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Abstract. Nowadays, the significance of keeping equipment function properly each time is obvious. If equipment fails during its use, it may have disastrous consequences. Estimating remaining useful life (RUL) of equipment is a key to prevent such calamities, improve its reliability, provide security and reduce unnecessary maintenance and operational cost. The evolution and tracking of damage is the foundation of RUL predicting, and also is one of the most important content of mechanical fault diagnosis. Slow-time variable process of mechanical damage would lead the phase space reconstructed by fast-time variable vibrate signals warping. Search the dynamics characteristic law of damage evolution analysis in the phase space, and build the relationship between fast-time variable signals and slow-time variable damage, and then damage evolution tracking is possible. To validate the theory, simulation model of bearing damage evolution is built, the outer-race fault evolution signals is obtained, and the trend of evolution of degradation of bearing fault is described with Phase Space Warping (PSW) theory and Smooth Orthogonal Decomposition (SOD). The results proved the feasibility of the methodology of PSW in damage evolution tracking.

Keywords. Phase space warping; Slow-time variable damage; Damage evolution tracking; Fault diagnosis

1. Introduction

Damage detection, tracking, and remaining life prediction of equipment is current research focus in prognostics and health management (PHM) technologies. With the increase of machinery operating speed and precision degree in high-speed machinery, and due to the complexity and highly nonlinearity of internal dynamics of mechanical systems, uncertainty of operation condition and strong noise background, the dynamics of damage evolution of components becomes extremely complex. Linear vibration analysis methods are difficult to give a very reasonable explanation to the relationship between the inherent dynamic behavior of moving parts and vibration state, because of slow evolution and sudden deterioration are important features of nonlinear vibration system. Therefore, it is possible a better approach to solve the problem with nonlinear dynamics theory and methods.

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The theory of phase space warping (PSW) is provide an effective approach to damage identification, evolution tracking and prediction based on nonlinear dynamics. In 2000, David Chelidze presented a nonlinear observer for damage evolution tracking[1], PSW-based damage identification method, and applied it to a two-well electromechanical oscillator, which is a platform of fatigue damage simulation experiment, and then successfully tracked the fatigue damage evolution of experimental element. Besides, a multidimensional damage identification approach has been developed based on this concept and validated using numerical model simulation[2]. In addition, David B. Segala, Albert Adams and et.al applied the theory to track and predict the onset of physiologic fatigue in prolonged load carriage walking[3].

In this paper, simulated vibration signals are used to reconstruct damage evolution in rolling element bearing outer-race fault from incipient stage to severe stage in an hour. Damage is reconstructed in two steps: (1) phase space warping based feature vectors are estimated from vibration time series; and (2) smooth orthogonal decomposition is used to extract damage related trends from these features. The reconstructed dominant damage variable closely tracks vibration amplitude reflecting global evolution trend. The results show that PSW combined SOD can be used to track bearing damage evolution, and provide a possible approach to predict the RUL of rotating components.

2. Theory of Phase Space Warping

As structural damage in [4], mechanical damage is viewed as evolving in a hierarchical dynamical system where slow-time fatigue damage evolution and fast-time vibration dynamics are coupled through parameters of a fast-time subsystem:

$$\dot{x} = f(x, \mu(\phi), t)\ ,\quad \dot{\phi} = \varepsilon g(\phi, x)\ ,\quad y = h(x) \tag{1}$$

where x is a fast-time dynamic variable describing vibration signals; ϕ is a slow-time dynamic variable describing damage evolution, which alters a parameter vector μ in the fast-time system; t is time; and ε is a small rate constant describing time scale separation. A measurement function h generating a scalar time series y is based on the fast-time variable, x.

The objective is to reconstruct the slow-time evolution of the fatigue damage variable ϕ from the captured fast-time dynamics y. Given a time sequence of data records of scalar fast-time series fatigue damage is identified from the PSW-based feature vectors estimated for each fast-time data record using SOD.

2.1. PSW-Based Features

To estimate the extent of PSW, the recorded vibration acceleration time series are embedded into a reconstructed phase space trajectory. Given a scalar time series in one of the data records $\{y_i\}_{i=1}^{N}$, the corresponding fast-time phase space is reconstructed using *delay coordinate embedding*[5], by generating a d-dimensional state vector $\{y(i)\}_{i=1+(d-1)\tau}^{N}$

$$y(i) = [y_i\ |,\ y_{i-\tau}, \ldots, y_{i-(d-1)\tau}]^{\mathrm{T}} \tag{2}$$

where T represents matrix transpose, d is a sufficient embedding dimension, and τ is a *delay time*. The embedding dimension is estimated by the method of false nearest neighbors[6] and the time delay is estimated as the first local minimum of the average mutual information[7].

The reconstructed state vectors are governed by an unknown map P: $\mathbb{R}^d \rightarrow \mathbb{R}^d$

$$y(i+1) = P(y(i); \phi) \tag{3}$$

One way to quantify PSW is to consider the following *tracking function* for a point in the reconstructed phase space:

$$e_R(y;\phi) = P(y;\phi) - P(y;\phi_R) \tag{4}$$

where ϕ_R is the reference state of the damage variable and ϕ is the current value. In other words, this metric measures the change in the direction of a trajectory caused by the drift in the damage state. $P(y;\phi)$ in equation (4) is directly available for every data record and using equation (3) we get

$$e_R(y;\phi) = y(i+1) - P(y(i);\phi_R) \tag{5}$$

However, $P(y(i);\phi_R)$ is only available for the points in the reference data record, and for any other data record, it needs to be estimated from the reference data record. For this estimation, as in [4], a local linear model is used:

$$P(y(i);\phi_R) \approx A_i y(i) + b_i \tag{6}$$

where the modal parameter matrix $A_i \in \mathbb{R}^{d\times d}$ and a parameter vector $b_i \in \mathbb{R}^d$ are determined for each query point $y(i)$. More specifically, N_m nearest neighbors of $y(i)$ and their images one time-step later in the reference phase space are used to estimate A_i and b_i in the least-squares sense. Thus, the tracking function is estimated using this *single-time-step reference model prediction* (STRMP) error:

$$\hat{e}_R(y(i);\phi) = y(i+1) - A_i y(i) - b_i \tag{7}$$

The magnitude of estimates of the STRMP error is expected to vary from point to point depending on the local trajectory curvature since it is a measure of the distortions described by a nonlinear map P. Also, its accuracy depends on the local probability distribution of the reference phase space points. Depending on how the trajectory evolves in time, some portions of the phase space result in better estimates of the STRMP error. To compensate for all these factors, the phase space is partitioned into N_e disjoint hyper-cuboids, $\{\mathrm{B}_i\}_{i=1}^{N_e}$ containing approximately the same number of points. Then, the expected value of $\|e_R(y;\phi)\|$ is evaluated in each hyper-cuboid to get:

$$e_i(\phi) = \frac{1}{N_i} \sum_{y\in \mathrm{B}_i} \|\hat{e}_R(y;\phi)\| \tag{8}$$

where N_i is the number of points in $\mathrm{B_i}$. Now, for each data record j (j = 1, ⋯, M) the averaged errors were assembled into a N_e-dimensional feature vector

$$e^j = [e_1(\phi), e_2(\phi), ..., e_{N_e}(\phi)] \tag{9}$$

The estimated feature vectors e^j are then calculated for all N_r data records and *row-wise* concatenated (in time sequence) into a *tracking matrix*,

$$Y = [e^1; e^2; ...; e^{N_r}] \tag{10}$$

2.2. Smooth Orthogonal Decomposition

SOD is a multivariate data analysis tool that can be used to extract smooth trends in time from a multivariate time series[8]. We assume that mechanical damage is deterministic process, whose variable are evolving smoothly in time. Thus, if the tracking matrix $Y \in \mathbb{R}^{M\times Ne}$ contains information about the damage evolution, SOD should be able to extract corresponding smooth trends from the features. In particular, these trends can be identified by solving following generalized eigenvalue problem:

$$[Y^T Y]\psi_i = \lambda_i [(DY)^T DY]\psi_i \tag{11}$$

where D is the discrete differential operator (e.g., based on forward difference). In practice, equation (11) is solved using *generalized singular value decomposition* of the matrix pair Y and DY, yielding:

$$
\begin{aligned}
&Y = UCX^T, \\
&DY = VSX^T, \\
&C^T C + S^T S = I
\end{aligned}
\tag{12}
$$

where I is the identity matrix, the matrices U and V are unitary, X is a square matrix, and C and S are non-negative diagonal matrices. X^{-1} is a matrix composed of eigenvectors φ_i, or *smooth orthogonal modes*. The columns of matrix UC contains *smooth orthogonal coordinates* (SOCs), and the corresponding eigenvalues λ_i, or *smooth orthogonal values* (SOVs) are given by term-by-term division of $\mathrm{diag}(C^T C)^{1/2}/\mathrm{diag}(S^T S)^{1/2}$. The dominant eigenvalues and corresponding coordinates represent the smoothest trends in Y. Therefore, the higher the value of SOVs, the smoother in time is the corresponding SOCs and damage related trends are expected to be embedded in those SOCs.

3. Experimental Results and Analysis

Due to the greatly long experimental time and huge experimental data of real bearing damage evolution experiment, so in order to validate that the PSW method is effective in the identification for bearing damage degradation, we utilize the simulation model of bearing damage to produce the needed damage evolution signals in the paper. It is different from actual signals that, in order to cut down the data quantity and computational cost, we shorten the fault degradation process (incipient fault to severe fault) from possible thousands hours to one hour. Therefore, computational time is greatly shortened, and the whole trend of damage evolution would be more obvious.

3.1. Simulation model of bearing damage evolution

There are many simulation models for bearing fault signals, one of the models that better characterizes the vibration produced by defective rolling element bearings is the one produced by Randall et al. in [9-11]:

$$x(t) = \sum_{i=0}^{M} A_i \cdot s(t - iT - \tau_i) + n(t) \tag{13}$$

where T is the period of impulse, $s(t)$ is the vibration waveform and $n(t)$ is external noise, A_i is the amplitude modulator, which is used to simulated the possible modulated situation, and the modulated period is Q. $f_m = 1/Q$ is shaft speed for inner-race fault and the cage speed for rolling element fault. τ_i is the random fluctuation around average period T, Oscillation waveforms generally decay rapidly with action of the system damping.

Simplify the vibration waveform $s(t)$ to an exponential damping cosine signal

$$s(t) = e^{-Bt}\cos(2\pi f_n t + \phi_s) \tag{14}$$

Also simplify the amplitude modulator $\{A_i\}$ to a cosine signal

$$A_i = A_0 \cdot \cos(2\pi f_m t + \varphi_A) \tag{15}$$

where A_0 is the common amplitude of impulse, and assume A_0 is almost stationary in a second, $iT+\tau_i$ is the specific time of the impulse, f_m is the modulation frequency (outer-race fault: $f_m = 0$; inner-race fault: $f_m = f_r$; rolling element fault: $f_m = f_b$). It is assumed that the first impulse occurs at time $t = 0$ without slip, so $\tau_0 = 0$. The original phase is also reasonable assumed $\varphi_A = 0$. Substitutions of equation (14) and (15) into (13), and added Gaussian white noise $n(t)$, lead to the simplified model of fault signals of rolling element bearing

$$x(t) = \sum_{i=0}^{M} [A_0 \cdot \cos 2\pi f_m (iT + \tau_i)][e^{-B(t - iT - \tau_i)} \cos(2\pi f_n (t - iT - \tau_i))] + n(t) \tag{16}$$

Take the outer-race fault as an example, let $f_m = 0\text{Hz}$, the nature frequency of bearing $f_n = 2000\text{Hz}$, the characteristic frequency of outer-race fault $f_o = 50\text{Hz}$, so set $T = 0.02\text{s}$, and substitutions it into equation (16), compute with Matlab software, we could obtain the result shown as figure 1.

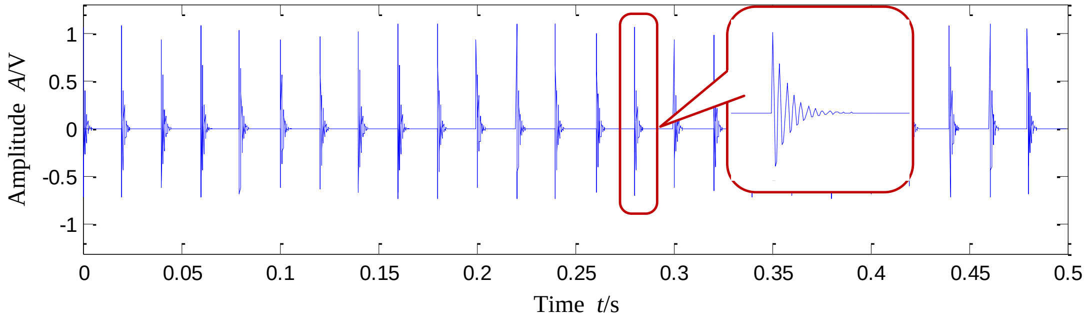

Figure 1. Simulated signals of bearing outer-race fault.

Based on the model above, simulate the process of outer-race fault of bearing from incipient stage to severe stage (failure) in an hour, which expressed as the variation of vibration impulse amplitude. Assume A_0 increases in power-law-type, shown as figure 2.

Take sample frequency $f_s = 10\text{kHz}$, and compute every second's simulated vibration signals with model (16), respectively. And then take another $f_s = 200\text{Hz}$ to resample the signals obtained above, we consequently get the needed damage evolution simulated signals, which can be shown as figure 3.

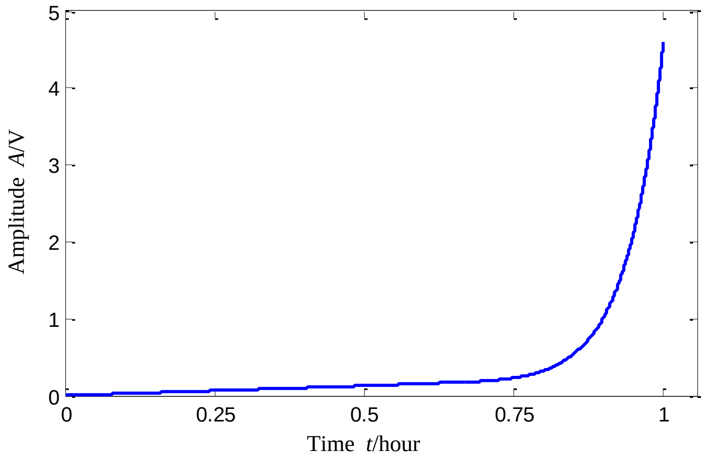

Figure 2. The hypothetic trend of amplitude of bearing damage evolution vibration signals.

3.2. Analysis of simulated signals

Calculate the damage tracking matrix Y of simulated signals with PSW first, and in order to extract the smooth trend, then take the SOD to Y, which can obtain the smooth orthogonal values and corresponding smooth orthogonal coordinates. Meanwhile, use average mutual information method and false nearest neighbors method to calculate the time delay $\tau = 7$, and embedding dimension $d = 8$, respectively. The results of signals analysis depicted in figure 4 and 5, where largest 32 SOVs and the first four dominant SOCs are shown. Results show that object signal has generally smoother trend in the dominant SOC as indicated by the larger corresponding SOV. The signals have one or two modes that are somewhat separate from the rest; The SOC_1 corresponding to the largest SOV indicate that object signal seem to have a monotonic increasing trend, but SOC_1 didn't tracked the actual trend very closely. It is smooth to track the trend in the first 40 minutes, whereas, the drastic increasing in last 15 minutes didn't captured.

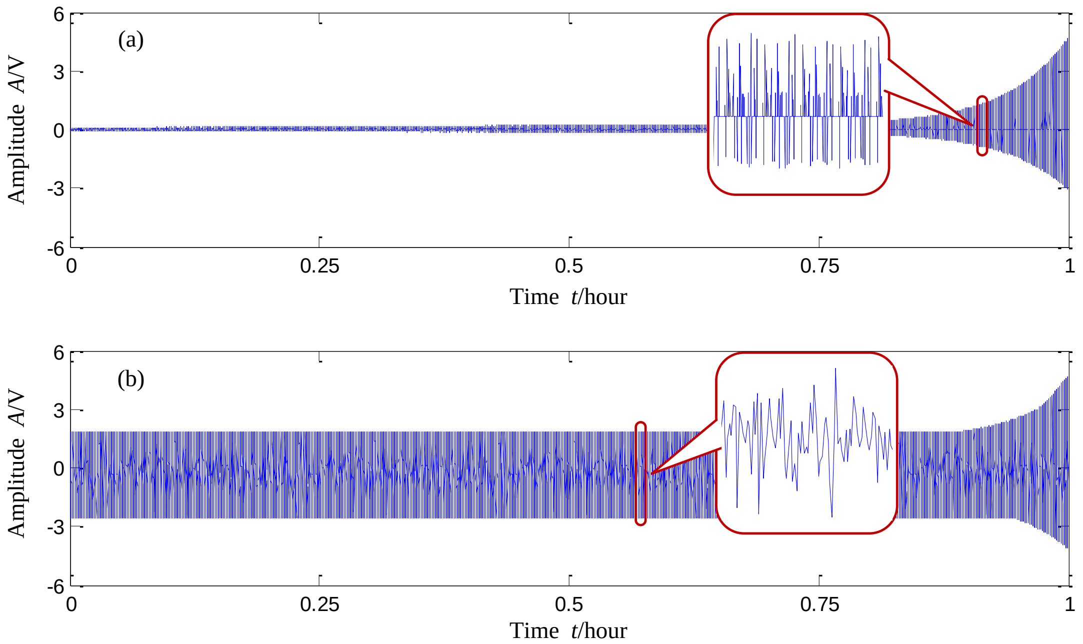

Figure 3. Simulated signals of bearing damage evolution (a) without noise (b) with white noise.

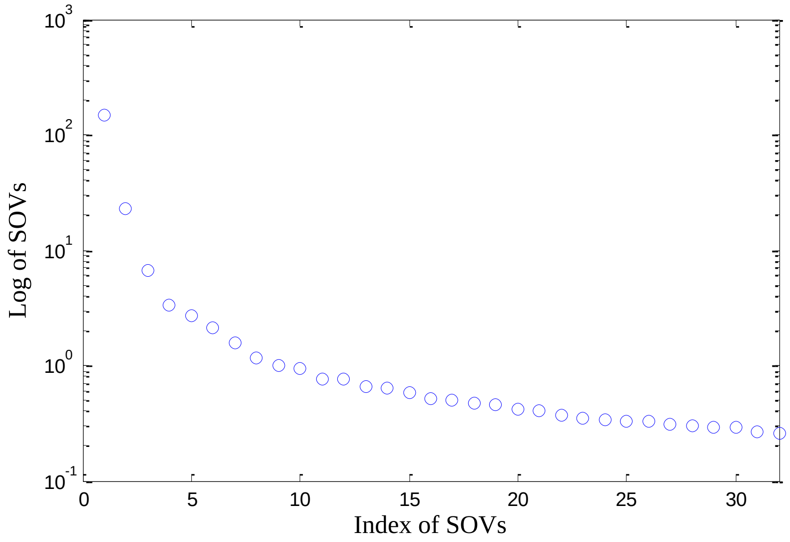

Figure 4. Largest 32 SOVs.

Therefore, we take first two dominant SOCs to be linearly combined and projected onto amplitude trend of vibration signal to show that information contained in the simulated signals. It is clear that the projected trends track the signals amplitude trend quite well, which can be shown as figure 6.The actual trend is captured by the SOCs.

On the base of damage tracking with PSW method, in addition, we could try to develop a reliable feature for evaluating damage extent, and build the RUL curves for object bearing, achieve RUL assessment of diagnostic object bearing.

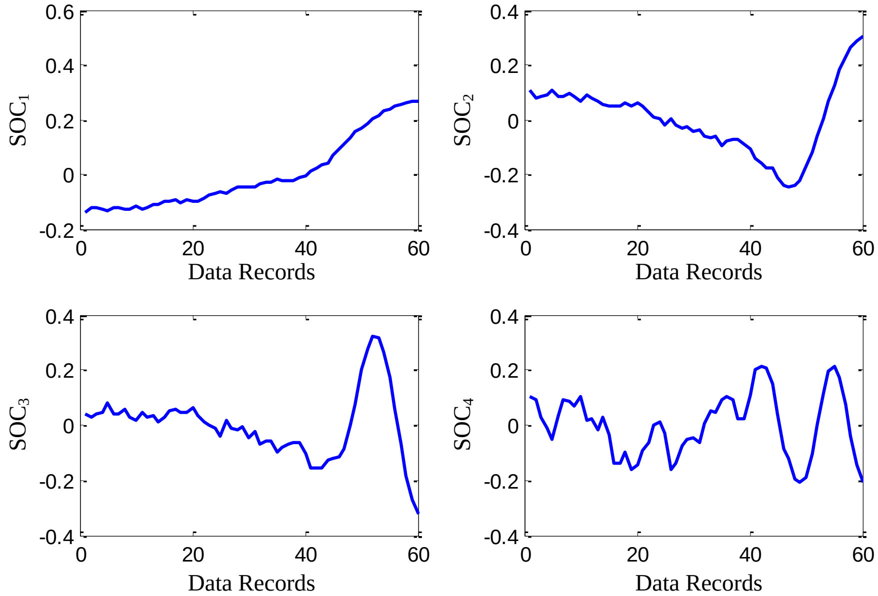

Figure 5. Corresponding dominant SOCs to largest four SOVs.

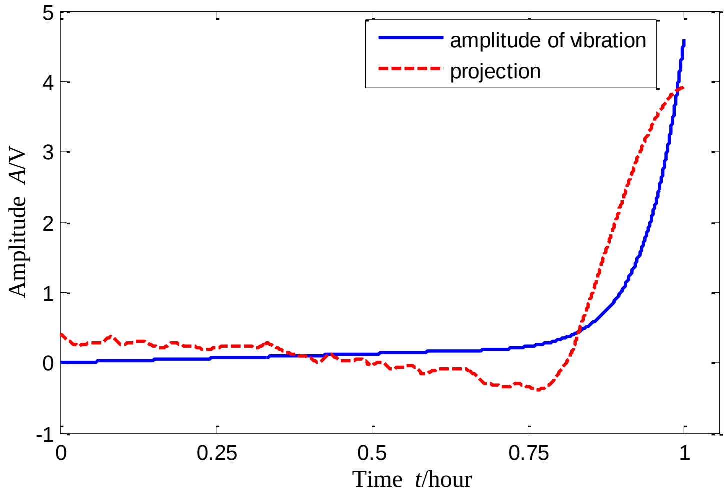

Figure 6. Comparison between the trend tracking result (dash line) and actual damage evolution trend (solid line).

4. Conclusion

In this paper, we applied the PSW method to the research of damage evolution tracking of bearing. After the introduction of PSW theory, firstly built the approved simulation model of bearing damage evolution signals, and obtained the simulation data of bearing outer-race damage short time span evolution. Then analyze the simulation data with PSW method, from the results, the trend of vibration signals amplitude is closely tracked with first two dominant SOCs which are linearly combined and projected onto original trend. It validates the feasibility of PSW in the research on bearing damage evolution tracking, and we could see its potential using in remaining useful life assessment of other rotating components, such as shaft, gear etc..

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References

[1] David Chelidze, 2000, A nonlinear observer for damage evolution tracking. PhD, The Pennsylvania State University

[2] Ming Liu, 2005, Multidimensional damage state identification using phase space warping. PhD, University of Rhode Island

[3] David B. Segala, David Chelidze & Albert Adams, 2008, Tracking Physiological Fatigue in Prolonged Load Carriage Walking Using Phase Space Warping and Smooth Orthogonal Decomposition, *International Mechanical Engineering Congress and Exposition*, Boston, Massachusetts

[4] Chelidze, D., and Liu, M., 2008. Reconstructing slow-time dynamics from fast-time measurements. *Philosophical Transaction of the Royal Society A*, **366**, pp. 729-3087.

[5] Sauer, T., Yorke, J. A., and Casdagli, M., 1991. Embedology. *J. Stat. Phys.*, **65**(3-4), pp. 579-616.

[6] Kennel, M., Brown, R., and Abarbanel, H., 1992. Determining embedding dimension for phase-space reconstruction using a geometrical construction. *Physical Reviews A*, **45** (6), pp. 3403-3411.

[7] Fraser, A., and Swinney, H., 1986. Independent coordinates for strange attractors from mutual information. *Physical Reviews A*, **33** (2), pp. 1134-1140.

[8] Chelidze, D., and Zhou, W., 2006. Smooth orthogonal decomposition based modal analysis. *Journal of Sound and Vibration*, **292**(3-5), pp. 461-473.

[9] Randall R B, et al., 2001. The relationship between spectral correlation and envelop analysis in the diagnostics of bearing faults and other cyclostationary machine signals, *Mechanical Systems and Signal Processing*, **15**: 945-962.

[10] Antoni J, Randall R B, 2005. A Stochastic model for simulation and diagnostics of rolling element bearings with localized faults, *Journal of Vibration and Acoustics*, Transactions of the ASME, **125**: 282-289.

[11] Yu Zhou, et al., Fault Diagnosis of Rolling Element Bearings Based on Cyclic Bispectrum, 2010. *Proc. 23rd Int. Conf. on Condition Monitoring and Diagnostic Engineering Management*, pp. 505-512