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Neutrosophic Goal Programming

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Abstract. In this paper, we introduce the goal programming in neutrosophic environment. The degrees of acceptance, indeterminacy and rejection of objectives are simultaneously considered. We propose two models for solving Neutrosophic Goal Programming Problem (NGPP), at one hand aiming to minimize the sum of the deviation (the Ist model), and at the other hand, transforming NGPP into a crisp programming model, using truth membership, indeterminacy membership, and falsity membership functions (the IInd model). Finally, an industrial design problem is given to illustrate the efficiency of the proposed models. The obtained results of the Ist model and of the IInd model are compared with other methods.

Keywords: Neutrosophic optimization; Goal programming problem.

1 Introduction

Goal Programming (GP) Models was originally introduced by Charnes and Cooper in early 1961 for a linear model. Multiple and conflicting goals can be used in goal programming. Also, GP allows simultaneous solution of a system of complex objectives, and the solution of the problem requires ascertaining among these multiple objectives. In this case, the model must be solved in such way, that each of the objectives to be achieved. Therefore, the sum of the deviations from the ideal should be minimized in the objective function. It is important that measure deviations from the ideal should have a single scale, because deviations with different scales cannot be collected. However, the target value associated with each goal could be of neutrosophic type in the real-world application.

In 1995, Smarandache [17] starting from philosophy (fretted to distinguish [8] between *absolute truth* and *relative truth* or between *absolute falsehood* and *relative falsehood* in logics, and respectively between *absolute membership* and *relative membership* or *absolute non-membership* and *relative non-membership* in set theory) began to use the non-standard analysis. Also, inspired from the sport games (winning, defeating, or tie scores), from votes (pro, contra, null/black votes), from positive/negative/zero numbers, from yes/no/NA, from decision making and control theory (making a decision, not making, or hesitating), from accepted/rejected/pending, etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, Smarandache [12] combined the non-standard analysis with a tri-component logic/set/probability theory and with philosophy. How to deal with all of them at once? Is it possible to unite them? [12].

Netrosophic Theory means Neutrosophy applied in many fields of sciences, in order to solve problems related to indeterminacy. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every entity <A> together with its opposite or negation <antiA>, and with their spectrum of neutralities <neutA> in between them (i.e. entities supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (which is based on <A> and <antiA> only). According to this new theory, every entity <A> tends to be neutralized and balanced by <antiA> and <nonA> entities - as a state of equilibrium. In a classical way, <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA>, of course) have common parts two by two, or even all three of them as well. Whilst the Neutrosophic Theory is based on the triad <A>, <neutA>, and <antiA>, it also studies the indeterminacy, labeled as I, with $In = I$ for $n \geq 1$, and $mI + nI = (m+n)I$ in neutrosophic structures developed in algebra, geometry, topology etc.

The most advanced fields of Netrosophic Theory are Neutrosophic Set, Neutrosophic Logic, Neutrosophic Probability, and Neutrosophic Statistics - that started in 1995, and recently Neutrosophic Precalculus and

Neutrosophic Calculus, together with their applications in practice. Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively of the fuzzy logic (especially of intuitionistic fuzzy set and of intuitionistic fuzzy logic). In neutrosophic logic, a proposition has a degree of truth (*T*), a degree of indeterminacy (*I*), and a degree of falsity (*F*), where *T,I,F* are standard or non-standard subsets of $]^-0, 1^+[$.

An important method for multi-objective decision making is the goal programming approach in practical decision making of real life. In a standard GP formulation, goals and constraints are precisely defined, but sometimes the system's aim and conditions include some vague and undetermined situations. In particular, mathematically expressing the decision maker's unclear target levels for the goals, and the need to optimize all goals at the same level, leads to complicated calculations.

The neutrosophic approach for goal programming tries to solve this kind of unclear difficulties.

The structure of the paper is as follows: the next section is a preliminary discussion; the third section describes the formation of the problem, by developing two models for neutrosophic goal programming; the fourth section presents an industrial design problem, to put on view how the approach can be applied; finally, the fifth section provides the conclusion.

2 Some preliminaries

Definition 1. [17] A real fuzzy number $\tilde{J}$ is a continuous fuzzy subset from the real line R whose triangular membership function $\mu_{\tilde{J}}(J)$ is defined by a continuous mapping from R to the closed interval [0,1], where

(1) $\mu_{\tilde{J}}(J)=0$ for all $J\in(-\infty,a_1]$,

(2) $\mu_{\tilde{J}}(J)$ is strictly increasing on $J\in[a_1,m]$,

(3) $\mu_{\tilde{J}}(J)=1$ for $J=m$,

(4) $\mu_{\tilde{J}}(J)$ is strictly decreasing on $J\in[m,a_2]$,

(5) $\mu_{\tilde{J}}(J)=0$ for all $J\in[a_2,+\infty)$.

This will be prompted by:

$$\mu_{\tilde{J}}(J)=\begin{cases}\dfrac{J-a_1}{m-a_1}, & a_1\le J\le m,\\ \dfrac{a_2-J}{a_2-m}, & m\le J\le a_2,\\ 0, & otherwise.\end{cases} \qquad (1)$$

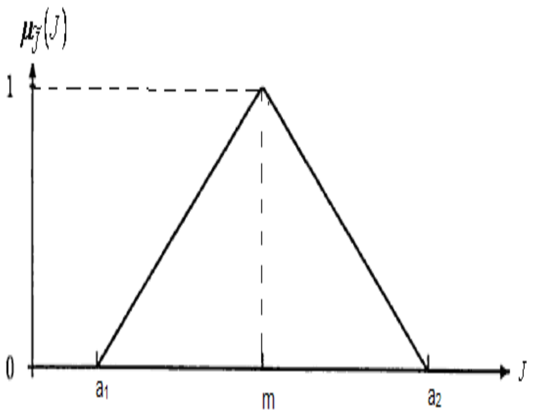

Figure 1: Membership Function of Fuzzy Number *J*.

where m is a given value a_1 and a_2 denoting the lower and upper bounds. Sometimes, it is more convenient to use the notation explicitly highlighting the membership function parameters. In this case, we obtain

$$\mu(J;a_1,m,a_2)=\text{Max}\left\{\text{Min}\left[\frac{J-a_1}{m-a_1},\frac{a_2-J}{a_2-m}\right],0\right\} \qquad (2)$$

In what follows, the definition of the α-level set or α-cut of the fuzzy number $\tilde{J}$ is introduced.

Definition 2. [1] Let $X=\{x_1, x_2, ..., x_n\}$ be a fixed non-empty universe; an intuitionistic fuzzy set IFS A in X is defined as

$$A=\{\langle x,\mu_A(x),\upsilon_A(x)\rangle | x\in X\}, \qquad (3)$$

which is characterized by a membership function

$\mu_A : X\to[0,1]$

and a non-membership function

$\upsilon_A : X\to[0,1]$

with the condition $0\le\mu_A(x)+\upsilon_A(x)\le1$ for all $x\in X$ where μ_A and υ_A represent, respectively, the degree of membership and non-membership of the element x to the set A. In addition, for each IFS A in X, $\pi_A(x)=1-\mu_A(x)-\upsilon_A(x)$ for all $x\in X$ is called the degree of hesitation of the element x to the set A. Especially, if $\pi_A(x)=0$, then the IFS A is degraded to a fuzzy set.

Definition 3. [4] The α-level set of the fuzzy parameters $\tilde{J}$ in problem (1) is defined as the ordinary set $L_\alpha(\tilde{J})$ for which the degree of membership function exceeds the level, α, $\alpha\in[0,1]$, where:

$$L_\alpha(\tilde{J})=\{J\in R \,|\, \mu_{\tilde{J}}(J)\ge\alpha\}, \qquad (4)$$

for certain values α_j^* to be in the unit interval.

Definition 4. [10] Let X be a space of points (objects) and $x\in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, as shown in *Figure 2*. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x):X\to]0^-,1^+[$, $I_A(x):X\to]0^-,1^+[$ and $F_A(x):X\to]0^-,1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so

$$0^-\leq\sup T_A(x)\leq\sup I_A(x)\leq F_A(x)\leq 3^+.$$

In the following, we adopt the notations $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively. Also we write SVN numbers instead of single valued neutrosophic numbers.

Definition 5. [10] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$$A=\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x\in X\}$$

where $\mu_A(x):X\to[0,1]$, $\sigma_A(x):X\to[0,1]$ and $\nu_A(x):X\to[0,1]$ with $0\leq\mu_A(x)+\sigma_A(x)+\nu_A(x)\leq 3$ for all $x\in X$. The intervals $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

For convenience, a SVN number is denoted by $A=(a,b,c)$, where $a,b,c\in[0,1]$ and $a+b+c\leq 3$.

Definition 6 Let $\tilde{J}$ be a neutrosophic number in the set of real numbers R, then its truth-membership function is defined as

$$T_{\tilde{J}}(J)=\begin{cases}\dfrac{J-a_1}{a_2-a_1}, & a_1\leq J\leq a_2,\\ \dfrac{a_2-J}{a_3-a_2}, & a_2\leq J\leq a_3,\\ 0, & otherwise.\end{cases} \tag{5}$$

its indeterminacy-membership function is defined as

$$I_{\tilde{J}}(J)=\begin{cases}\dfrac{J-b_1}{b_2-b_1}, & b_1\leq J\leq b_2,\\ \dfrac{b_2-J}{b_3-b_2}, & b_2\leq J\leq b_3,\\ 0, & otherwise.\end{cases} \tag{6}$$

and its falsity-membership function is defined as

$$F_{\tilde{J}}(J)=\begin{cases}\dfrac{J-c_1}{c_2-c_1}, & c_1\leq J\leq c_2,\\ \dfrac{c_2-J}{c_3-c_2}, & c_2\leq J\leq c_3,\\ 1, & otherwise.\end{cases} \tag{7}$$

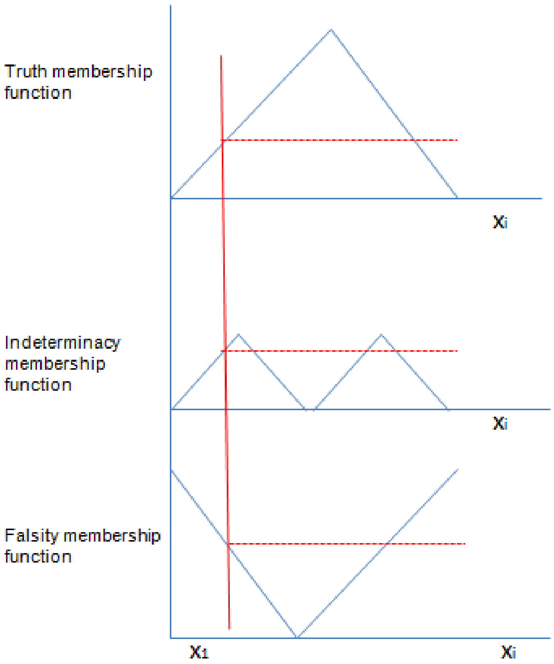

Figure 2: Neutrosophication process [11].

3 Neutrosophic Goal Programming Problem

Goal programming can be written as:

Find $x=(x_1,x_2,\ldots,x_n)^T$

To achieve:

$$z_i=t_i,\quad i=1,2,\ldots,k \tag{8}$$

subject to $x\in X$

where t_i, are scalars and represent the target achievement levels of the objective functions that the decision maker wishes to attain provided, X is feasible set of the constraints.

The achievement function of the (8) model is the following:

$$Min\ \sum_{i=1}^{k}(w_{1i}n_i+w_{2i}p_i) \tag{9}$$

Goal and constraints:

$z_i+n_i-p_i=t_i,\ i\in\{1,2,\ldots,k\}$

$x\in X,\quad n,p\geq 0,\ n\cdot p=0$

n_i, p_i are negative and positive deviations from t_i target.

The NGPP can be be written as:

Find $x=(x_1,x_2,\ldots,x_n)^T$

So as to:

Minimize z_i with target value t_i, acceptance tolerance a_i, indeterminacy tolerance d_i, rejection tolerance c_i, subject to

$$x \in X$$
$$g_j(x) \le b_j,\ j = 1,2,\ldots,m$$
$$x_i \ge 0, \qquad i = 1,2,\ldots,n$$

with truth-membership, indeterminacy-membership and falsity-membership functions

$$\mu_i^I(z_i) = \begin{cases} 1, & if \quad z_i \le t_i, \\ 1 - \dfrac{z_i - t_i}{a_i}, & if \quad t_i \le z_i \le t_i + a_i, \\ 0, & if \quad z_i \ge t_i + a_r \end{cases} \tag{10}$$

$$\sigma_i^I(z_i) = \begin{cases} 0, & if \quad z_i \le t_i, \\ \dfrac{z_i - t_i}{d_i}, & if \quad t_i \le z_i \le t_i + d_i, \\ 1 - \dfrac{z_i - t_i}{a_i - d_i}, & if \quad t_i + d_i \le z_i \le t_i + a_i, \\ 0, & if \quad z_i \ge t_i + a_i \end{cases} \tag{11}$$

$$\upsilon_i^I(z_i) = \begin{cases} 0, & if \quad z_i \le t_i, \\ \dfrac{z_i - t_i}{C_i}, & if \quad t_i \le z_i \le t_i + C_i, \\ 1, & if \quad z_i \ge t_i + C_i \end{cases} \tag{12}$$

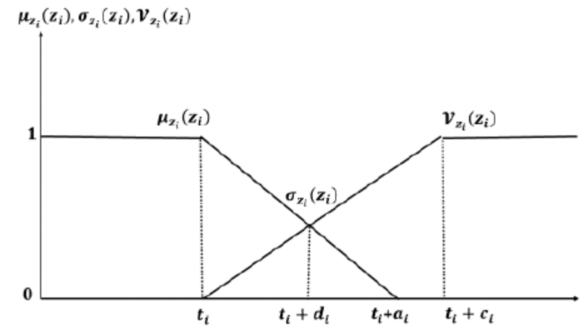

Figure 3: truth-membership, indeterminacy-membership and falsity-membership functions for z_i.

To maximize the degree the acceptance and indeterminacy of NGP objectives and constraints, also to minimize the degree of rejection of NGP objectives and constraints,

$$\begin{aligned} &Max \ \ \mu_{z_i}(z_i),\ i = 1,2,\ldots,k \\ &Max \ \ \sigma_{z_i}(z_i),\ i = 1,2,\ldots,k \\ &Min \ \ \upsilon_{z_i}(z_i),\ i = 1,2,\ldots,k \end{aligned} \tag{13}$$

subject to

$$0 \le \mu_{z_i}(z_i) + \sigma_{z_i}(z_i) + \upsilon_{z_i}(z_i) \le 3,\ i = 1,2,\ldots,k$$
$$\upsilon_{z_i}(z_i) \ge 0,\ i = 1,2,\ldots,k$$
$$\mu_{z_i}(z_i) \ge \upsilon_{z_i}(z_i),\ i = 1,2,\ldots,k$$
$$\mu_{z_i}(z_i) \ge \sigma_{z_i}(z_i),\ i = 1,2,\ldots,k$$
$$g_j(x) \le b_j,\ j = 1,2,\ldots,m$$
$$x \in X$$
$$x_j \ge 0, \qquad j = 1,2,\ldots,n$$

where $\mu_{z_i}(z_i),\ \sigma_{z_i}(z_i),\ \upsilon_{z_i}(z_i)$ are truth membership function, indeterminacy membership function, falsity membership function of neutrosophic decision set respectively.

The highest degree of truth membership function is unity. So, for the defined truth membership function $\mu_{z_i}(z_i)$, the flexible membership goals having the aspired level unity can be presented as

$$\mu_{z_i}(z_i) + n_{i1} - p_{i1} = 1$$

For the case of indeterminacy membership function, it can be written as:

$$\sigma_{z_i}(z_i) + n_{i2} - p_{i2} = 0.5$$

For the case of rejection (falsity membership function), it can be written as:

$$\mu_{z_i}(z_i) + n_{i3} - p_{i3} = 0$$

Here $n_{i1}, p_{i1},\ n_{i2}, p_{i2},\ n_{i3}$ *and* p_{i3} are under-deviational and over-deviational varibles.

Our goals are to maximize the degree of the acceptance and indeterminacy of NGP objectives and constraints, and minimize the degree of rejection of NGP objectives and constraints.

Model (I). The minimization of the sum of the deviation can be formulated as:

$$Min\ \lambda = \sum_{i=1}^{k} w_{i1} n_{i1} + \sum_{i=1}^{k} w_{i2} n_{i2} + \sum_{i=1}^{k} w_{i3} p_{i3} \tag{14}$$

subject to

$$\mu_{z_i}(z_i) + n_{i1} \ge 1,\ i = 1,2,\ldots,k$$
$$\sigma_{z_i}(z_i) + n_{i2} \ge 0.5,\ i = 1,2,\ldots,k$$
$$\upsilon_{z_i}(z_i) - p_{i3} \le 0,\ i = 1,2,\ldots,k$$
$$\upsilon_{z_i}(z_i) \ge 0,\ i = 1,2,\ldots,k$$
$$\mu_{z_i}(z_i) \ge \upsilon_{z_i}(z_i),\ i = 1,2,\ldots,k$$
$$\mu_{z_i}(z_i) \ge \sigma_{z_i}(z_i),\ i = 1,2,\ldots,k$$
$$0 \le \mu_{z_i}(z_i) + \sigma_{z_i}(z_i) + \upsilon_{z_i}(z_i) \le 3,\ i = 1,2,\ldots,k$$
$$g_j(x) \le b_j,\ j = 1,2,\ldots,m$$
$$n_{i1}, n_{i2},\ p_{i3} \ge 0,\ i = 1,2,\ldots,k$$
$$x \in X$$
$$x_j \ge 0, \qquad j = 1,2,\ldots,n$$

On the other hand, neutrosophic goal programming NGP in Model (13) can be represented by crisp programming model using truth membership, indeterminacy membership, and falsity membership functions as:

$$Max\ \alpha,\ Max\ \gamma,\ Min\ \beta \quad (15)$$

$$\mu_{z_i}(z_i) \geq \alpha,\ i = 1,2,...,k$$

$$\sigma_{z_i}(z_i) \geq \gamma,\ i = 1,2,...,k$$

$$\upsilon_{z_i}(z_i) \leq \beta,\ i = 1,2,...,k$$

$$z_i \leq t_i,\ i = 1,2,...,k$$

$$0 \leq \alpha + \gamma + \beta \leq 3$$

$$\alpha, \gamma \geq 0,\ \beta \leq 1$$

$$g_j(x) \leq b_j,\ j = 1,2,...,m$$

$$x_j \geq 0, \quad j = 1,2,...,n$$

In the model (15) the $Max\ \alpha, Max\ \gamma$ are equivalent to $Min(1-\alpha), Min(1-\gamma)$ respectively where $0 \leq \alpha,\ \gamma \leq 1$

$$Min\ \beta(1-\alpha)(1-\gamma) \quad (16)$$

subject to

$$z_i \leq t_i + a_i(a_i - d_i)\beta(1-\alpha)(1-\gamma),\ i = 1,2,...,k$$

$$z_i \leq t_i,\ i = 1,2,...,k$$

$$0 \leq \alpha + \gamma + \beta \leq 3$$

$$\alpha, \gamma \geq 0,\ \beta \leq 1$$

$$g_j(x) \leq b_j,\ j = 1,2,...,m$$

$$x_j \geq 0, \quad j = 1,2,...,n$$

If we take $\beta(1-\alpha)(1-\gamma) = v$ the model (16) becomes:

Model (II).

$$Minimize\ v \quad (17)$$

subject to

$$z_i \leq t_i + a_i(a_i - d_i)v,\ i = 1,2,...,k$$

$$z_i \leq t_i,\ i = 1,2,...,k$$

$$0 \leq \alpha + \gamma + \beta \leq 3$$

$$\alpha, \gamma \geq 0,\ \beta \leq 1$$

$$g_j(x) \leq b_j,\ j = 1,2,...,m$$

$$x_j \geq 0, \quad j = 1,2,...,n$$

The crisp model (17) is solved by using any mathematical programming technique with v as parameter to get optimal solution of objective functions.

4 Illustrative Example

This industrial application is selected from [15]. Suppose the decision maker wanting to remove about 98.5% biological oxygen demand (BOD) and the tolerances of acceptance, indeterminacy and rejection on this goal are 0.1, 0.2 and 0.3 respectively. Also, the decision maker wants to remove the amount of $BODS_5$ within 300 (thousand \$) tolerances of acceptance, indeterminacy and rejection 200, 250, 300 (thousand \$) respectively. Then the neutrosophic goal programming problem is:

$$\min\ z_1(x_1,x_2,x_3,x_4) = 19.4x_1^{-1.47} + 16.8x_2^{-1.66} + 91.5x_3^{-0.3} + 120x_4^{-0.33},$$

$$\min\ z_2(x_1,x_2,x_3,x_4) = x_1 x_2 x_3 x_4,$$

$$s.t.:$$

$$x_i \geq 0,\ i = 1,2,3,4,$$

with target 300, acceptance tolerance 200 , indeterminacy tolerance 100 , and rejection tolerance 300 for the first objective z_1, and also with target 0.015, acceptance tolerance 0.1, indeterminacy tolerance 0.05, and rejection tolerance 0.2 for the second objective z_2, where x_i is the percentage BOD5 (to remove 5 days BOD) after each step. Then, after four processes, the remaining percentage of BOD5 will be x_i, $i=1,2,3,4$. The aim is to minimize the remaining percentage of BOD5 with minmum annual cost as much as possible. The annual cost of BOD5 removal by various treatments is primary clarifier, trickling filter, activated sludge, carbon adsorption . z_1 represent the annual cost, while z_2 represent the cost removed from the wastewater.

The truth membership, indeterminacy membership, falsity membership functions were considered to be neutrosophic triangular.

The truth membership functions of the goals are obtained as follows:

$$\mu_1^I(z_1) = \begin{cases} 1, & if \quad z_1 \leq 300, \\ 1 - \dfrac{z_1 - 300}{200}, & if \quad 300 \leq z_1 \leq 500, \\ 0, & if \quad z_1 \geq 500 \end{cases}$$

$$\mu_2^I(z_2) = \begin{cases} 1, & if \quad z_2 \leq 0.015, \\ 1 - \dfrac{z_2 - 0.015}{0.1}, & if \quad 0.015 \leq z_2 \leq 0.115, \\ 0, & if \quad z_2 \geq 0.115. \end{cases}$$

The indeterminacy membership functions of the goals are given:

$$\sigma_1^I(z_1) = \begin{cases} 0, & if \quad z_1 \leq 300, \\ \dfrac{z_1 - 300}{100}, & if \quad 300 \leq z_1 \leq 400, \\ 1 - \dfrac{z_1 - 300}{100}, & if \quad 400 \leq z_1 \leq 600, \\ 0, & if \quad z_1 \geq 600 \end{cases}$$

$$\sigma_2^I(z_2)=\begin{cases}0, & if \quad z_2 \le 0.015,\\ \frac{z_2-0.015}{0.05}, & if \quad 0.015 \le z_2 \le 0.065,\\ 1-\frac{z_2-0.015}{0.05}, & if \quad 0.065 \le z_2 \le 0.215,\\ 0, & if \quad z_2 \ge 0.215\end{cases}$$

The falsity membership functions of the goals are obtained as follows:

$$\upsilon_1^I(z_1)=\begin{cases}0, & if \quad z_1 \le 300,\\ \frac{z_1-300}{300}, & if \quad 300 \le z_1 \le 600,\\ 1, & if \quad z_1 \ge 600\end{cases}$$

$$\upsilon_2^I(z_2)=\begin{cases}0, & if \quad z_2 \le 0.015,\\ \frac{z_2-0.015}{0.2}, & if \quad 0.015 \le z_2 \le 0.215,\\ 1, & if \quad z_2 \ge 0.215\end{cases}$$

Methods	z_1	z_2	x_1	x_2	x_3	x_4
FG^2P^2 Ref[15]	363.8048	0.046924	0.7059559	0.7248393	0.1598653	0.5733523
IFG^2P^2 Ref[15]	422.1483	0.01504	0.6380199	0.662717	0.09737155	0.3653206
Model (I)	317.666	0.1323	0.7741823	0.7865418	0.2512332	0.8647621
Model (II)	417.6666	0.2150	2.628853	3.087266	0.181976E-01	1.455760

Table 1: Comparison of optimal solution based on different methods

The software LINGO 15.0 is used to solve this problem. *Table 1* shows the comparison results with the proposed models and the others methods.
It is to be noted that model (I) offers a better solution than the other methods.

5 Conclusion and Future Work

The main purpose of this paper was to introduce goal programming in neutrosophic environment, simultaneously considering the degrees of acceptance, indeterminacy and rejection of objectives, by two models for solving neutrosophic goal programming problem (NGPP). In the first model, our aim was to minimize the sum of the deviation, while in the second model, NGPP was transformed into a crisp programming model using truth membership, indeterminacy membership, and falsity membership functions. Finally, a numerical experiment is given to illustrate the efficiency of the proposed methods. Moreover, a comparative study between other methods and our results is discussed. In the future studies, the proposed algorithm can be solved by metaheuristic algorithms.

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