Beyond the implementation of a dynamic stochastic general equilibrium model

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Abstract. *The present paper is structured as a brief guide focused on the rendering of the output generated by Dynare, a specific tool of Matlab, when implementing a Dynamic Stochastic General Equilibrium model for impulse-response function analysis purpose. After a concise description of a standard Real Business Cycle model, the steady state of variables, the eigenvalues, the matrix of covariance of exogenous shocks, the policy and transition functions, the theoretical moments, the variance decomposition, the matrix of correlations and the coefficient of autocorrelation of simulated variables, as well as the graphical representation of the impulse-response functions are displayed and construed accordingly.*

Keywords: Dynamic Stochastic General Equilibrium model, Real Business Cycle model, impulse-response function, structural shocks, Dynare code.

JEL Classification: C18, C40, C62, C68, C87.

Introduction

As many theoretical papers dedicated to the study of Dynamic Stochastic General Equilibrium models deal only with the implementation side, more exactly with the modality of rendering the same under the form of a specific code, without capturing the essential issues going beyond implementation, this study undertakes to fill such gap.

Starting, for actual exemplification purpose, with the presentation, in brief, of the model depicted by Aguiar and Gopinath (2004) and transposed into Dynare by Villemot (2012), the paper continues with the decryption of the output generated by Dynare in Matlab, when analysing the impulse-response functions of the model variables, it being construed step by step, in a user friendly manner.

The correlation between the strictly theoretical information provided and the numerical data revealed turns this incursion in the Dynare world into a pleasant journey for a complete understanding of its final effects.

Model

This basic Real Business Cycle model, briefly rendered hereinafter, is developed for a small economy, considering a single good and a single asset and introducing a transitory shock as well as a permanent shock to productivity.

The model utility function is of Cobb-Douglas type:

$$u_t = \frac{\left[C_t^\gamma \times (1 - L_t)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$$

where C_t represents the consumption of households, $L_{,t}$, the labour force provided by households, γ, the elasticity of consumption in relation to utility, and σ, the inverse of the elasticity of consumption and leisure time in relation to the related marginal disutility

There is also a Cobb-Douglas productivity function:

$$Y_t = e^{z_t} \times K_t^{1-\alpha} \times (\Gamma_t \times L_t)^\alpha$$

where K_t is the capital of the firm, α, the elasticity of labour in relation to output, z_t, the transitory shock to productivity, and Γ_t, the overall product of the permanent shock to productivity
with z_t represented by a first order autoregressive process:

$$z_t = \rho \times z_{t-1} + \varepsilon_t^z$$

with ε_t^z following a normal distribution of zero mean and variance σ_z^2

and with Γ_t taking the form:

$$\Gamma_t = e^{g_t} \times \Gamma_{t-1}$$

$$g_t = (1 - \rho_g) \times \mu_g + \rho_g \times g_{t-1} + \varepsilon_t^g$$

where μ_t is the long-run mean growth rate of productivity and g, the permanent shock to productivity, the realisation of which permanently influences Γ

with ε_t^g following a normal distribution of zero mean and variance σ_g^2

The capital movement equation is rendered by:

$$K_{t+1} = (1 - \delta) \times K_t - \frac{\phi}{2} \times \left(\frac{K_{t+1}}{K_t} - e^{\mu_g} \right)^2 \times K_t$$

where δ is the capital depreciation rate and $\frac{\phi}{2} \times \left(\frac{K_{t+1}}{K_t} - e^{\mu_g} \right)^2 \times K_t$, the capital adjustment

cost or, after normalisation:

$$e^g \hat{K}' = (1 - \delta) \times \hat{K} + \hat{X} - \frac{\phi}{2} \times \left(\frac{\hat{K}'}{\hat{K}} \times e^g - e^{\mu_g} \right)^2 \times \hat{K}$$

where symbol $\wedge$ renders the normalised form of the variable and symbol $'$ the related variable at time $t+1$

It is to be mentioned that the stochastic trend variables of the model are rendered stationary by dividing the same to the one lag overall product of the permanent shock to productivity.

The resource constraint, given the capital movement equation, takes the form:

$$C_t + K_{t+1} = Y_t + (1 - \delta) \times K_t - \frac{\phi}{2} \times \left(\frac{K_{t+1}}{K_t} - e^{\mu_g} \right)^2 \times K_t - B_t + q_t \times B_{t+1}$$

where B_t is the level of debt and q_t, the price of debt due in the next period
with

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + \psi \times \left(e^{\frac{B_{t+1} - b}{\Gamma_t}} - 1 \right)$$

where r_t is the domestic interest rate, r^*_t, the foreign interest rate, ψ, the interest rate elasticity in relation to changes of indebtedness, and b, the debt steady state level
with

$$(1 + r^*) \times \beta = e^{\mu_g \times [1 - \gamma \times (1 - \sigma)]}$$

where β is the subjective discount factor

with

$$\beta \times g_t^{\gamma \times (1 - \sigma)} = f$$

The resource constraint, after normalisation, becomes:

$$\hat{C} + e^g \times \hat{K}' = \hat{Y} + (1-\delta) \times \hat{K} - \frac{\phi}{2} \times \left(e^g \times \frac{\hat{K}'}{\hat{K}} - e^{\mu_g} \right)^2 \times \hat{K} - \hat{B} + e^g \times q \times \hat{B}'$$

The maximisation of the utility function, given the production function and the budget constraint, all of them used in their normalised form, is represented as follows:

$$V(\hat{K}, \hat{B}, z, g) = \max_{\hat{C}, L, \hat{K}', \hat{B}'} \left\{ \frac{\left[C_t^\gamma \times (1-L_t)^{1-\gamma} \right]^{1-\sigma}}{1-\sigma} + \beta \times e^{g \times \gamma \times (1-\sigma)} \times E \times V(\hat{K}', \hat{B}', z', g') \right\}$$

resulting in the related first order conditions, subsequently log-linearised by expansion in Taylor series around their deterministic steady-state.

Before actually implementing the arising equations in their final form, several model parameters were calibrated, for the rest of them being set the priors (used as in the original analysis), which are not rendered in the present paper, considering that this is not the aim of our study.

Before displaying the implementation-related output, we should mention two more issues:
1. u^C and u^L denote the utility of consumption, respectively the utility of leisure time
2. C_t^Y, I_t^Y and TB_t^Y represent the ratio of consumption, investments, respectively trade balance, in relation to gross domestic product, being described by the following equations:

$$C_t^Y = \frac{C_t}{Y_t}$$

$$I_t^Y = \frac{\left[g_t \times K_t - (1-\delta) \times K_{t-1} \right]}{Y_t}$$

$$TB_t^Y = \frac{\left[B_{t-1} - g_t \times q_t \times B_t \right]}{Y_t}$$

these last three variables being analysed from the perspective of their reaction on the occurrence of the shock to the productivity level and of the one to the productivity growth.

Dynare output: display and comments

Below is rendered what really happens beyond the implementation of such model, meaning the raw output actually displayed by Matlab, when resorting to an impulse-response function analysis (we specify this, as, when used for estimation purposes, such output becomes more complex).

First of all, a model summary is presented, it taking the following form:

MODEL SUMMARY	
Number of variables:	15
Number of stochastic shocks	2
Number of state variables	4
Number of jumpers	4
Number of static variables	9

In our example, the model variables, transposed into Dynare, are: *c*, *k*, *y*, *b*, *q*, *g*, *l*, *u*, *z*, *uc*, *ul*, *f*, *c_y*, *i_y*, *tb_y*, while the transitory and permanent stochastic shocks to productivity are: *eps_z* and *eps_g*, all these being declared at the beginning of the Dynare code implemented in Matlab, as:
var c k y b q g l u z uc ul f c_y tb_y i_y;
varexo eps_z eps_g;

Thereafter, the long-run equilibrium value of the model variables, determined by a non-linear Newton-type solver, is rendered below:

STEADY-STATE RESULTS:	
c	0.863041
k	5.41335
y	1.06051
b	0.1
q	0.968345
g	1.006
l	0.492446
z	0
u	-2.42527
uc	5.88191
ul	-8.61355
f	0.974155
c_y	0.813801
i_y	0.00243704
tb_y	0.183762

Usually, in order to help the virtual system to obtain the model steady state, mainly when considering a non-linear problem, an initialisation is needed. In this case, the anticipated value of variables is taken over by the said solver which undergoes an iterative procedure.

For the analysed model, this block, followed by the *steady* command, looks like:
initval;
z = 0;
c = 0.583095;
k = 4.02387;
y = 0.721195;
b = b_star;
q = 1/(1+r_star);
g = mu_g;
l = 0.321155;
z = 0;

u = (c^gamma*(1-l)^(1-gamma))^(1-sigma)/(1-sigma);
uc = gamma*u/c*(1-sigma);
ul = -(1-gamma)*u/(1-l)*(1-sigma);
f = beta*g^(gamma*(1-sigma));
c_y = c/y;
i_y = (g*k-(1-delta)*k)/y;
tb_y = (b-g*q*b)/y;
end;
steady;

The following issues displayed are the eingen values. According to the related theory, the Blanchard-Kahn conditions are met, therefore the convergence being reached, if we obtain the equality between the number of jumpers and that of eigenvalues greater than 1.

EIGENVALUES:		
Modulus	Real	Imaginary
0.72	0.72	0
0.9225	0.9225	0
0.94	0.94	0
0.9947	0.9947	0
1.032	1.032	0
1.113	1.113	0
Inf	-Inf	0
Inf	Inf	0

As it is reflected by the table above, there are 4 eigenvalues exceeding, in modulus, 1: *1.032*, *1.113*, *Inf* and *Inf* for the 4 jumpers of the model, that is the time *t+1* variables: *k*, *y*, *g*, *uc*, this standing for the verification of the rank condition.

The matrix of covariance of the model transitory and permanent shocks to productivity is also rendered:

MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS		
Variables	eps_z	eps_g
eps_z	1.000000	0.000000
eps_g	0.000000	1.000000

This form was already known, considering the distribution of shocks and the σ^2_z, respectively σ^2_g values declared within the Dynare code implemented in Matlab.
shocks;
var eps_z = 1;
var eps_g = 1;
end;
The policy and transition functions are nothing else but a recursive representation of the model, generating time series able to meet the said model rational expectation-related hypothesis.

POLICY AND TRANSITION FUNCTIONS			
	c_y	i_y	tb_y
Constant	0.808391	0.180925	0.010684
(correction)	-0.005410	-0.002837	0.008247
b(-1)	-0.029585	-0.011279	0.040864
z(-1)	-0.322810	0.497432	-0.174622
k(-1)	-0.011080	-0.046544	0.057624
g(-1)	0.535099	0.659559	-1.194658
eps_z	-0.001408	0.002170	-0.000762
eps_g	0.008149	0.010045	-0.018194
b(-1),b(-1)	-0.001826	-0.004444	0.006222
z(-1),b(-1)	0.057932	0.017410	-0.068659
z(-1),z(-1)	0.172566	-0.623451	0.214570
k(-1),b(-1)	0.005172	0.003017	-0.008705
k(-1),z(-1)	-0.004840	0.079540	-0.038210
k(-1),k(-1)	0.001200	0.001769	-0.004378
g(-1),b(-1)	-0.036002	-0.041693	0.083218
g(-1),z(-1)	-0.287474	-0.476052	0.372814
g(-1),k(-1)	-0.013312	0.019668	0.023810
g(-1),g(-1)	0.076139	-0.177279	-0.060357
eps_z,eps_z	0.000003	-0.000012	0.000004
eps_g,eps_z	-0.000019	-0.000032	0.000025
eps_g,eps_g	0.000079	0.000035	-0.000152
b(-1),eps_z	0.000253	0.000076	-0.000299
b(-1),eps_g	-0.000548	-0.000635	0.001267
z(-1),eps_z	0.001505	-0.005439	0.001872
z(-1),eps_g	-0.004378	-0.007250	0.005678
k(-1),eps_z	-0.000021	0.000347	-0.000167
k(-1),eps_g	-0.000203	0.000300	0.000363
g(-1),eps_z	-0.001254	-0.002076	0.001626
g(-1),eps_g	0.010420	0.004585	-0.019924

More exactly, by solving a rational expectation model taking the form:

$$E\{g[a_{t+1}, a_t, a_{t-1}, u_t]\} = 0$$

we obtain the function:

$$a_t = h[a_{t-1}, u_t]$$

which, once introduced into the model, meets the related expectations.

The expansion in Taylor series leads, for the first order approximation, to:

$$a_t = \overline{a} + h_a \times \hat{a}_{t-1} + h_u \times u_t$$

where $\overline{a}$ represents the steady state of a, $\hat{a}$, the deviation of a from its steady state value, and h_y and h_u, the equation parameters

with $h_x = \dfrac{\partial h}{\partial x}$

However, in many cases, the second order approximation is used, as in the present model, it taking the form:

$$a_t = \bar{a} + h_\Delta \times \Delta^2 + h_a \times \hat{a}_{t-1} + h_u \times u_t + h_{a,a} \times \hat{a}^2{}_{t-1} + h_{u,u} \times u^2{}_t + h_{a,u} \times \hat{a}_{t-1} \times u_t$$

where Δ^2 represents the shift effect of the future shock variance

Here we have the second order approximation, exemplified for c_t:

$$\frac{C_t}{y_t} = 0.808391 - 0.005410 \times \Delta^2 - 0.029585 \times \hat{B}_{t-1} - 0.322810 \times \hat{z}_{t-1} -$$

$$- 0.011080 \times \hat{K}_{t-1} + 0.535099 \times \hat{g}_{t-1} - 0.001408 \times \varepsilon_t^z + 0.008149 \times \varepsilon_t^g -$$

$$- 0.001826 \times \hat{B}_{t-1}^2 + ... + 0.010420 \times g_{t-1} \times \varepsilon_t^g$$

Then follows the description of some moments for the simulated variables: the mean, representing in fact the variable steady state value, the standard deviation, standing for the variable volatility, and the variance, square of the latter.

THEORETICAL MOMENTS			
Variable	Mean	Std. dev.	Variance
c_y	0.8758	0.0517	0.0027
i_y	0.1850	0.0260	0.0007
tb_y	-0.0612	0.0607	0.0037

The following three tables render the variance decomposition (expressed in percents), the matrix of correlations, respectively the coefficients of autocorrelation (for 5 lags), for the model simulated variables: c_y, i_y and tb_y. This information is displayed provided that such variables have a non-negative variance.

VARIANCE DECOMPOSITION (%)		
	eps_z	eps_g
c_y	3.90	96.10
i_y	2.72	97.28
tb_y	2.65	97.35

MATRIX OF CORRELATIONS			
Variables	c_y	i_y	tb_y
c_y	1.0000	0.1256	-0.9051
i_y	0.1256	1.0000	-0.5356
tb_y	-0.9051	-0.5356	1.0000

COEFFICIENTS OF AUTOCORRELATION					
Order	1	2	3	4	5
c_y	0.9845	0.9719	0.9613	0.9522	0.9442
i_y	0.9185	0.8430	0.7731	0.7085	0.6489
tb_y	0.9506	0.9089	0.8733	0.8423	0.8151

The graphical representation, rendered hereinafter, reflects the impulse-response functions, more precisely the reaction of the model simulated variables: c_y, i_y and tb_y when hit by the model shocks: eps_z, respectively eps_g, it being generated, as all previous results, by the Dynare command:

stoch_simul c_y i_y tb_y;
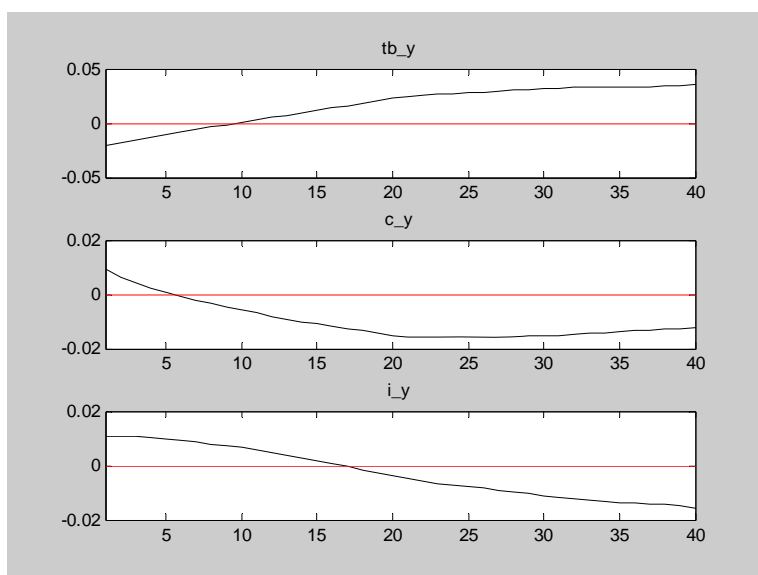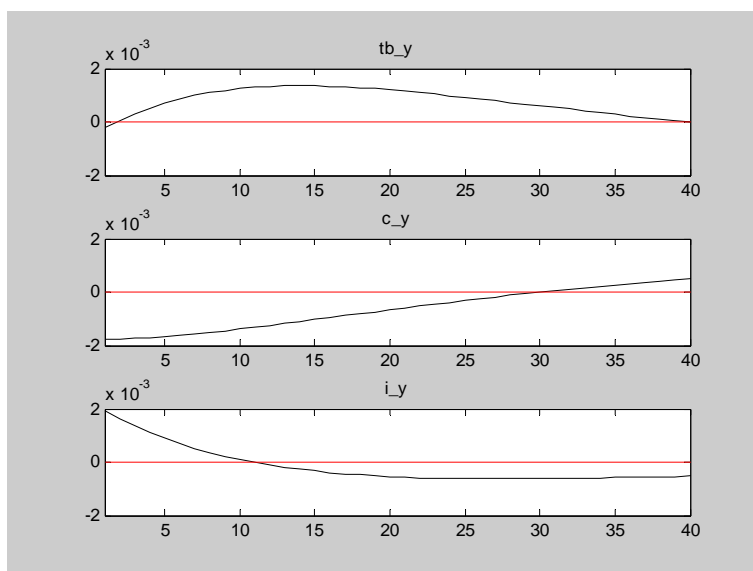
It is to be mentioned that the impulse-response functions are displayed only if the response of the variable exceeds 1e-10.

As revealed by these graphs, the related variables reach their initial steady state after 2 to 30 periods, as the case may be. If their return to equilibrium is not visually captured, it is necessary either to set longer analysis time when launching the related command or to check the model stationarity.

Conclusions

The aim of the paper was to render some important aspects related to the interpretation of the results arising from the implementation in Matlab, under the form of a Dynare code, of a Dynamic Stochastic General Equilibrium model centred on the analysis of impulse-response functions. After presenting the model summary, with its 15 variables and 2 stochastic shocks, the steady state of variables is approached, being underlined the role of the initialisation block, followed by the description of the modality of verifying the Blanchard-Kahn conditions, given by the equality between the number of jumpers and the number of eigenvalues greater, in modulus, than 1. The study continues with the rendering of the matrix of covariance of exogenous shocks, the variation of which is initially set by the code writer, then with the description of the policy and transition functions, which are a recursive representation of the model, herein exemplified on numerical data, with the listing of the theoretical moments, mean, standard deviation and variance, as well as with the display of the variance decomposition, matrix of correlations and coefficient of autocorrelation of the model simulated variables, these last three items being accessible provided that such variables have a non-negative variance. The paper ends with the graphical representation and the related interpretation of the impulse-response functions, under the impact of the structural shocks considered, the model simulated variables crossing their initial equilibrium level within the analysed period.

Acknowledgements

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