pi f_m}{\Gamma(m)} \left( \frac{m}{\Omega_i} \right)^{m-\frac{1}{2}} x_i^{2m-1} e^{-\frac{m}{\Omega_i} x_i^2} \times \frac{1}{\Gamma(m)} \gamma \left( m, \frac{m}{\Omega_i} x_i^2 \right) \end{aligned} \quad (4)$$

where  $\gamma(n, x)$  is the incomplete Gamma function (Gradshteyn, & Ryzhik, 2000), (Prudnikov et al., 2015).

Random variable  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$  follow correlated Gamma distribution:

$$\begin{aligned} p_{\Omega_1, \Omega_2, \Omega_3}(\Omega_1, \Omega_2, \Omega_3) &= \frac{1}{\Gamma(c) (1-\rho^2)^2 \rho^{c-1} \Omega_0^{c+2}} \times \\ &\times e^{-\frac{\Omega_1 + (1+\rho^2)\Omega_2 + \Omega_3}{2\Omega_0(1-\rho^2)}} \sum_{i=0}^{\infty} \left( \frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i+c-1} \times \end{aligned}$$

$$\begin{aligned} & \times \frac{1}{i_1! \Gamma(i_1 + c)} \sum_{i_2=0}^{\infty} \left( \frac{\rho}{\Omega_0 (1 - \rho^2)} \right)^{2i_2 + c - 1} \times \\ & \times \frac{1}{i_2! \Gamma(i_2 + c)} \Omega_1^{i_1 + c - 1} \Omega_2^{i_1 + i_2 + c - 1} \Omega_3^{i_2 + c - 1} \end{aligned} \quad (5)$$

where  $c$  is Gamma long term fading severity parameter,  $\rho$  is Gamma long term fading correlation coefficient and  $\Omega_0$  is signal envelope average power. Previously expression is derived for case when correlation matrix is exponential.

### 3. LEVEL CROSSING RATE OF MACRODIVERSITY SC RECEIVER OUTPUT SIGNAL

Average level crossing rate of output of macrodiversity SC receiver is:

$$\begin{aligned} N_x &= \int_0^{\infty} d\Omega_1 \int_0^{\Omega_1} d\Omega_2 \int_0^{\Omega_1} d\Omega_3 N_{x_1/\Omega_1} p_{\Omega_1\Omega_2\Omega_3}(\Omega_1\Omega_2\Omega_3) + \\ &+ \int_0^{\infty} d\Omega_2 \int_0^{\Omega_2} d\Omega_1 \int_0^{\Omega_2} d\Omega_3 N_{x_2/\Omega_2} p_{\Omega_1\Omega_2\Omega_3}(\Omega_1\Omega_2\Omega_3) + \\ &+ \int_0^{\infty} d\Omega_3 \int_0^{\Omega_3} d\Omega_1 \int_0^{\Omega_3} d\Omega_2 N_{x_3/\Omega_3} p_{\Omega_1\Omega_2\Omega_3}(\Omega_1\Omega_2\Omega_3) = \\ &= I_1 + I_2 + I_3 \end{aligned} \quad (6)$$

The integral  $I_1$  is:

$$\begin{aligned} I_1 &= \int_0^{\infty} d\Omega_1 \int_0^{\Omega_1} d\Omega_2 \int_0^{\Omega_1} d\Omega_3 N_{x_1/\Omega_1} p_{\Omega_1\Omega_2\Omega_3}(\Omega_1\Omega_2\Omega_3) = \\ &= \frac{4\pi f_m}{\Gamma(m)} m^{2m-3/2} x^{4m-1} \sum_{j_1=0}^{\infty} \frac{1}{(m+1)_{(j_1)}} m^{j_1} x^{2j_1} \times \\ & \times \sum_{i_1=0}^{\infty} \left( \frac{\rho}{\Omega_0 (1 - \rho^2)} \right)^{2i_1 + c - 1} \frac{1}{i_1! \Gamma(i_1 + c)} \times \\ & \times \sum_{i_2=0}^{\infty} \left( \frac{\rho}{\Omega_0 (1 - \rho^2)} \right)^{2i_2 + c - 1} \frac{1}{i_2! \Gamma(i_2 + c)} \times \\ & \times \frac{1}{i_1 + i_2 + c} \sum_{j_2=0}^{\infty} \frac{1}{(i_1 + i_2 + c + 1)_{(j_2)}} \left( \frac{1 + \rho^2}{\Omega_0 (1 - \rho^2)} \right)^{j_2} \times \\ & \times \frac{1}{i_2 + c} \sum_{j_3=0}^{\infty} \frac{1}{(i_2 + c + 1)_{(j_3)}} \left( \frac{1}{\Omega_0 (1 - \rho^2)} \right)^{j_3} \times \end{aligned}$$

$$\begin{aligned} & \times \left( \frac{2mx^2 \Omega_0 (1 - \rho^2)}{3 + \rho^2} \right)^{-m + i_1 + i_2 - \frac{j_1}{2} - \frac{j_2}{2} - \frac{j_3}{2} + \frac{3c}{2} + \frac{1}{4}} \times \\ & \times K_{-2m + 2i_1 + 2i_2 - j_1 + j_2 + j_3 + 3c + \frac{1}{2}} \left( 2 \sqrt{\frac{2mx^2 (3 + \rho^2)}{\Omega_0 (1 - \rho^2)}} \right) \end{aligned} \quad (7)$$

where  $(a)_n$  denoting the Pochhammer symbol (Gradshteyn, & Ryzhik, 2000) and where  $K_n(x)$  is the modified Bessel function of the second kind, order  $n$  and argument  $x$ .

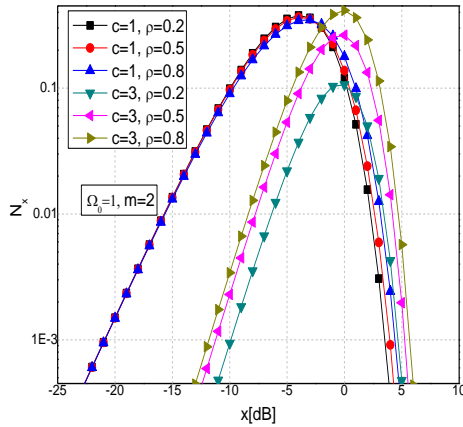
The integral  $I_2$  is:

$$\begin{aligned} I_2 &= \int_0^{\infty} d\Omega_2 \int_0^{\Omega_2} d\Omega_1 \int_0^{\Omega_2} d\Omega_3 N_{x_2/\Omega_2} p_{\Omega_1\Omega_2\Omega_3}(\Omega_1\Omega_2\Omega_3) = \\ &= \frac{4\pi f_m}{\Gamma(m)} m^{2m-3/2} x^{4m-1} \sum_{j_1=0}^{\infty} \frac{1}{(m+1)_{(j_1)}} m^{j_1} x^{2j_1} \times \\ & \times \sum_{i_1=0}^{\infty} \left( \frac{\rho}{\Omega_0 (1 - \rho^2)} \right)^{2i_1 + c - 1} \frac{1}{i_1! \Gamma(i_1 + c)} \times \\ & \times \sum_{i_2=0}^{\infty} \left( \frac{\rho}{\Omega_0 (1 - \rho^2)} \right)^{2i_2 + c - 1} \frac{1}{i_2! \Gamma(i_2 + c)} \times \\ & \times \frac{1}{i_1 + c} \sum_{j_2=0}^{\infty} \frac{1}{(i_1 + c + 1)_{(j_2)}} \left( \frac{1}{\Omega_0 (1 - \rho^2)} \right)^{j_2} \times \\ & \times \frac{1}{i_2 + c} \sum_{j_3=0}^{\infty} \frac{1}{(i_2 + c + 1)_{(j_3)}} \left( \frac{1}{\Omega_0 (1 - \rho^2)} \right)^{j_3} \times \\ & \times \left( \frac{2mx^2 \Omega_0 (1 - \rho^2)}{3 + \rho^2} \right)^{-m + i_1 + i_2 - \frac{j_1}{2} - \frac{j_2}{2} - \frac{j_3}{2} + \frac{3c}{2} + \frac{1}{4}} \times \\ & \times K_{-2m + 2i_1 + 2i_2 - j_1 + j_2 + j_3 + 3c + \frac{1}{2}} \left( 2 \sqrt{\frac{2mx^2 (3 + \rho^2)}{\Omega_0 (1 - \rho^2)}} \right) \end{aligned} \quad (8)$$

### 4. NUMERICAL RESULTS

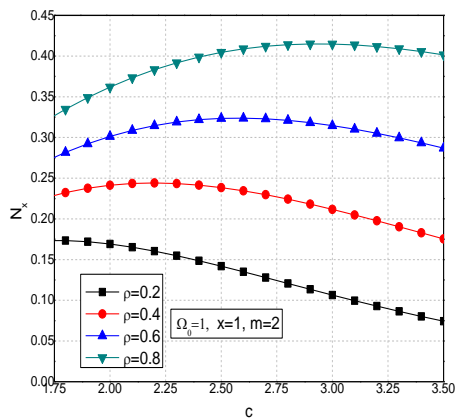
Average level crossing rate of macrodiversity reception with macrodiversity SC receiver and three microdiversity SC receivers versus resulting signal for several values of Gamma parameter  $c$  and correlation coefficient  $\rho$  is presented on Fig. 2. Level crossing rate increases for lower values of resulting signal and level crossing rate decreases for higher values resulting signal. The influence of resulting signal on level crossing rate is higher for lower values of resulting

signal. As Gamma parameter  $c$  increases average level crossing rate decreases resulting in system performance improvement. The influence of Gamma parameter  $c$  on average level crossing rate is higher for higher values of resulting envelope. When Gamma parameter  $c$  increases, maximum of curve goes to higher values of resulting signal and maximum has higher values.



**Fig. 2** Level crossing rate of macrodiversity SC receiver output signal envelope.

As correlation coefficient increases, average level crossing rate increases in system performance degradation. When correlation coefficient goes to one, the least signal occurs at both microdiversity receivers simultaneously. The influence of correlation on average level crossing rate is higher for higher values of Gamma parameter  $c$ .



**Fig. 3** Level crossing rate of macrodiversity SC receiver output signal envelope versus Gamma shadowing severity parameter  $c$ .

Normalized average level crossing rate versus Gamma long term fading severity parameter  $c$  for several values of correlation coefficient and for average power  $\Omega_0=1$ , resulting signal  $x=1$  and Nakagami parameter  $m=2$  is shown on Fig. 3. When parameter increases, curves have maximum. For higher values of parameter  $c$ , average level crossing rate decreases as parameter  $c$  increases. Average level crossing rate increases as correlation coefficient increases. The influence coefficient  $\rho$  on average level crossing rate is higher for higher values of parameter  $c$ .

## 5. CONCLUSION

Macrodiversity system with microdiversity selection combining receiver (SC) and three microdiversity selection combining receivers operating over correlated Gamma shadowed Nakagami- $m$  multipath fading environment is considered. Macrodiversity SC reception mitigates Gamma long term fading effects and microdiversity SC reception reduces Nakagami- $m$  short term fading effects on system performance. Long term fading is correlated due to three base stations are shadowed by the same obstacle. Short term fading is independent because antenna spacing at microdiversity is achieved that correlation coefficient of multipath fading goes to zero (Krstic et al., 2014). In this paper closed form expressions for average level crossing rate of macrodiversity SC receivers output signals are calculated and these expressions are used for calculation average level crossing rate of macrodiversity SC receiver output signal. By using this formula can be calculated average fade duration of proposed macrodiversity system as ratio of outage probability and average level crossing rate. By using derived expression for average level crossing rate can be evaluated average fade duration and average level crossing rate of macrodiversity techniques with three microdiversity SC receivers in composite Gamma long term fading and short Nakagami- $m$  fading channel. The system performance is better for lower values of level crossing rate. When Nakagami- $m$  multipath fading severity parameter increases, average level crossing rate decreases resulting in system performance improvement. Also, system performance is better for higher values of Gamma long term fading severity parameter. When correlation coefficient goes

to one macrodiversity system becomes microdiversity system.

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